

# Comparative analysis of multi-criteria decision making methods for the assessment of optimal SVC location

Faruk AYDIN<sup>1\*</sup> and Bilal GÜMÜŞ<sup>2</sup>

<sup>1</sup> Department of Electrical and Electronics Engineering, Faculty of Technology, Marmara University, İstanbul 34722, Turkey

<sup>2</sup> Department of Electrical and Electronics Engineering, Faculty of Engineering, Dicle University, Diyarbakır 21680, Turkey

**Abstract.** The goal of multi-criteria decision making (MCDM) is to select the most appropriate of the alternatives by evaluating many conflicting criteria together. MCDM methods are widely available in the literature and have been used in various energy problems. The key problems studied in electrical power systems in recent years have included voltage instability and voltage collapse. Different flexible alternating current transmission systems (FACTS) equipment has been used for this purpose for decades, increasing voltage stability while enhancing system efficiency, reliability and quality of supply, and offering environmental benefits. Finding the best locations for these devices in terms of voltage stability in actual electrical networks poses a serious problem. Many criteria should be considered when determining the most suitable location for the controller. The aim of this paper is to provide a comparative analysis of MCDM techniques to be used for optimal location of a static VAR compensator (SVC) device in terms of voltage stability. The ideal location can be determined by means of sorting according to priority criteria. The proposed approach was carried out using the Power System Analysis Toolbox (PSAT) in MATLAB in the IEEE 14-bus test system. Using ten different MCDM methods, the most appropriate locations were compared among themselves and a single ranking list was obtained, integrated with the Borda count method, which is a data fusion technique. The application results showed that the methods used are consistent among themselves. It was revealed that the integrated model was an appropriate method that could be used for optimal location selection, providing reliable and satisfactory results to power system planners.

**Key words:** Borda count method; FACTS; MCDM; optimal location; PSAT; voltage stability.

## 1. INTRODUCTION

Growing energy demand and the increase in electricity consumption rates force power systems to operate in regions close to stability limits. Although voltage instability happens in a critical area, which is mainly caused by the lack of reactive power in the network, this affects the whole power grid. Therefore, voltage stability plays a critical role in the stability of power systems [1]. The power electronics technology has offered the opportunity to develop flexible alternating current transmission systems (FACTS) equipment for stable power system operations. Especially in the last thirty years, many power electronics based controllers have been developed. FACTS controllers are used extensively for voltage control as well as for controlling load flow, reducing harmonics, minimizing losses and improving transient stability [2].

In addition to their benefits, such as enhancing the voltage profile and reducing power losses of the interconnected grid, shunt capacitors are mainly used for reactive power compensation. The loading margin and the power transmission capability can be increased by using shunt capacitors, i.e. the static

VAR compensator (SVC) and static synchronous compensator (STATCOM). Although SVC and STATCOM cost more than a basic shunt capacitor, they perform better in terms of voltage profile enhancement and loss reduction.

It is difficult and unnecessary to install shunt controllers on all buses, for economic and environmental reasons [3]. Identifying the optimal location for compensation devices requires calculating the stability requirements of the grid. Nevertheless, the problem is extremely complicated due to the nonlinearity of the power flow formulas and thorough analysis must be done to overcome it [4].

Learning and intelligent optimization methods are generally used in calculating the optimum capacity and location of FACTS devices, where multi-objective approaches have become widespread in recent years. In the fitness function, where the best result is attempted to be achieved for many goals, minimum satisfaction should be provided for each objective. Also, appropriate weights should be chosen to show the relative importance of the objectives. The weight coefficients of the criteria are selected equally in the literature or approximate values are derived by the trial and error method. Multi-criteria decision making (MCDM) techniques have never been used in previous studies to determine these weights [4–10]. Although MCDM methods are being used more and more frequently in energy selection problems, they have never been applied in the selection

\*e-mail: faruk.aydin@marmara.edu.tr

Manuscript submitted 2021-10-18, revised 2021-12-31, initially accepted for publication 2022-02-02, published in April 2022.

of suitable locations for FACTS devices, especially in terms of voltage stability [11–13].

MCDM problems can be examined under three main headings. These problems are choice, sorting and ranking. The goal of the decision maker in each type of problem is different. Whereas the aim of the decision maker in the selection problem is to find the best option, in the ranking problem, the aim is to rank all the alternatives from best to worst. In the sorting problem, the decision maker classifies the options according to the purpose. Decision making becomes difficult when there are too many criteria in this process and uncertainty renders the decision making process more complex. Many methods and computer programs have been developed by scientists to facilitate decision making and to make more effective decisions. Further studies on this subject are ongoing.

When faced with a particular problem, there is no clear guideline about which method to use for the solution. This is a subject that has been studied and discussed for many years in the literature. Depending on the MCDM method applied, the results may differ, especially when the alternatives are similar. Therefore, a comparative analysis can be made between some MCDM methods to better understand the similarities and differences. This way, in the long term guidelines are obtained to support the decision maker on which method to use [14].

In view of the above, this study provides a comparative analysis of MCDM methods to be used for the optimal placement of SVC devices. For this purpose, the multi-objective optimization problem is converted into a single objective function using MCDM techniques. Using ten different methods, the most appropriate locations were compared among themselves and a single ranking was obtained, integrated with the Borda count method, which is a data fusion technique. Ten different MCDM methods used in this study are listed below:

- AHP (analytic hierarchy process).
- TOPSIS (technique for order preference by similarity to ideal solution).
- VIKOR (VIseKriterijumsa Optimizacija I Kompromisno Resenje – multi-criteria optimization and compromise solution).
- WASPAS (weighted aggregated sum product assessment).
- GTMA (graph theory and matrix approach).
- PROMETHEE II (preference ranking organization method for enrichment of evaluation).
- GRA (grey relational analysis).
- MULTIMOORA (multi-objective optimization by a ratio analysis).
- ARAS (additive ratio assessment).
- COPRAS (complex proportional assessment).

In this suggested technique, three goals are accomplished simultaneously when the SVC controller is in the appropriate location, taking into account the operation and load constraints: increasing the loading margin, reducing the deviation of bus voltage and minimizing the power losses. The simulation results in the IEEE sample network have shown the efficiency of the suggested method.

## 2. FACTS DEVICES

Since traditional concepts and applications of energy systems have changed in recent years, it is necessary to increase the existing capacity of the grids by using FACTS devices [8]. In fact, power electronics controlled devices such as SVCs have been used in electrical power systems for many years. However, N. Hingorani developed the FACTS concept to control the power flow in a network and to ensure maximum loading of the transmission line [15, 16].

The FACTS controllers can be categorized as shunt, series, combined series-shunt and combined series-series. Basically, at a certain operating point, all compensators have a positive effect on stability. However, especially shunt compensators significantly increase the stability margin [16]. Since this study focuses on voltage stability, SVC has been chosen as the shunt FACTS device to be applied. These compensators, defined as shunt-connected static reactive power absorbers or generators, are most commonly modeled to consist of a thyristor-controlled reactor and a capacitor [17].

## 3. MCDM

In today's difficult and complex decision processes, MCDM can be defined as the issues that will enable the decision maker to make more efficient, fast and accurate decisions, where multiple criteria are optimized and sorted, and the best alternative is selected.

