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EIGEN VALUE APPROACH TO TWO DIMENSIONAL PROBLEM IN GENERALIZED MAGNETO MICROPOLAR THERMOELASTIC MEDIUM WITH ROTATION EFFECT

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In this study an eigen value approach has been employed to examine the mechanical force applied along with a transverse magnetic field in a two dimensional generalized magneto micropolar thermoelastic infinite space. Results have been obtained by treating rotational velocity to be invariant. Integral transforms have been applied to solve the system of partial differential equations. Components of displacement, normal stress, tangential couple stress, temperature distribution, electric field and magnetic field have been obtained in the transformed domain. Finally numerical inversion technique has been used to invert the result in the physical domain. Graphical analysis has been done to described the study.

Key words: magneto micropolar, thermoelasticity, eigen value, elasticity, integral transforms.

1. Introduction

The micropolar theory of elasticity was developed with the possibilities of its wide-ranging practical applications in diverse fields such as geophysics, optics and acoustics and so on. Contemporary engineering materials are usually made up of constituents possessing internal structures. Some of the material in this category are polycrystalline materials, materials with fibrous or coarse grain. Classical elasticity is inadequate to represent the behaviour of such materials. An analysis of these type of materials requires a special theory "Micropolar Elasticity" developed by Eringen [1] which deals with deformation of oriented particles. Basically a micropolar continuum is a collection of interconnected particles in the form of small rigid bodies which can undergo both translational and rotational motions. Classic examples of such materials are granular media and multimolecular bodies, whose microstructure act as an evident part in their macroscopic responses.

The current area of study namely: magneto micropolar thermoelasticity is an extension of this theory. This theory deals with the effects of the magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, in addition to elastic and electro-magnetic fields, thermal field is also present. Each of these fields contributes to the total deformation of the body and interacts with each other. Maxwell's equations still govern the electro-magnetic field while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Due to superposition of the electromagnetic field

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on the elastic field, the elastic-stress relation gets modified with the introduction of Lorentz's force as body force and in turn the elastic field influences the electro-magnetic field by modifying Ohm's law.

Basic equations of magneto micro thermoelasticity were obtained by Kaliski [2]. Also, the wave type of the above equations were studied by Kaliski and Nowacki [3]. Paria [4], Knopoff [5], Banos [6], Chadwick [7], and Purushothama [8] contributed to magneto elasticity theories. Nowacki [9] studied a simplified two dimensional problem of magneto-micropolar elasticity. Ezzat and Youssef [10] investigated the problem of micropolar thermal elasticity in perfectly conducting media. Bakasi *et al*. [11] studied magneto thermal elastic problems with thermal relaxations and heat sources in a three dimensional infinite rotating elastic media. A problem of generalized magneto thermoelasticity in a conducting medium with variable material properties was also studied by Youssef [12]. The effect of rotation was analyzed by Kumar and Rupender [13] by using a two dimensional model in an electromagnetic micropolar generalized thermoelastic medium in the presence of a transverse magnetic field subjected to a mechanical force or thermal source and observed that the application of a thermal source is more significant than the mechanical force. Ezzat and Bary [14] compared the one-temperature theory with the two temperature theory in a generalized magneto thermoelastic medium in a perfectly conducting medium using the state space approach subjected to a thermal shock and traction-free surface and found that the two-temperature generalized theory describes the behavior of the particles of an elastic body more accurately than the one-temperature theory. Ezzat and Awad [15] introduced the modified Ohm's law, including the temperature gradient and charge density effects, and the generalized Fourier's law including current density effect to the equations of the linear theory of micropolar generalized magneto thermoelasticity. A normal mode analysis is used to obtain the solution. He and Cao [16] used the generalized thermoelastic theory with thermal relaxation in the context of L-S theory to investigate the magneto thermoelastic problem of a thin slim strip placed in a magnetic field and subjected to a moving plane of heat source and found that the magnetic field significantly influences the variations of non-dimensional displacement and stress but has no effect on the non-dimensional temperature. Singh and Kumar [17] studied the interaction of the electromagnetic field with the elastic field in the presence of temperature by applying the mechanical force and thermal source by using modified Fourier and Ohm's law.

Increasing attention is devoted to the interaction between magnetic fields and strain in a micropolar thermoelastic solid due to many applications in the fields of geophysics, plasma physics and related areas. The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It may also find application in various engineering problems, crystal physics and solid-earth geophysics. The present study can be regarded as a better representation of the elastic model for studying the earth's planetary motion as it involves rotational velocity in addition to its thermal and electromagnetic field. The scope of the present study is to examine the interaction in the magneto micropolar thermoelastic material due to a mechanical source.

2. Basic equations

Following Baksi *et al*. [11], the linear equations of electrodynamics of a slowly moving medium for a homogenous and perfectly conducting elastic solid in the simplified form along with field equations of motion and constitutive relations in the theory of micropolar generalized thermoelasticity, taking into account the Lorentz force are given by Eqs (2.1)-(2.9)

$$\text{Curl}\,\boldsymbol{h} = \text{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}, \tag{2.1}$$

$$\text{Curl}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{h}}{\partial t}, \tag{2.2}$$

$$\boldsymbol{E} = -\mu_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}_o \right), \tag{2.3}$$

$$\operatorname{div}\boldsymbol{h}=0\,, \tag{2.4}$$

$$\begin{gathered}(\lambda+2\mu+K)\nabla(\nabla.\boldsymbol{u})-(\mu+K)\nabla\times(\nabla\times\boldsymbol{u})+K(\nabla\times\phi)+\boldsymbol{F}-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T=\\=\rho\left[\frac{\partial^2\boldsymbol{u}}{\partial t^2}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})+2\boldsymbol{\Omega}\times\frac{\partial\boldsymbol{u}}{\partial t}\right],\end{gathered} \tag{2.5}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\phi)-\gamma\nabla\times(\nabla\times\phi)+K(\nabla\times\boldsymbol{u})-2K\phi=\rho j\left[\frac{\partial^2\phi}{\partial t^2}+\boldsymbol{\Omega}\times\frac{\partial\phi}{\partial t}\right], \tag{2.6}$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_o\left(\frac{\partial}{\partial t}+\tau_o\eta_o\frac{\partial^2}{\partial t^2}\right)(\nabla.\boldsymbol{u})\,, \tag{2.7}$$

$$\sigma_{ij}=\lambda u_{k,k}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\in_{ijk}\phi_k\right)-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}\,, \tag{2.8}$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i} \tag{2.9}$$

where $\upsilon=(3\lambda+2\mu+K)\alpha_t$ and $\boldsymbol{F}=\mu_0(\boldsymbol{J}\times\boldsymbol{H}_o)$, (2.10)

