

# BENCH TESTING AND SIMULATION MODEL OF A COGENERATION SYSTEM WITH A STIRLING ENGINE

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## Abstract

Cogeneration systems using a Stirling engine [2] are currently being developed and promoted by the European Union [1]. On the other hand, the EU climatic package obligates Polish energy producers to buy 100% CO<sub>2</sub> emission rights from 2020 onwards. At that time, according to the experts a ton of CO<sub>2</sub> could reach the price of 63.5 Euro. Attention is drawn to the fact that it is only then that nuclear power plants will become profitable, in the same way as wind power plants. This could result in substantial growth of energy prices, both for the economy and for households [3].

The paper presents the experimental research of a cogeneration system with a one-way Alpha type Stirling engine. The impact of the upper heat source and the type of working gas (helium and nitrogen) on the load capacity of the cogeneration system was studied. Based on a Sankey diagram, the power spread of the cogeneration system was analysed. On the basis of experimental research, a model was created of a cogeneration system consisting of the following submodels: Stirling engine of the second order, belt transmission, electric motor and an electrochemical battery. The Stirling engine was installed in a testbed, in the mechatronic lab at the Faculty of Automotive and Construction Machinery Engineering (SIMR) of Warsaw University of Technology.

Furthermore, the impact of regenerator efficiency on the efficiency of the Stirling Engine was examined, which in turns impacts the efficiency of the entire cogeneration system.

**Keywords:** Stirling engine, Alpha, Simulink, Sankey diagram, Helium and Nitrogen, Cogeneration

## 1. Introduction

Stationary cogeneration systems are currently being developed [2] in water heating system that are capable of producing electricity from waste heat [4-6] (the so-called trigeneration systems). A major advantage of cogeneration systems using a Stirling engine is the possibility of using various types of alternative fuels (including biogas [7], LPG, CNG, methane [8]) as well as coal, coke, pea coal etc. In order to start a cogeneration system, it is necessary to reach a  $T_{emin}$  temperature in the expansion space (in tests, cogeneration systems have been started for temperatures exceeding 750 K – with nitrogen as the working gas). The requirements that Stirling engines face include, among others: high efficiency of a regenerator; start temperature as low as possible, as measured in the expansion space; high heat conductivity heater material, low longitudinal conductivity of the regenerator cartridge while maintaining a high heat conductivity of the cartridge, efficient cooling (a proper selection of waste spaces of the regenerator, cooler and heater [9-12]).

## 2. Experimental research of a cogeneration system

The research of a cogeneration system with a Stirling engine, as depicted in Fig. 1, were conducted with the help of Labview software, as well as National Instruments measurement sheets. The purpose of a measurement-analysis system written in Labview was the recording of temperatures ( $T_e$  – in expansion space,  $T_{rh}$  – on the hot side of the regenerator,  $T_{rc}$  – on the cold side of the regenerator and  $T_c$  – in compression space). In addition, the current and voltage of a direct current electric motor were recorded – Fig. 2 depicts the user interface that the authors of the paper used during the research.

The research was experimental in nature; its purpose was to determine the impact of the temperature of a heat source, the pressure of the working gas as well as its type on the load capacity of the cogeneration system, as well as the volume of the resulting electric power that may be efficiently used e.g. in households, biomass incinerators as well as anywhere that an unused waste heat flow exists. The measurements were conducted for two types of working gas – helium and nitrogen.

