State of the Art in Predictive Control of Wheeled Mobile Robots

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Abstract:
The paper is concerned with the problem of tracking control of wheeled mobile robots (WMRs) using predictive control systems. Various kinematic structures of WMRs important from the point of view of motion control are discussed. A hierarchical approach to the problem of motion control of this kind of robots is presented. The problems of trajectory tracking and path following control of WMRs are briefly discussed. The methods of predictive control of WMRs are described in detail and the following aspects relevant to predictive control are considered: kinematic structures of robots, slip of wheels and its compensation, assumed constraints, methods of optimization of the objective function, problems of model nonlinearity, linearization and discretization, stability of the control system and use of the state observers.

Keywords: *wheeled mobile robot, motion control, predictive control, model-based control, optimization*

1. Introduction

Wheeled mobile robots (WMRs) are vehicles designed to help humans with handling repetitive tasks, sometimes in hazardous or difficult-to-access environments. They find practical applications in various domains including: military, anti-terrorist, manufacturing, civil engineering, logistics and transport, agriculture, space exploration, healthcare and in other fields of science and technology [48].

The important task which WMRs have to perform is possibly most accurate realization of motion in changeable conditions, the changes being associated with a robot (e.g. displacement of mass of cargo), environment (e.g. type of ground and terrain inclination), but ultimately resulting from wheel-ground interaction. Important problems associated with motion of WMRs are slip phenomenon and ground deformation (in case of soft grounds like soil, sand, etc.). Especially the slip of wheels has significant influence on robot motion and requires adequate control strategy. The WMRs should also operate correctly in case of noisy or delayed input signals, temporary lack of signals from control system (e.g. in case of teleoperation) or sensors (e.g. from Global Navigation Satellite Systems) and permanent failure of selected sensors or other devices.

Model-based predictive control (MPC) is advanced control technique (usually understood as any technique more advanced than a standard PID control) which was tremendously successful in practical applications in recent decades, exerting great influence on directions of development of industrial control systems as well as on research in this area [37].

From the point of view of the WMRs, the predictive control is attractive technique which main advantage is connected with realization of accurate motion in previously mentioned changeable conditions.

Despite numerous works regarding particular solutions of predictive control of WMRs, there is lack of publications describing in a comprehensive way current state of the art, that is, taking into account: kinematic structures of robots, slip of wheels and its compensation, assumed constraints, methods of optimization of objective function, problems of model nonlinearity, linearization and discretization, stability of the control system and use of the state observer. Therefore the objective of this work is detailed review of work associated with predictive control of the WMRs while taking into consideration the mentioned aspects.

This review is mainly limited to the most recent works, that is, from the 2010–2015 period, with few exceptions. The paper focuses on the problem of direct control of the WMR movement, that is, excluding some issues related to path or trajectory planning and obstacle avoidance using MPC. However, on the occasion of the review it was noticed that often tackled issue in the research works is the problem of path or trajectory planning of the WMRs using the MPC algorithms.

2. Wheeled Mobile Robots and Motion Control

In general, it is possible to distinguish the following kinematic structures of the WMRs:

- differentially driven (e.g. three-wheeled Pioneer 2-DX robot with two non-steered driven wheels and a castor),
- skid-steered (e.g. six-wheeled IBIS robot [47] by PIAP with all wheels non-steered and independently driven),
- car-like (e.g. four-wheeled robot with two steered and driven wheels and two non-steered free wheels),
- omnidirectional robot (e.g. three-wheeled robot with non-steered mecanum wheels).

From the point of view of motion control of the WMRs, important phenomenon is the slip of wheels. Neglecting slip of wheels is reasonable when robot moves with small speeds and accelerations, which results in longitudinal slips of limited magnitude. In turn, neglecting of side slips can be justified when robot moves with small speeds, has steered, caster or mecanum wheels and turning radius is large with respect to velocity of motion. However, it is worth emphasizing that in case of wheeled robots with all non-steered wheels, which is the most popular design type in commercial solutions so far (see e.g. [23, 47]), the slip of wheels always takes place during change of direction of motion. Robots like that are called skid-steered mobile robots and are objects of research, for example, in the works [25, 39].

