

A graphical model to determine the influence of surface currents on small objects immersed in water

Monika Kijewska

Maritime University of Szczecin

1–2 Waly Chrobrego St., 70-500 Szczecin, Poland, e-mail: m.kijewska@am.szczecin.pl

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Abstract

This paper proposes a model of the interactions between surface currents and small, moving objects. These objects are immersed in water so that the part extending above the water is no larger than a human head. These interactions are defined as the weighted-directed graph. The basis for determining the edge weights are the directions of the surface currents. The speeds of these currents are used to calculate the time of moving objects. According to the modelling method of the surface-current influence on small objects, presented in this paper, it is possible to implement an application supporting search-and-rescue-operation planning. This method can be used to locate small objects, such as survivors, when planning search-and-rescue operations. Thus, the routes of these objects moving together with surface-water masses can be predicted using this method.

Introduction

The International Aeronautical and Marine Search and Rescue Manual (IAMSAR, as amended) (ICAO & IMO, 2013) recommends the computing of a search area where survivors may be located by finding two datum positions based on the total water current and leeway from the last-known position. However, this method in coastal waters may indicate an incorrect search area. This error is due to the fact that the method takes into account sea currents, which are the main, large-scale flow of ocean waters caused by, among others, large-scale winds. Therefore, the method presented in this manual is based on the natural phenomena occurring in large water reservoirs with characteristics of ocean basins. IAMSAR (ICAO & IMO, 2013) recommends using data from short-term, reliable weather forecasts in coastal areas. These forecasts can be generated by hydrodynamic numerical models with high resolution. The method for describing the surface-current influence on small objects presented in this paper can narrow down the search area of these objects to

restricted waters, e.g., in coastal areas. Moreover, this approach can minimize the time required to locate a search object.

Hydrodynamic numerical models generate discretization grids with nodes of associated data, including information on their location, the dynamics of the marine area (currents, waves), and hydrological parameters (water temperature, salinity, sea level). In this paper, the location data are associated with the nodes of the considered grid by using geographical coordinates. In addition, directions and speeds of sea currents are determined. The discretization-grid nodes might be represented within a certain data structure. We can move between different grid nodes. The movement from one node to another is carried out along a certain route.

We describe the movement of small objects immersed in water based on the part extending above water level no larger than a human head. The movement of such objects in water is caused by surface currents. We consider currents in the layer down to the depth of one meter. Such objects will move, following the directions of surface currents.

We will establish a radius of a circle encompassing the grid nodes. The object will be able to move from the current node only to these nodes. We assume that the object will move along the shortest path from the current node at the circle centre in accordance with the surface-current direction assigned to this node to another node located inside this circle. Therefore, this path will be the line segment between these two points. The radius of the circle should be designed so that the gradient of the line segment along which the object will move is the best possible approximation of the surface-current direction assigned to the current node. That is, the discretization grid can be represented in the form of a graph whose vertices correspond to the grid nodes and directed edges correspond to line segments which join the corresponding grid nodes; this is possible because the object can move from one vertex to another and get back.

A given area into which the object moves is an infinite set of points. However, the discretization grid of this area is a finite set. We can make a finite number of different movements along line segments between the grid nodes. However, in an unrestricted area we can make an infinite number of these movements. Therefore, the edges of the graph will assign weights to achieve the best approximation of the object's movement in an unrestricted area. The edge weight along which the object will move describes the difference between the gradient of this edge and the direction of the surface current associated with the grid node in which the object is located. The edge weights are nonnegative real numbers. If the weight of some edge is equal to zero, the gradient of this edge (along which the object is moving) will be the same as the surface-current direction associated with the grid node of the outgoing edge.

It can be observed that the vector field of surface currents is not a constant field. Both directions and speeds of these currents associated with the grid nodes can be different. Thus, incoming and outgoing edges with a given vertex can have different weights. As a result, the discretization grid of a given area, together with the data on the currents, can be used to define the weighted directed graph. This graph is a data structure that reflects the surface current's influence on small objects.

