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Modified small-signal models of BUCK, BOOST and BUCK-BOOST DC-DC converters

Key words: BUCK, BOOST, BUCK-BOOST, line-to-output, control-to-output, parasitic resistance, PWM converters, small signal model, DC-DC converters

Introduction

DC-DC converters are circuits consisting of both linear, and nonlinear elements. Additionally converters work as a switching circuits, therefore even though they consist of few elements the process of modeling is not as simple as in common circuits.

The first method of modeling the converters was presented in year 1976 and it was based on averaging of state space equations [1]. Since then converters have been described using various techniques e.g. switch averaging [2], [3] and separation of variables [4], [5], which was used to derive models presented in this paper. All of those methods have one in common - they are based on averaging signals over one switching cycle. The most popular models which can be found in the literature describe ideal converters, some of them include few parasitic resistances [2], [3], [6]. One can also find models considering parasitic resistances, which describe most losses in converter circuit [7], [8].

The authors who neglect some of parasitic resistances, probably are assuming that the losses are so small that ignoring them won't make a change in the model. Such assumption is true if parasitic resistances are sufficiently small [9]. However in some cases it is not possible to use the simpler model and thus the full model needs to be used [10].

In order to model the behavior of a converter, besides all other parasitic resistances, one can use static or dynamic model of diode resistance. The model considering dynamic resistance of a diode is more universal because, if needed, it is much easier to switch to the model with static resistance than opposite. Most models that can be found in literature do not include the dynamic resistance and voltage offset of a diode [1], [7], [11]. Therefore those models are limited in use. However there are some papers that include models considering both the diode resistance and voltage offset [8], [12] which allows to analyze an influence of additional parameter.

This paper contains models of converters working in the continuous conduction mode (CCM). The models consider the dynamic value of diode resistance and a voltage offset, created after linearization of diode characteristic. First chapter describes basic terms and nomenclature used in further part of this paper.

From the second to the fourth chapter one can find derived models of BUCK, BOOST and BUCK-BOOST converters, which can be used to simulate work of the converters. The models consider dynamic value of diode resistance and the voltage offset, as it has been mentioned previously.

The fifth chapter is used to present some Scilab simulations of models presented in this paper in comparison to known models [12].

The fifth chapter is followed by conclusion and references.

1. Static and dynamic diode resistance

When modeling an ideal DC-DC converter (figure 1) one doesn't need to consider parasitic resistances of its electronic components. When considering non ideal power converter one needs to specify values of parasitic resistances, which are a simple representation of power loses.

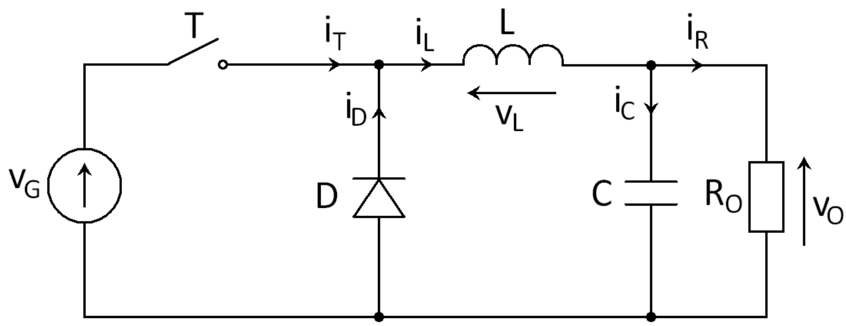

Fig. 1. An ideal step-down converter (BUCK) consisting of ideal transistor T, diode D, inductor L and capacitor C.

