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Production Scheduling under Fuzziness for the Furnace - Casting Line System

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Abstract

The problem considered in the paper is motivated by production planning in a foundry equipped with a furnace and a casting line, which provides a variety of castings in various grades of cast iron/steel for a large number of customers. The goal is to create the order of the melted metal loads to prevent delays in delivery of goods to customers. This problem is generally considered as a lot-sizing and scheduling problem. However, contrary to the classic approach, we assumed the fuzzy nature of the demand set for a given day. The paper describes a genetic algorithm adapted to take into account the fuzzy parameters of simultaneous grouping and scheduling tasks and presents the results achieved by the algorithm for example test problem.

Keywords: Application of information technology to the foundry industry, Production planning, Scheduling, Fuzzy numbers

1. Introduction

The main goal of an efficient production planning is to correctly distribute production tasks in the periods of time while taking into account customer needs and economic factors. A common way to support a production management is to use hierarchical information systems built on the basis of different decision support tools integrated with a common database.

As production planning and scheduling is the key factor of companies' success, it is necessary to develop new methods and algorithms solving such problems. Appropriate tool for planning and scheduling can become a link between management, marketing, production process, inventory management, logistics, distribution and technology. One of the newest approaches to solve scheduling problems is the application of fuzzy sets that belong to Computational Intelligence (CI) tools. Intelligent planning and scheduling allows the enterprise to be less sensitive to the changes in demand and its structure, to shorten the production cycles, to decrease inventory levels and simultaneously to keep a high level of service.

A short-term planning problem in foundries is especially complex, because production processes are of a continuous-discrete type. Their production programs are characterized by a very high level of quality requirements for particular products and simultaneously a large number of relatively small orders. In order to solve such problem a lot sizing model with additional technological constraints related to the casting production process is commonly used.

Lot sizing with fuzzy parameters has been rather rarely studied in the literature, especially when compared to the shop scheduling problems. Yan et al. [8] examined lot sizing production planning problem with profits, customer demands and production capacity characterized by fuzzy variables with trapezoidal membership functions. To solve the problem they proposed standard genetic algorithm hybridized with fuzzy simulation. Rezaei and Davoodi [5] studied a lot-sizing problem with supplier selection under fuzzy demand and costs (price, transaction cost and holding cost) with triangular membership functions. Also in this case a standard genetic algorithm was used to determine upper and lower bound for production quantities. Most recently Sahebjamnia and Torabi [6] considered a multi-level capacitated lot sizing problem with uncertain setup, holding, and backorder costs expressed as a fuzzy numbers with trapezoidal membership functions. They proposed a heuristic in which uncertain constraints as well as imprecise objective functions are converted into the crisp values by using the expected interval and value of the ill-known parameters, respectively. Then they solved such problem using a standard branch and bound solver.

The aim of this paper is to present the effective genetic algorithm for production planning and scheduling in the single furnace-single casting line system, when some parameters are of fuzzy nature. Section 2 provides a mixed integer programming (MIP) model for this problem. In Section 3, the details of proposed heuristic are given. The computational experiments are described in Section 4, and finally, the conclusions are drawn in Section 5.

2. Fuzzy lot-sizing and scheduling model

The MIP model presented in this section is an extension of Araujo et al. [1] lot sizing and scheduling model for automated foundry. The extended model takes into account the assumption that the demands can be sometimes expressed as fuzzy numbers. The fuzziness of the demand may originate mainly in faulty castings.

We use the following notation:

Indices

i=1,…,I - produced items; k=1,…,K - produced alloys

t=1,…,T - working days; n=1,…,N - sub-periods

Parameters

$\tilde{d}_{it}$ – fuzzy demand for item i in day t; w_i - weight of item i

a_i^k = 1, if item i is produced from alloy k, otherwise 0

s - setup penalty; C - loading capacity of the furnace

h_{it}^-, h_{it}^+ - penalty for delaying (–) and storing (+) production of item i in day t

Variables

$\tilde{I}_{it}^-, \tilde{I}_{it}^+$ - fuzzy number of items i delayed (–) and stored (+) at the end of day t

z_n^k = 1, if there is a setup (resulting from a change) of alloy k in sub-period n, otherwise 0

y_n^k = 1, if alloy k is produced in n in sub-period, otherwise 0

x_{in} - number of items i produced in sub-period n.

