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*Original research paper*

# New approach to isometric transformations in oblique local coordinate systems of reference

**Grzegorz Stępień*, Ewa Zalas, Tomasz Ziębka**

Maritime University of Szczecin
Institute of Geoinformatics
46 Żołnierska Street, 71–250 Szczecin, Poland
e-mails: g.stepien@am.szczecin.pl; ewa.zalas@gmail.com; tziebka@geometr.biz

Grzegorz Stąpień: https://orcid.org.0000-0002-1080-6388
Tomasz Ziębka: https://orcid.org.0000-0002-6580-7660
Ewa Zalas: https://orcid.org.0000-0001-5032-1177

*Corresponding author: Grzegorz Stępień

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**Abstract:** The research article describes a method of isometric transformation and determining an exterior orientation of a measurement instrument. The method is based on a designation of a "virtual" translation of two relative oblique orthogonal systems to a common, known in the both systems, point. The relative angle orientation of the systems does not change as each of the systems is moved along its axis. The next step is the designation of the three rotation angles (e.g. Tait-Bryan or Euler angles), transformation of the system convoluted at the calculated angles and moving the system to the initial position where the primary coordinate system was. This way eliminates movements of the systems from the calculations and makes it possible to calculate angles of mutual rotation angles of two orthogonal systems primarily involved in the movement. The research article covers laboratory calculations for simulated data. The accuracy of the results is $10^{-6}$ m ($10^{-3}$ regarding the accuracy of the input data). This confirmed the correctness of the assumed calculation method. In the following step the method was verified under field conditions, where the accuracy of the method raised to 0.003 m. The proposed method enabled to make the measurements with the oblique and uncentered instrument, e.g. total station instrument set over an unknown point. This is the reason why the method was named by the authors as Total Free Station – TFS. The method may be also used for isometric transformations for photogrammetric purposes.

**Key words**: oblique orthogonal system, isometric transformation, big rotation angles, Total Free Station

## 1. Introduction

To describe a spatial position of objects (physical bodies) referential orthogonal systems defined in three-dimensional Euclidean space are usually adopted. The selection of the system is free and dictated by practical reasons. The Rectangular Cartesian coordinate system which is associated with the agreed beginning (material body) is called an inertial coordinate system (constant) or laboratory coordinate system. A system associated with a microsystem (an atom or molecule) of a particular body is called a non inertial or molecular coordinate system (Kielich, 1977), while in the field of geodesy and photogrammetry this type of system is called an oblique system (Zalas et al., 2016).

The traditional system used in surveying is unequivocally defined by planes and axes of the levelled instrument, where the axis of rotation coincides with the local vertical line. The horizontal system is a specific type of the oblique system in which the rotation angles around the X and Y axes are equal to 0. This kind of system is very convenient as only four parameters of the transformation are necessary to define it unequivocally (assuming that the scale would not change): three movement vector components and angle of rotation around the vertical line of the system. Thus, only three co-ordinates of one point and one flat co-ordinate (x, y) of a different point must be known. However, sometimes a horizontal system is not sufficient or even not suitable to perform particular tasks that surveyors have to face, and then the spatial transformation becomes necessary. Measurements performed in unstable environments, such as surveying on floating objects, high chimneys or surfaces subjected to strong vibrations (close to generators, working machines and other equipment), often require unconventional approaches to deal with this problem. In such a situation, when it is impossible to associate the surveying coordinate system with the vertical line it is necessary to work with the compensator instrument switched off. Moreover, it is crucial to build upon the reference system realized by main planes and axes of the instrument. The obtained measurement data must be calculated with spatial transformation methods for the final coordinate system or the local coordinate system of the measured object.

