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ONE APPROACH TOWARDS ANALYTICAL DETERMINATION OF THE CNC MACHINE TOOL SERVO DRIVES POSITION LOOP GAIN

One of the most important factors which influences the dynamical behaviour of the servo drives for CNC machine tools is position loop gain or Kv-factor. It directly influences the contouring accuracy of the machine tool. Usually position loop gain is experimentally tuned on the already assembled CNC machine tool. This paper gives one approach towards its analytical calculation. The difference between analytical calculated and experimentally obtained Kv-factor is smaller than 5%, which is completely acceptable.

1. INTRODUCTION

The most important variable, which describes the behavior of a position control loop for CNC machine tools servo drives, is position loop gain or Kv-factor. This is the ratio of the command velocity (feed rate) v to the position control deviation (following error, tracking error, lag) Δx [2,7,8]

$$\mathrm{Kv}[\mathrm{s}^{-1}] = \frac{\mathrm{v}[\mathrm{mm/s}]}{\Delta \mathrm{x}[\mathrm{mm}]} \qquad \text{or} \qquad \mathrm{Kv}\left[\frac{\mathrm{m/min}}{\mathrm{mm}}\right] = \frac{\mathrm{v}[\mathrm{m/min}]}{\Delta \mathrm{x}[\mathrm{mm}]} \tag{1}$$

$$\mathrm{Kv}[\mathrm{s}^{-1}] = \frac{1000}{60} \cdot \mathrm{Kv}\left[\frac{\mathrm{m/min}}{\mathrm{mm}}\right] \tag{2}$$

From the magnitude of the Kv-factor depends tracking or following error. In multi-axis contouring the following errors along the different axes may cause form deviations of the machined contours. Generally position loop gain Kv should be high for faster system response and higher accuracy, but the maximum gains allowable are limited due to undesirable oscillatory responses at high gains and low damping factor.

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Usually Kv factor is experimentally tuned on the already assembled machine tool [2,8].

This paper presents approach for analytically calculation of the position loop gain Kv. A combined 6-th order digital-analog model of the position loop is presented. In order to ease the calculation, the 6-th order system is simplified with a second order model. With this approach it is very easy to calculate the Kv factor for necessary position loop damping. The difference of the replacement of the 6-th order system with second order system is presented with the simulation program MATLAB. Analytically calculated Kv factor is function of the nominal angular frequency ω and damping D of the servo drive electrical parts (motor and regulator), nominal angular frequency ω_m and damping D_m of the mechanical transmission elements, as well as sampling period T.

2. COMBINED DIGITAL-ANALOG MODEL OF THE SERVO DRIVE POSITION CONTROL LOOP AND ANALYTICAL CALCULATION OF THE Kv FACTOR

Fig. 1 presents digital-analog model of the CNC machine tool servo drive position control loop, where s represents Laplace operator.

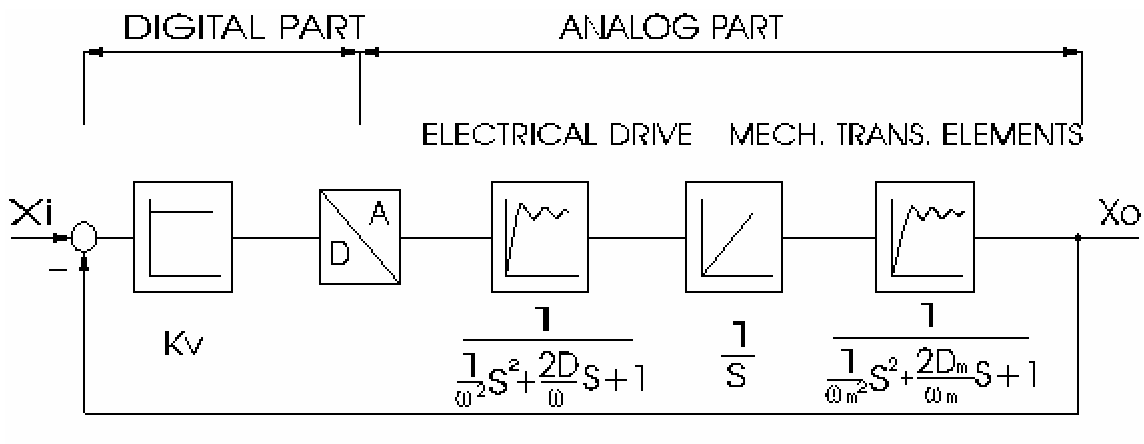

Fig. 1. Combined digital-analog model of the servo drive position control loop

Similar models are presented in [2,4,6], but the transfer function of mechanical transmission elements is not taken in consideration. Because of the existence of the digital part in the presented model we must use z-transformation for analysis. With some approximations and substitutions it is possible to analyze presented model in s-domain (with Laplace transformation). Digital-analog converter is substituted with zero order holder (z.o.h.) and sampler [3]. The new model is presented in Fig.2.