$\boldsymbol{H}_o$ is the external applied magnetic field intensity vector, $\boldsymbol{h}$ the induced magnetic field vector, $\boldsymbol{E}$ the induced electric field vector, $\boldsymbol{J}$ the current density vector, $\boldsymbol{u}$ the displacement vector, μ_0 and $\in_0$ the magnetic and electric permeabilities, respectively, and δ_{ij} – the Kroneker delta.

3. Formulation and solution of the problem

We consider a homogenous, isotropic, perfectly conducting micropolar generalized thermoelastic medium, permeated by an initial magnetic field $\boldsymbol{H}_o$ acting along the x_2 -axis.

For a two dimensional problem we take the displacement vector $\boldsymbol{u}$, rotation vector $\boldsymbol{\Omega}$ and microrotation vector ϕ as (by assuming $\boldsymbol{\Omega}$ to be invariant)

$$\begin{gathered}\boldsymbol{u}=(u_1,0,u_3),\quad \phi=(0,\phi_2,0),\quad \boldsymbol{\Omega}=(0,\Omega_2,0),\quad \boldsymbol{E}=(E_1,0,E_3),\\ \boldsymbol{h}=(0,h,0),\quad \boldsymbol{H}_0=(0,H_{02},0).\end{gathered} \tag{3.1}$$

Using expressions mentioned in Eq.(3.1) in Eqs (2.1)–(2.4), (2.8)-(2.9) we get

$$\begin{gathered}(\lambda+2\mu+K)\frac{\partial^2u_1}{\partial x_1^2}+(\lambda+\mu)\frac{\partial^2u_3}{\partial x_1\partial x_3}+(\mu+K)\frac{\partial^2u_1}{\partial x_3^2}-K\frac{\partial\phi_2}{\partial x_3}+\\-\mu_0H_{02}J_3-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\rho\left[\frac{\partial^2u_1}{\partial t^2}-3\Omega_2^2u_1\right],\end{gathered} \tag{3.2}$$

$$\left(\lambda+\mu\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\left(\lambda+2\mu+K\right)\frac{\partial^2 u_3}{\partial x_3^2}+\left(\mu+K\right)\frac{\partial^2 u_3}{\partial x_1^2}+K\frac{\partial\phi_2}{\partial x_3}+$$
$$+\mu_0 H_0 J_1-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\rho\left[\frac{\partial^2 u_3}{\partial t^2}-3\Omega_2^2 u_3\right], \tag{3.3}$$

$$-\gamma\left(\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_3^2}\right)+K\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2K\phi_2=\rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{3.4}$$

$$K^*\nabla^2 T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_o\left(\frac{\partial}{\partial t}+\tau_o\eta_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right), \tag{3.5}$$

$$\sigma_{31}=\mu\frac{\partial u_3}{\partial x_1}+\left(\mu+K\right)\frac{\partial u_1}{\partial x_3}+K\phi_2, \tag{3.6}$$

$$\sigma_{33}=\lambda\frac{\partial u_1}{\partial x_1}+\left(\lambda+2\mu+K\right)\frac{\partial u_3}{\partial x_3}-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3.7}$$

$$m_{32}=\gamma\frac{\partial\phi_2}{\partial x_3}, \tag{3.8}$$

$$E_1=\mu_0 H_{02}\frac{\partial u_3}{\partial t}, \tag{3.9}$$

$$E_3=-\mu_0 H_{02}\frac{\partial u_1}{\partial t}, \tag{3.10}$$

$$h=-H_{02}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right). \tag{3.11}$$

We define the dimensionless quantities as

$$x_i^*=\frac{\bar\omega}{c_1}x_i,\quad u_i^*=\frac{\rho c_1\bar\omega}{\nu T_0}u_i,\quad t^*=\bar\omega t,\quad \tau_0^*=\bar\omega\tau_0,\quad \tau_1^*=\bar\omega\tau_1,$$

$$J_i^*=\frac{\eta_0}{\sigma_0^2\mu_0^2 H_{02}c_0}J_i,\quad h^*=\frac{\eta_0}{\sigma_0\mu_0 H_{02}}h,\quad \sigma_{ij}^*=\frac{1}{\nu T_0}\sigma_{ij},\quad m_{ij}^*=\frac{\bar\omega}{c_1\nu T_0}m_{ij}, \tag{3.12}$$

$$E_i^*=\frac{E_i}{\mu_0 H_{02}c_1},\quad \Omega_2^*=\frac{\Omega_2}{\bar\omega},\quad T^*=\frac{\nu T}{\rho c_0^2},\quad \phi_2^*=\frac{\rho c_1^2\bar\omega}{\nu T_0}\phi_2,\quad \text{for}\quad i=1.3$$

where

$$c_1^2 = \frac{\lambda + 2\mu + \kappa}{\rho} \qquad \text{and} \qquad \bar{\omega} = \frac{\rho c^* c_1^2}{K} .$$

