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Probabilistic Approach to Determination of Oil Spill Domains at Port and Sea Water Areas

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ABSTRACT: A new method of oil spill domains' determination, based on a probabilistic approach, is recommended. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed. To describe the oil spill domain central point position a two-dimensional stochastic process is used. Parametric equations of oil spill domain central point drift trend curve for different kinds of hydro-meteorological conditions are determined. The general model of oil spill domain determination for various hydro-meteorological conditions is proposed. Moreover, approximate expected stochastic prediction of the oil spill domain movement in constant and changing hydro-meteorological conditions is proposed.

1 INTRODUCTION

One of the important duties in port activities and shipping is the prevention of oil release from port installations and ships and the spread of oil spills that often have dangerous consequences for port and sea water areas (Bogalecka & Kołowrocki 2018, Dąbrowska & Kołowrocki 2019A, NOAA). Thus, as the first step, there is a need for methods of oil spill domain movement modelling based on determination of the oil spill central point drift curve determination and the oil spill domain probable placement at any moment after the accident that could be the tools for increasing the shipping safety and effective port and sea environment protection. Even if, the real trajectory of the oil spill central point and the oil spill domain movement are different from those determined by the proposed methods, they can be useful in the port and sea environment protection.

The oil spill central point drift trend, the oil spill domain shape and its random position distribution fixed for different hydro-meteorological conditions allow us to construct the model of determination of the area in which, with the in advance fixed probability, the oil spill domain is placed (Dąbrowska & Kołowrocki 2019A). This way, the area determined for oil spill allow us to mark the domain where the actions of mitigating the oil release consequences should be performed. This approach is proposed to make oil releases at the sea prevention and mitigation actions more effective.

The general model of the oil spill domain determination based on the probabilistic approach may be practically applied in the oil spill consequences mitigation actions at the sea after its unknown parameters' statistical identification. Statistical experiments should be performed according to the methods of the model unknown parameters estimation. Thus, the methods of evaluation of unknown parameters of the oil spill central point drift curve and the joint density function should be proposed. Moreover, the procedures of their practical evaluations should be done as well (Dąbrowska & Kołowrocki 2019A).

2 MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

We denote by $A(t)$ the process of changing hydro-meteorological conditions at the sea water areas where the oil spill happened and distinguish m its states from the set $A = \{1,2,...,m\}$ in which it may stay at the moment t, $t \in <0,T>$, where $T > 0$. Further, we assume a semi-Markov model of the process $A(t)$ and denote by θ_{ij} its conditional sojourn time in the state i while its next transition will be done to the state j, where $i, j = 1,2,...,m$, $i \neq j$ (Dąbrowska & Kołowrocki 2019A). Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska & Kołowrocki 2019A):

– the vector of probabilities of its initial states at the moment t = 0

$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)]; \quad (1)$$

– the matrix of probabilities of its transitions between the particular states

$$[p_{ij}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \quad (2)$$

where $p_{ii} = 0$ for $i = 1,2,...,m$;

– the matrix of distribution functions of its conditional sojourn times θij at the particular states

$$[W_{ij}(t)] = \begin{bmatrix} W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\ W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}(t) & W_{m2}(t) & \cdots & W_{mm}(t) \end{bmatrix}, \quad (3)$$

where $W_{ii}(t) = 0$ for $i = 1,2,...,m$;

– the expected values (mean values) of its conditional sojourn times θij at the particular states

$$M_{ij} = E[\theta_{ij}] = \int_0^\infty t dW_{ij}(t), \; i, j = 1,2,...,m, \; i \neq j. \quad (4)$$

Having the above parameters of the process of changing hydro-meteorological conditions $A(t)$, $t \in <0,T>$, $T > 0$, this process following characteristics can be determined (Dąbrowska & Kołowrocki 2019A):

– the distribution functions of the unconditional sojourn time θi of the process of changing hydro-meteorological conditions at the particular states i, i = 1,2,...,m,

$$W_i(t) = \sum_{j=1}^{m} p_{ij} W_{ij}(t), \; i = 1,2,...,m; \quad (6)$$

– the mean values of the unconditional sojourn time θi of the process of changing hydro-meteorological conditions at the particular states i, i = 1,2,...,m,

$$M_i = E[\theta] = \sum_{j=1}^{m} p_{ij} E[\theta_{ij}], \; i = 1,2,...,m. \quad (7)$$