One of the most common tasks in the WMRs control is that a robot should move from the known initial position to the desired goal position. The control systems performing this task often have a hierarchical structure. The highest layer of the control system is responsible for global path planning in which complete knowledge about robot environment is usually assumed. The next layer performs local path planning, which enables avoidance of any previously unknown obstacles detected by robot environmental sensors during motion. The lowest layer of the control system, which is analyzed in this paper in detail, is responsible for trajectory tracking or less often path following.

The trajectory tracking control (or simply tracking control) is the kind of control where the chosen point of a robot has to move on desired motion trajectory. Similar problem but less often analyzed is path planning control in which the chosen point of a robot has to move on desired path. Main difference between those problems is that a trajectory is parametrized by time, which allows to compute all desired motion parameters of the robot. The path following is considered in applications in which spatial errors are more critical than temporal errors. Both kinds of problems are solved using the model predictive control technique, for instance, in article [29].

Tracking control (or path following) of the WMRs using a conventional PID controller is sensitive to variable conditions of operation such as change of transported mass or change of forces of motion resistance. Therefore, control systems of the WMRs are designed so as to guarantee high accuracy and stability of motion in variable conditions. The objective like that requires implementation of complex control techniques such as robust, adaptive or predictive control or their combinations. The control systems governing WMRs motion often use artificial intelligence (or soft computing) techniques which include artificial neural networks, fuzzy logic systems, evolutionary algorithms, etc., and various combinations of these techniques like neural-fuzzy control. Stability of the control systems of the WMRs is usually studied using Lyapunov method whose advantage is that as one of few methods it can be applied in case of non-linear systems.

3. Predictive Control Methods

3.1. General Idea of Predictive Control

In order to make the considerations which follow easier, short description of the predictive control algorithm is provided. The description is based mainly on the work [37].

General principle of predictive control is calculation at each sampling instant k of the algorithm (i.e. at time $t = k\Delta t$, where Δt is the controller sampling period) optimal control inputs (signals) sequence $\mathbf{u}(k + p|k)$, for $p = N_1, ..., N$ and $N_1 \geq 1$, so as to minimize differences between the desired future outputs (set-points) trajectory $\mathbf{y}^{sp}(k + p|k) \equiv \mathbf{y}_d(k + p|k)$ and predicted controlled outputs $\mathbf{y}(k + p|k)$ trajectory over the assumed horizon of prediction N. Prediction is performed based on: model of controlled object with assumed model of disturbances (uncontrolled inputs) and models of constraints, measurements of current and past outputs together with past values of control inputs, known or assumed controlled outputs [37].

Main advantage of this class of algorithms is that effects of control are satisfactory in the presence of object constraints and delays in control system. In comparison to the classic PID algorithm, the control error is defined as a difference between desired trajectory, and predicted trajectory. It follows that the MPC algorithms can respond to set point changes without need of occurrence of control error resulting from actual measurement, only based on the predicted output values. Important consequence of this approach is possibility of correct operation of the control system in case of temporary unavailability of the measured quantities. This is particularly important in case of autonomous control of vehicles using satellite navigation and loss of signals from satellites, e.g. as a result of motion in a tunnel, through terrain with trees or in a city with dense concentration of tall buildings.

The following symbols and terminology are introduced for the purpose of carrying out further analysis:

- $\mathbf{y}^{sp}(k + p|k) \equiv \mathbf{y}_d(k + p|k)$ – desired future trajectory,
- $\mathbf{y}(k)$ – output measurement,
- $\mathbf{d}(k) = \mathbf{y}(k) - \mathbf{y}(k|k-1)$ – unmeasured disturbance, defined as a difference between measured and predicted output value for a current sampling instant,
- $\mathbf{y}(k + p|k)$ – predicted controlled output trajectory calculated at a current sampling instant k for sampling instant $k + p$,
- $\mathbf{y}^0(k + p|k)$ – free component of the predicted output trajectory, corresponding to the situation where the control inputs are kept constant over the entire prediction horizon, i.e., $\mathbf{u}(k + p - 1) = \mathbf{u}(k - 1)$, dependent on current control signals,
- $\Delta\mathbf{y}(k + p|k)$ – forced component of the predicted output trajectory, dependent on future control signals,
- $\mathbf{u}(k + p)$ – control inputs trajectory,
- $\Delta\mathbf{u}(k + p)$ – sequence of control inputs changes,

where vectors of control inputs have dimension nu whilst vectors of controlled outputs and their predictions have dimension n_y.