Already in the seventies of the last century and for the needs of the shipbuilding industry, an optical levelling method was developed and used for the technological construction cycle of a ship on the ramp. The method allowed orienting planes and level axes of optical instruments in arbitrarily oblique systems. Commonly, especially in field of industrial geodesy, the data obtained in measurements in a horizontal system must be transformed to a local coordinate system of a measured object that usually is an oblique system. In particular cases, it is necessary to perform a transformation of data from an oblique system to another one – just to control geometric dependences of a measured object (Niebylski, 1977; 1984).

In the following article a new algorithm for transforming the space in an oblique system is presented. The system is based on the transformation through similarity method used in photogrammetry (1). However, a different approach to the designation of the system movement was proposed. The approach was based on so called “virtual movement”, i.e. movement between systems known in advance (virtual translation). This algorithm was implemented into traditional tacheometric surveying. The goal of the article is developing of free station method to the applications in non-stable places (e.g. floating vessels) were levelling of an instrument may not be possible.

## 2. Photogrammetric transformations in an oblique system

In photogrammetry and geodesy the basic and most frequently used transformation type is transformation through similarity. The method includes changing the scale, so called homotetia, translation and rotation – isometry (Kurczyński, 2014). A matrix that may be used to describe the isometric transformation using three angles (Tait-Bryan angles) connected to a movement (Baranowski, 2013). In photogrammetric transformations, the following transformation is used:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\omega & sin\omega \\ 0 & -sin\omega & cos\omega \end{bmatrix} \begin{bmatrix} cos\varphi & 0 & -sin\varphi \\ 0 & 1 & 0 \\ sin\varphi & 0 & cos\varphi \end{bmatrix} \begin{bmatrix} cos\kappa & sin\kappa & 0 \\ -sin\kappa & cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \tag{1}$$

where:

- $\omega$ – rotation around the X axis, the roll angle,
- $\varphi$ – rotation around the Y axis, the pitch angle,
- $\kappa$ – rotation around the Z axis, the yaw angle,
- X, Y, Z – coordinates in the primary coordinate system (without the rotation),
- X’, Y’, Z’ – coordinates in the secondary coordinate system (oblique one),
- $X_0$, $Y_0$, $Z_0$ – translation vector (movement of the primary coordinate system),
- $\lambda$ – factor of the scale change.

After multiplying matrices containing the angles of rotation around particular axes and recording that transformation using the A rotation matrix, the transformation can be recorded (1) as following:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \lambda \cdot A \cdot \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\omega & sin\omega \\ 0 & -sin\omega & cos\omega \end{bmatrix} \begin{bmatrix} cos\varphi & 0 & -sin\varphi \\ 0 & 1 & 0 \\ sin\varphi & 0 & cos\varphi \end{bmatrix} \begin{bmatrix} cos\kappa & sin\kappa & 0 \\ -sin\kappa & cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where:

$a_{11}$, $a_{12}$, $a_{13}$, ..., $a_{33}$ – factors of the A rotation matrix,
other signs as in (1).

The rotation matrix may be also expressed with Euler angles – nutation, precession and clear rotation – and then it takes the form of the B matrix.

$$B = \begin{bmatrix} cos(x',x'') & cos(x',y'') & cos(x',z'') \\ cos(y',x'') & cos(y',y'') & cos(y',z'') \\ cos(z',x'') & cos(z',y'') & cos(z',z'') \end{bmatrix} \tag{3}$$

where: the rotation matrix (3) contains angles of axis rotation relative to each other.

The transformations (1), (2), (3) are relatively unequivocal and are the transformations through similarity. They map an ordered section to another ordered section, i.e. vector to vector. The isometry does not change the lengths of the vectors. It also does not change the distance between points. The above statements are correct both for planes and spaces. Analogously, it takes place for the movement and rotation, but beside the rotations around a point, rotations around a straight line are available. According to Euler: Every rotation around a point is a rotation around a straight line passing through this point (Stark, 1974).

Matrix A (2) can be consider for the small rotation angles and was applied in the Bursa-Wolf (Bursa, 1962; Wolf, 1963) and the Molodensky-Badekas (Molodensky et al., 1960; Badekas, 1969) transformation.

