Vertical gravity gradient determination for the needs of contemporary absolute gravity measurements – first results

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Abstract. Current development of gravimetric measurement technologies gives an opportunity to determine absolute gravity value with high precision, also in field conditions. In particular, it concerns gravity determination with the use of the A10 free-fall gravimeter manufactured by Micro-g Solutions. Surveys with the A10 as well as with FG5 gravimeters require precise determination of vertical gravity gradient in order to properly reduce the measured gravity value, e.g. to the benchmark level. A special stand of multiple levels (20 cm interval) for vertical gradient determination had been designed and constructed in the Institute of Geodesy and Cartography (IGiK), Warsaw. The stand enables the determination of the vertical gravity gradient on every station where the measurement with the A10 can be performed. First results of vertical gradient determinations at the Polish National Gravity Control points in Borowa Gora Geodetic-Geophysical Observatory (two laboratory and one field station) with the use of the stand developed are presented in the paper. Results of measurements on multiple levels were fitted with linear and nonlinear model and compared with vertical gravity gradients determined in the year 2000, based on the measurements using a standard tripod. Measurement methodology with the use of the stand has been proposed.

Keywords: absolute gravity determination, free-fall gravimeter, vertical gravity gradient

1 Introduction

Currently available gravimetric measurement technologies provide gravity with accuracy that makes use of normal vertical gravity gradient (3.086

μGal/cm = 3086 E ($1\,E = 10^{-9} \cdot s^{-2}$)) for gravity measurement reductions highly unsatisfying. Vertical gravity gradients determined in Poland at the absolute gravity stations vary substantially; the smallest one of -2.230 μGal/cm (Olszak, 2009) was observed at one of meridional gravimetric calibration base line stations, while the biggest one of -4.944 μGal/cm (Sas *et al.*, 2009) - at Kasprowy Wierch vertical gravimetric calibration baseline station (altitude of nearly 2000 m) in Tatra Mountains. Those values differ from normal gravity gradient value up to 60-70%. Calculation of gravity reduction from a height of 1 metre to the benchmark level using normal vertical gravity gradient can cause an error exceeding 100 μGal. That error increases with height as the reduction increases. Differences between measured and normal vertical gravity gradient are mainly caused by regional and local topography and density anomalies in subsurface layers of the Earth crust. Vertical gravity gradient values smaller than normal one mostly occur at points located below ground level (building basements) or in valleys; values bigger than normal occur in hilly, mountainous areas.

The change of the vertical gravity gradient with height is also influenced by local topography. Biggest variations of vertical gravity gradient can occur within first tens of centimetres above benchmark level and can achieve values up to 0.055 μGal/cm (Torge, 1989). Measurements conducted in a high - rise building showed that starting from the height of 40 metres above topography vertical gravity gradient value stabilizes.

Subject concerning the determination of vertical gravity gradient is still open in terms of methods and instruments used. There are no specified regulations how to perform and interpret results of vertical gravity gradient measurements. The use of vertical gravity gradient in reduction of absolute gravity measurements was the motivation for designing and constructing a special stand in the Institute of Geodesy and Cartography. Its application is supposed to allow efficient determination of vertical gravity gradient with sufficient accuracy.

2 Vertical gravity determinations during the establishment of the polish national gravity control

Vertical gravity gradients were determined at 12 zero-order points of the Polish National Gravity Control (POGK) and their eccentric points in 1995, 1996, and 2000. They were determined from gravity measurements on two levels (Fig. 1) of 1 m height difference using standard tripod with 2 or 3 gravimeters following a measurement scheme of ABBAAB. . . BAABBA. Accuracy of those determinations with LaCoste&Romberg (LCR) gravimeters was estimated at ± 3 μGal/m (Sas-Uhrynowski, 2002). Observed vertical

Figure 1. *Vertical gravity gradient measurement on two levels*

gravity gradients varied from -2.40 to -3.94 μGal/cm. A linear change of gravity with height had been assumed giving a constant vertical gravity gradient value. Absolute gravity measurements were performed on those points with several absolute gravimeters (FG5, IMGC, JILAg, ZZG). Actual measurement height for those gravimeters varied from 36 cm (ZZG) to 130 cm (FG5), what in terms of gravity reduction with the use of constant vertical gradient value could cause an additional error in gravity reduced to the benchmark level.