There are numerous MCDM techniques discussed in the literature. In addition, many methods are used in determining the criteria weights, such as ranking, rating or pair-wise comparison. In decision-making problems solved according to many criteria, which method is used in determining the criteria weights depends on the decision maker's priorities. For instance, if ease of use and fast solution are desired in obtaining weights, one of the ranking or scoring methods can be used. On the other hand, if accuracy and theoretical structure are the main consideration, pair-wise comparison or/and trade-off analysis would be appropriate [18]. Experimental studies show that pair-wise comparison is one of the most effective methods in weight calculations. Similarly, AHP is the most popular method of MCDM as it provides significant convenience to users in terms of computability and comprehensibility. Therefore in this article, AHP and the pair-wise comparison method were used to assess the weights of the parameters.

Although studies on MCDM are much older, techniques used today began to take shape in the 1970s. The ten MCDM techniques used in this study, and their emergence dates, are presented in Fig. 1. Each technique has its own limitations and strengths. The advantages and disadvantages of MCDM methods, as well as relative complexity for each method, are summarized in Table 1 [19–21, 23, 24].

In the remainder of this section, MCDM methods and the Borda count method are introduced and brief information is given without the calculation steps in the application.

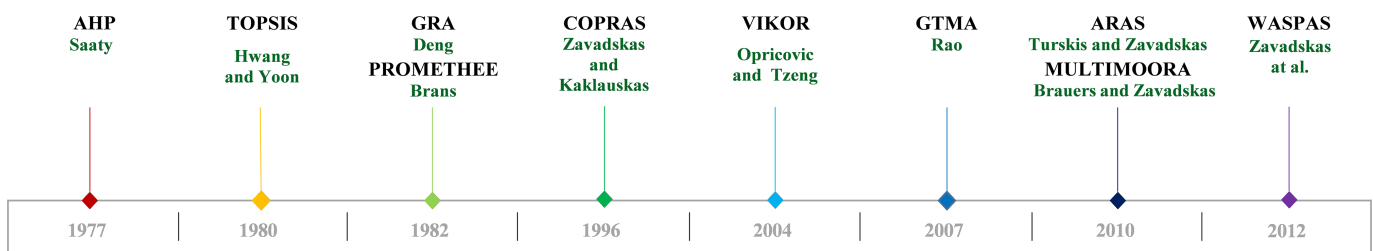
## Comparative analysis of multi-criteria decision making methods for the assessment of optimal SVC location

**Table 1**  
Advantages, disadvantages and relative complexities of MCDM Methods

Method	Advantages	Disadvantages	Complexity
AHP	Faster, and computation process is quite simple as compared with other methods. The method has a comprehensible logic. Can be easily applied to solve different problems. Since the method is based on a hierarchical structure, it focuses better on each criterion. One of the most popular methods, often combined with other methods.	Further analysis is needed to verify the results. Additional analysis is required to verify the results. Different hierarchies of criteria may influence the difference in allocation of weights. Complex decisions require higher pair-wise matrices. Interdependence between alternatives and objectives can lead to inaccurate/wrong result.	Low
TOPSIS	The method has a rational and comprehensible logic and the concept is in a quite simple mathematical form. The computation process is straightforward. Consistency and reliability. The method completely uses up allocated information and this information does not need to be independent. Results are obtained quite quickly as compared to other methods. The number of steps remains the same regardless of the number of attributes.	A strong deviation of one indicator from the ideal solution strongly influences the results. The method is suitable when the indicators of alternatives do not vary very strongly. Its use of Euclidean distance does not consider the correlation of attributes. Difficult to weight and keep consistency of judgment.	Low
VIKOR	It is quite suitable for combining with other methods. The method is tolerant of deviations of values in the assessment period. It can calculate not only a single ranking but also compromise solutions of the rankings.	Possible errors in calculations. Needs initial weights. The compromise must be in a form that can be approved to solve the problem.	Low
WASPAS	The method consists of two mathematically based techniques with short calculation steps. It is especially useful for the complete ranking of alternatives. Seeks to reach the highest accuracy. The method weights the beneficial and non-beneficial criteria in the problem separately.	Possible errors in calculations. The method takes into consideration only minimum (for non-beneficial attributes) and maximum (for beneficial attributes) values, and does not consider all the performance values.	Low
GTMA	The method provides a logical and systematic decision-making approach. For modeling and visual assessment, digraph representation is helpful.	The removal or addition of alternatives may change the final ranking (rank reversal problem). Possible errors in calculations.	High
PROMETHEE 2	The method is particularly useful when alternatives that are difficult to reconcile occur. Uncertain and fuzzy information can be incorporated into calculations. The method works with qualitative and quantitative information. Easy to use and does not require the assumption that criteria are proportionate	The computation process is quite long as compared with other MCDM methods. Possible errors in calculations because of a quite sophisticated computation process; therefore, the method is only suitable for experts. Does not provide a clear method by which to assign weights.	Medium
GRA	Perfect information has a unique solution. Calculations are simple and straightforward.	Optimal solution is difficult to obtain. Multiplying two grey numbers makes the grey number interval larger, which reduces the accuracy of the calculations and decisions.	High
MULTIMOORA	The method consists of a strongly mathematically-based technique. High level of consistency and reliability. It is tolerant of deviations of values in the assessment period.	Quite long computation process as compared with other MCDM methods. Possible calculation errors.	Medium

**Table 1** [cont.]  
Advantages, disadvantages and relative complexities of MCDM Methods

Method	Advantages	Disadvantages	Complexity
ARAS	Simple computational steps with less complexity. Can operate under a compromising situation. A relative measurement can be made in terms of ratio.	ARAS works reasonably well only when the number of alternatives is limited.	Low
COPRAS	Evaluates influence of maximizing and minimizing criteria separately. Mathematical formulation of this method eliminates the rank reversal problem. Simple calculation.	Provides unstable results in case of data variation, and the results might not reveal the true nature of the data. Requires another MCDM method to calculate the criteria weights.	Low



**Fig. 1.** Summary timeline of selected MCDM methods

### 3.1. AHP

AHP, an MCDM method developed by T.L. Saaty in the 1970s, aims to compare alternatives pair-wise for criteria weights. This technique determines the comparative importance and meaning of each element in the hierarchy by evaluating several data simultaneously and systematically. The numerical values obtained from the calculations made represent the weights or priorities [25, 26].

The first step in this technique is the creation of a hierarchical model. In the second step, according to expert guidance, pair-wise comparison matrices are constructed to show the comparative importance of each variable [27]. After the pair-wise comparisons are calculated, decision consistency is checked. If this value is below a certain precision value, the decision matrix can be accepted. The decision matrix is inconsistent for other cases. In these cases, decisions must be revised and improved to achieve a consistent matrix [28].

### 3.2. TOPSIS

TOPSIS is one of the MCDM methods developed by Yoon and Hwang in 1980 and applied in many fields. It is frequently preferred by decision makers both in terms of being easy to understand and not involving complex mathematical calculations [29].

In the TOPSIS method, the two basic concepts, called a positive and negative ideal solution, are very important. The method emphasizes that the optimal decision option should be close to the first and far from the second. Therefore, calculations are based on the distances from these points. The decision alternatives are sorted by comparing the distances [30].

### 3.3. VIKOR

The VIKOR method, developed by Tzeng and Opricovic in 2004, is used for ordering decision alternatives or selecting the best alternative among them in decision problems consisting of criteria with different units. The basis of the VIKOR technique is to define a compromise solution among the decision alternatives by considering the evaluation criteria. The compromise solution expression can be defined as the one nearest to the optimal solution. Thus, the decision closest to the optimal solution is made under certain conditions with a ranking index of decision alternatives [31, 32].

### 3.4. WASPAS

WASPAS is one of the MCDM techniques presented to the literature by Zavadskas et al. [33]. This method was developed by integrating the weighted sum model and weighted product model. By using these two techniques together, it is aimed to increase the reliability of the solution results and to correctly rank the decision alternatives [34].