3 MODELLING TREND OF OIL SPILL CENTRAL POINT DRIFT

First, for each fixed state k, $k \in \{1,2,\ldots,m\}$, of the process $A(t)$ and time $t \in <0,T>$, where T is time we are going to model the behaviour of the oil spill domain $\overline{D}^k(t)$, we define the central point of this oil spill domain as a point $(x^k(t), y^k(t))$, $t \in <0,T>$, $k \in \{1,2,\ldots,m\}$, on the plane Oxy that is the centre of the smallest circle, with the radius $r^k(t)$, $t \in <0,T>$, $k \in \{1,2,\ldots,m\}$, covering this domain (Figure 1). Thus, for the fixed oil spill domain $\overline{D}^k(t)$, we have

$$x^k(t) = \frac{x_1^k(t) + x_2^k(t)}{2}, \quad y^k(t) = \frac{y_1^k(t) + y_2^k(t)}{2},$$

$$t \in <0,T>, k \in \{1,2,\ldots,m\}, \quad (8)$$

where the $P_1(x_1^k(t), y_1^k(t))$ and $P_2(x_2^k(t), y_2^k(t))$ are the most distant points of the oil spill domain $\overline{D}^k(t)$, $t \in <0,T>$, $k \in \{1,2,\ldots,m\}$, and the radius $r^k(t)$, called the radius of the oil spill domain $\overline{D}^k(t)$, is given by

$$r^k(t) = \frac{1}{2}\sqrt{[x_1^k(t) - x_2^k(t)]^2 + [y_1^k(t) - y_2^k(t)]^2},$$

$$t \in <0,T>, k \in \{1,2,\ldots,m\}. \quad (9)$$

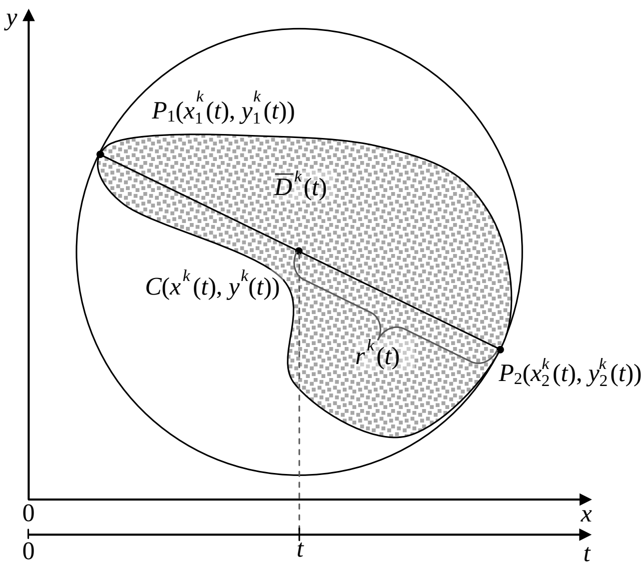

Figure 1. Interpretation of central point of oil spill definition.

Further, for each fixed state k, $k = 1,2,\ldots,m$, of the process $A(t)$ and time t, $t \in <0,T>$, we define a two-dimensional stochastic process

$$(X^k(t), Y^k(t)), \quad t \in <0,T>,$$

such that

$$(X^k, Y^k) : <0,T> \ \to R^2,$$

where $X^k(t)$, $Y^k(t)$ respectively are an abscissa and an ordinate of the plane Oxy point, in which the oil spill central point is placed at the moment t while the process $A(t)$, $t \in <0,T>$, is at the state k. We set deterministically the central point of oil spill domain in the area in which an accident has happened and an oil release was placed in the water as the origin $O(0,0)$ of the co-ordinate system Oxy. The value of a parameter t at the moment of accident we assume equal to 0. It means that the process $(X^k(t), Y^k(t))$, is a random two-dimensional co-ordinate (a random position) of the oil spill central point after the time t from the accident moment and that at the accident moment $t = 0$ the oil spill central point is at the point $O(0,0)$, i.e.

$$(X^k(0), Y^k(0)) = (0,0).$$

After some time, the central point of the oil spill starts its drift along a curve called a drift curve. In further analysis, we assume that processes

$$(X^k(t), Y^k(t)), \quad t \in <0,T>, k \in \{1,2,\ldots,m\},$$

are two-dimensional normal processes

$$N(m_X^k(t), m_Y^k(t), \rho_{XY}^k(t), \sigma_X^k(t), \sigma_Y^k(t)),$$

with varying in time expected values

$$m_X^k(t) = E[X^k(t)], \quad m_Y^k(t) = E[Y^k(t)],$$

standard deviations

$$\sigma_X^k(t), \quad \sigma_Y^k(t), \ t \in <0,T>, k \in \{1,2,\ldots,m\},$$

and correlation coefficients

$$\rho_{XY}^k(t),$$

i.e. with the joint density functions

$$\varphi_t^k(x,y) = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}}$$

$$\exp\{-\frac{1}{2(1-(\rho_{XY}{}^k(t))^2)}[\frac{(x-m_X{}^k(t))^2}{(\sigma_X{}^k(t))^2}$$

$$-2\rho_{XY}{}^k(t)\frac{(x-m_X{}^k(t))(y-m_Y{}^k(t))}{\sigma_X{}^k(t)\sigma_Y{}^k(t)}$$