The transformations through similarity were also applied to other solutions for nonlinear inverse problems (Málek et al., 2007). The transformations through similarity are also commonly used in close range photogrammetry, in which the rotation angles are much larger in comparison to the traditional aerial survey (Jue, 2008). For large rotation angles it is possible to define them if only two adjust points located on the primary coordinate system's axes are known (Zalas et al., 2016). For the transformation from the oblique system to a plane, the direct linear transformation (DLT) is applied, which is also based on the transformation (1), especially for data collected by unmanned aerial vehicles. This type of transformation is very popular due to the linear connections that link coordinates in both coordinate systems (Seedahmed and Habib, 2016).

Rotation directions and signs were assumed in the formula (1) and (2) according to signs on the Figure 1 which shows the xy-plane.

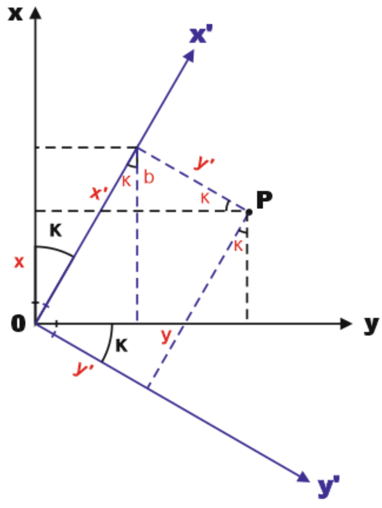


Fig. 1 The transformation through torsion – the rotation in the xy-plane at κ angle – torsion around the z axis (yaw angle, drift angle)

## 3. Proposed approach

The following article suggests a different approach to the transformation of coordinates in two relative oblique coordinate systems. To perform the transformation of the coordinates, it is obligatory to know four common points in both coordinate systems. In case of moving and rotating the system at three angles, the rotations are connected with the movements along particular axes. The traditional methods of surveying dictate to calculate them together with the movements. The proposed method is about defining the movement between systems (translation vector), which are called "virtual" ones, at first and then define three rotation angles. The virtual translation vector is designated based on the knowledge of one point in both coordinate systems. It is designated by moving two mutually oblique orthogonal systems to a common, known in the both systems, point. Every system is moved along its axis so the mutual angle orientation of the systems does not change (Figure 2), what is written as relations (Equation 4). After moving the systems their common centre is in the same point, what allows to calculate three rotation angles of the systems independently.

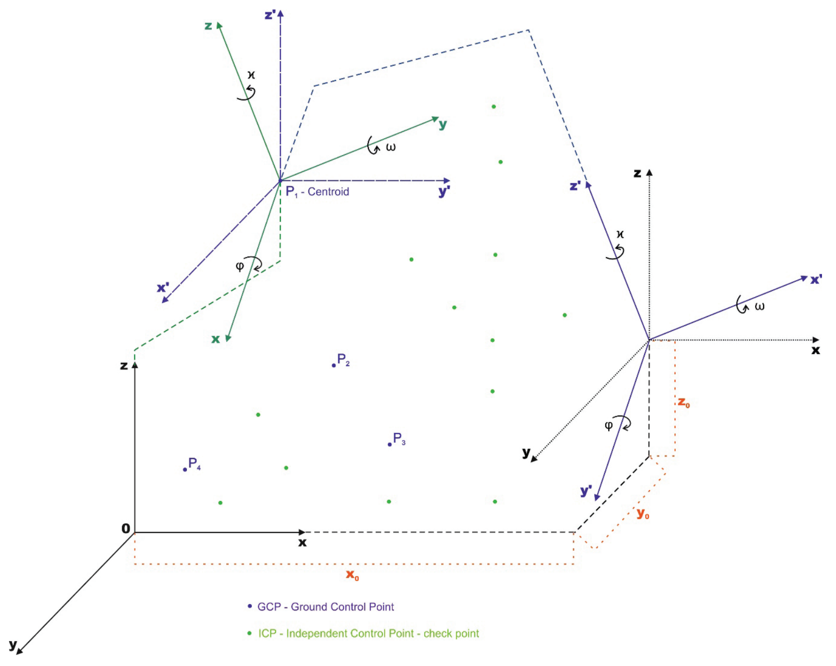


Fig. 2 The virtual movement of two relative oblique systems

$$\begin{aligned} P' - P'_i &= D' \\ P - P_i &= D \end{aligned} \tag{4}$$

where:

$$P' = \begin{bmatrix} X'_1 & \cdots & X'_n \\ Y'_1 & \dots & Y'_n \\ Z'_1 & \cdots & Z'_n \end{bmatrix}$$
$$P'_i = \begin{bmatrix} X'_i & \cdots & X'_i \\ Y'_i & \dots & Y'_i \\ Z'_i & \cdots & Z'_i \end{bmatrix}$$
$$P = \begin{bmatrix} X_1 & \cdots & X_n \\ Y_1 & \dots & Y_n \\ Z_1 & \cdots & Z_n \end{bmatrix} \tag{5}$$
$$P_i = \begin{bmatrix} X_i & \cdots & X_i \\ Y_i & \dots & Y_i \\ Z_i & \cdots & Z_i \end{bmatrix}$$

where in formulas (4), (5):

P – the matrix of points in the translated and convoluted system,

$P'_i$ – the matrix of points composed of one point (vector starting with the point 0 and ending with the point indexed with “i”) $P'_i$,

P – the matrix of points in the primary coordinate system – without translation and convolution,
$P_i$ – the matrix of points in the primary coordinate system composed of one point $P_i$,
i – point number that becomes the common centre for both systems,
n – natural number – number of points in the primary coordinate system (secondary coordinate system),
D – the matrix in the primary coordinate system after moving to the common point with the "i" index,
D'– the matrix in the translated and convoluted system (secondary coordinate system) after moving to the common point with the "i" index.

The next step is to determine three rotation angles (Euler angles and Tait-Bryan angles) and designate a matrix of the A rotation with the following relation:

$$D = A \cdot D' \tag{6}$$

where: signs are assumed as in the relations (Equation 2), (Equation 4), (Equation 5).

To designate the matrix of the A rotation in the relation (Equation 5), it is obligatory to know at minimum three points in both reference system. These points are included in the D and D' matrices. Regarding the fact that the A matrix fulfils the orthogonality conditions (Equation 7) Jue L. (2008).

$$A \cdot A^T = A^T \cdot A = I \tag{7}$$

where:
A – the rotation matrix defined in analogy to the relation (2),
I – the identity matrix.

After calculating the A matrix the next step is to translate the D' system along the $P_i$ vector (matrix). In practice, this means the "return" of the "convoluted" by the well-known path, after which the primary coordinate system was moved (not convoluted one). The relation may be recorded as following:

$$P_T = D' + P_i \tag{8}$$

where:
$P_T$ – the matrix of the points transformed from the duplicated system to the primary coordinate system, other signs are assumed as in the relations (Equation 6), (Equation 4), (Equation 5).

The next step is to compare the transformed points in the $P_T$ and P matrices. The comparison is performed for points which were used for the transformation as well as for the check points. The last step is to calculate the average real errors for the compared points using the equation (9):

$$m_x = \sqrt{\frac{\sum_{i=1}^{n} V_x \cdot V_x}{n}}$$
$$m_y = \sqrt{\frac{\sum_{i=1}^{n} V_y \cdot V_y}{n}}$$
$$m_z = \sqrt{\frac{\sum_{i=1}^{n} V_z \cdot V_z}{n}} \tag{9}$$
$$m_p = \sqrt{m_x^2 + m_y^2 + m_z^2}$$

where:

$m_x$, $m_y$, $m_z$, $m_p$ – average real errors respectively for the cooperates: x, y, z and the point,

$V_x$, $V_y$, $V_z$ – deviations after the transformation respectively for the cooperates: x, y, z.