The most important advantages of the method are that the application process is shorter and easier as compared to other MCDM methods, and it provides more accurate results while not requiring specific computer programs for calculations.

### 3.5. GTMA

GTMA is used in the literature as a multi-criteria decision making method. Graph theory is a systematic and logical approach. A graph model is used to model and analyze problems and systems in many areas such as economics, sociology, mathematics and engineering. In order to obtain index and system functions

to achieve goals, the matrix approach is used to analyze graph models [35].

### 3.6. PROMETHEE II

PROMETHEE is a MCDM method developed by Brans in 1982 and later expanded as PROMETHEE II, III, IV, V and VI with the contribution of other authors [36].

PROMETHEE can provide only partial sorting of options, whereas PROMETHEE II provides a complete sorting of options using pair-wise comparison. Extended versions of the PROMETHEE method can be applied in this study, so PROMETHEE II was used in the calculations.

### 3.7. GRA

Grey system theory is a methodology developed by Deng to solve problems involving incomplete and uncertain information. GRA is a decision making grading and classification method, developed using GST [37]. GRA can be used for decision problems with complex relationships, either alone or in the form of hybrid models with other methods.

### 3.8. MULTIMOORA

MOORA, an optimization and decision-making method based on proportional analysis, was developed by Zavadskas and Brauers [38]. Afterwards, they developed the more robust MULTIMOORA method by using the full multiplicative form, the reference point, and the ratio system approaches together [39]. In the MULTIMOORA method, the results obtained with these three approaches are evaluated according to the dominance theory and then the final ranking is obtained [39].

### 3.9. ARAS

The ARAS technique was presented by Zavadskas and Turskis in 2010 as a new approach to the solution of MCDM problems [40]. Unlike other methods, it compares the utility function values of alternatives in decision problems with the benefit function value of the optimal alternative determined by the decision maker [41].

### 3.10. COPRAS

COPRAS is an MCDM method developed in 1996 by Zavadskas and Kaklauskas [42]. The most important feature that distinguishes this method from other MCDM techniques; it selects the most suitable alternative considering the ideal best and the worst solutions [43, 44].

### 3.11. Borda count method

The Borda count method, which has a major share in the development of modern electoral systems, was developed by Jean-Charles de Borda in 1784. It is a technique that aims to rank alternatives by the sum of decision makers' individual preferences. In addition, the Borda count method is one of the data aggregation techniques that reduces two or more sorting formats to a more rational one. This method, which accepts each class of equal importance, is also quite simple in terms of applicability. The Borda method assigns points to each alternative

according to the rankings in the class under consideration. For a set of  $m$  decision criteria,  $N_A - 1$  points are given to the most preferred option,  $N_A - 2$  points are given for the second most preferred, down to zero points for the least preferred option. Finally, the values assigned to the alternatives in all classes are summed up, the Borda score is obtained and the ranking is performed.  $r_{ik}$  is the number of options  $i$  being at the  $k$ th rank,  $N_A$  is the total number of options and the Borda score of option  $i$  is calculated by the formula in equation (1) [45]:

$$B_i = \sum_{k=1}^{N_A} (N_A - r_{ik}). \quad (1)$$

## 4. PROPOSED METHODOLOGY

### 4.1. Continuation power flow method

Continuation power flow (CPF) method was used in this study to determine the system's maximum loadability limit. The CPF method is widely used for load flow problems in power systems, and unlike other techniques, it can obtain the entire nose curve even beyond the voltage collapse point (critical point). The value called the maximum loadability margin  $\lambda_{\max}$ , corresponding to the critical point in the plotted curve, can also be calculated without any problem. The critical point in the PV curve indicates the system's maximum loadability. The CPF technique uses the predictor-corrector approach to obtain the PV or  $\lambda V$  nose curve, as shown in Fig. 2 [46, 47]. At normal initial load conditions,  $\lambda$  can be increased in order to estimate an approximate solution by a tangent predictor. The correction step for a conventional load flow determines the complete solution [48, 49].

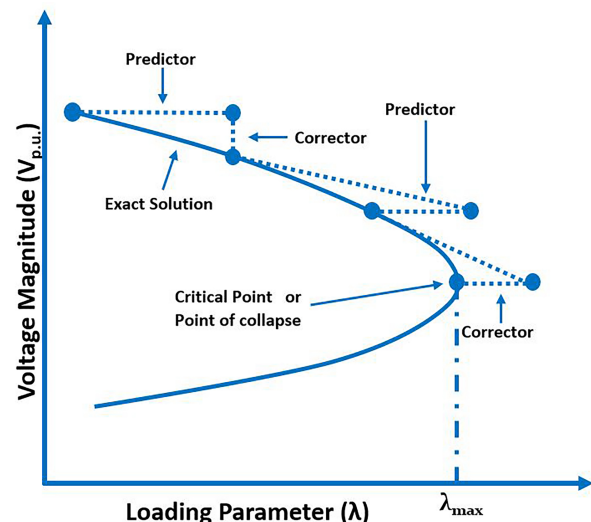


Fig. 2. Predictor-corrector approach used in the CPF

### 4.2. Objective function

Generally, in a multi-objective optimization problem, a series of objectives are tried to be optimized at the same time, taking into account the constraints of equality and inequality [49–51]. The objectives set for this paper are listed below.



- Loading margin

The improvement of voltage stability is accomplished by the commonly used voltage collapse proximity index, called the voltage stability margin (VSM) or load margin maximization, which refers to the largest load change that can be sustained by the power system at the reference operating point in a bus or a group of buses [49]. In this study, since the highest value of VSM and the lowest value of the objective function are sought in terms of stability, the inverse of maximum loadability should be taken:

$$f_1 = 1/\lambda_{\text{critical}}, \quad (2)$$

where  $\lambda_{\text{critical}}$  indicates the loading factor value at the critical point. This value corresponds to the  $\lambda_{\text{max}}$  value in the CPF method [52].

- Voltage deviation

The FACTS controllers should be connected to optimal placements to improve the voltage profile of the system and prevent voltage collapse because unacceptable service quality may result from overly low voltages on the buses. Therefore, minimizing the bus voltage deviation has been identified as the second aim [49, 53]. This objective function can be calculated as shown in equation (3):

$$f_2 = \sum_{m=1}^N |V_{m\text{ref}} - V_m|, \quad (3)$$

where  $N$  is the number of buses,  $V_{m\text{ref}}$  is the nominal voltage of bus  $m$  and  $V_m$  is the voltage magnitude at bus  $m$ . In this study, acceptable bus voltage range is selected in the range of 0.90–1.10 p.u.

- Active power losses

From a financial perspective, active power losses ( $P_{\text{loss}}$ ) should also be minimized [49, 52].  $P_{\text{loss}}$  can be represented as follows:

$$f_3 = P_{\text{loss}} = \sum_{m=1}^N g_m [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)], \quad (4)$$

where,  $g_m$  is the conductance of a transmission line,  $i$ -th bus and  $j$ -th bus voltages are  $V_i$  and  $V_j$ , while  $i$ -th bus and  $j$ -th bus voltage angles are  $\delta_i$  and  $\delta_j$ .