$$+\frac{(y-m_Y{}^k(t))^2}{(\sigma_Y{}^k(t))^2}]\},$$

$$(x,y) \in R^2, \ t \in <0,T>, k \in \{1,2,\ldots,m\}. \quad (10)$$

where $X^k(t)$, $Y^k(t)$ respectively are an abscissa and an ordinate of the plane Oxy point, in which the oil spill central point is placed at the moment t while the process $A(t)$, $t \in <0,T>$, is at the state k. We set deterministically the central point of oil spill domain in the area in which an accident has happened and an oil release was placed in the water as the origin $O(0,0)$ of the co-ordinate system Oxy. The value of a parameter t at the moment of accident we assume equal to 0. It means that the process $(X^k(t), Y^k(t))$, is a random two-dimensional co-ordinate (a random position) of the oil spill central point after the time t from the accident moment and that at the accident moment $t = 0$ the oil spill central point is at the point $O(0,0)$, i.e.

$$(X^k(0), Y^k(0)) = (0,0).$$

Thus, the points

$$(m_X^k(t), m_Y^k(t)), \ t \in <0,T>, k \in \{1,2,\ldots,m\},$$

create a curve K^k called an oil spill central point drift trend (Figure 2) which may be described in the parametric form

$$K^k : \begin{cases} x^k = x^k(t) \\ y^k = y^k(t), t \in <0,T>. \end{cases} \quad (11)$$

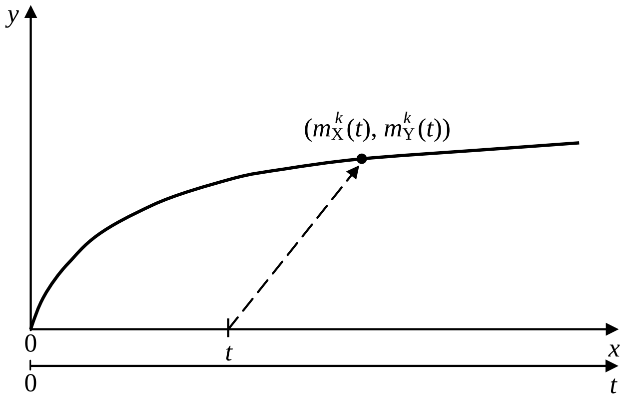

Figure 2. Oil spill central point drift trend.

4 MODELLING OIL SPILL DOMAIN

4.1 *Probabilistic approach*

We are interested in finding the search domain $D^k(t)$, $t \in <0,T>$, $k \in \{1,2,\ldots,m\}$, such that the central point of oil spill domain is placed in it with a fixed probability p. More exactly, we are looking for c such that

$$P((X^k(t),Y^k(t)) \in D^k(t)) = \iint_{D^k(t)} \varphi_t^k(x,y)dxdy = p,$$

$t \in <0,T>$, $k \in \{1,2,\dots,m\}$, (12)

where

$$D^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2}$$

$$- 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)}$$

$$+ \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}] \le c^2\},\ t \in <0,T>,$$

$k \in \{1,2,\dots,m\}$, (13)

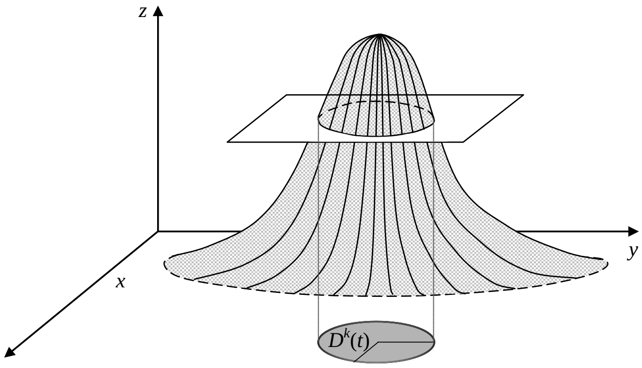

Figure 3. Domain $D^k(t)$ of integration bounded by an ellipse.

is the domain bounded by an ellipse being the projection on the plane $0xy$ (Figure 4) of the curve rising as the result of intersection (Figure 3) of the density function surface

$$\pi_1^k = \{(x,y,z): z = \varphi_t^k(x,y),\ (x,y) \in R^2\}, \quad (14)$$

and the plane

$$\pi_2^k = \{(x,y,z):$$

$$z = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}}\exp[-\frac{1}{2}c^2],$$

$(x,y) \in R^2\}$, $t \in <0,T>$, $k \in \{1,2,\dots,m\}$. (15)