Production planning problem is defined as follows:

Minimize $$\sum_{i=1}^{I}\sum_{t=1}^{T}(h_{it}^-\tilde{I}_{it}^- + h_{it}^+\tilde{I}_{it}^+) + \sum_{k=1}^{K}\sum_{n=1}^{N}(s\cdot z_n^k) \tag{1}$$

subject to:

$$\tilde{I}_{i,t-1}^+ - \tilde{I}_{i,t-1}^- + \sum_{n=1}^{N}\sum_{k=1}^{K} x_{in}a_i^k - \tilde{I}_{it}^+ + \tilde{I}_{it}^- \geq \tilde{d}_{it}, \quad i=1,...,I, t=1,...,T \tag{2}$$

$$\sum_{i=1}^{I} w_i x_{in} a_i^k \leq C\, y_n^k, \quad k=1,...,K, n=1,...,N \tag{3}$$

$$z_n^k \geq y_n^k - y_{n-1}^k, \quad k=1,...,K, n=1,...,N \tag{4}$$

$$\sum_{k=1}^{K} y_n^k = 1, \quad n=1,...,N \tag{5}$$

$$x_{it} \geq 0, \quad x_{it} \in \Im, \quad I_{it}^-, I_{it}^+ \in \mathrm{F}\ , \ i=1,...,I \tag{6}$$

The goal (1) is to find a schedule that minimizes the sum of the costs of delayed production, storage costs of finished goods and the setup cost, if the alloy is changed during furnace load. As the numbers of stored and delayed products are expressed as fuzzy numbers the objective function is also expressed in a fuzzy way.

Equation (2) balances fuzzy inventories, delays and the volume of production of each item in each period. Constraint (3) ensures that the furnace capacity is not exceeded in single load. Constraint (4) sets variable z_n^k to 1, if there is a change in alloys in the subsequent periods, while constraint (5) ensures that only one alloy is produced in each sub-period. Variable x_{it} is an integer number, while variables $\tilde{I}_{it}^-, \tilde{I}_{it}^+$ are fuzzy variables.

3. Solution heuristic

Genetic algorithms (GA) have been successfully applied to a wide range of lot-sizing problems, including some fuzzy lot-sizing problems described in Section 1. An extensive review of another genetic algorithms applications to solve the lot-sizing problem can be found in [4]. As we underlined earlier, most of the authors have used a standard version of genetic algorithm with some modifications that tailor the algorithm to the specific of the lot-sizing problem. In [3] we have presented three different strategies on how to represent a schedule for a foundry as a chromosome. Later [7] we have focused on the most efficient one regarding the memory complexity.

i	1	2	3	4	5	6	7	8	9	10
$x1_i$	9	97	6	20	32	49	30	89	10	34
$x2_i$	50	3	66	28	64	28	62	16	43	73
$x3_i$	33	35	61	81	15	41	13	36	4	27
$o1_i$	3	8	5	6	1	9	1	9	3	7
$o2_i$	4	6	3	8	2	10	3	8	1	10
$o3_i$	2	9	2	10	4	7	5	7	4	6
a_i	1	2	1	2	1	2	1	2	1	2

Fig. 1. Solution representation used in proposed GA

In the representation that is shown in Figure 1 the vector coding the solution consists of three parts (segments in a chromosome): vectors ***x*** representing the quantity of items that are produced in a given subperiod, vectors ***o*** representing the orders' numbers of the produced items, and vector ***a*** representing alloy type that is produced in this subperiod. Such representation enables for a direct application of the standard one-point crossover, however mutation operators require the application of special approach. We defined three different mutation operators: a mutation operating on items that adds or subtracts value of 1.0 in a randomly chosen element of a vector ***x***, a mutation that operates on orders and changes the order number in a randomly chosen element of vector ***o*** to another order produced from the same alloy, and finally a mutation that operates on alloy and changes alloy type in a randomly chosen element of vector ***a***. Different mutations allow the genetic algorithm for a very precise exploitation of the solution space. The outline of the algorithm is shown in Figure 2.

```
Initialize population P with random values
Evaluate population P and print the best solution
while terminal_condition not met
  Select solutions for recombination with binary tournament
  Perform one-point crossover with probability 0.5
  Perform mutation on vector x with probability 0.2
  Perform mutation on vector o with probability 0.02
  Perform mutation on vector a with probability 0.02
  Evaluate population P and print the best solution
while end
```

Fig. 2. Outline of genetic algorithm used in experiments.

The above representation of solutions can be used also for the fuzzified version of the lot-sizing problem in a foundry. The only problem is how to evaluate the objective function expressed in a fuzzy way. There are many approaches that can be used to solve this problem, including probabilistic, ranging and centroid ones [2]. We have chosen the method basing on deffuzification, as it is the fastest and easy to implement. In the experiments described in the following section we will express demand as fuzzy numbers with a triangular membership function. Thus we will use the following formula for the deffuzification of the objective function:

$$M_{\tilde{A}} = \frac{a+4b+c}{6} \tag{7}$$

where:

$\tilde{A} = (a, b, c)$ is a fuzzy function and its membership function is expressed as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, \; if \;\; a \le x \le b \\ \frac{c-x}{c-b}, \; if \;\; b \le x \le c \\ 0, \qquad otherwise \end{cases} \tag{8}$$

In the genetic algorithm we have used a binary tournament in order to select solutions to a new population. Thus we first have to defuzzify two objective functions and on that basis we choose the better of the two solutions.