Using the dimensionless quantities as defined in Eq.(3.12), the system of Eqs (3.2)-(3.11) after suppressing the asterisks can be rewritten as

$$\begin{aligned}&(\alpha_1 + \alpha_5)\frac{\partial^2 u_1}{\partial x_1^2} + (\alpha_2 + \alpha_5)\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \alpha_3 \frac{\partial^2 u_1}{\partial x_3^2} - \alpha_4 \frac{\partial \phi_2}{\partial x_3} + \\ &-\alpha_7\left(1 + \tau_1 \bar{\omega}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = (\alpha_6 + \alpha_7)\frac{\partial^2 u_1}{\partial t^2} - \alpha_9 u_1,\end{aligned} \tag{3.13}$$

$$\begin{aligned}&\alpha_3 \frac{\partial^2 u_3}{\partial x_1^2} + (\alpha_2 + \alpha_5)\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (\alpha_1 + \alpha_5)\frac{\partial^2 u_3}{\partial x_3^2} + \alpha_4 \frac{\partial \phi_2}{\partial x_1} + \\ &-\alpha_7\left(1 + \tau_1 \bar{\omega}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = (\alpha_6 + \alpha_7)\frac{\partial^2 u_3}{\partial t^2} - \alpha_9 u_3,\end{aligned} \tag{3.14}$$

$$-\alpha_{11}\left(\frac{\partial^2 \phi_2}{\partial x_1^2} + \frac{\partial^2 \phi_2}{\partial x_3^2}\right) + \alpha_{12}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - \alpha_{13}\phi_2 = \alpha_{14}\frac{\partial^2 \phi_2}{\partial t^2} , \tag{3.15}$$

$$\alpha_{15}\nabla^2 T = \alpha_{16}\left(\frac{\partial}{\partial t} + \tau_o \bar{\omega}\frac{\partial^2}{\partial t^2}\right)T + \alpha_{17}\left(\frac{\partial}{\partial t} + \tau_o \eta_o \bar{\omega}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right), \tag{3.16}$$

$$\sigma_{31} = \alpha_{20}\frac{\partial u_3}{\partial x_1} + \alpha_{21}\frac{\partial u_1}{\partial x_3} + \alpha_{22}\phi_2 , \tag{3.17}$$

$$\sigma_{33} = \alpha_{18}\frac{\partial u_1}{\partial x_1} + \alpha_{19}\frac{\partial u_3}{\partial x_3} - \alpha_{20} T , \tag{3.18}$$

$$m_{32} = \alpha_{23}\frac{\partial \phi_2}{\partial x_3} , \tag{3.19}$$

$$E_1 = \alpha_{24}\frac{\partial u_3}{\partial t} , \tag{3.20}$$

$$E_3 = -\alpha_{24}\frac{\partial u_1}{\partial t} , \tag{3.21}$$

$$h = -H_{02}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) \tag{3.22}$$

where

$$\alpha_1=\frac{(\lambda+2\mu+K)}{\rho c_1^3}\nu T_0\bar{\omega},\quad \alpha_2=\frac{(\lambda+\mu)}{\rho c_1^3}\nu T_0\bar{\omega},\quad \alpha_3=\frac{(\mu+K)}{\rho c_1^3}\nu T_0\bar{\omega},$$

$$\alpha_4=\frac{K\nu T_0\bar{\omega}}{\rho c_1^3},\quad \alpha_5=\frac{\mu_0 H_{02}^2\nu T_0\bar{\omega}}{\rho c_1^3},\quad \alpha_6=\frac{\varepsilon_0\mu_0^2 H_{02}^2\nu T_0\bar{\omega}}{\rho c_1},\quad \alpha_7=\frac{\nu T_0\bar{\omega}}{c_1},$$

$$\alpha_8=\frac{\nu T_0\bar{\omega}}{c_1},\quad \alpha_9=\frac{3\Omega_2^2 K\nu T_0\bar{\omega}}{\rho c_1^3},\quad \alpha_{11}=\frac{\gamma\nu T_0\bar{\omega}^2}{\rho c_1^4},\quad \alpha_{12}=\frac{K\nu T_0}{\rho c_1^2},\tag{3.23}$$

$$\alpha_{13}=\frac{2K\nu T_0}{\rho c_1^2},\quad \alpha_{14}=\frac{j\nu T_0\bar{\omega}^2}{c_1^2},\quad \alpha_{15}=\frac{K^* T_0\bar{\omega}^2}{c_1^2},\quad \alpha_{16}=\rho\bar{\omega}T_0,\quad \alpha_{17}=\frac{\nu^2 T_0^2\bar{\omega}}{\rho c_1^2},$$

$$\alpha_{18}=\frac{\lambda}{\rho c_1^2},\quad \alpha_{19}=\frac{(\lambda+2\mu+K)}{\rho c_1^2},\quad \alpha_{20}=\frac{\mu}{\rho c_1^2},\quad \alpha_{21}=\frac{\mu+K}{\rho c_1^2},\quad \alpha_{22}=\frac{K}{\rho c_1^2},$$

$$\alpha_{23}=\frac{\gamma\bar{\omega}^2}{\rho c_1^4},\quad \alpha_{24}=\frac{\nu T_0}{\rho c_1^2},\quad \alpha_{25}=\frac{\nu T_0}{\rho c_1^2\bar{\omega}}.$$

We take the Laplace and Fourier transform as

$$L\{f(x_1,x_3,t)\}=\int_0^\infty e^{-st}f(x_1,x_3,t)\,dt=\bar{f}(x_1,x_3,s),\tag{3.24}$$

$$F\{\bar{f}(x_1,x_3,s)\}=\int_{-\infty}^{\infty} e^{-ix_1\xi}f(x_1,x_3,t)\,dt=\tilde{f}(\xi,x_3,t)\tag{3.25}$$