Since

$$P((X^k(t),Y^k(t)) \in D^k(t)) = 1-\exp[-\frac{1}{2}c^2],\ t \in$$

$<0,T>$,
$k \in \{1,2,\dots,m\}$, (16)

then for a fixed probability p, the equality

$$p = P((X^k(t),Y^k(t)) \in D^k),$$

$t \in <0,T>$, $k \in \{1,2,\dots,m\}$, (17)

holds if

$$c^2 = -2\ln(1-p). \quad (18)$$

Thus, the domain in which at the moment t the central point of oil spill is placed with the fixed probability p is given by

$$D^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2}$$

$$- 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}]$$

$\le -2\ln(1-p)\}$, $t \in <0,T>$, $k \in \{1,2,\dots,m\}$. (19)

Considering the above and the assumed in Section 3 definition of the central point of oil spill, for each fixed state k, $k \in \{1,2,\dots,m\}$, of the process $A(t)$ and time $t \in <0,T>$, we define the oil spill domain

$$\overline{D}^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\overline{\sigma}_X^k(t))^2}$$

$$- 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\overline{\sigma}_X^k(t)\overline{\sigma}_Y^k(t)}$$

$$+\frac{(y-m_Y^k(t))^2}{(\overline{\sigma}_Y^k(t))^2}] \le -2\ln(1-p)\},$$

$t \in <0,T>$, $k \in \{1,2,\dots,m\}$, (20)

where

$$\overline{\sigma}_X^k(t) = \sigma_X^k(t) + r^k(t),\quad \overline{\sigma}_Y^k(t) = \sigma_Y^k(t) + r^k(t),$$

$t \in <0,T>$, $k \in \{1,2,\dots,m\}$, (21)

and

$r^k(t)$, $t \in <0,T>$, $k \in \{1,2,\dots,m\}$, (22)

is the radius of the oil spill domain $\overline{D}^k(t)$, $t \in <0,T>$, $k \in \{1,2,\dots,m\}$.

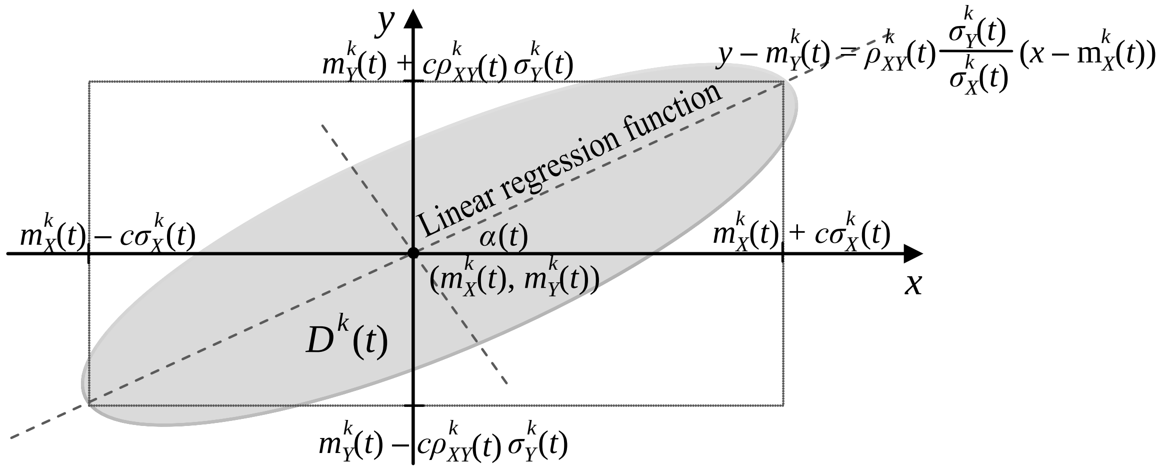

Figure 4. Domain $D^k(t)$ covering oil spill central point with probability p.