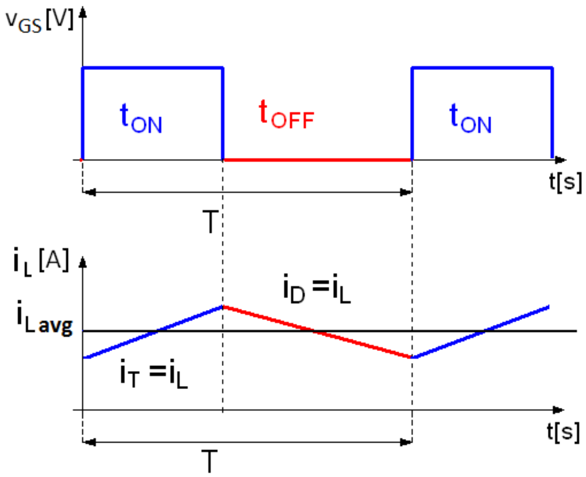

Fig. 2. Waveform of a transistor gate-to-source voltage and coil current during ON and OFF phase

Evaluation of those resistances for transistor, inductor and capacitor is simple and obvious. But on the other hand a lot of DC-DC converters contain a diode which can be represented by static or dynamic resistance. Diode current i_D changes with time in every switching cycle (figure 2). In CCM those changes are not significant if comparing with DC value of the current, but they can lead to significant changes in value of static resistance.

To deal with those changes one can use the characteristic of a diode after linearization, presented in fig 3b. In this case the slope represents the dynamic value of parasitic conductance of a diode, which is less vulnerable to small current changes than a static value. The value V_D is a representation of DC diode voltage. In converter's model the V_D is represented by an independent voltage source with constant value in fig 4, where all other parasitic resistances are included.

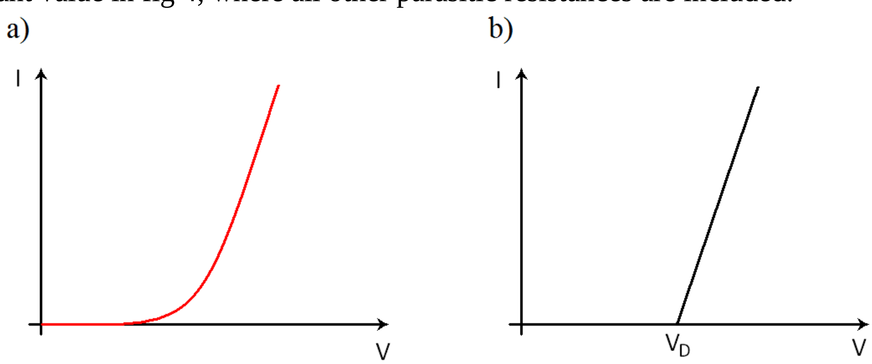

Fig. 3. Current-voltage characteristic of a diode: a) real; b) model based on linear approximation

2. Models of BUCK converter with all parasitic resistances

To create a mathematical model of a converter means to calculate it's transfer function. In this paper two important transfer functions are considered. One of them represent the response of output voltage to input voltage excitation and it is called line-to-output transfer function (1). The second transfer function represent dependency of output voltage on small-signal duty cycle and it is called control-to-output transfer function (2).

$$H_g(s) = \left. \frac{V_o(s)}{V_g(s)} \right|_{\theta(s)=0} \tag{1}$$

$$H_d(s) = \left. \frac{V_o(s)}{\theta(s)} \right|_{V_g(s)=0} \tag{2}$$

where:

$H_g(s)$ - transfer function line-to-output

$H_d(s)$ - transfer function control-to-output

$V_o(s)$ - small signal value of output voltage

$\theta(s)$ - small signal value of duty ratio

DC-DC converters are switching circuits hence when calculating the transfer function it is necessary to use one of averaging techniques presented in [2] [4] [5] [12]. All those techniques are based on averaging currents and/or voltages over one switching cycle. After averaging a linearization takes place where all signals are treated as a combination of a constant, and a small signal values as presented in (3).

$$x = X + x(t) \tag{3}$$

The linearization is followed by separation of the small signal values from the constant values. The small signal values are used to calculate the transfer functions accordingly to (1), (2).