The method described above was named the Total Free Station transformation (TFS) and it is the further development of the Free station method, in which the measurements are performed with a levelled instrument set in an unknown place (Schofield, 1984). In the proposed method an instrument is not levelled.

## 4. Surveying experiment

Within the experiment a few lab and field researches were performed. In the lab research the mathematics correctness of the proposed method was verified with simulation data. In the lab research a random set of points, recorded in the P matrix, was assumed:

$$P = \begin{bmatrix} 9.425 & 6.686 & -36.856 & 11.137 & 10.632 & 7.908 & 1.361 & -13.433 & -4.929 \\ 0.000 & 13.244 & 28.020 & -8.690 & -5.895 & 7.072 & 38.746 & 45.308 & -2.337 \\ 0.000 & 0.000 & -0.001 & 0.440 & 4.297 & 3.788 & 2.288 & 1.236 & -1.120 \end{bmatrix}$$

In the P matrix each of columns corresponds to one point (the vectors hooked at the origin). The first line records the X coordinate, the second line records the Y coordinate, and the third line records the Z coordinate. It was assumed that the coordinates are indicated in [m] and recorded with a precision of +-1 mm. Then, the P matrix was moved by the random vector (matrix):

$$displacement = \begin{bmatrix} 1555.555 & 1555.555 & 1555.555 & 1555.555 & 1555.555 & 1555.555 & 1555.555 & 1555.555 & 1555.555 \\ -100.675 & -100.675 & -100.675 & -100.675 & -100.675 & -100.675 & -100.675 & -100.675 & -100.675 \\ 154.321 & 154.321 & 154.321 & 154.321 & 154.321 & 154.321 & 154.321 & 154.321 & 154.321 \end{bmatrix}$$

and then the translated system of points was rotated by multiplying with the A matrix of the rotation:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.774 & -0.633 & -0.010 \\ 0.622 & 0.763 & -0.174 \\ 0.118 & 0.128 & 0.985 \end{bmatrix}$$

The A matrix was calculated according to the relation (Equation 2) for the assumed angles $\omega = 10.00000^0$, $\varphi = 0.57893^0$, $\kappa = 39.30716^0$. In this way the coordinates of the points in the translated and rotated system (at three angles) were obtained and recorded in the P matrix:

$$P' = \begin{bmatrix} 1273.0716 & 1262.563 & 1219.5139 & 1279.8964 & 1277.6962 & 1267.3799 & 1242.2657 & 1226.6732 & 1263.4573 \\ 870.5708 & 878.9725 & 863.1438 & 864.9286 & 866.0774 & 874.3655 & 894.7214 & 890.7025 & 860.0466 \\ 323.2771 & 324.6507 & 321.4169 & 322.799 & 326.8957 & 327.7344 & 329.5428 & 327.6059 & 320.1854 \end{bmatrix}$$

In the following step the inverse transformation was performed according to the TFS transformation. At first a displacement to the centroid of systems was performed (virtual translation) using the point no. 2 (i = 2) as the centroid. This allowed obtaining the matrices D and D' in both systems computed by applying (Equation 4):

$$\begin{bmatrix} 2.739 & 0.000 & -43.542 & 4.451 & 3.946 & 1.222 & -5.325 & -20.119 & -11.615 \\ -13.244 & 0.000 & 14.776 & -21.934 & -19.139 & -6.172 & 25.502 & 32.064 & -15.581 \\ 0.000 & 0.000 & -0.001 & 0.440 & 4.297 & 3.788 & 2.288 & 1.236 & -1.120 \end{bmatrix}$$

$$\begin{bmatrix} 10.509 & 0.000 & -43.049 & 17.333 & 15.133 & 4.817 & -20.297 & -35.890 & 0.894 \\ -8.402 & 0.000 & -15.829 & -14.044 & -12.895 & -4.607 & 15.749 & 11.730 & -18.926 \\ -1.374 & 0.000 & -3.234 & -1.852 & 2.245 & 3.084 & 4.892 & 2.955 & -4.465 \end{bmatrix}$$