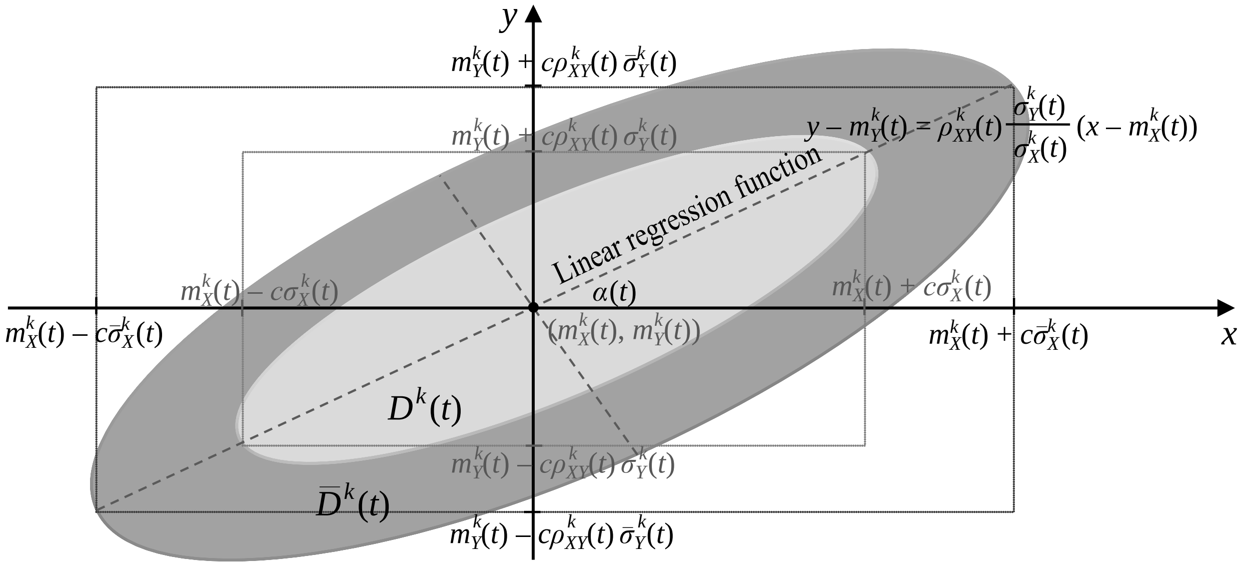

Figure 5. Oil spill domain $\overline{D}^k(t)$.

The graph of the oil spill domain $\overline{D}^k(t)$ is given in Figure 5.

To find the oil spill domain $\overline{D}^k(t)$ determined by (20)-(22) and presented in Figure 5, the statistical methods of its general model unknown parameters estimation are proposed in (Dąbrowska & Kołowrocki 2019A). These methods are presented in the form of algorithms giving successive steps which should be done to evaluate these unknown model parameters on the base of statistical data coming from experiments performed at the sea.

4.2 *Oil spill domain for fixed hydro-meteorological conditions*

We suppose that the process $A(t)$ for all $t \in <0,T>$, is at the fixed state k, $k \in \{1,2,\ldots,m\}$. Assuming a time step Δt and a number of steps s, $s \geq 1$, such that

$$(s-1)\Delta t < M_k \leq s\Delta t, \quad s\Delta t \leq T, \tag{23}$$

where

$$M_k = E[\theta_k],\ k \in \{1,2,\ldots,m\}, \tag{24}$$

are the expected value of the process $A(t)$, $t \in <0,T>$, sojourn times θ_k, $k = 1,2\ldots,m$, at the state k determined in Section 2, after multiple applying sequentially the procedure from Section 4.1, for

$$t = 1\Delta t, 2\Delta t, \cdots, s\Delta t, \tag{25}$$

we receive the following sequence of oil spill domains (Figure 6)

$$\overline{D}^k(\Delta t), \overline{D}^k(2\Delta t), \ldots, \overline{D}^k(s\Delta t). \tag{26}$$

Hence, the oil spill domain $\overline{D}^k$, $k \in \{1,2,\ldots,m\}$, is described by the sum of determined domains of the sequence (26)

$$\overline{D}^k = \bigcup_{i=1}^{s} \overline{D}^k(i\Delta t) = \overline{D}^k(1\Delta t) \cup \overline{D}^k(2\Delta t) \cup \ldots \cup \overline{D}^k(s\Delta t),$$
$$k = 1,2,\ldots,m, \tag{27}$$

and illustrated in Figure 6.

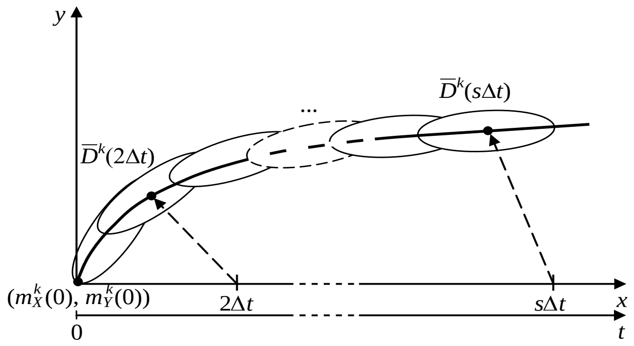

Figure 6. Oil spill domain for fixed hydro-meteorological conditions.

Remark 1. The oil spill domain $\overline{D}^k$ defined by (27) and illustrated in Figure 6 is determined for constant radius $r^k(t) = r^k$, $t \in <0,T>$, $k \in \{1,2,\ldots,m\}$. If the radius is not constant, we define the sequence of domains (Dąbrowska & Kołowrocki 2019A)

$$\overline{\overline{\boldsymbol{D}}}^{k}(b\Delta t) = \bigcup_{a=1}^{b} \overline{\overline{D}}^{k}(a\Delta t) = \overline{\overline{D}}^{k}(1\Delta t) \cup \overline{\overline{D}}^{k}(2\Delta t) \cup \ldots$$

$$\cup\, \overline{\overline{D}}^{k}(b\Delta t), \quad b = 1,2,...,s, \quad k \in \{1,2,\ldots,m\},$$