Control outputs are determined over control horizon $N_u \leq N$ by minimizing the selected objective function which describes the control quality over the prediction horizon. For this purpose, the following quadratic function is usually used:

$$J(k) = \sum_{p=N_1}^{N} \|\mathbf{y}_d(k+p|k) - \mathbf{y}(k+p|k)\|^2_{\mathbf{\Psi}(p)} + \\ + \sum_{p=0}^{N_u-1} \|\mathbf{\Delta u}(k+p|k)\|^2_{\Lambda(p)}, \tag{1}$$

where $\Lambda(p)$, $\Psi(p)$ – matrices of weights with positive coefficients.

In equation (1) the first sum concerns the difference beetween desired and predicted output trajectory. The second sum represents magnitude of future control signal increments, which can be regarded as the measure of control energy. Optimization of this function consists in minimization of the control error, taking into account energy consumption.

Optimization can be conducted by means of analytical method (in case of linear model) or by means of numerical calculations. Use of linear models is particularly recommended, because the optimal sequence of control values is obtained in a simple and unique way. In practical considerations also constraints of output signals, control signals and increments of control signals should be taken into account, which makes the process of optimization more complicated.

Values predicted for the given time instant k usually differ from the measured value for that time instant by $\mathbf{d}(k) = \mathbf{y}(k) - \mathbf{y}(k|k-1)$.

For the linear object model, according to the superposition principle it is possible to treat the output signal $\mathbf{y}(k+p|k)$ as a sum of free and forced trajectories:

$$\mathbf{y}(k+p|k) = \mathbf{y}^0(k+p|k) + \mathbf{\Delta y}(k+p|k) \quad (2)$$

The forced trajectory depends only on future increments of control signals, which results from the following dependency:

$$\begin{aligned} \mathbf{\Delta y}(k+p|k) &= \mathbf{M}_p \left[\mathbf{\Delta u}(k|k)^T \right. \\ &\left. \mathbf{\Delta u}(k+1|k)^T \ldots \mathbf{\Delta u}(k+p-1|k)^T\right]^T. \end{aligned} \quad (3)$$

From this dependency, it follows that knowing matrix of robot dynamics $\mathbf{M}_p$ it is possible to determine sequence of control signals based on the optimization problem (most often quadratic programming).

3.2. Predictive Control Algorithms

There exist many variants of the predictive control algorithm, which differ in form of the object model and the method of solution of the optimization problem. Types of the predictive control algorithm include [37]:

- DMC (Dynamic Matrix Control) – algorithm based on the object model in a form of step response,
- GPC (Generalized Predictive Control) – algorithm based on the object model in the form of difference equations,
- MPCS (Model Predictive Control with State-space Equations) – algorithm which uses linear model of object dynamics in state space,
- NMPC (Nonlinear Model Predictive Control) – method in which nonlinear dynamics model of the controlled object is used,
- FMPC (Fuzzy Model Predictive Control) – method based on fuzzy nonlinear dynamics model of the controlled object,
- NNPC (Neural Network Predictive Control) – algorithm with modelling or optimization based on artificial neural networks.

Dynamics Matrix Control. DMC algorithm is often used in case of lack of knowledge of mathematical model of the object or difficulties in its implementation (large industrial objects). In case of WMRs with known model of dynamics it is rarely used.

Generalized Predictive Control. In GPC algorithm a model of object in the following form of a difference equation is used:

$$\mathbf{A}(z^{-1})\mathbf{y}(k) = \mathbf{B}(z^{-1})\mathbf{u}(k-1) + \mathbf{C}(z^{-1})\frac{\mathbf{v}(k)}{1-z^{-1}} \quad (4)$$

where: $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are polynomial matrices, $\mathbf{v}(k)$ is a vector of white noises with zero mean, z^{-1} denotes the operator of a unit time delay, denotes the backward-difference operator.

Model like that can be transformed using the Bézout identity to the form useful for the objective function. Object transfer function can be obtained based on known model or by conducting object identification.