4. Computational experiments

4.1. Test problems

Experiments have been conducted using similar procedure that has been described by de Araujo [1]. However, this time demand has been fuzzified for some part of orders. In order to demonstrate the use of fuzzy parameters we considered one planning problem with 50 items made from 10 different alloys. Five different sets of experiments have been conducted, in each different number of orders has been fuzzified. In the first variant demand was crisp for all orders. For the remaining ones, the demand was fuzzified by 5%, 10% and 20%, respectively. The characteristic of the problem is given in Table 1.

The values for demand, weight and delaying cost were determined using uniform distribution within a given range. For the variants in which the demand was expressed by fuzzy numbers original value was replaced with the fuzzy number with triangular membership function.

Table 1.
Test problems characteristics.

Parameter	Value
number of items (*I*), number of alloys (*K*)	(50,10)
number of days (*T*)	5
number of subperiods (*N*)	10
furnace capacity *C* [kg]	5,000
demand ($\tilde{d}_{it}$) [items/subperiod]	[10, 60]
weight of item (w_i) [kg]	[2, 50]
setup penalty (*s*) [PLN]	100
delaying cost (h_i^-) [PLN/item]	[3.00, 9.00]
holding cost (h_i^+) [PLN/item]	w_i * 0.02 + 0.05
fuzziness of the demand (triangular)	0%, 5%, 10%, 20%

4.2. Results of the experiments

Genetic algorithm was run for 20 times for each variant of the problem. The following parameters have been used:

– population size: 50 individuals,
– number of generations: 50,000.

A single run for the algorithm took 2 minutes for the crisp version of the algorithm and 4 minutes for the fuzzy version. The results for 20 runs have been collected in Table 2. Column ‘costs’ represents the penalty function expressed in PLN, column ‘production’ shows the production volume of castings scheduled for all periods, while column ‘furnace utilization’ provides the ratio of sum of castings’ weights scheduled to the overall furnace capacity (C).

Table 2.
Results of the experiments for crisp and fuzzy demand

demand		costs	produc-tion	furnace utilization
crisp	***average***	**63,453**	**7,672.7**	**0.918**
	std.dev.	3,366	225.9	0.018
fuzzy 5%	***average***	**55,763**	**7,689.6**	**0.926**
	std.dev.	3,890	238.1	0.013
fuzzy 10%	***average***	**54,716**	**7,740.5**	**0.940**
	std.dev.	4,080	253.0	**0.014**
fuzzy 20%	***average***	**52,980**	**7,578,9**	**0.921**
	std.dev.	2,866	252.1	0.017

We can see that introducing fuzzy parameters into production scheduling process may bring significant benefits. First of all furnace utilization can be improved. From 91.8% baseline value it is possible to achieve 94% utilization of the furnaces, depending on the level of fuzziness that was set for the demand. Much less impact, almost negligible, can be observed for production volume. From the obvious reason the total costs decrease with the increase of fuzziness in the demand, as less number of castings is delayed or manufactured earlier. However, the most interesting observation is that the benefit from introduction of fuzzy demand, measured as the furnace utilization and castings production volume, can be observed to only certain point (in analysed case it was 10%). This phenomena should be examined in more detail, as it may be caused by the particular structure of the data used in the experiments, but when confirmed, it indicates that the impact of fuzziness on the parameters should be carefully studied before applying such solution into planning and scheduling practice.

5. Conclusions

In this paper, the mathematical programming model presented earlier [7] for a small foundry has been extended to more complex problem, in which demand is expressed in the form of fuzzy numbers. The model is based on a well-known lot-sizing problem extended to handle the fuzzy constraints. A dedicated version of a genetic algorithm has been used for the lot-sizing and scheduling problem in single furnace-single casting line environment. The genetic algorithm proposed by authors can achieve good results within few minutes, and it can potentially handle more complex problems in which more parameters are described as fuzzy-numbers or e.g. intervals.

A computational experiment conducted by the authors has shown that introduction of fuzzy sets into standard planning and scheduling models may bring significant improvement in the capacity utilization of furnaces and casting lines. It has also indicated that further research should be done in this field, as too extensive fuzzification of parameters may worsen the quality of generated schedules. Also some improvements should be introduced to the algorithm itself in order to speed up the schedule generation process, as it takes twice as much as the schedule with crisp parameters only.

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