The task now is to find a minimum-weight edge that goes out of the vertex where the object is located. This approach allows us to find a method of predicting the route that a small object immersed in water follows together with surface-water masses. This approach allows narrowing down the object's search area. Moreover, it minimizes a single operation time

to search the edge along which the object will move. We do not perform calculations on the entire base of nodes. Therefore, this approach allows us to reduce the computational complexity and memory complexity of algorithms for searching the nodes database.

The graph enables determining the relationships between the grid nodes where data values exist. These nodes can be connected using arrows labeled with directions to model the influence of surface currents on a small object immersed in water. Searching the data in the grid nodes on the basis of graph searching is much more efficient.

The graph theory is widely used in scientific research, including static-network analysis (analysis of a network whose characteristic does not change over time) and dynamic-network analysis (a network analysis which describes time-varying relationships). The static-network analysis provides a basis for many GIS analyses. These networks describe spatial relationships between geographic objects such as road systems, transportation networks, water and land areas, and buildings (e.g., Cichociński, 2012).

The dynamic-network analysis describes time-varying phenomena; an example might be the analysis of traffic stream variability in a number of crossroads (e.g., Alivand, Alesheikh & Malek, 2008). In this case, an object location and the time at which the object has reached its location are important. The characteristic of this network varies over time in either a predictable or an unpredictable way. If properties of the network change over time predictably, then the edge weights are a function of time. This method requires a lot of traffic data from different times and locations of the network. Thus, the weight function must be re-calculated. On the other hand, if properties of the network change over time in an unpredictable way, then the edge weights are re-calculated at each time step. The network is divided into subnetworks and the optimization is done locally in each subnetwork (the search of the shortest path). This method is based on partitioning space-time. One method for doing such partitions is based on the discretization of continuous time to smaller time intervals. At each time interval, the traffic condition is constant. Further, the space is divided according to the time intervals. Therefore, the optimal solution at each time and space interval is provided independently. In order to find the optimal solution for the entire network, this method is integrated with heuristic methods derived from the graph theory. The dynamic-network analysis uses Mobile Geospatial Information Systems (MGIS, as

amended) as a variety of conventional GIS and its main task is to examine a non-geographic object moving in the geographic space.

Hydrodynamic models generate data on sea currents over certain time intervals. Therefore, these data should be updated in the graphical model. Hence, the edge weights of the graph have to be re-calculated whenever up-to-date data on surface currents are provided from the hydrodynamic model to the graphical model.

Essential input data for building a graphical model

We assume that input data that are downloaded to run the model are derived from hydrodynamic models for a given area. These models generate data on currents – specifically their directions and speeds over certain time intervals and in different layers. For instance, in the Polish area, such data can be obtained from the M3D (Kowalewski, 1997) and the HIROMB model (Funquist & Kleine, 2007). These data are computed in nodes of a square-discretization grid of a considered area. Suppose that the data are organized in the form of an array and may be delivered as a file. In the given layer, we get successive data on:

- geographical coordinates of the grid node;
- current direction (in the range from -180° to 180°);
- current speed (in cm/s).

Let us recall that we consider surface currents in the layer up to one meter deep. Therefore, we select a layer in which a chosen hydrodynamic model best reflects the values of these surface currents.

Definition of the graphical model

Let us introduce the following notation:

$(\lambda_i; \varphi_i)$ – geographical coordinates of the i -th node of the square-discretization grid consisting of the n nodes ($i = 1, 2, \dots, n$);

λ_i – longitude of the i -th node;

φ_i – latitude of the i -th node;

α_i – direction of the surface current at the i -th node with the chosen layer;

v_i – speed of the surface current at the i -th node with the chosen layer.

In the *weighted directed graph* G : a triple (V, E, f) , where $V \neq \emptyset, E \subseteq V^2$, and $f: E \rightarrow R_+ \cup \{0\}$, the set V is called the *vertex set* of the graph G , and $x \in V$ is the *vertex* of the graph G . In turn, the set E is called the *edge set* of the graph G , and $xy \in E$ is

the *directed edge* (briefly the *edge*) of G . The edge xy is called an *outgoing edge* of the vertex x and an *incoming edge* of the vertex y . Moreover, the vertex y is the *neighbour* of the vertex x . The function f is called the *weight function* of the graph G , and $w = f(xy)$ is the *weight* of the edge xy in G .