Model Predictive Control with State-space Equations. MPCS algorithm uses linear model of object dynamics in state space of the form:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k), \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{x}(k) + \mathbf{D}\,\mathbf{u}(k), \end{aligned} \quad (5)$$

where: $\mathbf{x}(k)$ is a state vector, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are respectively state (system) matrix, input (control) matrix, output matrix and feedforward matrix.

Model like that is used in majority of the reviewed works concerning predictive control. The advantage in comparison to the above mentioned algorithms is no of need of storing data about previous values of control signals.

Vector of optimal increments of control signals can be determined using transformations described in works [28, 37, 46]. It is also analogous for the other mentioned earlier algorithms based on linear model. Two types of the MPCS algorithm can be distinguished:

- Algorithm with Measured State, in which entire state vector is available for measurements,
- Algorithm with Estimated State, in which state vector is estimated using state observer or Kalman filter.

Nonlinear Model Predictive Control. In NMPC algorithms, nonlinear model of dynamics of controlled object is used. There exist various approaches to this problem. Often model linearization is used in the neighborhood of the operating point, which ultimately boils down to use of algorithms relying on linear model. In contrast, the NMPC uses advanced methods of objective function optimization, which was discussed in the works: [3, 7, 10, 19, 28, 29, 31, 34, 38, 44].

Fuzzy Model Predictive Control. FMPC method relies on fuzzy nonlinear object model. Example of this approach is described in [33]. In this case, the approaches based on linearization at the operating point or nonlinear optimization methods are possible.

Neural Network Predictive Control. NNPC method uses algorithms with optimization based on artificial neural networks, and relevant examples are in works [11, 43].

4. Predictive Control of Wheeled Mobile Robots

4.1. Wheeled Mobile Robots and Slippage

In the present work, kinematic structures of the robots that appear as objects of research in the reviewed articles were analyzed. It was noticed that, in the works concerning predictive control, differentially driven robots and three-wheeled omnidirectional robots are predominant. The works [6, 7, 12, 22, 24, 28, 31, 32, 43] concern two-wheeled robots, and work [6] concerns inverted pendulum robot. In turn, works [2, 5, 10, 20, 21, 29, 38] describe three-wheeled omnidirectional robots, whereas [1, 4, 18, 35], three-wheeled robots with two fixed driven wheels and castor.

Works [13, 17, 27, 30] cover research involving four-wheeled robots of car-like steering type, the work [11] is concerned with differentially driven four-wheeled robot with two wheels driven and two castors, whereas works [9, 45, 46] describe four-wheeled omnidirectional robot. Apart from the works [3, 42], no other works were specifically concerned with predictive control of skid-steered robots. In the work [42] however, authors do not take into account model of robot dynamics, neither they explicitly mention that robot wheels are not steered. In turn, authors of the paper [3] introduce a simplified model of the robot reducing it to a two-wheeled version. It should be pointed out, that in case of robots like that, because slip of wheels is inherent property of their motion during turning, advantages of the predictive control associated with high accuracy of the realized motion become the most important.

As far as slip of wheels is concerned, in a couple of reviewed works, that is, [13–15, 26, 27, 30, 36], information of modeling slip phenomenon and control of the object in conditions of slip of wheels was explicitly stated. In several works, topic of methods of compensation of this kind of disturbances for three-wheeled omnidirectional robots was discussed. Detailed description of controller structure for the case of compensation of friction results for robots like that can be found in works: [2, 5, 9, 28]. In particular, in work [2] example description of model transformations is contained, where in a separate matrix dependencies of measurable (or modelled) disturbances are given.

After taking into account total number of reviewed publications about predictive control it can be noticed, that slip of wheels and its compensation are rarely discussed problems. It follows from the mentioned earlier deficiency of works on skid-steered mobile robots.

Advantages of predictive control can be revealed also in case of robot control with large delays, e.g. during teleoperation of planetary rovers. In such cases, use of other methods of control would lead to large errors and be hazardous for the robot and its surroundings.

4.2. Constraints of Controlled Objects and Optimization Criteria

In modeling of every real controlled object, physical constraints should be taken into account. The constraints may be associated with:

- minimum and maximum values of control, state and output signals of the robot (e.g., magnitudes of control voltages, rates of change of signals, etc.),
- control and prediction horizons at which calculations can be conducted in real time,
- sampling frequency (length of sampling period).

