Arch. Min. Sci., Vol. 57 (2012), No 1, p. 169–177

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/v10267-012-0012-7

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ANGULAR OSCILLATIONS OF VIBRATORY MACHINES OF INDEPENDENT DRIVING SYSTEMS CAUSED BY A NON-CENTRAL DIRECTION OF THE EXCITING FORCE OPERATIONS

WAHANIA KĄTOWE DWUNAPĘDOWYCH MASZYN WIBRACYJNYCH WYKORZYSTUJĄCYCH ZJAWISKO SAMOSYNCHRONIZACJI, WYWOŁANE NIECENTRALNYM KIERUNKIEM DZIAŁANIA SIŁY WYMUSZAJĄCEJ

It was proved in the paper that commonly used dependencies determining the angular oscillations of the vibratory machine bodies, caused by a non- central force application, lead to significant errors in case of machines with self-synchronising driving systems. Phase shifting of vibrators angles of rotation caused by a non-central direction of the excitation force was indicated as the reason of such situation. It was shown that the real amplitudes values of the body rocking vibrations are – in typical systems – several times smaller than the ones determined by the currently used methods.

Keywords: vibratory machines, self-synchronisation, machine bodies oscillations, eccentric direction of exciting force

W pracy zbudowano model matematyczny dwunapędowej maszyny wibracyjnej wykorzystującej zjawisko samosynchronizacji, obejmujący przypadek gdy oś symetrii zespołu napędowego nie przechodzi przez środek masy maszyny. W modelu uwzględniono wpływ zawieszenia sprężystego korpusu maszyny. Określono wpływ niecentralnego działania siły na przebieg synchronizacji wibratorów i sformułowano przydatne dla praktyki inżynierskiej zależności określające wahania kątowe korpusu maszyny i trajektorie dowolnego punktu korpusu maszyny.

Słowa kluczowe: maszyny wibracyjne, samosynchronizacja, niecentralne przyłożenie siły wymuszającej

1. Problem formulation

Utilised in mechanical processing plants of useful minerals and in other industrial branches, vibratory machines such as: vibrating screens, conveyers, vibrating tables or self-feeding shak-

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ing grids, require – for the proper functioning – a machine body translatory motion and a linear vibration trajectory.

Thus, the condition for the proper operation of such machines is the requirement that the vibrator exciting force has a constant direction of operation and that this direction is passing through the system mass centre (which protects the body against rocking motion).

Such drive is usually obtained by means of the set of two counter-running inertial vibrators, which symmetry line passes through the system mass centre, and the cophasal vibrators work is obtained by the self-synchronisation (Blechman, 1994).

The scheme of such machine is shown (in the case of $w = 0$) in Fig. 1.

where:

m — unbalanced mass [kg],
e — eccentric of a rotor unbalance [m],
M_k — machine body mass [kg],