An implementation of diode model considering the dynamic resistance R_D, and voltage offset V_D, into a BUCK converter consisting of real elements is presented in figure 4

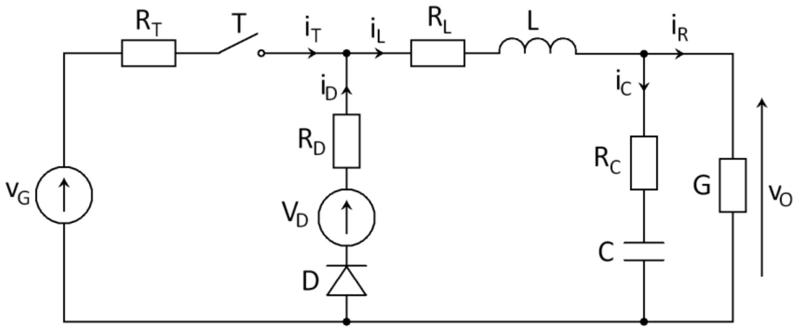

Fig. 4. Non ideal BUCK converter consisting of ideal components and their parasitic resistances

Using one of averaging techniques based on separation of variables the BUCK converter transfer functions has been derived:

$$H_{g(BUCK,CCM)}(s) = \frac{D_A(1+sCR_C)}{s^2C_ZL+(C_ZR_Z+LG+CR_C)s+GR_Z+1} \quad (4)$$

$$H_{d(BUCK,CCM)}(s) = \frac{(V_G - I_L(R_T - R_D) - V_D)(1+sCR_C)}{s^2C_ZL+(C_ZR_Z+LG+CR_C)s+GR_Z+1} \quad (5)$$

where:

$$V_O = \frac{D_AV_G + V_D(1-D_A)}{GR_Z+1} \quad (6)$$

$$I_L = \frac{D_AV_G + V_D(1-D_A)}{GR_Z+1}G \quad (7)$$

$$C_Z = C(1+GR_C) \quad (8)$$

$$R_Z = D_A(R_T - R_D) + R_L + R_D \quad (9)$$

The values V_O, V_G, V_D, I_L, and D_A are constant values of output voltage, input voltage, diode offset voltage, coil current and duty ratio respectively. Equations (6) – (7) have been derived after linearization and separation of the small signal values from the constant values.

Accordingly to (4) the value of diode voltage offset V_D in BUCK converter doesn't influence the transfer function $H_g(s)$. It means that only change of static to dynamic resistance affects this transfer functions. The equation (4) is not different from the one presented in [5][14] except that here the value of R_D refers to the dynamic resistance.

3. Models of BOOST converter

Circuit of a BOOST converter considering all parasitic resistances and model of a diode with dynamic resistance and voltage offset is presented in figure 5

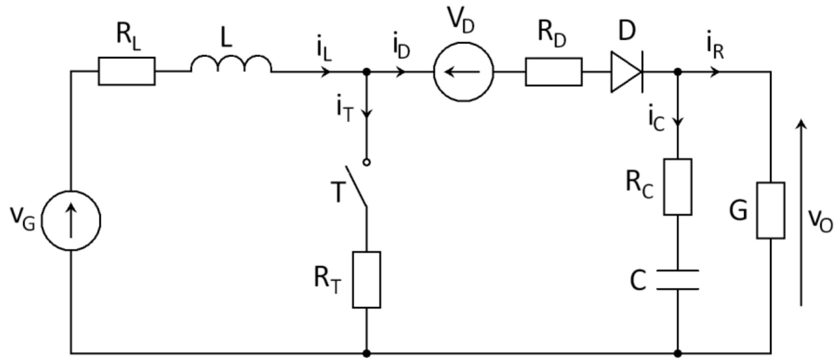

Fig. 5. Circuit of a BOOST converter considering all parasitic resistances and model of a diode with dynamic resistance and voltage offset

Transfer functions related to the circuit presented in figure 5 are presented in (10) and (11):

$$H_{g(BOOST,CCM)}(s) = \frac{(1+sCR_C)(1-D_A)}{s^2LC_Z + s(C_ZR_Z + GL + CR_C(1-D_A)^2) + GR_Z + (1-D_A)^2} \tag{10}$$

$$H_{d(BOOST,CCM)}(s) = \frac{(1+sCR_C)(1-D_A)(-I_L(R_T - R_D) + V_D + V_O - \frac{I_L(sL+R_Z)}{(1-D_A)})}{s^2LC_Z + s(C_ZR_Z + GL + CR_C(1-D_A)^2) + GR_Z + (1-D_A)^2} \tag{11}$$

where for BOOST converter:

$$V_O = \frac{V_G - V_D(1-D_A)}{R_ZG + (1-D_A)^2}(1-D_A) \tag{12}$$

$$I_L = \frac{V_G - V_D(1-D_A)}{R_ZG + (1-D_A)^2}G \tag{13}$$

Formulas related to C_Z and R_Z have been presented in (8) and (9).