The constraints can significantly complicate the procedure of optimization in predictive control. In the literature two main approaches connected to this problem are followed in general:

- introduction of limits for control signals and use of methods analogous to restriction of the integrating action (anti-windup) in the PID controller, while keeping simple methods of optimization (the solution is usually not optimal),
- solution of the optimization problem using more advanced numerical methods (not always possible).

In the reviewed literature concerning predictive control of WMRs, in the example works [2, 5, 11, 17, 18, 21, 22, 28, 44, 46] authors mainly use the second approach, because it is more promising for more effective solution of the problem.

Many works concerning predictive control, like [4, 9, 11, 12, 17, 19, 22, 29, 32–34, 36, 38, 41, 43–46], take up the topic of optimization of the objective function. This topic is especially elaborated in case of nonlinear algorithms of predictive control. Another factor that imposes use of sophisticated numerical methods is the need of taking into account hardware limitations (computer performance).

4.3. Linearization, Uncertainty and Discretization of the Model

In case of nonlinear control objects, important problem from the point of view of predictive control is determination of their mathematical models. In order to enable implementation of well-known algorithms for linear models, linearization of the model in the neighborhood of operating point is carried out. Approaches consisting in linearization of the model at each step of the algorithm, or calculation of the free trajectory based on nonlinear model and carrying out remaining calculations using the linearized model, are also known in theory. In work [4] comparison of methods of modeling of WMRs with linearization and using nonlinear optimization is presented. Other works concerned with the problem of linearization of the model of WMR to use it in predictive control include [2, 6–9, 15, 18, 22, 24, 28, 32, 42, 43, 45, 46].

An important problem is determination of discrete model of the object based on the model with continuous time. Worth attention is the method from work [28], where discretization based on solution of the system state equation is used. Its advantage is accurate reflection of system dynamics. Other works concerned with determination of the model of WMRs with discrete time include [5, 12, 19, 36].

4.4. Stability of the Control System

Another important problem is ensuring control system stability in order to guarantee correct operation of the controller. In [1] example of approach to stability study based on Lyapunov function is given. Other works on this problem include [3, 22, 31, 42]. One may notice that despite its significance, the problem of control system stability is rarely considered in the works concerning predictive control of wheeled mobile robots. It probably follows from difficulty of this kind of analysis for more complex objects and control systems.

4.5. Estimation of the State Vector

For development of the predictive control algorithm, the mathematical model of an object is required. In case of system model in state space with discrete time, information about actual state of the system is necessary. This information can be obtained by:

- measurement of physical quantities that belong to the state vector (e.g., angular velocities of robot wheels, driving torques),
- estimation of state variables of the system based on knowledge of approximate mathematical model and measurements of input (e.g., voltage control signals for drives) and output quantities of the object (e.g., velocity and orientation of the robot platform).

Because rarely there exists a possibility of measurement of all state variables, often methods of state vector estimation are used, like for example in works [4, 12, 13, 16, 27, 30, 40]. Often used methods of state vector estimation include state observer and Kalman filter.

State observer. Model of the state observer can have the form resulting from the object model, that is [37]:

$$\begin{aligned}\hat{\mathbf{x}}(k+1|k) &= \mathbf{A}\,\hat{\mathbf{x}}(k|k-1) + \\ &+\mathbf{B}\,\mathbf{u}(k) + \mathbf{L}(y(k) - \mathbf{C}\,\hat{\mathbf{x}}(k|k-1))\end{aligned} \tag{6}$$

where $\hat{\mathbf{x}}(k|k-1)$ denotes the estimate of the state vector $\mathbf{x}(k)$ evaluated on the basis of information available at the previous sampling instant $k-1$, while $\mathbf{L}$ is a gain matrix which defines observer dynamics.