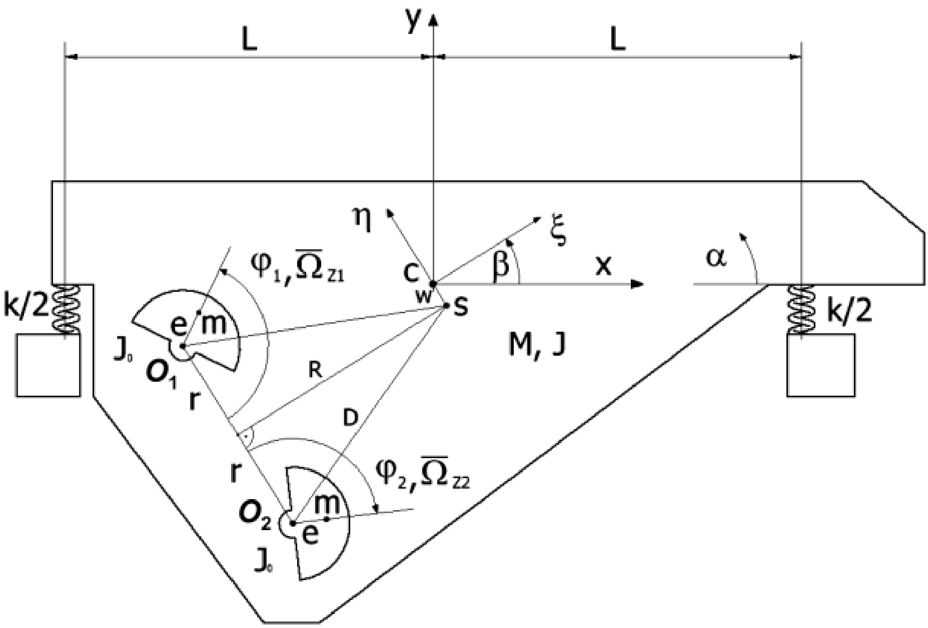

Fig. 1. Two-vibrator over-resonance vibratory machine calculation model

$M = M_k + 2m$ — mass of the vibrating part of the machine (body and vibrator) [kg],
C — centre of mass of the body with unbalanced masses brought to the vibrator axis of rotation,
S — centre of symmetry of the driving system,
w — eccentricity of the force direction,
J — central moment of the body inertia with unbalanced masses brought to the vibrator axis of rotation [kgm^2],

J_o — moment of inertia of the vibrator together with engine, calculated versus the axis of rotation [kgm^2],
ω — angular velocity [1/s],
k_x, k_y, k_ξ, k_η — coefficients of elasticity of the body supporting system – correspondingly – along axes: x, y and ξ, η [N/m],
$\overline{\Omega}_{z1}$, $\overline{\Omega}_{z2}$ — moments exerted to the rotating masses originated from the drive and resistance to motion [Nm],

Since due to constructional problems as well as manufacturing errors an accurate passing the line of action of the excitation force through the vibration system centre is not always possible, the need arises of the estimation of the resulting body rockings transferring themselves e.g. into a diversification of the vibration trajectory in various places of the body working surface and disturbances in the feedstock transportation. This problem is solved in the references (e.g. Banaszewski, 1990), however the derived there computational equations are based on the tacit assumption, that the deviation of the perpendicular bisector of the driving system from the mass centre does not influence the vibrators synchronisation. More adequate approach was proposed in (Banaszewski, 2010), where synchronization conditions for vertically oriented, non central system of counter-rotating inertial vibrators with independent drive was analysed ,but with not taking into considerations influence of elastic suspension system.

The aim of the hereby paper is to formulate mathematical model for the case of vibratory machine drive by a system of two independent inertial vibrators that cophasal work is obtained by the self-synchronisation, with the non-central application of the exciting force and taking into consideration influence of elastic suspension stiffness.

The model shown in Figure 1 was discussed, with taking into account the limited power of vibrators driving systems. The location of the mass centre C in the scheme does not correspond with the nominal line of action of the driving system but is shifted versus it by a distance $CS = w$.

2. Analysis of the influence of non-central positioning of vibrators on their cophasal running

The analysis of the vibrators running was performed by the averaging method (Hayasi, 1964). This allows to write equations of motion of the machine body with singling out slow-changing and fast-changing effects. Thus, assuming the equality of angular velocities – for synchronous running – of both vibrators $\dot{\varphi}_1 = \dot{\varphi}_2$ and their 'slow' variability ($\omega \approx$ const), we can write the body equations of motion in the absolute system ξ, η, in a form:

$$M\ddot{\xi} + k_\xi \xi = me\omega^2[\sin\varphi_1 + \sin\varphi_2] \tag{1a}$$

$$M\ddot{\eta} + k_\eta \eta = me\omega^2[\cos\varphi_2 - \cos\varphi_1] \tag{1b}$$

$$J\ddot{\alpha} + k_y l^2 \alpha = me\omega^2 r[\sin\varphi_2 - \sin\varphi_1] + me\omega^2 R[\cos\varphi_1 - \cos\varphi_2] + \\ + me\omega^2 w[\sin\varphi_1 + \sin\varphi_2] \tag{1c}$$

where:

ξ, η — absolute coordinates determining a location of the body mass centre, $\xi \| R$,

α — body angle of rotation,

$$k_\xi = k_x \cos^2\beta + k_y \sin^2\beta \tag{2a}$$

$$k_\eta = k_y \cos^2\beta + k_x \sin^2\beta \tag{2b}$$

– remaining notations as in Figure 1

When calculating moments $\overline{\Omega}_{ci}$ from excitation forces $\overline{P}_i$, $i = 1,2$ of vibrators versus the body mass centre C, shifted versus the driving system centre of symmetry S by a directed segment $\overline{CS}$ of a length w, the following dependence was used:

$$\overline{\Omega}_{ci} = \overline{CO_i} \times \overline{P_i} = (\overline{CS} + \overline{SO_i}) \times \overline{P_i} = \overline{CS} \times \overline{P_i} + \overline{SO_i} \times \overline{P_i} \tag{3}$$

Denoting: $\varphi_1 - \varphi_2 = \Delta\varphi = \text{const}$ and assuming: $\varphi_2 = \omega t$, $\omega = \text{const}$, equations (1a, 1b, 1c) can be presented, after rearrangements, in a form:

$$M\ddot{\xi} + k_\xi \xi = me\omega^2 \sqrt{2(1+\cos\Delta\varphi)}\,\sin(\omega t + \delta) \tag{4a}$$

where: $\sin\delta = \dfrac{\sin\Delta\varphi}{\sqrt{2(1+\cos\Delta\varphi)}}$, $\cos\delta = \dfrac{1+\cos\Delta\varphi}{\sqrt{2(1+\cos\Delta\varphi)}}$

$$M\ddot{\eta} + k_\eta \eta = me\omega^2 \sqrt{2(1-\cos\Delta\varphi)}\,\sin(\omega t + \kappa) \tag{4b}$$

where: $\sin\kappa = \dfrac{1-\cos\Delta\varphi}{\sqrt{2(1-\cos\Delta\varphi)}}$, $\cos\kappa = \dfrac{\sin\Delta\varphi}{\sqrt{2(1-\cos\Delta\varphi)}}$

$$J\ddot{\alpha} + k_y l^2 \alpha = me\omega^2 D\sqrt{2(1-\cos\Delta\varphi)}\,\sin(\omega t + \lambda) + \\ + me\omega^2 w\sqrt{2(1+\cos\Delta\varphi)}\,\sin(\omega t + \theta) \tag{4c}$$

where: $\sin\lambda = \dfrac{R(\cos\Delta\varphi - 1) - r\sin\Delta\varphi}{D\sqrt{2(1-\cos\Delta\varphi)}}$, $\cos\lambda = \dfrac{r(1-\cos\Delta\varphi) - R\sin\Delta\varphi}{D\sqrt{2(1-\cos\Delta\varphi)}}$

and, as can be easily verified: $\theta = \delta$.

Particular integrals of these equations, describing the steady state, are of a form:

$$\xi(t) = \frac{me\omega^2\sqrt{2(1+\cos\Delta\varphi)}}{k_\xi - M\omega^2}\sin(\omega t + \delta) \tag{5a}$$

$$\eta(t) = \frac{me\omega^2\sqrt{2(1-\cos\Delta\varphi)}}{k_\eta - M\omega^2}\sin(\omega t + \kappa) \tag{5b}$$

$$\alpha(t)=\frac{me\omega^2 D\sqrt{2(1-\cos\Delta\varphi)}}{k_y l^2-J\omega^2}\sin(\omega t+\lambda)+\frac{me\omega^2 w\sqrt{2(1+\cos\Delta\varphi)}}{k_y l^2-J\omega^2}\sin(\omega t+\theta) \tag{5c}$$

Dynamic equations, analysed so far, described body vibrations at the assumption that the vibrators angular motion in the steady state can be considered to be uniform. Such assumption is equivalent to omitting the body vibrations influence on the vibrators running. Presently, we will write vibrators equations of motion taking into account these couplings, it means in a not inertial system related to the machine body.