After applying the transformation as defined in Eqs (3.24)-(3.25) on Eqs (3.13)–(3.16), we get

$$D^2\tilde{u}_1=\frac{1}{\alpha_3}\Big[\Big\{(\alpha_1+\alpha_5)\xi^2+(\alpha_6+\alpha_8)s^2-\alpha_9\Big\}\tilde{u}_1-i\xi(\alpha_2+\alpha_5)D\tilde{u}_3+ \\ +\alpha_4 D\tilde{\phi}_2-i\xi\alpha_7(1+\bar{\omega}\tau_1)\tilde{T}\Big],\tag{3.26}$$

$$D^2\tilde{u}_3=\frac{1}{\alpha_1+\alpha_5}\Big[-i\xi(\alpha_2+\alpha_5)D\tilde{u}_1+\Big\{\alpha_3\xi^2+(\alpha_6+\alpha_8)s^2-\alpha_9\Big\}\tilde{u}_3+ \\ -i\xi\alpha_4\tilde{\phi}_2+\alpha_7(1+\bar{\omega}\tau_1 s)D\tilde{T}\Big],\tag{3.27}$$

$$D^2\tilde{\phi}_2=\frac{1}{\alpha_{11}}\Big[\alpha_{12}(D\tilde{u}_1-i\xi\tilde{u}_3)+\Big(\alpha_{11}\xi^2-\alpha_{13}-\alpha_{14}s^2\Big)\tilde{\phi}_2\Big],\tag{3.28}$$

$$D^2\tilde{T} = \frac{1}{\alpha_{15}}\left[i\xi\alpha_{17}\left(s+\tau_0\eta_0\bar{\omega}s^2\right)\tilde{u}_1 + \alpha_{17}\left(s+\tau_0\eta_0\bar{\omega}s^2\right)D\tilde{u}_3 + \left(\alpha_{15}\xi^2 + s + \bar{\omega}\tau_0 s^2\right)\tilde{T}\right], \quad (3.29)$$

$$\tilde{\sigma}_{31} = if_{52}\tilde{u}_3 + \alpha_{21}D\tilde{u}_1 + \alpha_{22}\tilde{\phi}_2, \quad (3.30)$$

$$\tilde{\sigma}_{33} = if_{51}\tilde{u}_1 + \alpha_{19}D\tilde{u}_3 - \alpha_{20}\tilde{T}, \quad (3.31)$$

$$\tilde{m}_{32} = \alpha_{23}D\tilde{\phi}_2, \quad (3.32)$$

$$\tilde{E}_1 = f_{53}\tilde{u}_3, \quad (3.33)$$

$$\tilde{E}_3 = -f_{53}\tilde{u}_1, \quad (3.34)$$

$$\tilde{h} = -if_{54}\tilde{u}_1 - \alpha_{25}D\tilde{u}_3 \quad (3.35)$$

where $D = \frac{d}{dz}$.

Equations (3.26)-(3.29) can be written in a matrix form as

$$DW(\xi, x_3, s) = AW(\xi, x_3, s) \quad (3.36)$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \qquad U = \begin{bmatrix} \tilde{u}_1 & \tilde{u}_3 & \tilde{\phi}_2 & \tilde{T} \end{bmatrix}', \quad (3.37)$$

$$A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \; A_1 = \begin{bmatrix} 0 & -ig_{12} & g_{13} & 0 \\ ig_{21} & 0 & 0 & g_{24} \\ g31 & 0 & 0 & 0 \\ 0 & g_{42} & 0 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} f_{11} & 0 & 0 & -if_{14} \\ 0 & f_{22} & if_{23} & 0 \\ 0 & -if_{32} & f_{33} & 0 \\ if_{41} & 0 & 0 & f_{44} \end{bmatrix}, \quad (3.38)$$

I is identity matrix of order 4, O is null matrix of order of 4 and $[\;]'$ is the transpose of matrix.
Where

$$f_{11} = \frac{1}{\alpha_3}\left[(\alpha_1+\alpha_5)\xi^2 + (\alpha_6+\alpha_8)s^2 - \alpha_9\right], \qquad f_{14} = \frac{\xi\alpha_7}{\alpha_3}(1+\tilde{\omega}\tau_1 s),$$

$$f_{22} = \frac{\alpha_3\xi^2 + \alpha_6 s^2 + \alpha_8 s^2 - \alpha_9}{\alpha_1+\alpha_5}, \qquad f_{23} = -\frac{\alpha_4\xi}{\alpha_1+\alpha_5}, \qquad f_{32} = \frac{\xi\alpha_{12}}{\alpha_{11}},$$

$$f_{33} = \frac{\alpha_{11}\xi^2 - \alpha_{14}s^2 - \alpha_{13}}{\alpha_{11}}, \qquad f_{41} = -\frac{\xi\alpha_{17}}{\alpha_{15}}\left(s+\tau_0\eta_0\bar{\omega}s^2\right), \quad (2.39)$$

$$f_{44}=\frac{1}{\alpha_{15}}\left(\alpha_{15}\xi^2+s+\tau_0\bar{\omega}s^2\right),\qquad g_{12}=-\frac{\xi\left(\alpha_2+\alpha_5\right)}{\alpha_3},\qquad g_{13}=\frac{\alpha_4}{\alpha_3},$$

$$g_{21}=-\frac{\xi\left(\alpha_5+\alpha_2\right)}{\alpha_1+\alpha_5},\qquad g_{24}=\frac{\alpha_7}{\alpha_1+\alpha_5}(1+\tau_1\bar{\omega}s),\qquad g_{31}=\frac{\alpha_{12}}{\alpha_{11}},\ g_{42}=\frac{\alpha_{17}}{\alpha_{15}}\left(s+\tau_0\eta_0\bar{\omega}s^2\right).$$

To solve the above equation, we take $W\left(\xi,x_3,s\right)=X\left(\xi,s\right)e^{qx_3}$, for some parameter q.

Using this value in Eq.(3.36), we get

$$AW(\xi,x_3,s)=qW(\xi,x_3,s)\,, \tag{3.40}$$

which leads to the eigen value problem.