The objective function can be formulated as shown in equation (5) by considering the equality and inequality constraints:

$$\text{Minimize } F = [f_1, f_2, f_3]. \quad (5)$$

Functions  $f_1$ ,  $f_2$  and  $f_3$  are defined above. The most critical stage of the decision making process is the determination of the criterion weights, in other words, the level of importance. In multi-criteria selection problems, the criteria are of equal importance in some studies, whereas in others, the criteria weights are determined by one of the weighting methods with expert opinions, and the final solution is reached with MCDM methods. The criterion weights can be used in all MCDM methods after using pair-wise comparison as described below. The equation given in equation (6) is also used when final ranking is

performed on the last stage of the AHP method. The numerical values of the functions obtained using the AHP method and detailed in the previous study are given in Chapter 5 [49]. Numerical values obtained by other methods are not included and only the rankings are shown.

$$F = \omega_1 f_1 + \omega_2 f_2 + \omega_3 f_3. \quad (6)$$

Subject to:

$$\omega_1 + \omega_2 + \omega_3 = 1, \quad (7)$$

$$0 < \omega_1, \omega_2, \omega_3 < 1, \quad (8)$$

where,  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  are weighting factors of objective functions VSM, voltage deviation and power losses, respectively. VSM exerts the greatest influence on voltage stability, while voltage deviation and active power losses have less influence on voltage stability [49]. Coefficients (weights)  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  are calculated as 0.724, 0.193 and 0.083, respectively, using the AHP method, and the pair-wise comparison matrix showing the criterion importance is given in Table 2.

**Table 2**

Pair-wise comparison matrix of the criteria

	Loading margin	Voltage deviation	Real power losses	Priorities
Loading margin	1	5	7	0.724
Voltage deviation	1/5	1	3	0.193
Real power losses	1/7	1/3	1	0.083

## 5. SIMULATION RESULTS AND DISCUSSION

In order to improve voltage stability, the most suitable location for SVC in MCDM-based programming was determined by placing the compensator at various buses in the test system. In simulation studies performed for this purpose, CPF technique was used in the Power System Analysis Toolbox (PSAT) software in MATLAB [54]. The proposed method has been tested in the IEEE 14 bus system shown in Fig. 3.

This system has five synchronous machines, generators on bus 1 and 2, and synchronous capacitors for reactive power support on buses 3, 6 and 8. The test system consists of 20 transmission lines and 14 buses. A total of 259 MW active load and 77.4 MVAR reactive load are distributed across 11 load buses [49].

Since the reactive power limits of the generator are taken into account, objective function  $F$  is calculated at a loading factor near to the critical point. Fig. 4 demonstrates SVC's optimal location for three objectives.

The criteria values in the decision matrix were converted into proportional values in order not to dominate each other, and Table 3 indicates the calculation results of the proposed approach. When the final results are examined, it is seen that although the values are very close to each other, the best place for SVC in this test system is bus 9. The value presented in bold font shows the most suitable location where the objective function is minimum.

Comparative analysis of multi-criteria decision making methods for the assessment of optimal SVC location

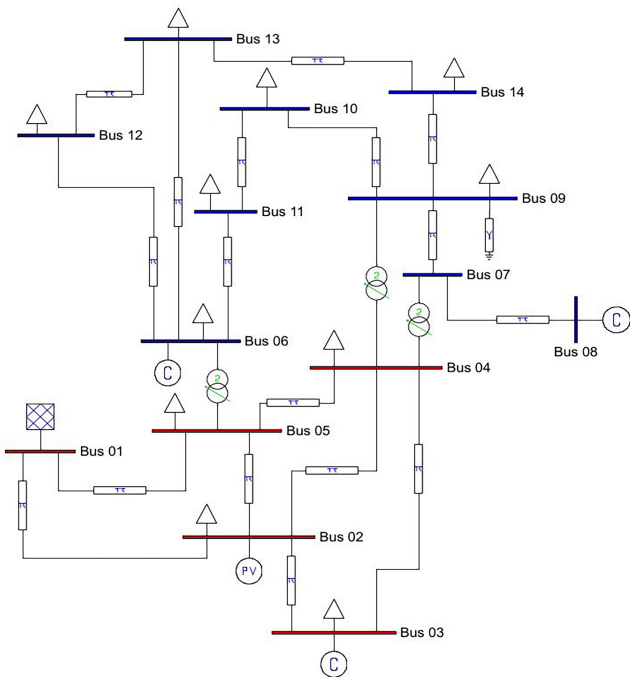


Fig. 3. IEEE 14-bus system's PSAT model

Table 3

Minimum values of objective function in IEEE 14-bus test system

Bus No.	$f_1$	$f_2$	$f_3$	$F$
Bus04	0.1109	0.1225	0.1091	0.1130
Bus05	0.1108	0.1253	0.1089	0.1135
Bus07	0.1104	0.1110	0.1124	0.1107
Bus09	0.1097	0.1006	0.1135	<b>0.1082</b>
Bus10	0.1100	0.0998	0.1134	0.1083
Bus11	0.1117	0.1077	0.1119	0.1109
Bus12	0.1134	0.1212	0.1080	0.1144
Bus13	0.1125	0.1134	0.1096	0.1124
Bus14	0.1106	0.0984	0.1132	0.1085

As can be understood from the results in Table 4, most of the methods manage to give the same rankings overall. According to the findings, it is seen that bus 9 is in most cases the best place for SVC. There is also a consistency in the lower ranks, and bus 12 is the worst location. Furthermore, it is noteworthy that bus 5 finds itself in the last row in some methods and in the middle rows in others. When the test system is analyzed as a whole, it is remarkable that the most suitable location is on the low voltage side and near the weakest bus. Likewise, the buses on the high voltage side are always in the lower ranks. AHP, which is the most preferred method in MCDM problems, is in the same rank with the BORDA solution and its compatibility with other methods indicates that this technique can be used for the election problem described. However, given the proximity between the numerical values found, the rankings obtained by other methods cannot be ignored or considered completely wrong.

The radar chart in Fig. 5 gives the Borda value corresponding to the buses for each method used. According to the Borda count method, since the alternative with the highest total score

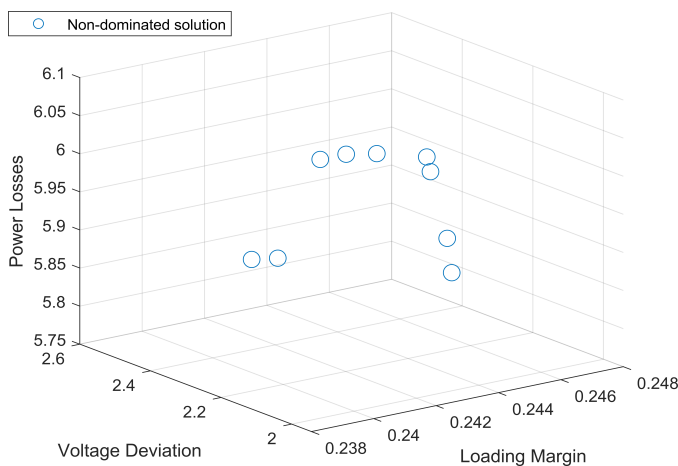


Fig. 4. SVC's optimal location for three objectives

Table 3 obviously demonstrates the importance of the MCDM technique used in this study for combining identified objectives and sustaining the minimum level of satisfaction for the objective function. From Table 3, it can be seen that bus 12 provides minimum power losses but its loading margin is not acceptable. Similarly, bus 14 gives the best value in voltage deviation and a desirable value in the loading margin but active power loss is very high.

The results given above are numerical values obtained by the AHP method. Similarly, necessary calculations were made with other MCDM methods and the rankings are determined. Then the Borda count method was used to evaluate the accuracy of all these methods and calculate the compromise solution. Table 4 summarizes the AHP, TOPSIS, VIKOR, WASPAS, GTMA, PROMETHEE II, GRA, MULTIMOORA, ARAS, COPRAS and BORDA results and ranking.

