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The sea bottom surface described by Coons pieces

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Abstract

In this paper, a method of mathematical description of a surface, which can be used for modeling the sea bottom and detecting underwater objects using sonar (a side scan sonar or a front one) or a multibeam echosounder, is presented. The method is based on Coons plates and is described in four steps, which can be used for determination of the sea bottom for spatial presentation and volume calculation. A new sounding vessel and its equipment were used for the collection of geospatial data, and the results of a geospatial model of the sea bottom on the basis of the collected data are shown. The sea bottom is presented using Coons surfaces and a triangulated irregular network.

Introduction

Modeling of surfaces in navigation, hydroacoustics and hydrography has many applications, e.g. for modeling of the sea bottom and surfaces of constant sound speed in water (Makar & Zellma, 1999; Makar, 2005; 2007; 2008; 2009a,b; 2010a,b; 2011a,b; 2012a,b; Makar & Sassais, 2011). It can also be used for prediction of the distribution of the sound speed in water and for modeling of meteorological and oceanographic processes and other constant (i.e. the land) or fluctuating surfaces.

Well-known methods and new algorithms are used in computer graphics (Stieczkin & Subbotin, 1976; Ramesh, Rangachar & Schunck, 1995; Piegl & Tiller, 1997; Kiciak, 2000; Salomon, 2006; Wolter, Reuter & Peinecke, 2007) and other applications, such as dynamic systems identification (Makar & Zellma, 2000a,b; 2001; 2003).

The Coons surface

We start with a linear Coons surface (Coons, 1964; 1967), which is a generalization of lofted surfaces. This type of surface patch is defined by its four boundary curves. All four boundary curves are given, and none must be a straight line. Naturally, the boundary curves must meet at the corner points, so these points are implicitly known.

Coons decided to search for an expression $P(x, y)$ of the surface that (1) is symmetric in x and y and (2) is an interpolation of $P(x, 0)$ and $P(x, 1)$ in one direction and of $P(0, y)$ and $P(1, y)$ in the other direction. He found a surprisingly simple, two-step solution.

The first step is to construct two lofted surfaces from two sets of opposite boundary curves. These surfaces are (Kiciak, 2000; Salomon, 2006):

$$P_a(x, y) = P(0, y)(1 - x) + P(1, y)\,x \qquad (1)$$

and

$$P_b(x, y) = P(x, 0)(1 - y) + P(x, 1)\,y \qquad (2)$$

The second step is to tentatively attempt to create the final surface $P(x, y)$ as the sum $P_a(x, y) + P_b(x, y)$. It is clear that this is not the expression we are looking for because it does not converge to the right curves at the boundaries. For $x = 0$, for example, we want $P(x, y)$ to converge to the boundary curve $P(0, y)$. The sum above, however, converges to (Kiciak, 2000; Salomon, 2006):

$$P(0, y) + P(0, 0)(1-y) + P(0, 1)\,y \tag{3}$$

We, therefore, have to subtract:

$$P(0, 0)(1-y) + P(0, 1)\,y \tag{4}$$

Similarly, for $x = 1$, the sum converges to (Kiciak, 2000; Salomon, 2006):

$$P(1, y) + P(1, 0)(1-y) + P(1, 1)\,y \tag{5}$$

so we have to subtract:

$$P(1, 0)(1-y) + P(1, 1)\,y \tag{6}$$

For $y = 0$, we have to subtract (Kiciak, 2000; Salomon, 2006):

$$P(0, 0)(1-x) + P(1, 0)\,x \tag{7}$$

and for $y = 1$, we have to subtract:

$$P(0, 1)(1-x) + P(1, 1)\,x \tag{8}$$

The expressions $P(0, 0)$, $P(0, 1)$, $P(1, 0)$ and $P(1, 1)$ are simply the four corner points. A better notation for them may be P_{00}, P_{01}, P_{10} and P_{11}.