where

$$\overline{\overline{D}}^{k}(a\Delta t) := \overline{D}^{k}(a\Delta t), \quad a = 1,2,...,b,\ b = 1,2,...,s,$$

$$k = 1,2,...,m,$$

defined by (20) with the following substitutions:

$$m_X^k(t) := m_X^k(a\Delta t),$$

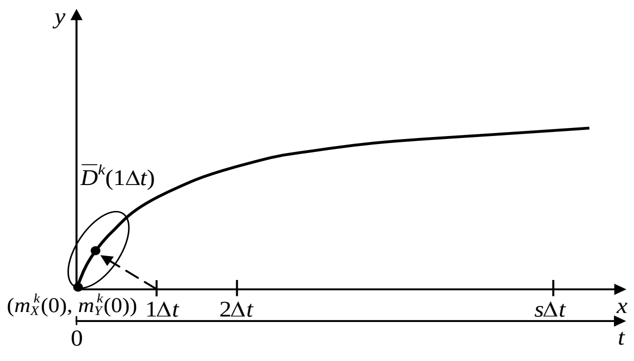

Figure 7. Oil spill domain at the time 1Δ*t*.

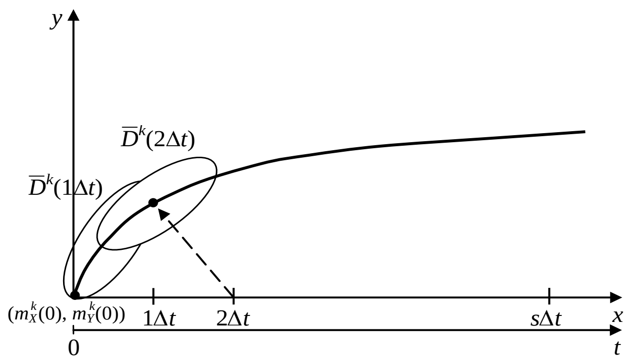

Figure 8. Oil spill domain at the time 2Δ*t*.

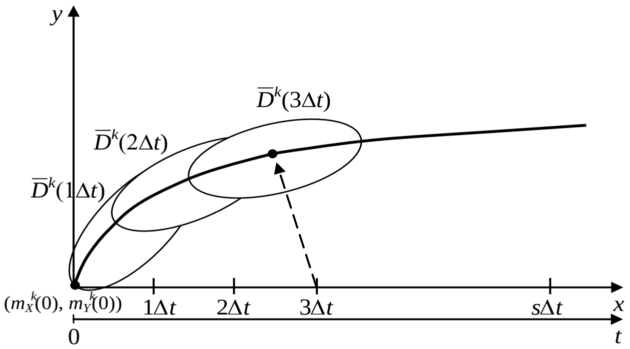

Figure 9. Oil spill domain at the time 3Δ*t*.

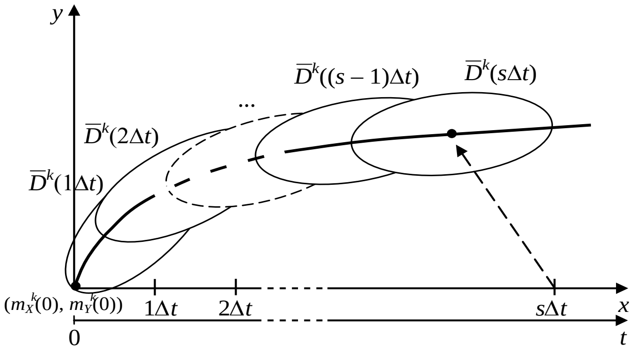

Figure 10. Oil spill domain at the time *s*Δ*t*.

$$m_Y^k(t) := m_Y^k(a\Delta t),$$

$$\overline{\sigma}_X^k(t) := \overline{\overline{\sigma}}_X^k(b\Delta t) = \sigma_X^k(b\Delta t) + r^k(b\Delta t),$$

$$\overline{\sigma}_Y^k(t) := \overline{\overline{\sigma}}_Y^k(b\Delta t) = \sigma_Y^k(b\Delta t) + r^k(b\Delta t),$$

$$a = 1,2,...,b,\ b = 1,2,...,s, \quad k \in \{1,2,\ldots,m\}.$$

This oil spill domain movement is illustrated in Figures 7-10.