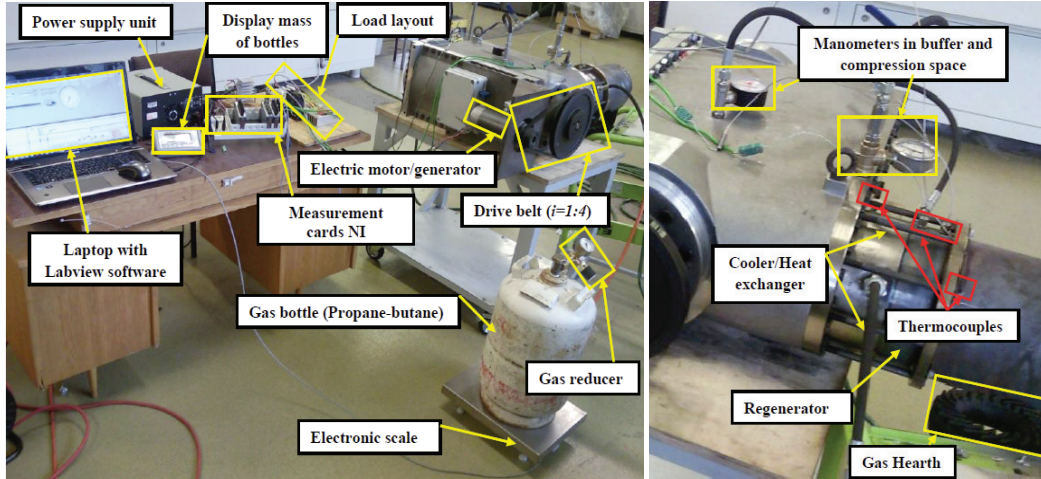


Fig. 1. Scheme of the measurement system used in the research

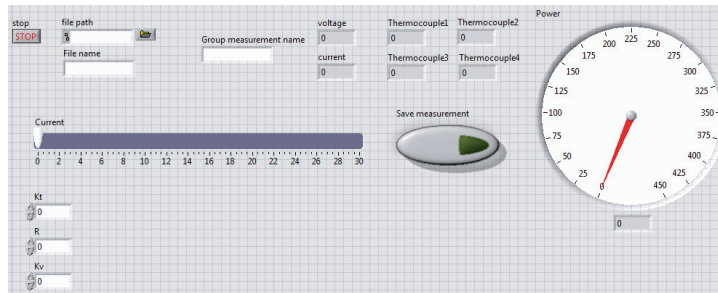


Fig. 2. User interface written in Labview 2011 software – measurement recording

Figure 3 depicts the impact of temperature (measured in the expansion space) on the volume of cogenerated power. The volume of cogenerated power increased with the rise of temperature in the expansion space (measured with pressure  $p=1$  MPa).

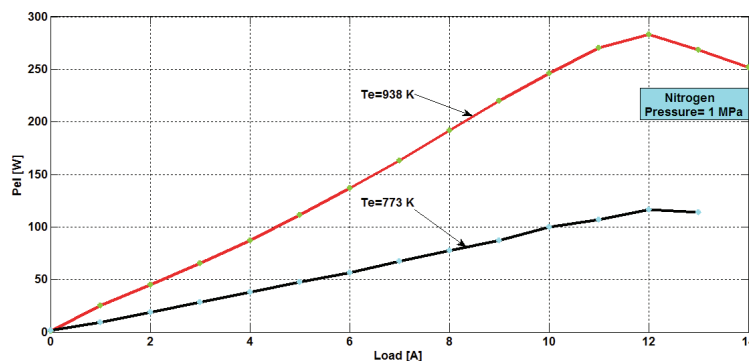


Fig. 3. The impact of temperature of an upper heat source (expansion space) on the volume of cogenerated power. Working gas: nitrogen. Temperature in the compression space  $T_c=299$  K

Figure 4 depicts the comparative course for the following working gases: nitrogen and helium, with the pressure 0.7 MPa. For nitrogen as a working gas, a higher load capacity of the cogeneration system has been obtained.

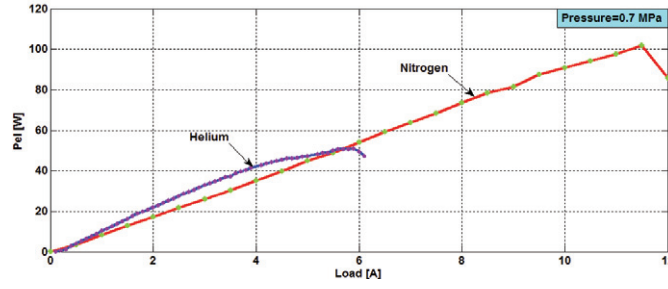


Fig. 4. Comparative course of nitrogen and helium with pressure  $p=7$  bar. Temperatures:  $T_{eN_2}=787$  K,  $T_{eHe}=900$  K,  $T_{eN_2}=787$  K. Temperature in the compression space  $T_c=299$  K

### 3. Analysis of change in energy form in the cogeneration system

The measurement and analysis of power spreading in the cogeneration system was possible thanks to the employment of a Sankey diagram, as depicted in [13].