3 Construction of the stand for the determination of vertical gravity gradient

The team of the Institute of Geodesy and Cartography undertook the design and construction of a special stand for measuring vertical gravity gradient. The idea of the stand was to allow gravity measurement with a static gravimeter on multiple, equally distributed levels covering the range of heights of gravity measurement with both A10 and FG5 gravimeters. Among different concepts of its construction the one consisting of separated segments piled over each other, has been accepted. The stand is made from aluminium alloy. Each consecutive segment connects with the lower one by three pins and a screw keeping segments tight together. Such tight connections between segments assures good stability during the measurement on each level. The stand had been constructed mainly

Figure 2. *LaCoste&Romberg G-1012 with all segments of the stand*

for the use at the stations surveyed with the A10 absolute gravimeter. Spacing of the setting legs and stroke of the adjustment screws correspond to the one of tripod used for field measurements with the A10. The stand consists of six elements (Fig. 2):

- 3 segments of 40 cm in height
- 1 segment of 20 cm in height
- 1 plate for setting the gravimeter
- 1 tripod with the possibility of levelling in field conditions

Combination of segments allows gravity measurement every 20 cm of height above the benchmark within the range from 20 cm to 160 cm. The stand allows measurement precisely centred above the benchmark. Its construction is similar to the one used for vertical gravity gradient measurements at BIPM (Bureau International des Poids et Mersures) during the International Comparison of Absolute Gravimeters campaigns. The total weight of all segments is about 25 kg what makes the whole construction very handy and portable. Precise dimensioning of each segment allows equal interval of height change between consecutive measurement levels which was difficult to achieve with the use of a standard tripod. Change in levelling of the relative gravimeter on consecutive levels is negligible, causing changes in height by no more than 1 mm.

Figure 3. *LCR G-1012 on the constructed stand (left) at A-BG station, A10 measurement at 156 field station (right)*

4 Vertical gradient measurement with the use of the stand developed

The measurements for the determination of the vertical gravity gradient had been performed with three LCR gravimeters: G-1012, G-1084 and G-1078 at absolute gravity (POGK) point A-BG (Fig. 3 - left) at the Borowa Gora Geodetic-Geophysical Observatory and its two eccentric points (BG-G2 and 156) using the stand developed. G-1012 and G-1084 gravimeters are equipped with LRFB-300 feedback system which allows to acquire data with 1 second time resolution and 1 μGal data resolution (L and R Meter Service User's Manual, 2009).

G-1078 gravimeter is equipped with short range analogue feedback system designed for micro-gravimetric surveys. All gravimeters used in the experiment had been calibrated on the central gravimetric calibration base line in Poland in August 2011.

A-BG and BG-G2 stations are located inside gravimetric laboratories 2 m and 1.6 m below ground level, respectively. Measurements at those two stations were performed on 9 (described as 0-1-2-3-4-5-6-7-8) levels: 6 - 21 - 41 - 61 - 81 - 101 - 121 - 141 - 161 cm above the benchmark level with profile measurement method ABCDEFGHI. . . IHGFEDCBA (Barlik and Pachuta, 2007). The survey had been performed across the whole measurement range of the stand in order to determine vertical gravity

Figure 4. *Vertical gradient measurement at 156 field station*

gradient above measurement level of the FG5 (130 cm). Instrumental drift was determined linear as weighted mean. Measurements at 156 field point had been conducted on 6 levels (described as 1-2-3-4-5-6): 21 - 41 - 61 - 81 - 101 - 121 cm also with profile measurement method (Fig. 4). For practical reasons measuring on higher levels had not been performed as the measurement level of the A10 in field conditions is around 84 cm (Fig. 3 - right) and FG5 field measurements are rarely practiced. The survey had been carried out inside a tent to minimize wind effect. All mentioned above measurements are referred to as reference measurements.

5 Reference measurement results

Three LCR gravimeters used at each point provide three vertical gradient determinations. Results obtained, with drift and tides removed, were then fitted with linear and nonlinear model (2^{nd} order polynomial).