4. Models of BUCK-BOOST converter

In figure 6 one can find a circuit of BUCK-BOOST converter which was used to derive transfer functions (14) and (15). The circuit contains all parasitic resistances and model of a diode considering dynamic resistance and voltage offset.

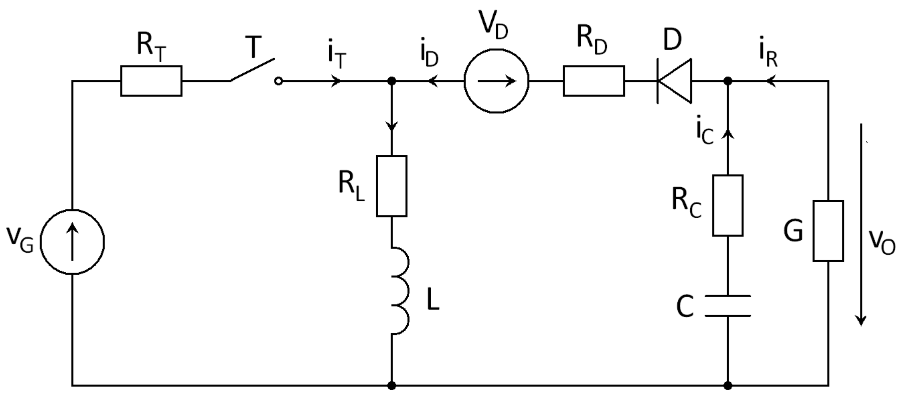

Fig. 6. Circuit of a BUCK-BOOST converter, including all parasitic resistances and model of a diode considering dynamic resistance and voltage offset

Small signal models related to the circuit in figure 6 are presented in (14) and (15)

$$H_{g(BUCK-BOOST,CCM)} = \frac{(1+sCR_C)(1-D_A)D_A}{s^2LC_z + s(C_zR_Z + LG + CR_C(1-D_A)^2) + GR_Z + (1-D_A)^2} \tag{14}$$

$$H_{d(BUCK-BOOST,CCM)} = \frac{(1+sCR_C)(1-D_A)\left(-\dfrac{I_L(sL+R_Z)}{(1-D_A)} + V_G - I_L(R_L+R_D) + V_O + V_D\right)}{s^2LC_z + s(C_zR_Z + LG + CR_C(1-D_A)^2) + GR_Z + (1-D_A)^2} \tag{15}$$

Constant values of output voltage and coil current can be calculated with (16) and (17):

$$V_O = \frac{-V_GD_A(1-D_A) + V_D(1-D_A)^2}{GR_Z - (1-D_A)^2} \tag{16}$$

$$I_L = \frac{-V_GD_A + V_D(1-D_A)}{GR_Z - (1-D_A)^2}G \tag{17}$$

As it was mentioned before, formulas related to C_Z and R_Z have been presented in (8) and (9).

5. Simulations of BUCK converter

In this chapter one can find simulations of a BUCK converter. Values of the elements, used in following simulations, have been chosen to be in compliance with values of real, measured elements. All those values have been presented in tab. 1.

Table 1

V_G=5 V	C=44,7 µF	D_A=0,5
R_T=4 mΩ	R_C=35 mΩ	G=0,2 S
R_{D_STAT}=364 mΩ	L=23,5 µH	V_D=0,8 V
R_{D_DYN}=59 mΩ	*R_L=62 mΩ	

* Resistance R_L includes the value of a resistor used to measure the coil current

5.1. Comparison with other model

A large signal model considering diode voltage offset has already been presented in [12]. The model is presented in figure 7. The differences in nomenclature are explained in tab. 2.