The initial power of the cogeneration system is a result of the calorific value of propane-butane gas as well as hour expenditure, reduced by output losses, losses due to heat convection and losses resulting from the heating of air by the steel elements.

$$P_{in} = G \cdot W_u - (P_{wyl} + P_{kon} + P_{air}), \quad (1)$$

where:

$G \cdot W_u$  – product of the calorific value and hourly fuel consumption (for LPG  $W_u = 46 \text{ MJ/kg}$ ),

$P_{wyl}$  – output loss, presented in detail in [13],

$P_{kon}$  – loss resulting from heat convection through the pipe wall,

$P_{air}$  – loss due to heating the air volume by the construction elements and the gas, for LPG it is, according to [13], 0.5-0.8% of the value of  $GW_u$  flux for gas furnaces.

The thermodynamic power of the working gas includes the losses resulting from heat convection in the heater, regenerator, heat exchanger and the Stirling engine (second-order model) balance, which can be expressed with the following equation:

$$P_{term} = \eta_{term} \cdot P_{in}, \quad (2)$$

$\eta_{term}$  – the efficiency of the Stirling engine (in the case of thermal efficiency of the Stirling engine with no losses)  $\eta_{term} = 1 - q_e / q_c$  for the first-order model [14]. For the purpose of the research, a model of a Stirling engine was constructed, which was further expanded and eventually served as a basis for building a second-order model. For the second-order model, including the loss balance, the useful efficiency of a Stirling engine is as follows [13]:

$$\eta_{term} = \eta_e = \frac{N_e}{\dot{Q}_e}, \quad (3)$$

where, based on [9, 13]:

$N_e$  – Stirling engine Net Power [13],

$\dot{Q}_e$  – Heat flux with improved performance [13].

The mechanical power is the thermodynamic power reduced by the efficiency of the belt transmission, which can be expressed with the following equation:

$$P_{mech} = \eta_{term} \cdot \eta_{mech} \cdot P_{in}, \quad (4)$$

where:

$\eta_{mech}$  – efficiency of the belt transmission [13]  $\eta_{mech} \cong 95\%$ .

The electric power is the mechanical power, reduced by the electric motor losses (voltage loss on the electrical machine – the efficiency of the electrical motor is included).

$$P_{el} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} \cdot P_{in} , \quad (5)$$

where:

$$\eta_{el} = \frac{P_{el}}{P_{mech}} , \quad (6)$$

The actual (cogenerated) power is the electrical power, including the losses of the control system.

$$P_{rzecz} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} \cdot \eta_{rzecz} \cdot P_{in} , \quad (7)$$

where, based on [13]:

$\eta_{rzecz}$  – efficiency of transmission and control in the electronic elements ( $\eta_{rzecz} \approx 1$ ).

The output power is the cogenerated power reduced by the losses in the electrochemical battery (loss of voltage on the battery terminals, internal resistance of the battery – the electrodes and the electrolyte).

$$P_{cog} = P_{out} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} \cdot \eta_{rzecz} \cdot \eta_{ak} \cdot P_{in} , \quad (8)$$

where:

$$\eta_{ak} = \left( \frac{i_a(t)}{I_n} \right)^{-\beta} , \quad (9)$$

where:

$i_a(t)$  – charging / discharging current of the battery,

$I_n$  – rated current of the battery,

$\beta$  – constant, based on Peukert equation [16].

Cogeneration efficiency, including the storage of energy in the electrochemical battery is expressed by the following equation:

$$\eta_{kog} = \eta_{out} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} \cdot \eta_{rzecz} \cdot \eta_{ak} = \frac{P_{out}}{P_{in}} . \quad (10)$$

If there is not electrochemical battery in the cogeneration system – the electrical power is used in real time (it is then valid to assume  $\eta_{ak} = 1$ ), then the power of cogeneration is as follows:

$$P_{cog} = P_{el} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} \cdot P_{in} . \quad (11)$$

While the cogeneration efficiency is as follows:

$$\eta_{cog} = \eta_{out} = \eta_{term} \cdot \eta_{mech} \cdot \eta_{el} = \frac{P_{out}}{P_{in}} . \quad (12)$$