5.1 Linear approximation

Results of measurements on consecutive levels were first fitted with linear model:

$$g(h) = a \cdot h + b \tag{1}$$

where g(h) - gravimeter reading on measurement height h with respect to the benchmark, a, b - fit coefficients. Vertical gradients were determined using the following formula:

$$W_{zz}(h_i) = \frac{g(h_i) - g(h_0)}{h_i} = a \tag{2}$$

Table 1. *Linear approximations of vertical gravity gradient*

	2011		2000
	W_{zz} $[10^{-9} \cdot s^{-2}]$	$m_{W_{zz}}$ $[10^{-9} \cdot s^{-2}]$	W_{zz} $[10^{-9} \cdot s^{-2}]$
A-BG $_{G1012}$	2695	13	
A-BG $_{G1084}$	2692	20	2751
A-BG $_{G1078}$	2701	22	
BG-G2 $_{G1012}$	2778	15	
BG-G2 $_{G1084}$	2786	17	2817
BG-G2 $_{G1078}$	2821	9	
156 $_{G1012}$	3108	11	
156 $_{G1084}$	3115	13	2928
156 $_{G1078}$	3176	12	

where h_i is the height of reduction (measurement height) and h_0 is equal 0 as reference height. As change of gravity with height is calculated as linear, the vertical gravity gradient is a constant value. Results of the fit in terms of determined vertical gradients and fit error are presented in Table 1. On the right hand side of the table results from the year 2000 are presented. They were obtained from four measurements taken with two LCR gravimeters with the use of a standard tripod.

Vertical gravity gradients obtained from three determinations at every point agree with each other at the level of 50-60 E (5-6 μGal/m). Compared to results from the year 2000, values of gravity gradients at laboratory points are smaller by 4-5 μGal/m which results in 4 μGal (A10) and 6 μGal (FG5) differences in reduction of measured gravity to the level of the benchmark. Significant difference of around 200 E (20 μGal/m) at 156 field station could be caused by a gross error in height measurement (5-6 cm) during the survey in the year 2000. It is also clearly visible that vertical gravity gradient below ground level is significantly smaller than normal gravity gradient - its value at the field point is much closer to normal vertical gravity gradient value.

5.2 Nonlinear approximation

Results of measurements on consecutive levels were also fitted with a nonlinear model, i.e. 2^{nd} order polynomial:

$$g\left(h\right) = a \cdot h^2 + b \cdot h + c \tag{3}$$

where g(h) - gravimeter reading on measurement height h with respect to the benchmark, a, b, c - fit coefficients. Vertical gradients were determined

Table 2. *Nonlinear fit approximation coefficients*

	a	m_a	b	m_b	c	m_c	m_{fit} $[10^{-9} s^{-2}]$
A-BG $_{G1012}$	0.0029	0.0013	-0.2742	0.0022	-0.0161	0.0008	9
A-BG $_{G1084}$	-0.0006	0.0030	-0.2681	0.0053	-0.0128	0.0018	20
A-BG $_{G1078}$	-0.0032	0.0031	-0.2648	0.0054	-0.0138	0.0019	21
BG-G2 $_{G1012}$	-0.0000	0.0015	-0.2778	0.0026	-0.0162	0.0009	10
BG-G2 $_{G1084}$	0.0056	0.0018	-0.2879	0.0032	-0.0180	0.0011	12
BG-G2 $_{G1078}$	-0.0017	0.0011	-0.2792	0.0020	-0.0175	0.0007	8
156 $_{G1012}$	0.0067	0.0031	-0.3204	0.0045	-0.0665	0.0014	7
156 $_{G1084}$	-0.0025	0.0058	-0.3079	0.0044	-0.0640	0.0026	14
156 $_{G1078}$	-0.0090	0.0028	-0.3048	0.0041	-0.0638	0.0013	7

using the following formula:

$$W_{zz}\left(h_i\right) = \frac{g(h_i) - g(h_0)}{h_i} = a \cdot h_i + b \tag{4}$$

Results of the fit are presented in Table 2, showing all fit coefficients along with error of their determination and overall fit error. As change of gravity with height is calculated with 2^{nd} order polynomial, the calculated vertical gravity gradient changes linearly with height.

The calculated a coefficient determines the scale of the vertical gradient variation with height. Its negative sign indicates the increase of vertical gravity gradient while positive one - the decrease with height. Since vertical gravity gradient above ground level is bigger than below ground, it could be expected that it increases with height within the range of the stand at laboratory stations. Values of a coefficient for performed measurements have different signs. For each laboratory point, measurements with two out of three gravimeters show the increase of vertical gravity gradient value with height and agree with each other in terms of a coefficient determination error. But at the same time for A-BG station the value of the coefficient is of the same order of magnitude as the error of its determination therefore any conclusive interpretation is at this point impossible. As for the field station 156 values of a coefficient differ at a much bigger scale exceeding errors of determination.