Then, applying moments from forces of transportation to vibrators, their equations of motion obtain the following forms:

$$J_0\ddot{\varphi}_1=\Omega_{z1}-me\ddot{\xi}_1\cos\varphi_1-me\ddot{\eta}_1\sin\varphi_1 \tag{6a}$$

$$J_0\ddot{\varphi}_2=\Omega_{z2}-me\ddot{\xi}_2\cos\varphi_2+me\ddot{\eta}_2\sin\varphi_2 \tag{6b}$$

where:

$\Omega_{z1,2}$ — external moments (difference between the driving moment and anti-torque),
J_0 — as in Figure 1.

Let us denote as **vibratory moments** Ω_{wi}, $i-1,2$, the following expressions:

$$\Omega_{w1}=-me(\ddot{\xi}_1\cos\varphi_1+\ddot{\eta}_1\sin\varphi_1) \tag{7a}$$

$$\Omega_{w2}=-me(\ddot{\xi}_2\cos\varphi_2-\ddot{\eta}_2\sin\varphi_2) \tag{7b}$$

On the basis of the previously determined solutions of the machine body motion (not taking into account influence of vibratory moments on the vibrators running) we will determine components of acceleration of both vibrators axes. Axis acceleration $\overline{a}_i$ of an individual vibrator equals:

$$\begin{aligned}\overline{a}_i&=\overline{a}_c+\overline{\varepsilon}_\alpha\times\overline{CO}_i-\overline{CO}_i\cdot\omega_\alpha^2 =\overline{i}\cdot\ddot{\xi}+\overline{j}\cdot\ddot{\eta}+\overline{n}\cdot\ddot{\alpha}\times\overline{CS}+\\&+\overline{n}\cdot\ddot{\alpha}\times\overline{SO}_i-(\overline{CS}+\overline{SO}_i)\cdot\dot{\alpha}^2\end{aligned} \tag{8}$$

where: $\overline{i},\overline{j},\overline{n}$ — versors of axis ξ,η,ζ of the central reference system, respectively.

Omitting – in the above given formula – expressions containing $\dot{\alpha}^2$ and the component with $\overline{CS}$, as being at the lower order than the remaining ones, it is possible to calculate components of the acceleration vectors of axes of both vibrators:

$$\ddot{\xi}_1\approx\ddot{\xi}-\ddot{\alpha}\cdot r \tag{9a}$$

$$\ddot{\eta}_1\approx\ddot{\eta}-\ddot{\alpha}\cdot R \tag{9b}$$

$$\ddot{\xi}_2\approx\ddot{\xi}-\ddot{\alpha}\cdot r \tag{9c}$$

$$\ddot{\eta}_2\approx\ddot{\eta}-\ddot{\alpha}\cdot R \tag{9d}$$

After substituting the second derivatives of dependence (5) into (9), it is possible to calculate the vibratory moments values acc. to (7) and obtain :

$$\Omega_{w1} = -\sqrt{2}m^2e^2\omega^4 \cdot \left[\frac{\sqrt{1+\cos\Delta\varphi}}{M\omega^2 - k_\xi} \sin(\omega t + \delta)\cdot\cos(\omega t + \Delta\varphi) - \right.$$
$$\frac{rD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda)\cdot\cos(\omega t + \Delta\varphi) - \frac{rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \theta)\cdot\cos(\omega t + \Delta\varphi) +$$
$$\frac{\sqrt{1-\cos\Delta\varphi}}{M\omega^2 - k_\eta} \sin(\omega t + \kappa)\cdot\sin(\omega t + \Delta\varphi) - \frac{RD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda)\cdot\sin(\omega t + \Delta\varphi) -$$
$$\left. \frac{Rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \theta)\cdot\sin(\omega t + \Delta\varphi) \right] \tag{10a}$$