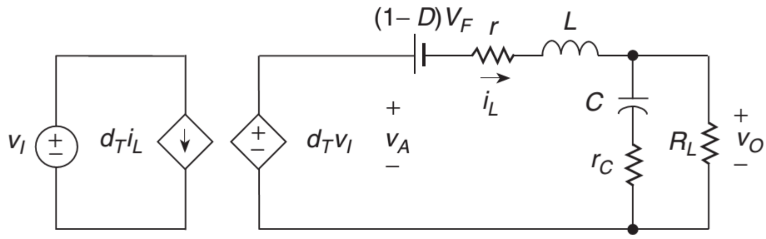

Fig. 7. Large signal model of a BUCK converter presented in [12]

where:

$$r = Dr_{DS} + (1 - D)R_F + r_L \tag{18}$$

Table 2

Symbol used in [12]	Description	Equivalent symbol used in this paper
v_I	Input voltage	v_G
V_F	Forward voltage of a diode	V_D
r_L	ESR of a inductor	R_L
r_C	ESR of a capacitor	R_C
r_{DS}	$R_{DS(ON)}$ resistance of a transistor	R_T
R_F	Forward resistance of a diode	R_D
R_L	Load resistance	R_O=1/G

According to the model from figure 7 the line-to-output and control-to-output transfer functions are as follows:

$$H_{g(BUCK\_CCM\_1)}(s) = \frac{(1+sCR_C)D_A}{s^2LC_Z + s(C_ZR_Z + LG + CR_C) + GR_Z + 1} \tag{19}$$

$$H_{d(BUCK\_CCM\_1)}(s) = \frac{V_G(1+sCR_C)}{s^2LC_Z + s(C_ZR_Z + LG + CR_C) + GR_Z + 1} \tag{20}$$

The nomenclature of equations (19) and (20) has been changed accordingly to tab 2, so that they could be easily compared to equations (4), (5). According to this, only the control-to-output transfer function (20) is different from that presented in (5). To assess the differences a simulation has been done.

First simulation presented in figure 8 compares two models in frequency domain. The characteristic corresponds to a situation where duty ratio was periodically changing, and the amplitude of the variations was equal to 0,1. Dashed curve was calculated based on model from figure 7 [12, page 409] which will be called model 1 from now on. The solid curve was simulated according to equation (5) which will be called model 2. All parameters used for the simulation are presented in tab. 1.

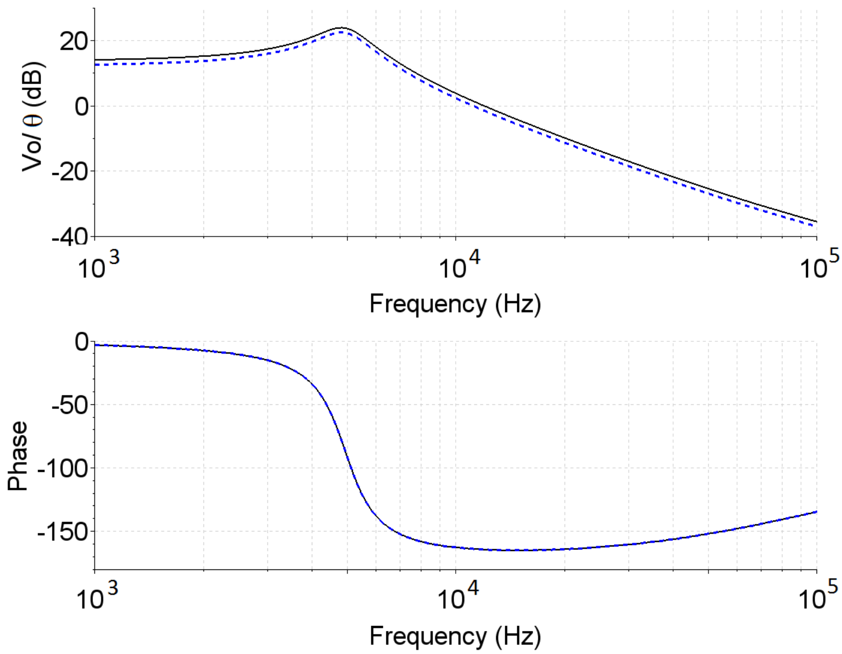

Fig. 8. Bode characteristic of control-to-output transfer function of a BUCK converter: dashed - model 1, solid – model 2.