Internal consistency of the results, confirmed by the overall fit error suggests an instrumental factor influencing the results, as both measurement and computation for each gravimeter followed the same strategy. Further tests with LCR gravimeters are required to further investigate differences obtained in vertical gradient determinations. Similar problems with vertical gravity gradient determinations using LCR gravimeters were

Table 3. *Vertical gravity gradient values on different heights calculated from nonlinear approximation* $[10^{-9} \cdot s^{-2}]$

Height	0.15 m (LCR)	0.72 m (A10 lab)	0.84 m (A10 field)	1.00 m	1.30 m (FG5)	linear fit
A-BG$_{G1012}$	2738	**2721**	2718	2713	**2704**	2695
A-BG$_{G1084}$	2682	**2685**	2686	2687	**2689**	2699
A-BG$_{G1078}$	2653	**2671**	2675	2680	**2690**	2701
average	2691	**2692**	2693	2693	**2694**	2698
BG-G2$_{G1012}$	2778	**2778**	2778	2778	**2778**	2778
BG-G2$_{G1084}$	2871	**2839**	2832	2823	**2806**	2786
BG-G2$_{G1078}$	2795	**2804**	2806	2809	**2814**	2821
average	2815	**2807**	2805	2803	**2799**	2795
156$_{G1012}$	3194	3156	**3148**	3137	-	3108
156$_{G1084}$	3083	3097	**3100**	3104	-	3115
156$_{G1078}$	3062	3113	**3123**	3138	-	3176
average	3113	3122	**3124**	3126	-	3133

addressed in literature (e.g. Csapó and Völgyesi, 2004). The aspect of agreement between multiple vertical gravity gradient determinations will be mentioned later in the paper.

Table 3 presents values of vertical gravity gradients calculated on typical heights of gravity measurements with LCR, A10 (lab and field) and FG5 gravimeters. Last column presents the value obtained from linear fit.

Bold and underscored numbers in Table 3 correspond to the most important values in terms of absolute measurement on the respective point. Variations of vertical gravity gradient values on each level agree with each other within 60 E (6 μGal/m) in absolute determination cases. What should be emphasized, however, is that linear fit values differ from nonlinear determinations as much as 100 E depending on the height. Figures 5, 6, and 7 present variations of vertical gravity gradient from nonlinear determinations with respect to linear determinations (from Table 3) at A-BG, BG-G2, and 156 station, respectively.

6 Optimisation of vertical gravity gradient measurement

The A10 gravimeter is capable of performing rapid absolute gravity determinations in laboratory and field conditions (Sekowski *et al.*, 2011). Each of those determinations requires knowledge about vertical gravity gradient of possibly high accuracy, but in the same time its determina-

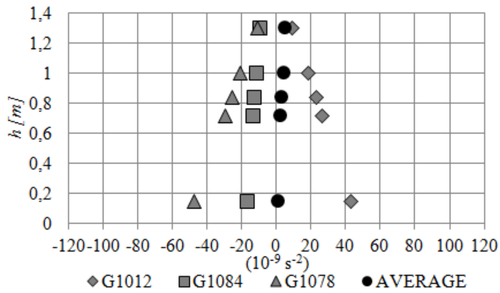

Figure 5. *Deviations of vertical gravity gradient values from linear determination at A-BG station*

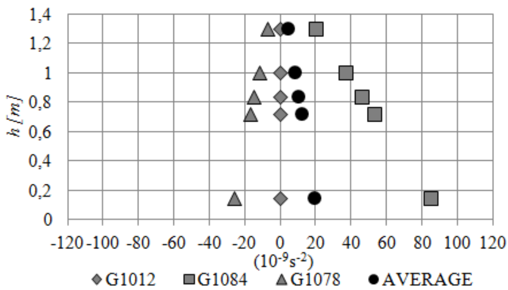

Figure 6. *Deviations of vertical gravity gradient values from linear determination at BG-G2 station*

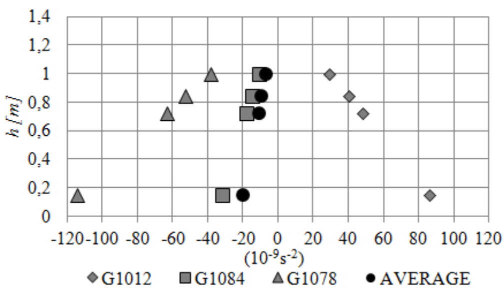

Figure 7. *Deviations of vertical gravity gradient values from linear determination at 156 station*

tion should be as efficient as possible. Efficiency of the vertical gravity gradient determination is especially important when the A10 is used for modernization of national gravity control where large number of points is to be measured.