For the observer, the state reconstruction error is defined as:

$$\mathbf{e}(k) = \hat{\mathbf{x}}(k) - \mathbf{x}(k). \tag{7}$$

For this error to tend to zero with time, the form of $\mathbf{L}$ matrix should be assumed such that the asymptotic stability of the system is guaranteed. The problem of determination of $\mathbf{L}$ matrix of the observer boils down to the basic problem of synthesis of a control system – changing positions of the poles of the transposed system. It follows that it is possible to determine the state observer if and only if the object described with the model in state space is observable. The possibility of influencing object dynamics by introduction of the observer adds to it yet another important property – possibility of stabilization of the control system [37].

The state observer may be complemented with model of a measurable or possible-to-model disturbance. After determination of the observer, synthesis of the control system with predictive controller can be carried out based on the estimated object model. The example of a system like that is described in [13].

Kalman filter. In case of the Kalman filter, the following object model can be assumed [37]:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k) + \mathbf{E}\,\mathbf{w}_x(k), \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{x}(k) + \mathbf{D}\,\mathbf{u}(k) + \mathbf{F}\,\mathbf{w}_y(k),\end{aligned} \tag{8}$$

where $\mathbf{w}_x(k)$ and $\mathbf{w}_y(k)$ are respectively vectors of non-measurable disturbances of the system and of the measurement.

The disturbances in general are assumed as probabilistic signals having character of white noise. The idea of the Kalman filter is determination of the optimal state estimate by minimization of the objective function [37]:

$$J(k) = \mathbf{E}(\mathbf{x}(k) - \hat{\mathbf{x}}(k))^T\,(\mathbf{x}(k) - \hat{\mathbf{x}}(k)). \tag{9}$$

The algorithm of determination of the filter model can rely on the observer equation. Examples of determination of system state based on the Kalman filter are provided in works [12, 13].

5. Conclusion

In the present paper, the state of the art concerning predictive control system for motion control of the WMRs was described in detail. Results of this review are presented in a compact way in Table 1. The table contains information about particular works including: authors, year of publication, reference, number of wheels and type of investigated vehicle, and description of the analyzed problem. The works are given in the order of publication (years). All mentioned works contain description of vehicle dynamics or kinematics model (except for [34] where, however, a reference is made to model from another paper), results of simulation and/or empirical research, therefore these information is not included in the table. After analysis of data presented in the table one can notice that:

- differentially driven (mainly three-wheeled) and omnidirectional robots are predominant,
- skid-steered robots are rarely analyzed but more often slip of wheels or friction compensation are taken into consideration,
- drive units of the WMRs are hardly ever taken into account in robot model,
- the problem of model identification of the WMRs is very rarely analyzed,
- very popular is the approach of description of model of the WMR in state space,
- in recent years more and more often the nonlinear MPC methods, state observer and Kalman filter are used,
- stability of predictive control systems and methods of tuning of the MPC controllers are seldom investigated,
- the problems of objective function optimization, model linearization or discretization as well as signal constraints are often considered,
- path or trajectory planning and obstacle avoidance problems are sometimes analyzed together with trajectory tracking (or less often path following) control,
- in selected works the problems of robot formation control or simultaneous tracking control and stabilization are analyzed.

Table 1. Selected works concerning predictive control of WMRs and ground vehicles