This type of surface is known as a linear Coons surface. Its expression is:

$$P(x, y) = P_a(x, y) + P_b(x, y) - P_{ab}(x, y) \tag{9}$$

where:

$$P_{ab}(x, y) = P_{00}(1-x)(1-y) + P_{01}(1-x)\,y + P_{10}\,x(1-y) + P_{11}\,x\,y \tag{10}$$

P_a and P_b are lofted surfaces, whereas P_{ab} is a bilinear surface. The final expression is:

$$P(x,y) = P_a(x,y) + P_b(x,y) - P_{ab}(x,y) = (1-x, x)\begin{bmatrix} P(0,y) \\ P(1,y) \end{bmatrix} + (1-y, y)\begin{bmatrix} P(x,0) \\ P(x,1) \end{bmatrix} \tag{11}$$

$$-(1-x, x)\begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}\begin{bmatrix} 1-y \\ y \end{bmatrix} = (1-x, x, 1)\begin{bmatrix} -P_{00} & -P_{01} & P(0,y) \\ -P_{10} & -P_{00} & P(1,y) \\ P(x,0) & P(x,1) & P(0,0,0) \end{bmatrix}\begin{bmatrix} 1-y \\ y \\ 1 \end{bmatrix} \tag{12}$$

Let us take into consideration the four corner points:

$$P_{00} = (-1, -1, 0) \qquad P_{01} = (-1, 1, 0)$$
$$P_{10} = (1, -1, 0) \quad \text{and} \quad P_{11} = (1, 1, 0) \tag{13}$$

These points lie on the xy plane. Calculating the four boundary curves of a linear Coons surface patch is realized in the following steps (Kiciak, 2000; Salomon, 2006):

1. Take the selection boundary curve $P(0, y)$ as the straight line from P_{00} to P_{01}:
   $P(0, y) = P_{00}(1 - y) + P_{01}\,y = (-1, 2y - 1, 0)$.
2. Place the two points $(1, -0.5, 0.5)$ and $(1, 0.5, -0.5)$ between P_{10} and P_{11}, and calculate the boundary curve $P(1, y)$ as the cubic Lagrange polynomial (Kiciak, 2000; Salomon, 2006) determined by these four points:

$$P(1,y) = \frac{1}{2}\left(y^3, y^2, y, 1\right)\begin{bmatrix} -9 & -27 & 27 & 9 \\ 18 & -45 & 36 & -9 \\ -11 & 18 & -9 & 2 \\ 2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} (1,-1,0) \\ (1,-0.5,0.5) \\ (1,0.5,-0.5) \\ (1,1,0) \end{bmatrix} = \left(1, \frac{-4 - y + 27y^2 - 18y^3}{4}, \frac{27y - 3y^2 + 2y^3}{4}\right) \tag{14}$$

3. Place the single point $(0, -1, -0.5)$ between points P_{00} and P_{10}, and calculate the boundary curve $P(x, 0)$ as the quadratic Lagrange polynomial (Kiciak, 2000; Salomon, 2006) determined by these three points:

$$P(x,0) = \left(x^2, x, 1\right)\begin{bmatrix} 2 & -4 & 2 \\ -3 & 4 & -1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} (-1,-1,0) \\ (0,-1,-0.5) \\ (1,-1,0) \end{bmatrix} = \left(2x^2 - 1, -1, 2x^2 - 2x\right) \tag{15}$$