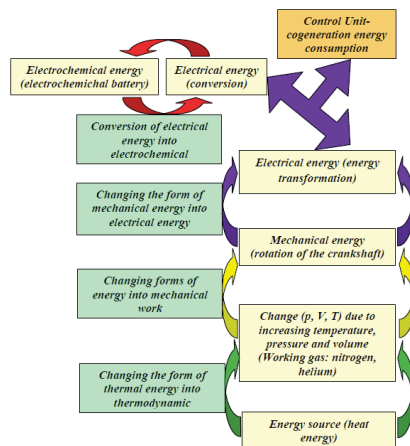


Fig. 5. Diagram presenting changes in energy form in the cogeneration system [13]

In summary – Fig. 5 depicts the changes in energy form resulting in the cogeneration system with a Stirling engine, which results in losses expressed by corresponding efficiencies (equations (1) through (12)).

#### 4. Simulation model of a cogeneration system

The simulation model of the cogeneration system consists of the following submodels: output losses and losses due to thermal convection; Stirling engine including the losses resulting on heat exchangers; belt transmission; electric motor; electrochemical battery. The model was created in the Matlab Simulink environment – Fig. 6 depicts a view of the simulation model of the cogeneration system.

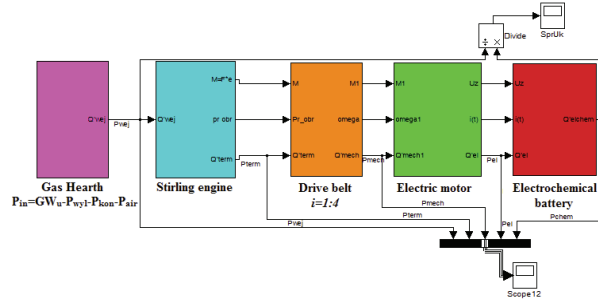


Fig. 6. Simulation model of the cogeneration system – created in the Matlab Simulink program [13]

On the basis of experimental research results, the change in temperature was simulated along the length of the regenerator – Fig. 7. Additionally, on the basis of a solid model of the cogeneration system [17] as well as first-order model of a Stirling engine [14], the change in regenerator efficiency during a single thermal circuit was simulated – Fig. 8. The thermodynamical analysis of the efficiency of the regenerator of the cogeneration system has been presented in [18].

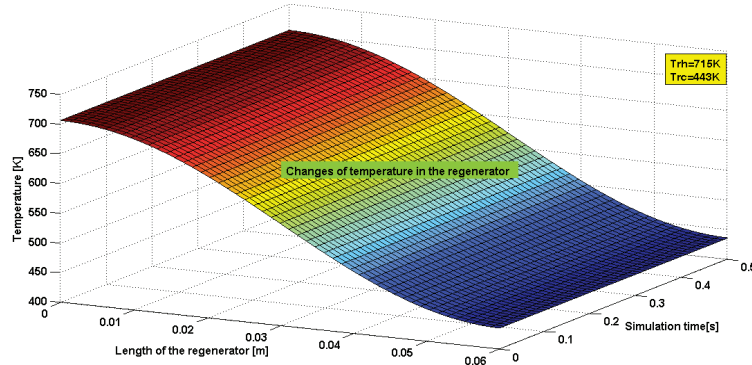


Fig. 7. The change in temperature along the length of the regenerator which was obtained on the basis of experimental research results ( $T_{rh}=715K$  and  $T_{rc}=442K$  [13])

The change of temperature along the length of the regenerator cartridge ( $x$  – length of the regenerator cartridge) has been simulated on the basis of equation [13] as well as measurement results:

$$T(x) = 1025 - \left( 455 + 135 \cdot \cos\left(\frac{\pi \cdot x}{0.38}\right) \right), \quad (13)$$

The efficiency of the regenerator is described by equation [8, 12]:

$$\eta_r = \frac{NTU_R}{1 + NTU_R}, \quad (14)$$

where:

$NTU_R$  – number of units of exchanged heat for both heat transfer periods (from the hot working gas to the regenerator cartridge and from the regenerator cartridge to the cold gas).