The simulation shows good consistence between those two models. Differences between magnitudes are small for element values presented in tab. 1. But in some cases (e.g. when the input voltage is relatively small) the difference in magnitude can be higher, which is presented in figure 9 where only input voltage V_G was changed from 5V to 2V. All other parameters are the same as in tab. 1. If the input voltage rises, the difference between magnitudes goes to zero and the two models are equivalent.

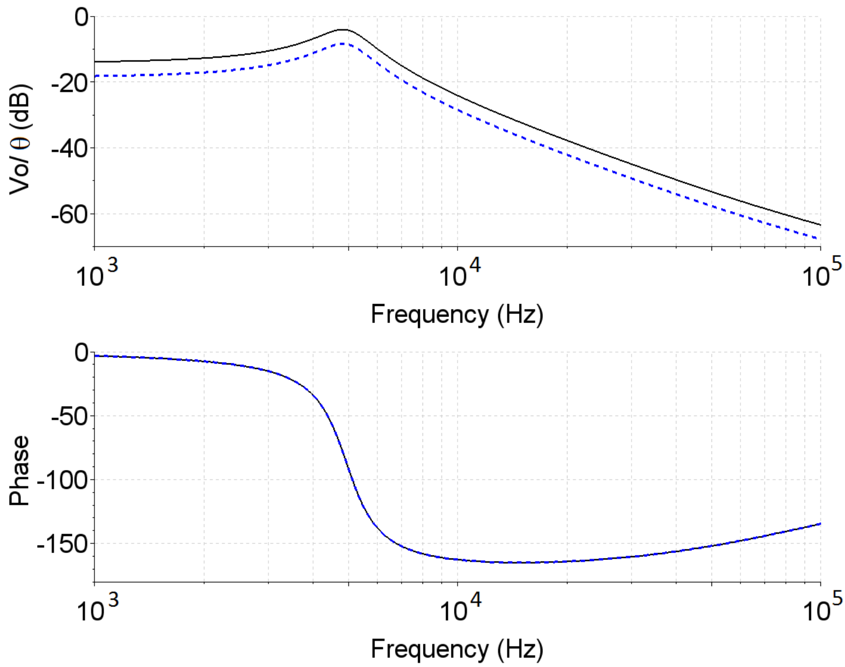

Fig. 9. Control-to-output transfer function of a BUCK converter with V_G=2V: dashed - model 1, solid – model 2.

5.2. Comparison of models with various combination of parasitic resistances

The second simulation presented in the figure 10 refers to bode characteristic of a control-to-output transfer function of a BUCK converter. This simulation shows differences in models including various parasitic resistances.