A characteristic equation corresponding to the matrix A is given as

$$\left|A-qI\right|=0\,, \tag{3.41}$$

which on expansion gives

$$q^8-\lambda_1q^6+\lambda_2q^4-\lambda_3q^2+\lambda_4=0 \tag{3.42}$$

where

$$\lambda_1=g_{24}g_4+f_{11}+f_{44}+f_{22}+f_{33}-g_{12}g_{21}+g_{13}g_{31}\,,$$

$$\begin{aligned}\lambda_2=&-f_{41}g_{12}g_{24}+f_{14}f_{41}+g_{13}g_{24}g_{31}g_{42}+f_{33}g_{24}g_{42}+f_{11}g_{42}-f_{14}f_{21}g_{42}+f_{11}f_{44}+\\&+f_{11f_{22}}+f_{11}f_{33}+f_{22}f_{44}+f_{33}f_{44}+f_{22}f_{33}+f_{23}f_{32}-f_{44}g_{12}g_{21}-f_{33}g_{12}g_{21}+\\&+f_{23}g_{12}g_{31}+f_{44}g_{13}g_{31}+f_{32}g_{13}g_{21}+f_{22}g_{13}g_{31},\end{aligned}$$

$$\begin{aligned}\lambda_3=&-f_{32}f_{41}g_{13}g_{24}-f_{33}f_{41}g_{12}g_{24}+f_{14}f_{33}f_{41}+f_{14}f_{22}f_{41}+f_{14}f_{23}g_{31}g_{42}+\\&+f_{11}f_{33}g_{42}-f_{14}f_{21}f_{33}g_{42}+f_{22}f_{11}f_{44}+f_{11}f_{33}f_{44}+f_{11}f_{22}f_{33}+\\&+f_{22}f_{33}f_{44}+f_{23}f_{32}f_{44}-f_{33}f_{44}g_{12}g_{21}+f_{23}f_{44}g_{12}g_{31}+f_{32}f_{44}g_{13}g_{21}+f_{22}f_{44}g_{13}g_{31},\end{aligned}$$

$$\lambda_4=f_{14}f_{23}f_{32}f_{41}+f_{14}f_{33}f_{41}f_{22}+f_{11}f_{22}f_{33}f_{44}+f_{11}f_{23}f_{32}f_{44}\,. \tag{3.43}$$

The eigen values of matrix A are the characteristic roots of Eq.(3.42). The eigen vectors $X(\xi,s)$ corresponding to eigen value q_p can be determined by solving the homogenous equations

$$\left[A-qI\right]X(\xi,s)=0\,, \tag{3.44}$$

which gives

$$X_p(\xi,s)=\begin{bmatrix}X_{p1}\\X_{p2}\end{bmatrix},\quad X_{p1}=\begin{bmatrix}a_pq_p\\b_p\\c_pq_p\\d_p\end{bmatrix},\quad X_{p2}=q_pX_{p1}\quad \text{for}\quad q=q_p,\quad p=1,2,3,4,$$

and $$X_j(\xi,s)=\begin{bmatrix}X_{j1}\\X_{j2}\end{bmatrix},\ X_{j1}=\begin{bmatrix}-a_pq_p\\b_p\\-c_pq_p\\d_p\end{bmatrix},\ X_{p2}=q_pX_{p1},\ \text{for } j=p+4,\quad q=-q_p,\quad p=1,2,3,4 \tag{3.45}$$

where

$$a_p=\left[\left\{-g_{12}\left(f_{44}-q_p^2\right)+if_{44}g_{42}\right\}\left(f_{33}-q_p^2\right)-g_{13}g_{31}q_p^2\left(f_{44}-q_p^2\right)\right],$$

$$b_p=\left[\left\{\left(f_{11}-q_s^2\right)\left(f_{44}-q_p^2\right)+if_{14}f_{41}\right\}\left(f_{23}-q_p^2\right)-g_{13}g_{31}q_p^2\left(f_{44}-q_p^2\right)\right],$$

$$c_p=\left[\left\{\left(f_{11}-q_s^2\right)\left(f_{44}-q_p^2\right)+f_{14}f_{41}\right\}f_{32}+g_{31}q_p^2\left\{-g_{12}\left(f_{44}-q_p^2\right)+if_{14}g_{42}\right\}\right],$$

$$d_p=-\frac{i\left(f_{41}a_p+g_{42}b_p\right)}{f_{44}-q_p^2}. \tag{3.46}$$

Thus a solution of Eq.(3.40) becomes

$$W(\xi,s)=\sum_{p=1}^{4}\left[B_pX_p(\xi,s)e^{q_px_3}+B_{p+4}X_{p+4}(\xi,s)e^{-q_px_3}\right] \tag{3.47}$$

where $B_i's$ are eight arbitrary constants.

Now after using Eqs (3.26)-(3.35), (3.37) and (3.47), we obtain values of $\tilde{u}_1,\tilde{u}_3,\tilde{\phi}_2,\tilde{T}$, $\tilde{\sigma}_{31}$, $\tilde{\sigma}_{33}$, $\tilde{m}_{32},\tilde{E}_1,\tilde{E}_3$ and $\tilde{h}$ as

$$\tilde{u}_1=\sum_{p=1}^{4}\left[a_pq_pB_pe^{q_px_3}-a_pq_pB_{p+4}e^{-q_px_3}\right], \tag{3.48}$$

$$\tilde{u}_3=\sum_{p=1}^{4}\left[b_pB_pe^{q_px_3}+b_pB_{p+4}e^{-q_px_3}\right], \tag{3.49}$$

$$\tilde{\phi}_2=-\sum_{p=1}^{4}\left[c_pB_pe^{q_px_3}+c_pB_{p+4}e^{-q_px_3}\right], \tag{3.50}$$