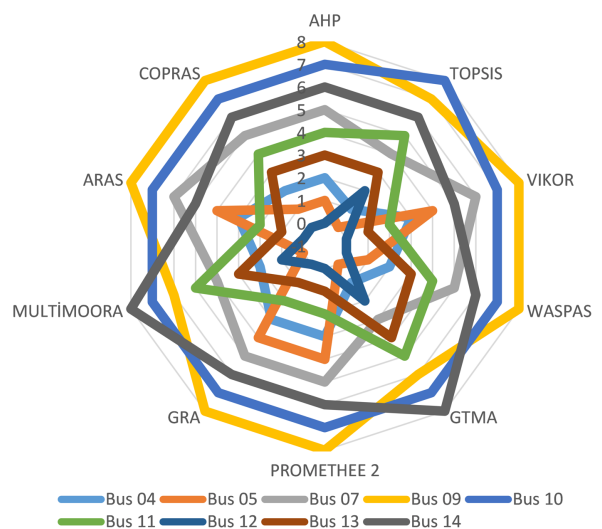


Fig. 5. Borda score comparison for the AHP, TOPSIS, VIKOR, WASPAS, GTMA, PROMETHEE II, GRA, MULTIMOORA, ARAS, COPRAS methods

**Table 4**  
 Comparison of the different rankings

AHP	TOPSIS	VIKOR	WASPAS	GTMA	PROMETHEE 2	GRA	MULTIMOORA	ARAS	COPRAS	BORDA
Bus09	Bus10	Bus09	Bus09	Bus14	Bus09	Bus09	Bus14	Bus09	Bus09	Bus09
Bus10	Bus09	Bus10	Bus10	Bus10	Bus10	Bus10	Bus10	Bus10	Bus10	Bus10
Bus14	Bus14	Bus07	Bus14	Bus09	Bus14	Bus14	Bus09	Bus07	Bus14	Bus14
Bus07	Bus11	Bus14	Bus07	Bus11	Bus07	Bus07	Bus11	Bus14	Bus07	Bus07
Bus11	Bus07	Bus05	Bus11	Bus13	Bus05	Bus05	Bus07	Bus05	Bus11	Bus11
Bus13	Bus13	Bus04	Bus13	Bus07	Bus04	Bus04	Bus13	Bus04	Bus13	Bus13
Bus04	Bus12	Bus11	Bus04	Bus12	Bus11	Bus11	Bus04	Bus11	Bus04	Bus04
Bus05	Bus04	Bus13	Bus05	Bus04	Bus13	Bus13	Bus12	Bus13	Bus05	Bus05
Bus12	Bus05	Bus12	Bus12	Bus05	Bus12	Bus12	Bus05	Bus12	Bus12	Bus12

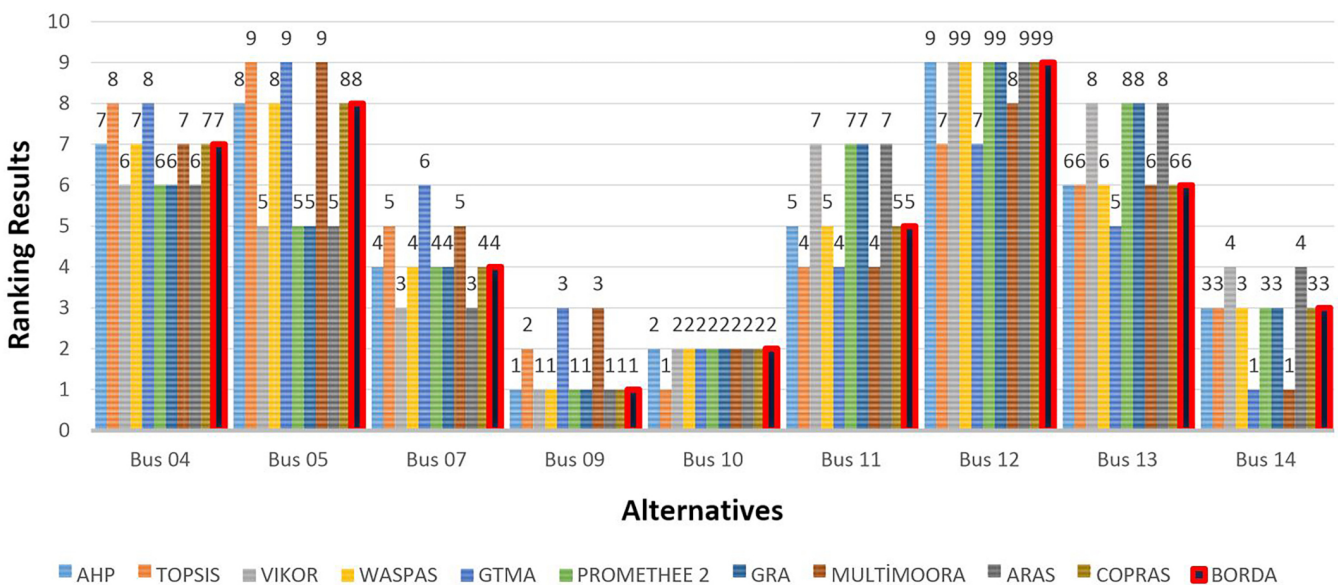
is the best choice, the most suitable location is bus 9. Likewise, it can readily be seen that bus 12 is the worst location.

From the comparative perspective of the methods used, a few general observations are as follows. The results obtained with AHP, WASPAS and COPRAS methods are exactly the same with the final ranking in the BORDA method. In the same way, the results of VIKOR and ARAS methods and PROMETHEE 2 and GRA methods are exactly the same. Particularly, the upper rows of GTMA and MULTIMOORA methods are very close to each other. TOPSIS ranking results show a higher deviation as compared to other methods. Generally, the ranking results are consistent, with some exceptions, and as can be seen in Fig. 6, in most cases, the methods provide similar solutions.

Similarity ratio can be examined to compare the consistency of the methods used. Statistical comparison was made between these ten methods in pairs using the Kendall correlation coefficient. Kendall rank correlation coefficient provides relational

and nonparametric measurement of ordered data. With this method, the details of which are not described in this paper, the dependency between the rankings was calculated statistically and high (%84) concordance was observed. This shows that there is a strong relationship between the rankings. The resulting Kendall correlation matrix is presented in Fig. 7, summarizing the correlation coefficients between MCDM methods.

From a comparative perspective, the rankings found by means of different techniques appear to be relatively stable. This means that method selection does not make much difference in terms of outcomes. Simpler methods can always be proposed for understandability, computability, interpretability and traceability, but comparative evaluation guarantees the robustness of the rankings. Consistency, noncontradiction and close values indicate that the MCDM methods used can be trusted in practice. However, it should be noted that all calculations and rankings are for the three specific objectives.