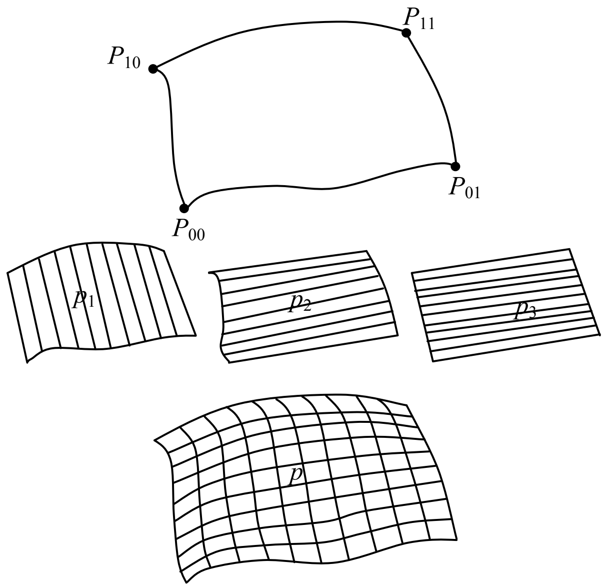

Figure 1. Construction of a bilinear Coons surface

4. Similarly, place a new point (0, 1, 0.5) between points P_{01} and P_{11}, and calculate the boundary curve $P(x, 1)$ as the quadratic Lagrange polynomial determined by these three points:

$$P(x,1) = \left(x^2, x, 1\right)\begin{bmatrix} 2 & -4 & 2 \\ -3 & 4 & -1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} (-1,1,0) \\ (0,1,-0.5) \\ (1,1,0) \end{bmatrix} = \\ = \left(2x-1, 1, -1, -2x^2+2x\right) \quad (16)$$

In Figure 1, the construction of a bilinear Coons surface is shown.

Hydrographic surveys and results

Hydrographic surveys were conducted by the new hydrographic vessels, one of which is presented in Figure 2. Each of them is equipped with:
- a singlebeam echosounder Simrad EA400;
- a multibeam echosounder Simrad EM2040;
- two sound speed in water profilers, one of which is mounted close to the multibeam transducer, presented below;
- a DGPS Trimble receiver;
- USBL underwater navigation.

The multibeam echosounder EM2040 works with the frequency range 200–400 kHz and a max ping rate of 50 Hz. The transducer is shown in Figure 3. The swath coverage sector is up to 140 degrees, 5.5 times water depth (single RX); and 200 degrees, 10 times water depth (dual RX). The sounding patterns are as follows:
- equiangular;
- equidistant;
- high density.

Figure 2. Hydrographic vessel and its multibeam transducer with a sound speed probe

Figure 3. Hydrographic vessel and its multibeam transducer with a sound speed probe

During the hydrographic surveys, the coverage sector was set up in the range of 65–70 degrees. The results of these surveys showing the sea bottom surface modeled using Coons plates are presented in Figures 4 and 5.

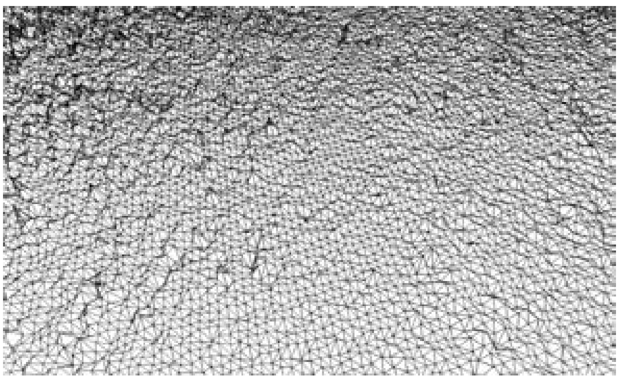

Figure 4. Visualization of a zoomed fragment of the sea bottom

Figure 5. Visualization of the sea bottom obtained during hydrographic surveys using Coons surfaces

Conclusions

For modeling a surface, there are many mathematical methods, such as basis B-splines, NURBS (non-uniform rational B-splines), Bezier, Hermite and Bernstein's pieces. Coons surfaces is another method that can be used in hydrography for modeling the sea bottom and other phenomena.

The presented method has been used successfully for presentation of the sea bottom on the basis of hydrographic surveys using a multibeam echosounder and seems to give equally positive results using a singlebeam echosounder.

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