$$\tilde{T}=\sum_{p=1}^{4}\left[d_pB_pe^{q_px_3}+d_pB_{p+4}e^{-q_px_3}\right], \tag{3.51}$$

$$\tilde{\sigma}_{33}=\sum_{p=1}^{4}\Big[\left(ia_pq_pf_{51}+\alpha_{19}b_pq_p-\alpha_{20}d_p\right)B_pe^{q_px_3}+ \\ +\left(-if_{51}a_pq_p-\alpha_{19}b_pq_p-\alpha_{20}d_p\right)B_{p+4}e^{-q_px_3}\Big], \tag{3.52}$$

$$\tilde{\sigma}_{31}=\sum_{p=1}^{4}\Big[\left(if_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p\right)B_pe^{q_px_3}+ \\ +\left(if_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p\right)B_{p+4}e^{-q_px_3}\Big], \tag{3.53}$$

$$\tilde{m}_{32}=-\alpha_{23}\sum_{p=1}^{4}\left[b_pc_pq_pB_pe^{q_px_3}-c_pq_pB_{p+4}e^{-q_px_3}\right], \tag{3.54}$$

$$\tilde{E}_1=f_{53}\sum_{p=1}^{4}\left[b_pB_pe^{q_px_3}+b_pB_{p+4}e^{-q_px_3}\right], \tag{3.55}$$

$$\tilde{E}_3=-f_{53}\sum_{p=1}^{4}\left[a_pq_pB_pe^{q_px_3}-a_pq_pB_{p+4}e^{-q_px_3}\right], \tag{3.56}$$

$$\tilde{h}=-if_{54}\sum_{p=1}^{4}\Big[\left(-ia_pq_pf_{54}+\alpha_{25}b_pq_p\right)B_pe^{q_px_3}+ \\ +\left(if_{54}a_pq_p+\alpha_{25}b_p-q_p\right)B_{p+4}e^{-q_px_3}\Big]. \tag{3.57}$$

4. Boundary conditions

We consider an infinite micropolar elastic space in which a concentrated force $F=-P_0\delta(x_1)\delta(t)$ where P_0 is the magnitude of the force, acting in the direction of the x_3 -axis at the origin. The boundary conditions for the present problem on the plane $x_3=0$ are

$$u_1\left(x_1,0^+,t\right)-u_1\left(x_1,0^-,t\right)=0, \tag{4.1}$$

$$u_3\left(x_1,0^+,t\right)-u_3\left(x_1,0^-,t\right)=0, \tag{4.2}$$

$$\phi_2\left(x_1,0^+,t\right)-\phi_2\left(x_1,0^-,t\right)=0\,, \tag{4.3}$$

$$T\left(x_1,0^+,t\right)-T\left(x_1,0^-,t\right)=0\,, \tag{4.4}$$

$$\frac{\partial T}{\partial x_3}\left(x_1,0^+,t\right)-\frac{\partial T}{\partial x_3}\left(x_1,0^-,t\right)=0\,, \tag{4.5}$$

$$\sigma_{31}\left(x_1,0^+,t\right)-\sigma_{31}\left(x_1,0^-,t\right)=0\,, \tag{4.6}$$

$$\sigma_{33}\left(x_1,0^+,t\right)-\sigma_{33}\left(x_1,0^-,t\right)=-P_0\delta\left(x_1\right)\delta\left(t\right), \tag{4.7}$$

$$m_{32}\left(x_1,0^+,t\right)-m_{32}\left(x_1,0^-,t\right)=0\,. \tag{4.8}$$

After solving Eqs (4.1)-(4.8), we get

$$B_1=B_5=\frac{P_0c_4}{l_{11}q_1}\left[\frac{l_{12}l_{23}-l_{13}l_{22}}{l_{22}l_{33}-l_{32}l_{23}}\right], \tag{4.9}$$

$$B_2=B_6=-\frac{P_0l_{23}c_4}{q_2\left(l_{22}l_{33}-l_{32}l_{23}\right)}\,, \tag{4.10}$$

$$B_3=B_7=\frac{P_0l_{22}c_4}{q_3\left(l_{22}l_{33}-l_{32}l_{23}\right)}, \tag{4.11}$$

$$B_4=B_8=-\frac{P_0c_4\left[a_1\left(l_{12}l_{23}-l_{13}l_{22}\right)-a_2l_{11}l_{23}+a_3l_{22}l_{11}\right]}{a_4q_4\left(l_{22}l_{33}-l_{32}l_{23}\right)}\,. \tag{4.12}$$

Using these values of $B_i{}'s$ in Eqs (3.48)-(3.57), we obtain transformed components of displacement, microrotation, temperature distribution, tangential and normal stress, induced electric field and magnetic field, where

$$l_{11}=a_2c_4-c_1a_4\,,\qquad l_{12}=a_2c_4-a_4c_2\,,\qquad l_{13}=a_3c_4-a_4c_3\,,$$

$$l_{22}=\left(d_2c_4-c_2d_4\right)-\frac{\left(d_1c_4-c_1d_4\right)}{\left(a_1c_4-c_1a_4\right)}\left(a_2c_4-c_2a_4\right)\;,$$

$$l_{23}=\left(d_3c_4-c_3d_4\right)-\frac{\left(d_1c_4-c_1d_4\right)}{\left(a_1c_4-c_1a_4\right)}\left(a_3c_4-c_3a_4\right)\;, \tag{4.13}$$

$$l_{32}=\left(b_2c_4-c_2b_4\right)-\frac{\left(b_1c_4-c_1b_4\right)}{\left(a_1c_4-c_1a_4\right)}\left(a_2c_4-c_2a_4\right)\;,$$

$$l_{33}=\left(b_3c_4-c_3b_4\right)-\frac{\left(b_1c_4-c_1b_4\right)}{\left(a_1c_4-c_1a_4\right)}\left(a_3c_4-c_3a_4\right).$$

5. Inversion of the transforms

The transformed components of displacement, microrotation, temperature distribution, tangential and normal stress, couple stress, induced electric field and magnetic field are dependent on x_3, s and ξ. To obtain them in the physical domain in the form of $f(x_1, x_3, t)$, we invert integral transforms by using the inversion technique as used by Singh *et al.* [18].