**Fig. 6.** Rankings of the alternatives for optimal SVC location in IEEE-14 bus test system



## Comparative analysis of multi-criteria decision making methods for the assessment of optimal SVC location

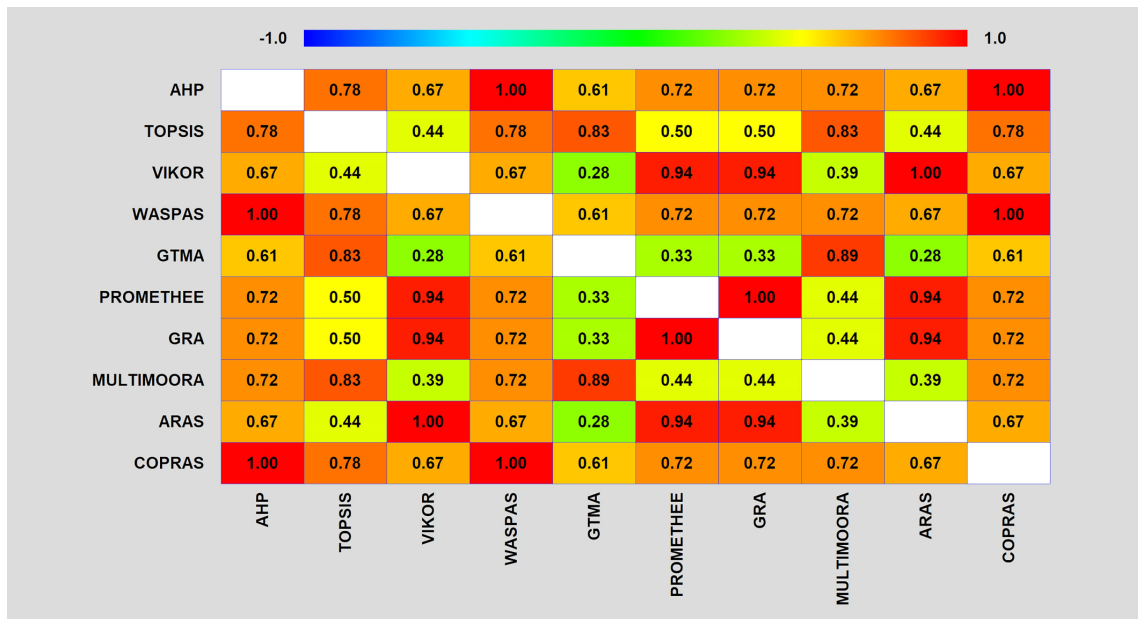


Fig. 7. Results of the Kendall correlation matrix between MCDM methods

## 6. CONCLUSIONS

Nowadays, it is important to use MCDM methods in terms of rational decision making, considering the rapidly and continuously changing criteria that often contradict one another. The development and diversity of MCDM techniques raises the question of which method the decision maker should choose. Although there is no strict rule in determining the ideal MCDM method, the structure of the problem and the characteristics of the technique to be used should be appropriate. Since these methods have different algorithms, it is common practice to compare them using several techniques. This enables more robust decision making. A data fusion technique was proposed in this paper using ten different methods to improve voltage stability. These methods were chosen because they are simple and do not require any special software in calculations. Then, the ten different sequences obtained were combined with the Borda count method. Finally, on the IEEE 14-bus network model, this method's efficacy has been indicated. With the approach developed to increase the loading margin and reduce voltage deviation and power losses, the placement of the FACTS controller is optimized for three goals. Using the objective function rankings among the weakest buses, it was found that the optimum SVC location in the IEEE 14-bus model is bus 9. When the general performance of the system was analyzed, it was seen that the ten MCDM methods used gave similar results and converged to similar solutions. In this context, it has been proven that MCDM methods can be used as a powerful tool to select optimal location of FACTS devices. By applying this approach to actual systems, power system operators can be provided with useful information for voltage stability and its improvement. The results obtained are promising and encouraging for potential applications in larger power systems. This ranking information can help the relevant institutions as regards the investments to be made and the policy to be followed.

There are many economic, technological and environmental factors that affect the efficiency and performance of the FACTS device, such as the power level, voltage level, system requirements, administrative obligations, land requirements, reliability, functionality, technological feasibility, ecological conditions and short-circuit power at the installation node. The present research focused only on three objectives. In future studies, it is possible to use multiple FACTS devices, such as UPFC, STATCOM and TCSC, to expand the number of controllers and establish hybrid configurations. In addition, cost minimization in generator units, reduction of FACTS installation cost, etc. can be added to the objective function.

Using the controllers at minimum capacity, minimizing the error rate in calculations, and optimum use of resources are other issues that can be studied. In addition, research on angular stability can also be done. In a number of potential future studies, besides voltage stability, the most suitable location can be chosen not only for angular stability but also for frequency stability.

## ACKNOWLEDGMENTS

This work was supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under Project 121E008.

## REFERENCES

- [1] G.N. Kumar and M.S. Kalavathi, "Cat Swarm Optimization for optimal placement of multiple UPFC's in voltage stability enhancement under contingency," *Int. J. Electr. Power Energy Syst.*, vol. 57, pp. 97–104, 2014, doi: [10.1016/j.ijepes.2013.11.050](https://doi.org/10.1016/j.ijepes.2013.11.050).