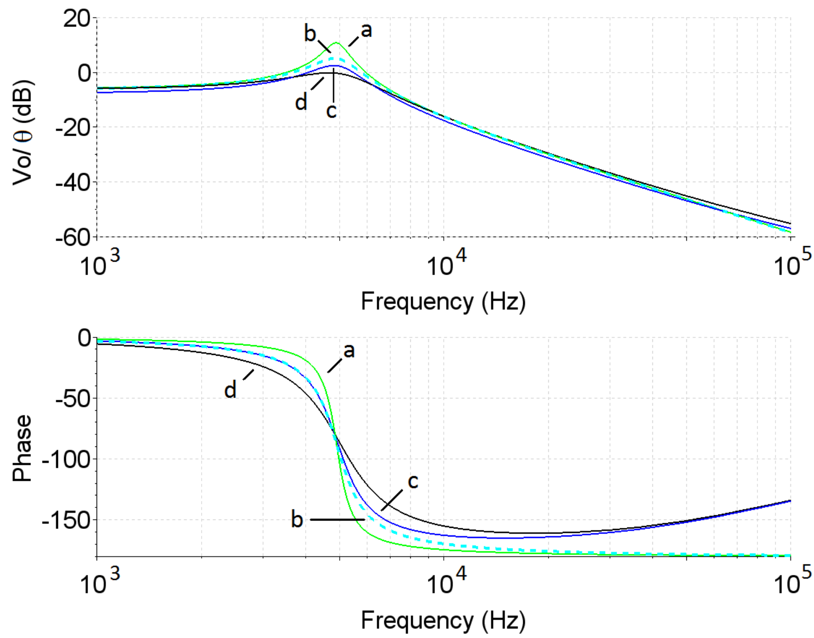

Fig. 10. Control-to-output transfer function of a BUCK converter: a) ideal converter; b) parasitic resistances of coil and capacitor only, c) all parasitic resistances and a dynamic resistance of a diode with voltage offset; d) all parasitic resistances and a static resistance of a diode

It can be seen from fig 10 that simulation of ideal converter (fig. 10a) differs even from the model which does not include all of parasitic resistances (fig.10b). Moreover it can be seen that in this particular case amplitudes of plots 'b' and 'c' are similar, but of course it does not mean that they are always going to be similar. Main differences between those two models can be seen in phase diagram. The plot in fig. 10d refers to a situation where static value of diode resistance has been used. Differences between models with static and dynamic resistance of a diode are large because of the differences between those resistances, which was mentioned at the beginning of this paper.

Regardless differences between those two resistances, there is a much more interesting feature. From phase chart in figure 10 one can notice a big differences between models which do, and do not include parasitic resistances of transistor and diode. Those differences appear in higher frequencies, nevertheless this simulation shows that parasitic resistances of switches need to be considered in the model of a converter.

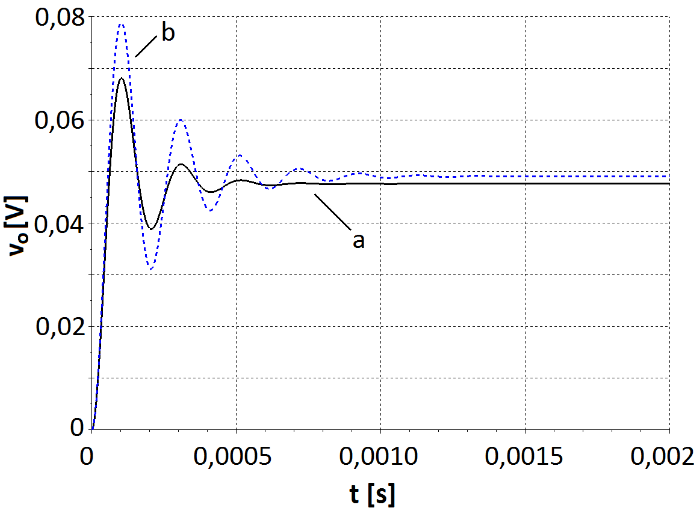

Fig. 11. Output voltage response to step change of input voltage by 1V according to model including: a) static resistance, b) dynamic resistance of a diode

From the table 1 one can read that value of static resistance of a diode is not only a few times bigger than the value of dynamic resistance, but it is also the biggest parasitic resistance in the whole circuit of a converter. It means that any change of this resistance will impact the characteristics of a converter. This can be seen in figure 11 where line-to-output (4) transfer function of BUCK converter was used to simulate transient response for step change of input signal. This transfer function is not different from the one presented in other papers [4][5]. The only exception is the value of diode resistance. In this example first static and later dynamic resistances have been used to calculate the numerator and the denominator of transfer function. The reader can see the difference in amplitude, time of oscillations and constant value of output voltage.

Conclusions

The extended models of two common transfer functions of a converter have been presented. The models include additional voltage source referring to the constant value of diode voltage, which appears after linearization of diode characteristic. Presented model of line-to-output transfer function is equivalent with model obtained with a different method [12]. The difference appears in the control-to-output transfer function. Simulations show that differences between those transfer functions increase when the input voltage is getting lower. When the input voltage is much higher than the offset voltage of a diode V_D, then the differences can be neglected. Moreover presented models show that the influence of the voltage offset is visible only with the control-to-output transfer function. Nevertheless regarding the simulations differences between using static and dynamic resistance of a diode are noticeable in all types of transfer functions.