After the translation of the systems to the joint point (centroid – the second column in the D and D' matrices – above) – the point became a common origin of both coordinate systems. This was obtained without a simultaneous change of the system angle orientation relative to each other. Then, sequentially according to the relations (Equation 4), (Equation 6) – and taking into account (Equation 7) a rotation of the secondary coordinate system was performed and in accordance with the relation (Equation 8) the system was moved along a vector (matrix) counter to the one which was used to move the primary coordinate system previously (virtual translation). The transformed coordinates that took the form of:

$$\begin{bmatrix} 9.425004 & 6.686 & -36.85602 & 11.137007 & 10.632008 & 7.908004 & 1.360993 & -13.433015 & -4.929001 \\ 0.000005 & 13.244 & 28.019995 & -8.689992 & -5.894991 & 7.072003 & 38.745991 & 45.307988 & -2.336994 \\ -0.000001 & 0.000 & -0.001006 & 0.439998 & 4.296996 & 3.787998 & 2.288001 & 1.236 & -1.120004 \end{bmatrix}$$

(the higher precision was assumed on purpose) were compared to the coordinates of the primary coordinate system. The following deviations in the specific points were achieved as follows (small values close to zero):

$$\begin{bmatrix} -000004 & 0 & 0.00002 & -0.000007 & -0.000008 & -0.000004 & 0.000007 & 0.000015 & 0.000001 \\ 0.000005 & 0 & 0.000005 & -0.000008 & -0.000009 & -0.000003 & 0.000009 & 0.000012 & -0.000006 \\ -0.000001 & 0 & 0.000006 & 0.000002 & 0.000004 & 0.000002 & -0.000001 & 0 & 0.000004 \end{bmatrix}$$

Then, according to the relations (Equation 10) the average real errors were calculated. The small values indicate a proper performance of the proposed algorithm. The following values were obtained:

$$m_x = 0.000004 \text{ [m]} \quad m_y = 0.000003 \text{ [m]} \quad m_z = 0.000001 \text{ [m]}$$

In the result the accuracy of the position of the points after the transformation amounted $10^{-6}$ m and the accuracy of the input coordinates was $10^{-3}$ m. The errors obtained for the simulated data were $10^3$ times lower than the accuracy of the input data. It was therefore concluded that the transformation method is mathematically correct and consequently the field researches were continued.

The tacheometric surveying was performed in two mutually oblique coordinate systems that were translated by an unknown value (Figure 3). In the first case the instrument used for the surveying was levelled and centred with the compensator switched on (P matrix), while in the second case the instrument was not levelled but moved and the surveying coordinate system was connected to the moved and oblique instrument. Additionally, the instrument had the compensator switched off (P' matrix). The field surveying was performed with the theodolite station (model Trimble M3) with the angle accuracy of 3" and 2 mm + 2 ppm distance measurement accuracy measured to reflective foil. Eight points distant of 10 – 50 m from the instrument were surveyed in two full series. The points were signalled with a reflective foil with a cross. After calculating the average coordinates, the average error of the typical concern was obtained and it amounted 0.5 – 3.2 mm. In this way two matrices of the coordinates were obtained:

$$P = \begin{bmatrix} 0.000 & 25723.3 & 15798.8 & 889.8 & -16239.5 & -33676.3 & -8293.8 & 27883 & 11769.3 \\ 0.000 & 2.0 & 6623.0 & 15720.3 & 25695.3 & 23131 & -43591.8 & -39376.5 & -2138.3 \\ 0.000 & 1648.0 & 6783.5 & 7168.4 & 2210.0 & 1422 & -56 & 15 & -23 \end{bmatrix}$$

$$P' = \begin{bmatrix} 3034.0 & 28488.0 & 17657.5 & 1669.8 & -16511.5 & -27305.5 & 585.0 & 35882.5 & 14983.8 \\ -3387.3 & 1.0 & 5207.3 & 12250.5 & 19913.5 & -30778.3 & -47688.8 & -38725.8 & -3950.5 \\ -1490.5 & 774.3 & 5706.0 & 5781.3 & 463 & -1023.5 & -2002.0 & -1026 & -1246.5 \end{bmatrix}$$