- [2] A. Gupta and P.R. Sharma, "Static and transient voltage stability assessment of power system by proper placement of UPFC with POD controller," *WSEAS Trans. Power Syst.*, vol. 8, no. 4, pp. 197–206, 2013.
- [3] D. Lu and X. Zhang, "Transient stability analysis and control of power systems with considering flux decay by energy function approach," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 60, no. 1, pp. 3–8, 2012, doi: [10.2478/v10175-012-0001-1](https://doi.org/10.2478/v10175-012-0001-1).
- [4] R. Sirjani, A. Mohamed, and H. Shareef, "Optimal allocation of shunt Var compensators in power systems using a novel global harmony search algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 43, no. 1, pp. 562–572, 2012, doi: [10.1016/j.ijepes.2012.05.068](https://doi.org/10.1016/j.ijepes.2012.05.068).
- [5] M. El-Azab, W.A. Omran, S.F. Mekhamer, and H.E.A. Talaat, "Allocation of FACTS devices using a probabilistic multi-objective approach incorporating various sources of uncertainty and dynamic line rating," *IEEE Access*, vol. 8, pp. 167647–167664, 2020, doi: [10.1109/ACCESS.2020.3023744](https://doi.org/10.1109/ACCESS.2020.3023744).
- [6] A.A. Sadiq, S.S. Adamu, and M. Buhari, "Optimal distributed generation planning in distribution networks: A comparison of transmission network models with FACTS," *Eng. Sci. Technol. Int. J.*, vol. 22, no. 1, pp. 33–46, 2019, doi: [10.1016/j.jestch.2018.09.013](https://doi.org/10.1016/j.jestch.2018.09.013).
- [7] A.A. Shehata, A. Refaat, and N.V. Korovkin, "Optimal allocation of FACTS devices based on multi-objective multi-verse optimizer algorithm for multi-objective power system optimization problems," in *International Multi-Conference on Industrial Engineering and Modern Technologies (FarEastCon)*, Vladivostok, Russia, 2020, doi: [10.1109/FarEastCon50210.2020.9271359](https://doi.org/10.1109/FarEastCon50210.2020.9271359).
- [8] S. Galvani, M.T. Hagh, M.B.B. Sharifian, and B. Mohammadi-Ivatloo, "Multiobjective predictability-based optimal placement and parameters setting of UPFC in wind power included power systems," *IEEE Trans. Ind. Inform.*, vol. 15, no. 2, pp. 878–888, 2019, doi: [10.1109/TII.2018.2818821](https://doi.org/10.1109/TII.2018.2818821).
- [9] Y. Amrane and N.E.L.Y. Kouba, "A multiobjective optimal VAR dispatch using FACTS devices considering voltage stability and contingency analysis," in *Predictive Modelling for Energy Management and Power Systems Engineering*, 1st ed., Elsevier, 2020, ch. 1, pp. 1–26, doi: [10.1016/B978-0-12-817772-3.00001-X](https://doi.org/10.1016/B978-0-12-817772-3.00001-X).
- [10] S.K. Mahmoudi, M.E.H. Golshan, and R. Zamani, "Coordinated voltage control scheme for transmission system considering objectives and constraints of network and control devices," *Electr. Power Syst. Res.*, vol. 192, 2020, doi: [10.1016/j.eprsr.2020.106908](https://doi.org/10.1016/j.eprsr.2020.106908).
- [11] J. Zhou, Y. Wu, C. Wu, F. He, B. Zhang, and F. Liu, "A geographical information system based multi-criteria decision-making approach for location analysis and evaluation of urban photovoltaic charging station: A case study in Beijing," *Energy Convers. Manag.*, vol. 205, p. 112340, 2020, doi: [10.1016/j.enconman.2019.112340](https://doi.org/10.1016/j.enconman.2019.112340).
- [12] J.M. Sanchez-Lozano, J. Teruel-Solano, P.L. Soto-Elvira, and M.S. Garcia-Cascales, "Geographical information systems (GIS) and multi-criteria decision making (MCDM) methods for the evaluation of solar farms locations: Case study in south-eastern Spain," *Renew. Sustain. Energy Rev.*, vol. 24, pp. 544–556, 2013, doi: [10.1016/j.rser.2013.03.019](https://doi.org/10.1016/j.rser.2013.03.019).
- [13] G. Villacreses, G. Gaona, J. Martinez-Gomez, and D.J. Jijon, "Wind farms suitability location using geographical information system (GIS), based on multi-criteria decision making (MCDM) methods: The case of continental Ecuador," *Renew. Energy*, vol. 109, pp. 275–286, 2017, doi: [10.1016/j.renene.2017.03.041](https://doi.org/10.1016/j.renene.2017.03.041).
- [14] B. Ceballos, M.T. Lamata, and D.A. Pelta, "A comparative analysis of multi-criteria decision-making methods," *Prog. Artif. Intell.*, vol. 5, no. 4, pp. 315–322, 2016, doi: [10.1007/s13748-016-0093-1](https://doi.org/10.1007/s13748-016-0093-1).
- [15] N. Hingorani and L. Gyugyi, *Understanding FACTS: Concepts and Technology Flexible AC Transmission Systems*. vol. 1, New York, NY, USA: IEEE Press, 2000, pp. 1–35.
- [16] H.R. Muhammed, *Power Electronics, Devices, Circuits and Applications*. 4th ed., Essex, England: Pearson Education, 2014, pp. 626–657.
- [17] N.A. Skaria, S. Baby, and D.M. Anumodu, "Genetic algorithm based optimal location of SVC in power system for voltage stability enhancement," in *Annual International Conference on Emerging Research Areas: Magnetics, Machines and Drives (AICERA/iCMMD)*, Kottayam, India, 2014, pp. 1–6, doi: [10.1109/AICERA.2014.6908188](https://doi.org/10.1109/AICERA.2014.6908188).
- [18] E. Aycin, *Multi Criteria Decision Making: Computer Applied Solutions*. Ankara, Turkey: Nobel Academic Publishing (in Turkish), 2020, pp. 4–10.
- [19] P.K. Pramanik, S. Biswas, S. Pal, D. Marinković, and P. Choudhury, "A Comparative Analysis of Multi-Criteria Decision-Making Methods for Resource Selection in Mobile Crowd Computing," *Symmetry*, vol. 13, no. 9, p. 1713, 2021, doi: [10.3390/sym13091713](https://doi.org/10.3390/sym13091713).
- [20] I. Siksnylyte-Butkiene, E.K. Zavadskas, and D. Streimikiene, "Multi-Criteria Decision-Making (MCDM) for the Assessment of Renewable Energy Technologies in a Household: A Review," *Energies*, vol. 13, no. 5, p. 1164, 2020, doi: [10.3390/en13051164](https://doi.org/10.3390/en13051164).
- [21] M. Velasquez and P. Hester, "An analysis of multi-criteria decision making methods," *Int. J. Oper. Res.*, vol. 10, no. 2, pp. 56–66, 2013.
- [22] I. Siksnylyte, E.K. Zavadskas, D. Streimikiene, and D. Sharma, "An Overview of Multi-Criteria Decision-Making Methods in Dealing with Sustainable Energy Development Issues," *Energies*, vol. 11, no. 10, p. 2754, 2018, doi: [10.3390/en11102754](https://doi.org/10.3390/en11102754).
- [23] A. Malek, S. Ebrahimnejad, and R. Tavakkoli-Moghaddam, "An Improved Hybrid Grey Relational Analysis Approach for Green Resilient Supply Chain Network Assessment," *Sustainability*, vol. 9, no. 8, p. 1433, 2017, doi: [10.3390/su9081433](https://doi.org/10.3390/su9081433).
- [24] S. Bakar, N. Harish, A. Kahartini, S. Agos, H. Nasir, E. Tahir, and E. Janisip, "Application of Graph Theory and Matrix Approach as Decision Analysis Tool for Smartphone Selection," *ASM Sci. J.*, vol. 12, no. 6, pp. 53–59, 2019.
- [25] H.H. Goh, B.C. Kok, H.T. Yeo, S.W. Lee, and A.A.M. Zin, "Combination of TOPSIS and AHP in load shedding scheme for large pulp mill electrical system," *Int. J. Electr. Power Energy Syst.*, vol. 47, pp. 198–204, 2013, doi: [10.1016/j.ijepes.2012.10.059](https://doi.org/10.1016/j.ijepes.2012.10.059).
- [26] T. Witkowski, P. Antczak, and A. Antczak, "Multi-objective decision making and search space for the evaluation of production process scheduling," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 57, no.3, pp. 195–208, 2009, doi: [10.2478/v10175-010-0121-4](https://doi.org/10.2478/v10175-010-0121-4).
- [27] S. Liu, Q. Zhao, M. Wen, L. Deng, S. Dong, and C. Wang, "Assessing the impact of hydroelectric project construction on the ecological integrity of the Nuozhadu Nature Reserve, southwest China," *Stoch. Environ. Res. Risk Assess.*, vol. 27, no. 7, pp. 1709–1718, 2013.
- [28] A. Awasthi and S.S. Chauhan, "Using AHP and Dempster-Shafer theory for evaluating sustainable transport solutions," *Environ. Model. Software*, vol. 26, no. 6, pp. 787–796, 2011, doi: [10.1016/j.envsoft.2010.11.010](https://doi.org/10.1016/j.envsoft.2010.11.010).