Presented equations can be easily implemented in a mathematical program such as Scilab or Matlab. To the knowledge of the author all equations of transfer functions presented in this work hadn't been presented before and thus are original extension of models presented in [4][5]. Moreover all models can be very easily modified to use with value of static diode resistance which makes them more practical and universal.

References

1. E.Dijk, H.J.N.Spruijt, D.M. O'Sullivan, J.B. Klaassens, *PWM-switch modeling of DC-DC converters*, IEEE Transactions On Power Electronics, Vol. 10, No.6, November (1995)
2. V.Vorperian, *Simplified Analysis of PWM converters using model of PWM switch part I: continuous conduction mode*, IEEE Transactions On Aerospace And Electronic Systems Vol. 26, No. 3 May (1990)
3. R.D.Middlebrook, S.Cuk, *A general unified approach to modeling switching-converter power stages*, IEEE Power Electronics Specialists Conference, Cleveland, OH., June 8-10 (1976)
4. W.Janke, *Equivalent circuits for averaged description of DC-DC switch-mode power converters based on separation of variables approach*, Bulletin Of The Polish Academy Of Sciences Technical Sciences, Vol. 61, No. 3, pp. 711-723 (2013)
5. W.Janke, *Averaged models of pulse-modulated DC-DC power converters. Part II: Models based on the separation of variables*, Archives Of Electrical Engineering, Vol. 61(4), pp. 633-654 (2012)
6. V.Biolkova, Z.Kolka, D.Biolek, S*tate-space averaging (SSA) revisited: on the accuracy of SSA-based line-to-output frequency responses of switched DC-DC converters*, WSEAS Transactions On Circuits And Systems , Issue 2, Vol. 9, February (2010)
7. V.Jha, P.Rai, *State space averaged modeling of basic converter topologies*, VSRD International Journal of Electrical, Electronics & Communication Engineering, Vol. 2 (6), 566-575 (2012)
8. C.T.Rim, G.B.Joung, G.H.Cho, *Practical switch based state-space modeling of DC-DC converters with all parasitics*, IEEE Transactions On Power Electronics, Vol. 6, No. 4, October (1991)

9. M.Walczak, *Symulacje charakterystyk wejściowych i wyjściowych impulsowych przetwornic napięcia stałego w trybie CCM*, XIV International Phd Workshop OWD, 20-23 October (2012)
10. M.Jinno, P.Y.Chen, Y.C.Lai, K. Harada, *Investigation on the ripple voltage and the stability of SR BUCK converters witch high output current and low output voltage*, IEEE Transactions On Industrial Electronics, Vol. 57. No 3. March (2010)
11. R.W.Erickson, D.Maksimovic, *Fundamentals of power electronics*, second edition, Sixth printing (2004)
12. M.K.Kazimierczuk, *Pulse-width modulated DC–DC power converters*, J.Wiley (2008)
13. W.Janke, *Impulsowe przetwornica napięcia stałego*, Wydawnictwo uczelniane Politechniki Koszalińskiej, Monografia nr 269, Koszalin (2014)

Abstract

Various models of DC-DC converters have been presented in many references. This paper provides extended version of a common transfer functions (line-to-output, control-to-output) of three most popular DC-DC converters BUCK, BOOST and BUCK-BOOST. The extension includes all parasitic resistances and additional voltage source created after linearization of diode characteristic.

Streszczenie

W dostępnej literaturze istnieje wiele modeli przetwornic napięcia stałego. Modele te różnią się między innymi rodzajem uwzględnianych rezystancji pasożytniczych. Niniejszy artykuł zawiera modele przetwornic napięcia stałego BUCK, BOOST oraz BUCK-BOOST, wyprowadzone przy pomocy techniki separacji zmiennych. Modele te uwzględniają zarówno rezystancje pasożytnicze wszystkich elementów przetwornicy jak i dodatkowe źródło napięcia powstałe po linearyzacji charakterystyki diody.

Słowa kluczowe: BUCK, BOOST, BUCK-BOOST, rezystancje pasożytnicze w przetwornicach napięcia stałego, modele małosygnałowe, przetwornice napięcia stałego