Reference measurements of vertical gravity gradient performed at laboratory points show that linear and nonlinear fit is insensitive to removing some measurement levels from the determination of vertical gravity gradient, as long as they are removed symmetrically. Determinations at field station 156 seem to be more sensitive to removing measurement levels, probably due to less stable measurement conditions.

Proposed change to reference measurement strategy reads as follows. Symmetrical removal of 4 measurement levels for laboratory measurements (measured levels 0-2-4-6-8—8-6-4-2-0 where 0 level is the measurement without the stand on a benchmark level). As for 156 field station optimisation concerns the resignation of survey on 3 levels 6, 4 and 2 when going down with surveying procedure (measured levels 1-2-3-4-5-6—5-3-1 where 1 level is the lowest level measured with the stand). The tested optimised strategy reduces time of single gravimeter measurement from 2 h to 1.25 h for laboratory points and from 1.25 h to less than 1 h for the field station. At the same time this strategy does not significantly affect the results. Differences in reduction referred to reference measurements do not exceed 1.5 μGal.

Initial plan of vertical gravity gradient determination concerns using at least two gravimeters, therefore the criterion of mutual agreement of the results needs to be specified. The idea of such criterion reads as follows. In this study which is focused on the gravity survey with the A10 gravimeter, difference between reduction from two or more vertical gradient determinations, has been assumed not exceeding 3 μGal (this value corresponds to the standard deviation of gravity obtained with the A10-020 gravimeter at the laboratory stations (Sekowski *et al.*, 2011)). As referred to vertical gravity gradient value at any given height this criterion yields:

$$\Delta W_{zz} < 30/h[E] \tag{5}$$

where h is given in metres. Figure 8 presents limits of tolerance for the proposed criterion for the difference between vertical gravity gradient determinations. It shows that vertical gravity gradient value strongly depends on height of the measurement. The higher the measurement is performed, the more accurate the vertical gravity gradient needs to be determined.

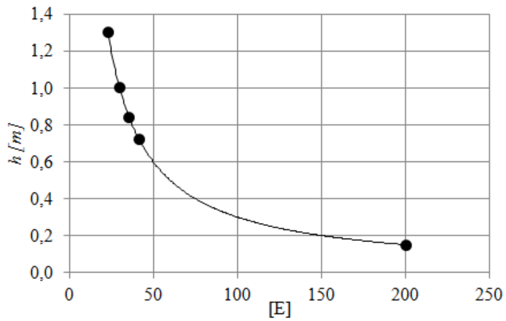

Figure 8. *Limit of tolerance for difference between vertical gravity gradient determinations with respect to height*

Further tests are planned to verify the value proposed in the criterion and to study instrumental issues affecting measurements. Test measurements will focus more on repeatability of gravimeters used for vertical gravity gradient determination.

7 Conclusions

The conducted experiments and the analysis of the results obtained allow to draw the following conclusions:

- The use of normal vertical gravity gradient in reduction of absolute gravity measurements can cause errors exceeding 100 μGal.
- Observed change of the vertical gradient with height may cause differences in reduction of gravity coming up to a few microgals.
- The use of at least two gravimeters for the determination of vertical gravity gradients allows to verify determinations with single gravimeter against each other as well as verify them in terms of nonlinear change of gravity with height.
- Result of a vertical gravity gradient determination should be given as constant value (linear change of gravity with height) and parameters for its calculation at arbitrary height (nonlinear change of gravity with height).
- The session of gravity measurement for vertical gravity gradient determination can be significantly shortened by reducing measurement levels. Caution is advised as not to cause decrease in accuracy of the gradient determined.

Acknowledgements

The research was done in the framework of the statutory project "Problems of geodesy and geodynamics" of the Institute of Geodesy and Cartography, Warsaw. The author would like to thank Prof. J. Krynski and Dr M. Sekowski for consulting the design and the use of the stand as well valuable discussions and suggestions in approaching the results. The stand had been constructed at the Institute of Geodesy and Cartography by M. Kolodziejczyk whose cooperation in the realization of the project is appreciated.

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