6. Numerical result and discussion

Following Eringen [19], we take the following values of relevant parameters for the case of magnesium crystal as

$$\lambda = 9.4\times 10^{10}\, N/m^2, \quad \mu = 4\times 10^{10}\, N/m^2, \quad K = 1\times 10^{10}\, N/m^2, \quad \rho = 1.74\times 10^{3}\, kg/m^3,$$

$$x_3 = 1, \quad j = 0.2\times 10^{-19}\, m^2, \quad K^* = 1.1753\times 10^{-19}\, m^2, \quad \omega^* = 0.0787\times 10^{-1}\, N\,\mathrm{sec}/m^2,$$

$$\tau_0 = 6.131\times 10^{-13} s, \quad \tau_1 = 8.765\times 10^{-13} s, \quad \varepsilon = 0.073, \quad T_0 = 296\, K,$$

$$\alpha_0 = 0.779\times 10^{-9}\, N, \quad \epsilon_0 = \frac{1}{36\pi}\times 10^{-9}\, Fm^{-1}, \quad \mu_0 = 4\pi\times 10^{-7}\, Hm^{-1}, \quad \Omega = 1 .$$

The computations are carried out for the non-dimensional time $t=1/2$ and range $0 \le x_1 \le 9$. The distribution of non-dimensional normal displacement u_3, non-dimensional normal stress σ_{33}, non-dimensional tangential couple stress m_{32} and non-dimensional temperature distribution T with non-dimensional distance x_1 have been shown in Figs 1–4.

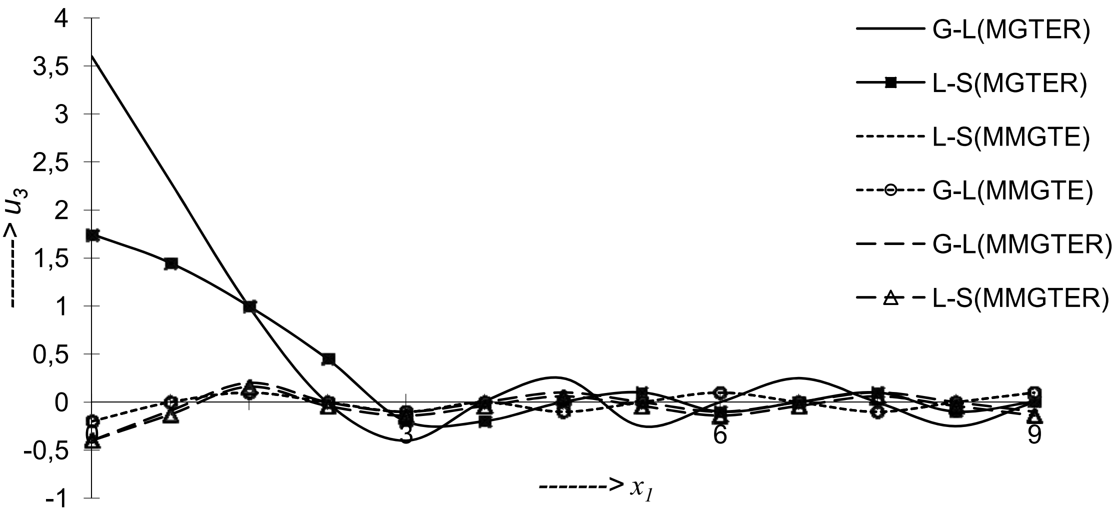

Fig.1. Variation in normal displacement u_3.

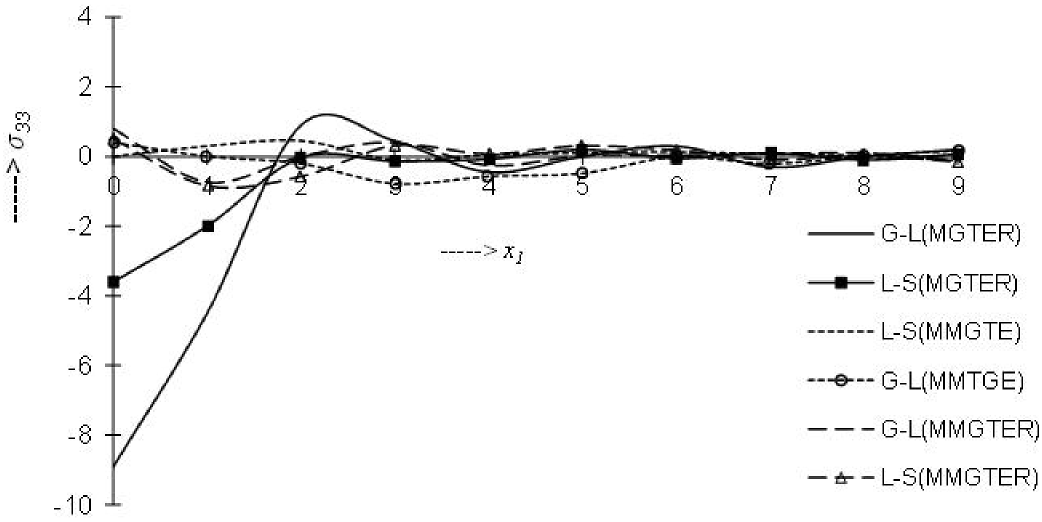

Fig.2. Variation in normal force stress σ_{33} .