We can now define the graph $G = (V, E, f)$ as a model of the impact of the surface currents on small objects considered in this paper. The set of vertices of the graph G is defined as the set:

$$V = \{1, 2, 3, \dots, n\} \quad (1)$$

where the vertex i corresponds to the node $(\lambda_i; \varphi_i)$ of the discretization grid of the considered area ($i = 1, 2, \dots, n$).

In order to determine the edge set of the graph, we need to make a certain cartographic transformation of the geographic-coordinate system by changing its centre. The centre of the new geographic-coordinate system is placed in the centre of the considered area (i.e., the translation of the equator and the prime meridian). Both the equator and the prime meridian are moved to the centre of the area. Then the line segment of one geographical degree will be identical along any meridian and along the circumference of the equator.

This system is replaced by the so-called *moved coordinate system* (Pietrek, 2006), which then is expressed in a certain linear system. The reader recalls that $(\lambda_i; \varphi_i)$ means the i -th discretization grid node described in the geographic system. First, we express the coordinates $(\lambda_i; \varphi_i)$ of this node in the Cartesian-coordinate system in the following way (without loss of generality we may put $r = 1$):

$$\begin{aligned} x &= \cos \varphi_i \cdot \cos \lambda_i \\ y &= \cos \varphi_i \cdot \sin \lambda_i \\ z &= \sin \varphi_i \end{aligned} \quad (2)$$

The next step is rotating this Cartesian coordinate system about Oz axis by the angle:

$$\alpha = 0.5 (\min L + \max L) \quad (3)$$

where L is the set of all the longitudes attached to the nodes of the discretization grid. The rotation is done in the following way:

$$O1 = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (4)$$

In the 3×1 matrix $O1$, there are Cartesian coordinates of the node (x, y, z) obtained by rotation.

Further, we rotate the obtained system about the Oy axis by the angle:

$$\beta = 0.5 (\min F + \max F) \quad (5)$$

where F is the set of all the latitudes attached to the nodes of the discretization grid. This rotation is done as follows:

$$O2 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \cdot O1 \quad (6)$$

In the 3×1 matrix $O2$, there are Cartesian coordinates of the node (x, y, z) obtained by two rotations. Let us denote them as follows: x_0, y_0, z_0 . Then, the obtained Cartesian-coordinate system is converted to the geographic system in the following way:

$$\lambda_p = \arctan \frac{y_0}{x_0}; \quad \varphi_p = \arcsin z_0 \quad (7)$$

In this way, we have obtained the coordinates (λ_p, φ_p) of the discretization grid node in the new, moved coordinate system. In this system, the grid of the area is square. Thus, all the distances between the grid nodes in vertical and horizontal directions are equal.

Next, we should transform this moved coordinate system as:

$$xm_i = \lambda_p \cdot 60 \cdot 1852; \quad ym_i = \varphi_p \cdot 60 \cdot 1852 \quad (8)$$

Then, the coordinates (xm_i, ym_i) of the i -th node of the grid are expressed in meters (1 Mm = 1852 m).

The edge set E of the graph G is in this form:

$$E = \{ij \in V^2: d((x_i, y_i)(x_j, y_j)) \leq r\} \quad (9)$$

where the vertex i of G corresponds to the node (xm_i, ym_i) of the discretization grid in the linear system, the vertex j corresponds to the node (xm_j, ym_j) , and $r > 0$. The distance d is the Cartesian distance. Therefore, to obtain all the neighbours of the vertex i , we should find the nodes (xm_j, ym_j) , whose maximum distance is r meters from the node (xm_i, ym_i) ; r is the radius of the circle at the centre in the node (xm_i, ym_i) (Figure 1).