The coordinates in the P and P' matrices were indicated in [mm]. Each column includes one point (a vector anchored to the origin). Next, the transformation of the coordinates from the P' oblique coordinate system to the P primary coordinate system was performed with the TFS method and in accordance with relations (Equation 4), (Equation 6) taking into account (Equation 8). Finally, the real errors were calculated based on (Equation 10) and the obtained values were as following:

Table 1. TFS errors in the site surveys

| Real errors | $m_x$ [mm] | $m_y$ [mm] | $m_z$ [mm] | $m_p$ [mm] |
|---|---|---|---|---|
| For points used for the transformation | 0.003 | 0.005 | 0.008 | 0.010 |
| For the check points | 2.6 | 0.5 | 2.1 | 3.4 |

## 5. Discussion on results and conclusions

During the calculations in the laboratory coordinate system the $10^{-3}$ accuracy of the input data was obtained. This proves the mathematical correctness of the proposed method – the TFS transformation. During the field surveys the typical observation error, resulting mostly from the accuracy of the instrument, was 0.5–3.2 mm.

For the TFS transformation, three points were used: points number 4, 6 and 7 (column number of the P matrix). For these points the average errors were the lowest. Various variants of points were tested and regardless the configuration the error did not exceed the value: mp $\leq$ 5.6 mm. The lowest errors obtained for points no. 4, 6 and 7 allow developing the hypothesis that the reason for such results are the points forming a triangle with the largest surface area of all triangles that would be possible to create and at the same time using the points of the P matrix (the levelled and centred coordinate system). This minimizes the effect of the extrapolation when rotating the points by the calculated angles (for the most distant points). The points 4, 6 and 7 are characterized by the fact that their highest absolute values of the coordinates are as respectively: x, y, z. At the same time the solution was tested – calculating the matrix of the rotation – with a selection of 4 points. In the case of points no. 4, 6, 7, the same errors were obtained and the accuracy did not increase. Moreover, in case of the calculations in three adjust points the calculations are possible, if none of three points is a beginning for the coordinate system. In this case to calculate the matrix of the rotations it is obligatory to add the fourth point. The point which is the beginning for the coordinate system, after the virtual translation, should be well traceable, designated very carefully and measured in at least two full survey series.

Comparing the obtained accuracies of the TFS transformation with the typical concern error obtained during surveying, it should be stated that they are at the same level. Therefore, the transformation, despite the double measurements (in the primary coordinate system and in the secondary coordinate system) and the summation of the errors, does not affect the reduction of the accuracy. It was finally stated that the developed TFS transformation method is useful for surveying. Thus, it is possible that performing the measurements with the oblique instrument set in an unknown place (with the compensator switched off) has no influence on the accuracy of the transformed coordinates. For the TFS transformation four adjust points are obligatory. They must be known in both coordinate systems. One of them is necessary for calculating the virtual translation, while the other three points are useful for calculating the rotation matrix. The use of modified calculations (by the virtual translation), used in photogrammetry for surveying, fulfilled the task. Additionally, in this case the method was useful also for high rotation angles what is not so often using in photogrammetric transformations. According to the authors, the method may be found useful for local, precise surveying as well as for other geodynamic and engineering purposes, especially in non-stable places (e.g. on floating vessels) were levelling of an instrument may not be possible.

Fig. 3. Oblique tachymeter during surveying

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