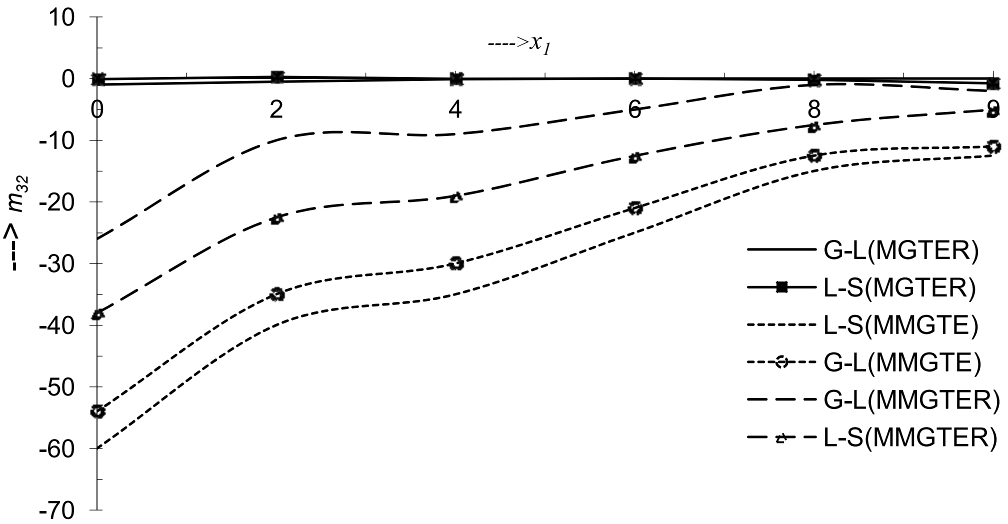

Fig.3. Variation in tangential couple stress m_{32} .

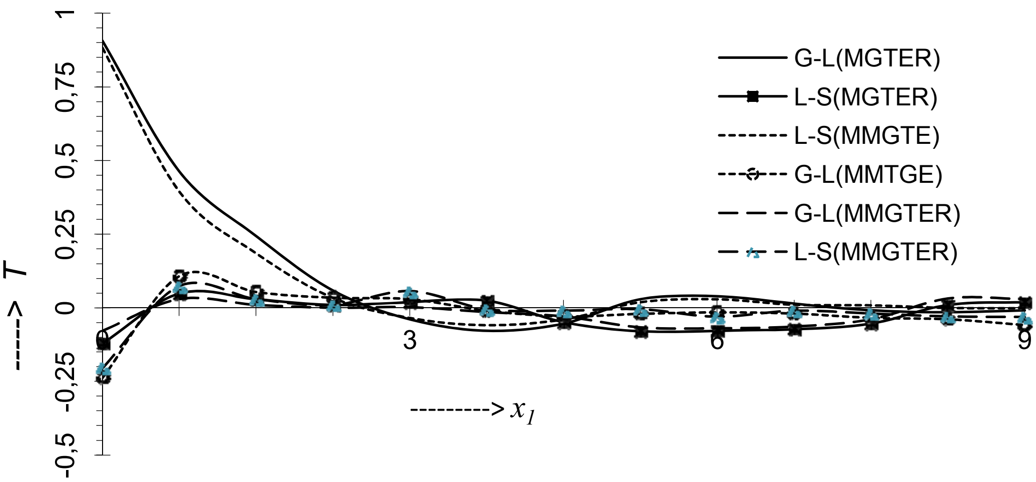

Fig.4. Variation in temperature field T.

The solid line and solid line with solid square represent a micropolar generalized thermoelastic medium, for Green and Lindsay [20] theory as G-L (MGTER) and for Lord and Shulman [21] theory as L-S (MGTER), respectively with the rotation effect. The small dashes line and small dashes line with circles represent a magneto micropolar generalized thermoelastic medium, for L–S theory as L-S (MMGTE) and for G–L theory as G-L (MMGTE), respectively. The large dashes line and large dashes line with triangles represent magneto micropolar generalized thermoelastic medium with the rotation effect, for G-L theory as G-L (MMGTER) and for L-S theory as L-S (MMGTER), respectively. The variations in normal displacement u_3, normal stress σ_{33}, tangential couple stress m_{32} and temperature distribution T with distance x_1 have been shown for mechanical force in Figs 1–4.

It is clear from Fig.1 that near the source u_3 has higher values for G-L (MGTER) and L-S (MGTER) theories as compared to its values for all other theories. Also, as x_1 increases, the electromagnetic and rotation effect tend to diminish. Figure 2 again shows that electromagnetism and the rotation effect have much less impact in the range $3 \leq x_1 \leq 9$ for normal stress σ_{33}. Figure 3 shows that tangential couple stress keeps on increasing as we move away from the point of application of the source for all theories. Finally, Fig.4 shows that variation in the temperature distribution T with the rotation effect, near the source has higher values and then keeps on decreasing with x_1 whereas without the rotation effect it has lower values near the source and then keeps on increasing with x_1.

7. Conclusion

From the above discussion it is evident that normal displacement, normal stress, tangential couple stress and temperature distribution T are affected significantly by the application of rotation and magnetic field. Significant difference can be obtained in the temperature distribution by including the rotation effect. Also, when the case of rotation effect is considered, normal stress shows opposite behaviour for L-S and G-L theories.

Nomenclature

c^* – specific heat at constant strain
$\boldsymbol{E}$ – induced electric field
$\boldsymbol{H_0}$ – external applied magnetic field
$\boldsymbol{h}$ – induced magnetic field
$\boldsymbol{J}$ – current density vector
j – microinertia
$\boldsymbol{u}$ – displacement vector
$\alpha,\beta,\gamma,\kappa$ – micropolar elastic constants
α_t – coefficient of linear thermal expansion
δ_{ij} – Kronecker delta
$\in_0$ – electric permeability
$\in_{ijk}$ – alternating tensor
λ,μ – Lame's constants
μ_{ij} – couple stress tensor
μ_0 – magnetic permeability
ρ – density
ρ_e – volume charge density
σ_{ij} – stress tensor
τ_0,τ_1 – relaxation times
ϕ – microrotation vector

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