4.3 *Oil spill domain in varying hydro-meteorological conditions*

We assume that the process of changing hydro-meteorological conditions in succession takes the states $k_1, k_2, \ldots, k_{n+1}$, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n+1$. For a fixed step of time Δt, after multiple applying sequentially the procedure from Section 4.1:

– for

$$t = 1\Delta t, 2\Delta t, \cdots, s_1\Delta t, \tag{28}$$

at the process $A(t)$ state k_1;

– for

$$t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t, \tag{29}$$

at the process $A(t)$ state k_2;

– for

$$t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t, \tag{30}$$

at the process $A(t)$ state k_n;

we receive the following sequence of oil spill domains (Figure 11):

$$\overline{D}^{k_1}(1\Delta t), \overline{D}^{k_1}(2\Delta t), \ldots, \overline{D}^{k_1}(s_1\Delta t), \tag{30}$$

$$\overline{D}^{k_2}((s_1+1)\Delta t), \overline{D}^{k_2}((s_1+2)\Delta t), \ldots, \overline{D}^{k_2}(s_2\Delta t), \tag{31}$$

$$\overline{D}^{k_n}((s_{n-1}+1)\Delta t), \overline{D}^{k_n}((s_{n-1}+2)\Delta t), \ldots, \overline{D}^{k_n}(s_n\Delta t), \tag{32}$$

where s_i, $i = 1,2,\ldots,n$, are such that

$$(s_i-1)\Delta t < \sum_{j=1}^{i} M_{k_j k_{j+1}} \le s_i\Delta t,\ i = 1,2,\ldots,n,$$
$$s_n\Delta t \le T, \tag{33}$$

and

$$M_{k_j k_{j+1}} = E[\theta_{k_j k_{j+1}}],\ j = 1,2\ldots,n, \tag{34}$$

are the expected value of the process $A(t)$, $t \in \langle 0,T\rangle$, conditional sojourn times $\theta_{k_j k_{j+1}}$, $j = 1,2\ldots,n$ at the states k_j, upon the next state is k_{j+1}, $j = 1,2\ldots,n$, $k_i \in \{1,2,\ldots,m\}$, $i = 1,2\ldots,n$, determined in Section 2.

Hence, the oil spill domain $\overline{D}^{k_1,k_2,\ldots,k_n}$, $k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\}$, is described by the sum of determined domains of the sequences (30)-(32), given by

$$\overline{D}^{k_1,k_2,\ldots,k_n} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{s_i - s_{i-1}} \overline{D}^{k_i}((s_{i-1} + j)\Delta t)$$
$$= [\overline{D}^{k_1}(1\Delta t) \cup \overline{D}^{k_1}(2\Delta t) \cup \ldots \cup \overline{D}^{k_1}(s_1\Delta t)]$$
$$\cup [\overline{D}^{k_2}((s_1+1)\Delta t) \cup \overline{D}^{k_2}((s_1+2)\Delta t) \cup \ldots \cup \overline{D}^{k_2}(s_2\Delta t)]$$
$$\cup [\overline{D}^{k_n}((s_{n-1}+1)\Delta t) \cup \overline{D}^{k_n}((s_{n-1}+2)\Delta t) \cup \ldots \cup \overline{D}^{k_n}(s_n\Delta t)]$$
$$\text{for } k_1, k_2, \ldots, k_n \in \{1,2,\ldots,m\},\quad s_0 = 0, \tag{35}$$

Remark 2. The oil spill domain $\overline{D}^{k_1,k_2,\ldots,k_n}$ defined by (35) and illustrated in Figure 11 is determined for constant radiuses $r^{k_i}(t) = r^{k_i}$, $t \in \langle 0,T\rangle$, $k_i \in \{1,2,\ldots,m\}$, $i = 1,2,\ldots,n$. If the radiuses are not constant, we define the sequence of domains for each sate k_i, $k_i \in \{1,2,\ldots,m\}$, $i = 1,2,\ldots,n$, in a way similar to that described in Remark 1 in Section 4.2, i.e. we define the sequence of domains

$$\overline{\overline{\boldsymbol{D}}}^{k_1,k_2,\ldots,k_n}(b_i\Delta t) = \bigcup_{i=1}^{n} \bigcup_{a_i=1}^{b_i} \overline{\overline{\boldsymbol{D}}}^{k_i}(s_{i-1} + a_i\Delta t)$$
$$= [\overline{\overline{D}}^{k_1}(1\Delta t) \cup \overline{\overline{D}}^{k_1}(2\Delta t) \cup \ldots \cup \overline{\overline{D}}^{k_1}(s_1\Delta t)]$$
$$\cup [\overline{\overline{D}}^{k_2}((s_1+1)\Delta t) \cup \overline{\overline{D}}^{k_2}((s_1+2)\Delta t) \cup \ldots \cup \overline{\overline{D}}^{k_2}(s_2\Delta t)]$$
$$\cup [\overline{\overline{D}}^{k_n}((s_{n-1}+1)\Delta t) \cup \overline{\overline{D}}^{k_n}((s_{n-1}+2)\Delta t) \cup \ldots \cup \overline{\overline{D}}^{k_n}(s_n\Delta t)]$$
$$\text{for } b_i = 1,2,\ldots, s_i - s_{i-1},\quad k_i \in \{1,2,\ldots,m\},\ i = 1,2,\ldots,n,$$