Now we will describe how to obtain the edge weights of the graph G . Consider the vertex i and its neighbour k . Recall that, in the linear system, the node (xm_i, ym_i) corresponds to the vertex i and the node (xm_k, ym_k) corresponds to the vertex k . We calculate certain direction angles of the outgoing edges of the vertex i . Let us consider the following cases:

1. Let $ym_k > ym_i$. Then the direction angle θ_i of the edge ik is defined as follows:

$$\theta_i = \arctan \frac{xm_k - xm_i}{ym_k - ym_i} \text{ (degrees)} \quad (10)$$

2. Let $ym_k = ym_i$ and $xm_k < xm_i$. Then $\theta_i = -90^\circ$.

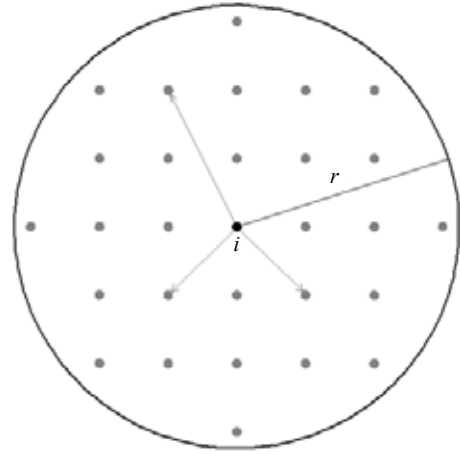


Figure 1. The vertex i , its neighbours, and three possible outgoing edges

3. Let $ym_k = ym_i$ and $xm_k > xm_i$. Then $\theta_i = 90^\circ$.

4. Let $ym_k < ym_i$ and $xm_k < xm_i$. Then

$$\theta_i = \arctan \frac{xm_k - xm_i}{ym_k - ym_i} - 180 \text{ (degrees)} \quad (11)$$

5. Let $ym_k < ym_i$ and $xm_k \geq xm_i$. Then

$$\theta_i = \arctan \frac{xm_k - xm_i}{ym_k - ym_i} + 180 \text{ (degrees)} \quad (12)$$

The direction angle θ_i of the outgoing edge of the vertex i is the angle between this edge and the north direction.

Recall that the direction of the surface current generated by the hydrodynamic model at the i -th node of the discretization grid is denoted by α_i . The i -th node of the grid corresponds to the vertex i of the graph G . For the edge ik of G , we set $b = |\theta_i - \alpha_i|$. The weight function $f: E \rightarrow R_+ \cup \{0\}$ is defined as follows:

$$f(ik) = \begin{cases} b, & b \leq 180 \\ 360 - b, & b > 180 \end{cases} \quad (13)$$

The number $w = f(ik)$ is called the weight of the edge ik .

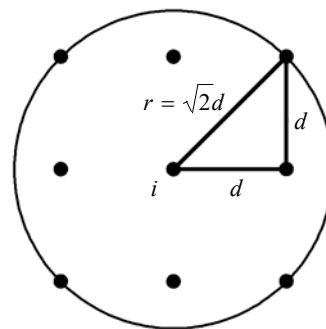


Figure 2. The vertex i and its neighbours in the first iteration

We will now describe how to select the radius r of the circle in which all the neighbours of the given vertex will be located. Let us assume that for the vertex i we determine all its neighbours. Recall that in the linear system, the grid node $(xm_i; ym_i)$ corresponds to the vertex i of the graph. In the first step, the radius of the circle is selected as follows:

$$r = \sqrt{2} \cdot d \tag{14}$$

where d is the discretization-grid step in the linear system (Figure 2). In this case, the vertex i has eight neighbours. The considered object will move from the vertex i along its outgoing edge with the minimum weight. Therefore, we would like to obtain the number b , associated to that edge, which is as small as possible. The maximum possible value b for such an edge is 22.5° . We assume the movement-direction accuracy δ , which is the maximum possible angular difference between the direction of this outgoing edge and surface-current direction assigned to the vertex i . Furthermore, we also assume that the data assigned by the hydrodynamic model are updated every Δt time units. If the weight of this outgoing edge is not greater than δ and the time needed to move along this edge is less than $0.1 \cdot \Delta t$, then $r = \sqrt{2} \cdot d$.