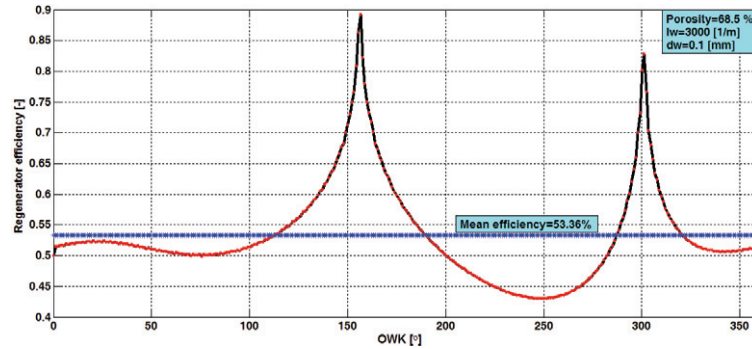


Fig. 8. Course of regenerator efficiency during one cycle

Figure 9 depicts the energy fluxes from the entry to the cogeneration system via analysis of the energy flux in the Stirling engine, belt transmission, electric motor, to the electrochemical battery. Horizontal lines mark the average values of energy fluxes. Tab. 1 presents the comparison of efficiencies of cogeneration system elements as measured, compared to the simulation model.

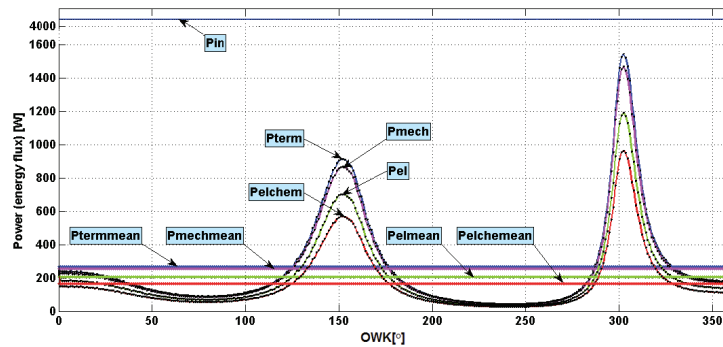


Fig. 9. Joint course of energy (power) fluxes from the entry to the cogeneration system  $P_{in}$  to the exit point of the system  $P_{elchem} = P_{cog}$ .

Tab. 1. Comparison of simulated and measured efficiencies

	Model	Measurements
Efficiency of the regenerator	53.36%	–
Thermal efficiency	15.36%	–
General efficiency of the Stirling engine	6.6%	7.3%
Efficiency of the cogeneration system with a battery	4.1%	4.7%
Efficiency of the cogeneration system without a battery	5.06%	5.8%
Efficiency of the electric motor	85%	–
Efficiency of the electrochemical battery	80.1%	–

## 5. Conclusions

Based on the results of the measurements, it can be concluded that with the rise in temperature in the expansion space, the volume of cogenerated power rises. For 0.7 MPa pressure, a clearly higher load capacity of the cogeneration system results for nitrogen (despite the higher temperature of helium).

The volume of the cogenerated power can be influenced by the increased efficiency of the regenerator, which in turn can be improved by a proper selection of the porosity of the regenerator, the heater material as well as by effective cooling (cooler).

For 1 MPa pressure and for equal temperatures ( $T_c=299$  K,  $T_e= 938$ K) and geometric dimensions of the Stirling engine, the results of measurements and the model were compared,

showing similar values of the heat flux, efficiency of components as well as the entirety of the cogeneration system.

An increase in the efficiency of the Stirling engine can be obtained via proper insulation of the heater from the environment (thereby reducing the loss via heat convection through the insulating walls). Consequently, it is possible to reduce the consumption of the gas flow into the furnace. As a result, the heat balance of the cogeneration system improves.

In the simulation model, the possibility of electricity storage in a secondary power source (electrochemical battery) was explored, whose efficiency (80.1%) impacts the efficiency of the entire cogeneration system. The use of the electrochemical battery allows for efficient storage of excess cogenerated energy and its reuse.

## Acknowledgement

This paper is co-financed by the European Union, as part of the European Social Fund, project „Program Rozwojowy Politechniki Warszawskiej” („Warsaw University of Technology Development Programme”), carried out by the Centre of Advanced Studies.



Centrum Studiów Zaawansowanych PW  
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