$$\Omega_{w2} = -\sqrt{2}m^2e^2\omega^4 \cdot \left[\frac{\sqrt{1+\cos\Delta\varphi}}{M\omega^2 - k_\xi} \sin(\omega t + \delta)\cdot\cos(\omega t) + \right.$$
$$\frac{rD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda)\cdot\cos(\omega t) + \frac{rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \theta)\cdot\cos(\omega t) -$$
$$\frac{\sqrt{1-\cos\Delta\varphi}}{M\omega^2 - k_\eta} \sin(\omega t + \kappa)\cdot\sin(\omega t) + \frac{RD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda)\cdot\sin(\omega t) +$$
$$\left. \frac{Rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \sin(\omega t + \theta)\cdot\sin(\omega t) \right] \tag{10b}$$

By *averaging the vibratory moments value for the period: T = 2π/ω* the following is obtained:

$$\widetilde{\Omega}_{w1} = \frac{1}{T}\int_0^T \Omega_{w1}(t)dt =$$
$$-\sqrt{2}m^2e^2\omega^4 \cdot \frac{\omega}{2\pi} \cdot \left[\frac{\sqrt{1+\cos\Delta\varphi}}{M\omega^2 - k_\xi} \int_0^{2\pi/\omega} \sin(\omega t + \delta)\cdot\cos(\omega t + \Delta\varphi)dt - \right.$$
$$\frac{rD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda)\cdot\cos(\omega t + \Delta\varphi)dt - \frac{rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \theta)\cdot\cos(\omega t + \Delta\varphi)dt +$$
$$\frac{\sqrt{1-\cos\Delta\varphi}}{M\omega^2 - k_\eta} \int_0^{2\pi/\omega} \sin(\omega t + \kappa)\cdot\sin(\omega t + \Delta\varphi)dt - \frac{RD\sqrt{1-\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda)\cdot\sin(\omega t + \Delta\varphi)dt -$$
$$\left. \frac{Rw\sqrt{1+\cos\Delta\varphi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \theta)\cdot\sin(\omega t + \Delta\varphi)dt \right] \tag{11}$$

After calculating the integrals, utilising the dependence (4), which determines values of angles $\lambda, \kappa, \delta, \theta$ and after performing certain transformations, the following was obtained:

$$\widetilde{\Omega}_{w1} = \frac{-m^2 e^2 \omega^4}{2}\left[\frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi}\right]\sin\Delta\varphi - \frac{m^2 e^2 \omega^4}{2}\left[\frac{wr}{J\omega^2 - k_y l^2}\sin\Delta\phi - \frac{wR}{J\omega^2 - k_y l^2}(1+\cos\Delta\varphi)\right] \tag{12a}$$

Acting in a similar fashion in reference to the vibratory moment of the second vibrator the analogous equation was obtained:

$$\widetilde{\Omega}_{w2} = \frac{m^2 e^2 \omega^4}{2}\left[\frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi}\right]\sin\Delta\varphi - \frac{m^2 e^2 \omega^4}{2}\left[\frac{wr}{J\omega^2 - k_y l^2}\sin\Delta\varphi + \frac{wR}{J\omega^2 - k_y l^2}(1+\cos\Delta\varphi)\right] \tag{12b}$$

The equality $\widetilde{\Omega}_{w1} = \widetilde{\Omega}_{w2}$ *is the condition that both vibrators will be running with the same rotational velocity.*

This equality, after reduction and trigonometric transformations leads to the equation:

$$\tan\left(\frac{\Delta\varphi}{2}\right) = \frac{\dfrac{wR}{J\omega^2 - k_y l^2}}{\dfrac{D^2}{J\omega^2 - k_y l^2} + \dfrac{1}{M\omega^2 - k_\eta} - \dfrac{1}{M\omega^2 - k_\xi}} \tag{13}$$

The above equation allows to determine the difference, $\Delta\varphi$, of angles of rotation of both vibrators, and then to determine – on the basis of (5c) – angular oscillations of the machine body.