Authors	Year	Ref.	Wheels no./ vehicle type	Analyzed issues
Armesto et al.	2015	[4]	3/differentially driven	Nonlinear MPC, model in state space, tracking control, linearization, Kalman filter, optimization
Bature et al.	2015	[6]	2/inverted pendulum	Model in state space, linearization, identification methods, tracking control
Garcia et al.	2015	[13]	4/car-like	Model in state space, Kalman filter, path planning, state observer, delays
Kanjanawanishkul	2015	[21]	3/omnidirectional	Model in state space, path following, virtual vehicle, constraints
Lucet et al.	2015	[30]	4/car-like	Model in state space, slip of wheels, state observer, tracking control, adaptive control, dynamic stabilization
Nascimento et al.	2015	[34]	–/omnidirectional	Nonlinear MPC, robot formation control, controller tuning, optimization
Yang et al.	2015	[43]	2/differentially driven	Nonlinear MPC, model in state space, hybrid chaotic optimization, tracking control, extreme learning machine, artificial neural network
Yu et al.	2015	[44]	–/–	Nonlinear MPC, model in state space, path following, stability, constraints, optimization
Barreto et al.	2014	[5]	3/omnidirectional	Model in state space, friction compensation, tracking control, discretization, constraints
Deng et al.	2014	[11]	4/differentially driven	Model in state space, constraints, optimization, tracking control, artificial neural network
Farrokhsiar & Najjaran	2014	[12]	2/differentially driven	Model in state space, Kalman filter, stochastic control, motion planning, statistical linearization, discretization, constraints, optimization
Li et al.	2014	[28]	2/differentially driven	Nonlinear MPC, model in state space, feedback linearization, tracking control, discretization, constraints
Teatro et al.	2014	[38]	3/omnidirectional	Nonlinear MPC, motion planning and tracking, obstacle avoidance, optimization
Zarghami et al.	2014	[45]	4/omnidirectional	Model in state space, tracking control, delays, linearization, optimization
Zarghami et al.	2014	[46]	4/omnidirectional	Model in state space, tracking control, delays, linearization, constraints, optimization
Cartade et al.	2013	[8]	4/car-like	Slip of wheels, robot formation control, adaptive control, state observer, linearization
Guillet et al.	2013	[16]	4/car-like	Model in state space, slip of wheels, robot formation control, adaptive control, state observer, linearization
Amoozgar & Zhang	2012	[1]	3/differentially driven	Model in state space, tracking control, state tracking, kinematic approach, stability
Lenain & Thuilot	2012	[27	4/car-like	Model in state space, slip of wheels, tracking control, adaptive control, Kalman filter, observer

Table 1 cont. Selected works concerning predictive control of WMRs and ground vehicles

Authors	Year	Ref.	Wheels no./vehicle type	Analyzed issues
Ma et al.	2012	[31]	2/differentially driven	Nonlinear MPC, tracking control, stabilization, time-varying system, T-S fuzzy model, constraints, stability
Panathula et al.	2012	[36]	3/differentially driven	Model in state space, slip of wheels, discretization, tracking control, optimization
Araújo et al.	2011	[2]	3/omnidirectional	Model in state space, tracking control, linearization, constraints, stability
González et al.	2011	[15]	–/differentially driven	Model in state space, slip of wheels, robust MPC, tracking control, linearization, constraints, stability
Hach et al.	2011	[17]	4/car-like	Model in state space, slip of wheels, path tracking, optimization, constraints
Hedjar et al.	2011	[18]	3/differentially driven	Nonlinear MPC, tracking control, approximation, obstacle avoidance, artificial potential field, constraints
Maurović et al.	2011	[32]	2/differentially driven	Model in state space, tracking control, linearization, optimization
Conceição et al.	2010	[9]	4/omnidirectional	Model in state space, friction compensation, tracking control, linearization, optimization
Kanjanawanishkul	2010	[20]	3/omnidirectional, 2/differentially driven	PhD thesis regarding many problems associated with predictive control of WMRs, including robot formation control and path following
Kanjanawanishkul et al.	2010	[22]	2/differentially driven	Model in state space, path following, path replanning, obstacle avoidance, optimization, linearization, constraints
Wang et al.	2010	[41]	3/differentially driven	Model in state space, motor model, path tracking, electromagnetism-like optimization mechanism
Argomedo et al.	2009	[3]	4/skid-steered	Model in state space, slip of wheels, discretization, tracking control, open-closed system, quadratic optimization, stabilization, stability
Brezak et al.	2009	[7]	2/differentially driven	Model in state space, linear/nonlinear control, discretization, linearization, comparison of tracking controllers
Lee & Yoo	2009	[26]	4/car-like	Model in state space, nonlinear MPC, slip of wheels, tracking control
Conceição et al.	2008	[10]	3/omnidirectional	Model in state space, nonlinear MPC, tracking control, optimization
Li et al.	2008	[29]	3/omnidirectional	Model in state space, nonlinear MPC, path following, tracking control, optimization, constraints
Pacheco et al.	2008	[35]	3/differentially driven	Path planning, tracking control, identification, optimization, artificial potential field
Xie & Fierro	2008	[42]	4/skid-steered	Model in state space, tracking control, linearization, stabilization, stability, optimization, constraints
Klančar & Škrjanc	2007	[24]	3/differentially driven	Model in state space, tracking control, linearization, state feedback, constraints

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