- [29] S.R. Marjani, V. Talavat, and S. Galvani, "Optimal allocation of D-STATCOM and reconfiguration in radial distribution network using MOPSO algorithm in TOPSIS framework," *Int. Trans. Electr. Energy Syst.*, vol. 29, no. 2, p. e2723, 2019, doi: [10.1002/etep.2723](https://doi.org/10.1002/etep.2723).
- [30] T. Mythili and R. Thanigaivelan, "Optimization of wire EDM process parameters on Al6061/Al2O3 composite and its surface integrity studies," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 68, no. 6, pp. 1403–1412, 2020, doi: [10.24425/bpasts.2020.135382](https://doi.org/10.24425/bpasts.2020.135382).
- [31] C. Li, M. Negnevitsky, X. Wang, W.L. Yue, and X. Zou, "Multi-criteria analysis of policies for implementing clean energy vehicles in China," *Energy Policy*, vol. 129, pp. 826–840, 2019, doi: [10.1016/j.enpol.2019.03.002](https://doi.org/10.1016/j.enpol.2019.03.002).
- [32] K.G. Saravanan, R. Thanigaivelan, and M. Soundarrajan, "Comparison of electrochemical micromachining performance using TOPSIS, VIKOR and GRA for magnetic field and UV rays heated electrolyte," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 69, p. e138816, 2021, doi: [10.24425/bpasts.2021.138816](https://doi.org/10.24425/bpasts.2021.138816).
- [33] E. Zavadskas, Z. Turskis, J. Antucheviciene, and A. Zakarevicius, "Optimization of weighted aggregated sum product assessment," *Elektron. ir Elektrotech.*, vol. 122, no. 6, pp. 3–6, 2012, doi: [10.5755/j01.eee.122.6.1810](https://doi.org/10.5755/j01.eee.122.6.1810).
- [34] T. Balezentis and D. Streimikiene, "Multi-criteria ranking of energy generation scenarios with Monte Carlo simulation," *Appl. Energy*, vol. 185, no. 1, pp. 862–871, 2017, doi: [10.1016/j.apenergy.2016.10.085](https://doi.org/10.1016/j.apenergy.2016.10.085).
- [35] N.K. Geetha and P. Sekar, "Graph theory matrix approach – a qualitative decision making tool," *Mater. Today*, vol. 4, no. 8, pp. 7741–7749, 2017, doi: [10.1016/j.matpr.2017.07.109](https://doi.org/10.1016/j.matpr.2017.07.109).
- [36] M.A. Nikouei, M. Orouzadeh, and S. Mehdipour-Ataei, "The PROMETHEE multiple criteria decision making analysis for selecting the best membrane prepared from sulfonated poly(ether ketone)s and poly(ether sulfone)s for proton exchange membrane fuel cell," *Energy*, vol. 119, pp. 77–85, 2017, doi: [10.1016/j.energy.2016.12.052](https://doi.org/10.1016/j.energy.2016.12.052).
- [37] E. Kose, S. Burmaoglu, and M. Kabak, "Grey relational analysis between energy consumption and economic growth," *Grey Syst.*, vol. 3, no. 3, pp. 291–304, 2013, doi: [10.1108/GS-06-2013-0010](https://doi.org/10.1108/GS-06-2013-0010).
- [38] W. Karel, W. Brauers, and E. Zavadskas, "The MOORA method and its application to privatization in a transition economy," *Control. Cybern.*, vol. 35, no. 2, pp. 445–469, 2006.
- [39] Z.H. Li, "An extension of the MULTIMOORA method for multiple criteria group decision making based upon hesitant fuzzy sets," *IMA J. Appl. Math.*, vol. 2014, no. 2, pp. 1–16, 2014, doi: [10.1155/2014/527836](https://doi.org/10.1155/2014/527836).
- [40] E.K. Zavadskas, Z. Turskis, and T. Vilutiene, "Multiple criteria analysis of foundation instalment alternatives by applying additive ratio assessment (ARAS) method," *Arch. Civ. Mech. Eng.*, vol. 10, no. 3, pp. 123–141, 2010, doi: [10.1016/S1644-9665\(12\)60141-1](https://doi.org/10.1016/S1644-9665(12)60141-1).
- [41] D. Streimikiene, J. Sliogeriene, and Z. Turskis, "Multi-criteria analysis of electricity generation technologies in Lithuania," *Renew. Energy*, vol. 85, pp. 148–156, 2016, doi: [10.1016/j.renene.2015.06.032](https://doi.org/10.1016/j.renene.2015.06.032).
- [42] E.K. Zavadskas, A. Kaklauskas, and T. Vilutiene, "Multicriteria evaluation of apartment blocks maintenance contractors: Lithuanian case study," *Int. J. Strateg. Prop. Manag.*, vol. 13, no. 4, pp. 319–338, 2009, doi: [10.3846/1648-715X.2009.13.319-338](https://doi.org/10.3846/1648-715X.2009.13.319-338).
- [43] G.N. Yucenur, S. Caylak, G. Gonul, and M. Postalcioglu, "An integrated solution with SWARA&COPRAS methods in renewable energy production: City selection for biogas facility," *Renew. Energy*, vol. 145, pp. 2587–2597, 2020, doi: [10.1016/j.renene.2019.08.011](https://doi.org/10.1016/j.renene.2019.08.011).
- [44] N. Kundakciand A. Isik, "Integration of MACBETH and COPRAS methods to select air compressor for a textile company," *Decis. Sci. Lett.*, vol. 5, no. 3, pp. 381–394, 2016, doi: [10.5267/j.dsl.2016.2.003](https://doi.org/10.5267/j.dsl.2016.2.003).
- [45] I. Topcu, F. Ulengin, O. Kabak, M. Isik, B. Unver, and S.O. Ekici, "The evaluation of electricity generation resources: The case of Turkey," *Energy*, vol. 167, pp. 417–427, 2019, doi: [10.1016/j.energy.2018.10.126](https://doi.org/10.1016/j.energy.2018.10.126).
- [46] V. Ajarapu and C. Christy, "The continuation power flow: A tool for steady state voltage stability analysis," *IEEE Trans. Power Syst.*, vol. 7, no. 1, pp. 416–423, 1992, doi: [10.1109/59.141737](https://doi.org/10.1109/59.141737).
- [47] M.M. Aman, G.B. Jasmon, A.H. A. Bakar, and H. Mokhlis, "Optimum network reconfiguration based on maximization of system loadability using continuation power flow theorem," *Int. J. Electr. Power Energy Syst.*, vol. 54, pp. 123–133, 2014, doi: [10.1016/j.ijepes.2013.06.026](https://doi.org/10.1016/j.ijepes.2013.06.026).
- [48] A.S. Telang and P.P. Bedekar, "Systematic approach for optimal placement and sizing of STATCOM to assess the voltage stability," in *International Conference on Circuit, Power and Computing Technologies (ICCPCT)*, Nagercoil, India, 2016, pp. 1–6, doi: [0.1109/ICCPCT.2016.7530132](https://doi.org/10.1109/ICCPCT.2016.7530132).
- [49] F. Aydin and B. Gumus, "Determining optimal SVC location for voltage stability using multi-criteria decision making based solution: Analytic hierarchy process (AHP) approach," *IEEE Access*, to be published, 2021, doi: [10.1109/ACCESS.2021.3121196](https://doi.org/10.1109/ACCESS.2021.3121196).
- [50] R. Benabid and M. Boudour, "Optimal location and size of SVC and TCSC for multi-objective static voltage stability enhancement," *RE&PQJ*, vol. 1, no. 6, pp. 175–180, 2008, doi: [10.24084/repqj06.256](https://doi.org/10.24084/repqj06.256).
- [51] R. Hari Kumar, N. Mayadevi, V.P. Mini, and S. Ushakumari, "Transforming distribution system into a sustainable isolated microgrid considering contingency," *Bull. Pol. Acad. Sci. Tech. Sci.*, vol. 67, no. 5, pp. 871–881, 2019, doi: [10.24425/bpasts.2019.130875](https://doi.org/10.24425/bpasts.2019.130875).
- [52] A. Laifa and A. Medoued, "Optimal FACTS location to enhance voltage stability using multi-objective harmony search," in *International Conference on Electric Power and Energy Conversion Systems (EPECS)*, Istanbul, Turkey, 2013, pp. 1–6, doi: [10.1109/EPECS.2013.6712990](https://doi.org/10.1109/EPECS.2013.6712990).
- [53] A.R. Phadke, M. Fozdar, and K.R. Niazi, "A new multi-objective fuzzy-GA formulation for optimal placement and sizing of shunt FACTS controller," *Int. J. Electr. Power Energy Syst.*, vol. 40, no. 1, pp. 46–53, 2012, doi: [10.1016/j.ijepes.2012.02.004](https://doi.org/10.1016/j.ijepes.2012.02.004).
- [54] F. Milano, "An open source power system analysis toolbox," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1199–1206, 2005, doi: [10.1109/TPWRS.2005.851911](https://doi.org/10.1109/TPWRS.2005.851911).