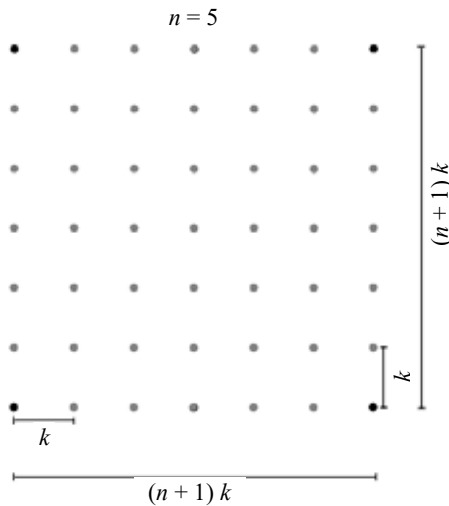


Figure 3. A part of the grid after interpolation

In another case, we should make an interpolation on the discretization grid. We then generate n additional grid nodes in the vertical and horizontal direction between each two old-grid nodes. In the new nodes, we assign surface-current speeds and directions. The bilinear interpolation is proposed to determine the surface currents at these additional nodes (Pyrchla, 2008). We first select a value for n (e.g., $n = 5$) in order to not increase computing

power too much, but to improve the direction of the predicted object's movement. If this direction is still incorrect, we should repeat the interpolation. For what is intended, at the least movement direction accuracy δ of the object at the given node, we establish $r = 2^{1/2} \cdot (n + 1) \cdot k \cdot a$, where k is the discretization step of the new grid (Figure 3) and a ($0 < a \leq 1$) is a scale of decreasing the radius of the circle (Figure 4). Initially, we can select $a = 0.5$. If the time of the moving object along the chosen edge is too long (longer than Δt), we can establish the scale a as, e.g., $a = 0.4$, $a = 0.3$, $a = 0.2$, $a = 0.1$.

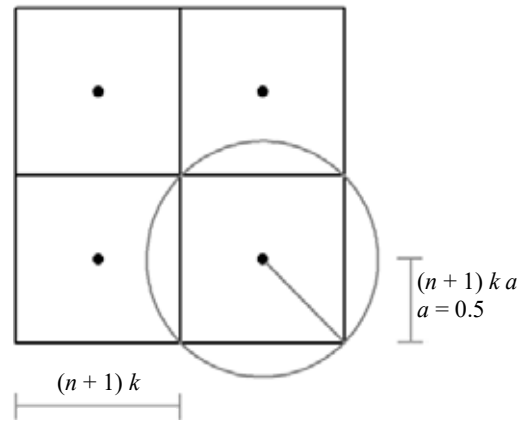


Figure 4. Selecting the scale a of decreasing the radius of the circle in the grid before the interpolation

Functional description of the graphical model

Suppose that the object is located at the i -th node of the discretization grid. Recall that the i -th node of the grid corresponds to the vertex i of the graph G . The movement of this object defined by the model will take place as follows:

- We select the outgoing edge of the vertex i with the minimum weight. The object will move along this edge.
- We determine the time required to move from the vertex i to another vertex along this edge. This time is determined on the basis of the surface-current speed assigned to the i -th node by the hydrodynamic model. Suppose that the object will move at a constant speed along the selected edge. Therefore, the time required to move along this edge equals s/v , where s is the length of the route along the chosen edge and v is the surface current speed assigned to the i -th node.
- Steps 1 and 2 are repeated and the times required to move along the chosen edges of the graph are summed. If the sum of the time lengths is equal

to Δt , then the graphical model updates the data on the surface currents and re-calculates the edge weights of the graph, and returns to step 1.

- The model terminates when it reaches the vertex corresponding to the grid node located on land or when it exceeds the simulation time of the object's movement.

Conclusions

The graphical model has been designed to provide a tool to reflect the influence of surface currents on small objects immersed in water, e.g. in coastal areas or harbours. The graphical model allows us to predict the route of small objects immersed in water that move together with surface water masses, e.g. a person in water who does not drift in the wind, but only with currents. As a consequence, this approach can narrow down the search area of the object. In addition, this solution may be used to determine the characteristics of water exchange based on the environmental-risk-management model methodology. The model is based on the directions and speeds of the surface currents generated by a hydrodynamic model. In the given time steps, the proposed model updates surface-current data. The graphical model

is characterized by its simplicity, which ensures that results are obtained very quickly. Furthermore, this approach can increase the productivity of grid-node searches.

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