A simpler, but sufficiently accurate, version of the dependence of the body angular oscillations amplitude can be recommended for the engineering practice. Taking advantage of the fact that usually:

$$wR \ll D^2 \tag{14}$$

and omitting – for over-resonance machines – elements: k_ξ, k_η and k_y respectively versus $M\omega^2$ and $J\omega^2$, we obtain equation (13) in a form:

$$\Delta\varphi \cong 2\frac{wR}{D^2} \tag{15}$$

As the author's investigations indicate (Michalczyk, 2010), (Michalczyk, Czubak, 2010), the similar disphasing effect occurs also in systems with the central direction of the exciting force, but at the asymmetry of vibrators resistance to motion or at the asymmetric influence of forces resulting from collisions with feedstock materials.

Presently the amplitude of the body angular oscillations will be determined, on the basis of (5c). In order to do that the approximate values, for $w << R, D$, of trigonometric functions of angles $\Delta\varphi, \lambda, \theta$ occurring in (5c), were determined on the basis of (4):

$$\sqrt{2(1-\cos\Delta\varphi)} \cong \frac{2wR}{D^2}\ ,\ \sqrt{2(1+\cos\Delta\varphi)} \cong 2\ ,\ \sin\lambda \cong -\frac{r}{D}\ ,$$
$$\cos\lambda \cong -\frac{R}{D}\ ,\ \sin\theta \cong \frac{wR}{D^2}\ ,\ \cos\vartheta \cong 1 \tag{16}$$

After substituting (16) into (5c) and carrying out several trigonometric transformations and simplifications (discussed above), it is possible to obtain the following dependence for the amplitude A_α of the body angular oscillations:

$$A_\alpha \cong \frac{2me\omega^2 w}{J\omega^2 - k_y l^2} \cdot \frac{\sqrt{r^4 + R^2(r-w)^2}}{D^2} \tag{17a}$$

If, in addition, $w \ll r$, the simpler version of this dependence can be applied:

$$A_\alpha \cong \frac{2m\omega^2 w}{J\omega^2 - k_y l^2} \cdot \frac{r}{D} \tag{17b}$$

It should be noticed, that – as can be easily proved on the basis of the analysis of (5c) – for the case of cophasal running of vibrators $\Delta\varphi = 0$, and the classic solution of the body rocking motion is described by the following expression:

$$A_{\alpha(\Delta\varphi=0)} \cong \frac{2m\omega^2 w}{J\omega^2 - k_y l^2} \tag{18}$$

Thus, finally:

$$A_\alpha \cong A_{\alpha(\Delta\varphi=0)} \cdot \frac{r}{D} \tag{19}$$

The dependences (13) and (5) with constants values (4) enable one to establish shape of trajectory for any point (x_{io}, y_{io}) of machine body:

$$x_i(t) = \xi(t)\cdot\cos\beta - \eta(t)\cdot\sin\beta - y_{io}\cdot\alpha(t) \tag{20a}$$

$$y_i(t) = \xi(t)\cdot\sin\beta + \eta(t)\cdot\cos\beta + x_{io}\cdot\alpha(t) \tag{20b}$$

3. Conclusions

1. It was proved in the paper, that the analysis of the machine body rocking vibrations caused by the non-central application of the exciting force, carried out at assuming the vibrators cophasal running, leads to a significant over-estimation of real amplitudes.
2. In systems of independent drives the vibrators are disphased by an angle $\Delta\varphi \cong 2\frac{wR}{D^2}$, which leads to the resulting body rocking amplitude given by the equation (17).
3. Such amplitude is smaller, in approximation at the ratio $\frac{r}{D}$ versus the amplitude estimated without taking into consideration this effect.

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Received: 16 September 2011