where

$$\overline{\overline{D}}^{k_i}(s_{i-1} + a_i\Delta t) := \overline{D}^{k}(s_{i-1} + a_i\Delta t),$$
$$a_i = 1,2,\ldots,b_i,\ b_i = 1,2,\ldots, s_i - s_{i-1},\quad k_i \in \{1,2,\ldots,m\},$$
$$i = 1,2,\ldots,n,$$

defined by (20) with the following substitutions:

$$m_X^k(t) := m_X^{k_i}(s_{i-1} + a_i\Delta t),$$
$$m_Y^k(t) := m_Y^{k_i}(s_{i-1} + a_i\Delta t),$$
$$\overline{\sigma}_X^k(t) := \overline{\overline{\sigma}}_X^{k_i}(s_{i-1} + b_i\Delta t)$$
$$= \sigma_X^{k_i}(s_{i-1} + b_i\Delta t) + \sum_{j=1}^{i} r^{k_j}(s_{j-1} + b_j\Delta t),$$
$$\overline{\sigma}_Y^k(t) := \overline{\overline{\sigma}}_Y^{k_i}(s_{i-1} + b_i\Delta t)$$
$$= \sigma_Y^{k_i}(s_{i-1} + b_i\Delta t) + \sum_{j=1}^{i} r^{k_j}(s_{j-1} + b_j\Delta t),$$
$$a_i = 1,2,\ldots,b_i,\ b_i = 1,2,\ldots, s_i - s_{i-1},\ k_i \in \{1,2,\ldots,m\},$$
$$i = 1,2,\ldots,n.$$

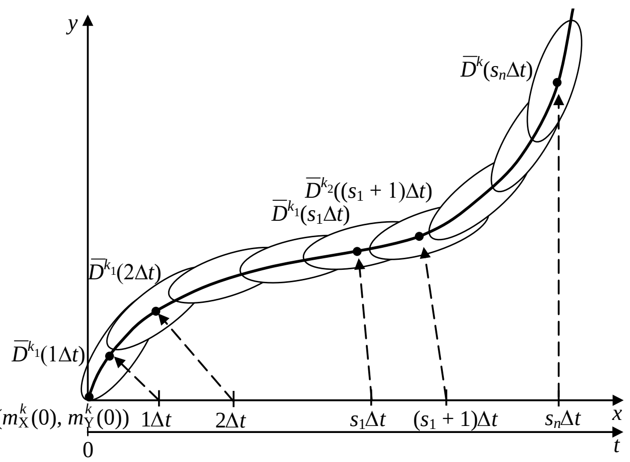

Figure 11. Oil spill domain for changing hydro-meteorological conditions.

The oil spill domain movement in this case can be illustrated in a similar way (a bit more complicated) to that given in Figures 7-10.

5 CONCLUSIONS

The improvement of the methods of the oil spill domains determination is the main real possibility of the identifying the pollution size and the reduction of time of its consequences elimination. Therefore, it seems to be necessary to start with the new and effective methods of the oil spill domains at port and sea waters determination in constant and changing hydro-meteorological conditions. The most important criterion of new methods should be the time of the oil spill consequences minimising. One of the essential factors that could ensure these criteria fulfilment is the accuracy of methods of the oil spill domain determination. Those methods should be the basic parts of the general problem of different kinds of pollution identification, their consequences reduction and elimination at the port and sea water areas to elaborate a complete information system assisting people and objects in the protection against the hazardous contamination of the environment. One of the new efficient methods of more precise determination of the oil spill domains determination could be a probabilistic approach to this problem presented in this paper and preliminarily in (Dąbrowska & Kołowrocki 2019A).

The oil spill domains determined for different hydro-meteorological conditions can be also done for other kind of spills, dangerous for the environment. The proposed probabilistic approach to oil spill domains determination would surely improve the efficiency of people activities in the environment protection. A weak point of the method is the time and cost of the experiments necessary to perform at the port and sea water areas in order to identify statistically particular components of the proposed models (Dąbrowska & Kołowrocki 2019A). Especially experiments needed to evaluate drift trends and parameters of the central point of oil spill position distributions can consume much time and be costly as they have to be done for different kind of spills and different hydro-meteorological conditions in various areas. A strong and positive point of the method is the fact that the experiments for the fixed port and sea water areas and fixed hydro-meteorological conditions have to be done only once and the identified models may be used for all environment protection actions at these regions and also transferred for other regions with similar hydro-meteorological conditions.

The proposed stochastic approach can be supplemented by the Monte Carlo simulation approach (Dąbrowska 2019) to the spill oil domain movement investigation proposed in (Dąbrowska & Kołowrocki 2019A, 2019B). These two approaches are the authors' primary original approaches to the oil spill domain determination which are intended to be significantly developed with the close considering the contents of publications cited in references below.

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