According [3] we can approximate sampler and zero order holder (z.o.h.) in Laplace domain with the following transfer function:

$$G(s) = \frac{1 - e^{-T \cdot s}}{T \cdot s} \tag{3}$$

With the Padè approximation of the first order for the $e^{-T \cdot s}$ we get:

$$e^{-T \cdot s} \approx \frac{1 - \frac{T}{2} \cdot s}{1 + \frac{T}{2} \cdot s} \tag{4}$$

where T is sampling time (period).

In that case G(s) becomes:

$$G(s) = \frac{1 - e^{-T \cdot s}}{T \cdot s} \approx \frac{1}{1 + \frac{T}{2} \cdot s} \tag{5}$$

With these simplifications servo drive position control loop may be presented with following model (Fig. 3).

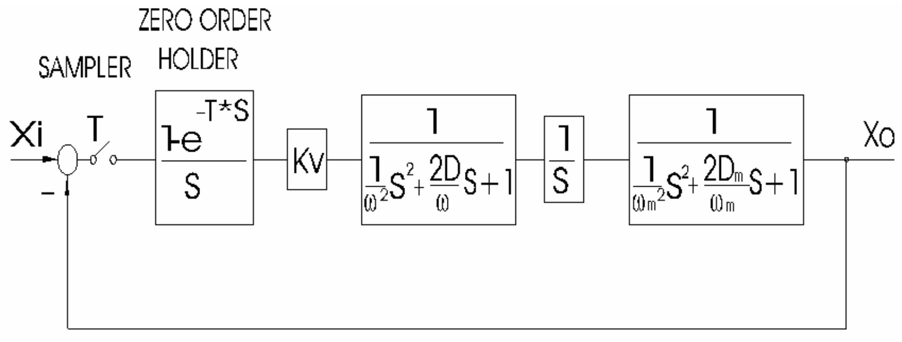

Fig. 2. Modified model of the servo drive position control loop presented in Fig. 1

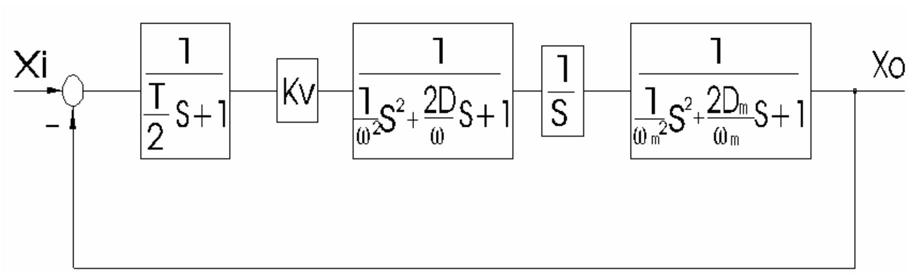

Fig. 3. Analog model of the servo drive position control loop

The model in Fig. 3 may be analyzed in s-domain with Laplace transformation. The transfer function of the servo drive position control loop presented in Fig. 3 is:

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}\cdot\frac{1}{\frac{1}{\omega_m{}^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1}}{1+Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}\cdot\frac{1}{\frac{1}{\omega_m{}^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1}} \tag{6}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{T}{2}\cdot s+1\right)\cdot\left(\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1\right)\cdot\left(\frac{1}{s}\right)\cdot\left(\frac{1}{\omega_m{}^2}\cdot s^2+\frac{2D_m}{\omega_m}\cdot s+1\right)+Kv} \tag{7}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_6\cdot s^6+a_5\cdot s^5+a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s^1+a_0} \tag{8}$$

where:

$$a_6=\frac{T}{2\omega^2\omega_m^2},\ a_5=\left[\frac{1}{\omega^2\omega_m^2}+\left(\frac{2D_m}{\omega^2\omega_m}+\frac{2D}{\omega\omega_m^2}\right)\cdot\frac{T}{2}\right],$$

$$a_4=\left[\left(\frac{2D_m}{\omega^2\omega_m}+\frac{2D}{\omega\omega_m^2}\right)+\left(\frac{1}{\omega^2}+\frac{4DD_m}{\omega\omega_m}+\frac{1}{\omega_m^2}\right)\cdot\frac{T}{2}\right],$$

$$a_3=\left[\left(\frac{1}{\omega^2}+\frac{4DD_m}{\omega\omega_m}+\frac{1}{\omega_m^2}\right)+\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}\right)\cdot\frac{T}{2}\right],\ a_2=\left[\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}\right)+\frac{T}{2}\right], a_1=1,$$

$a_0=Kv$ and $b_0=Kv$.

Having information's about the magnitude of the variables ω, D, ω_m, D_m and T in real servo drive position control loops, we can conclude that a_6, a_5, a_4, a_3 tends towards zero ($a_6, a_5, a_4, a_3 \rightarrow 0$). So in that case we can simplify 6-th order system with the second order system [1,2]. We will present servo drive position control loop with the simplified transfer function:

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_2\cdot s^2+a_1\cdot s^1+a_0} \tag{9}$$

where $a_2 = \left[\left(\frac{2D}{\omega} + \frac{2D_m}{\omega_m}\right) + \frac{T}{2}\right]$, $a_1 = 1$, $a_0 = Kv$ and $b_0 = Kv$.

In that case

$$\frac{Xo(s)}{Xi(s)} = \frac{Kv}{\left(\frac{2D}{\omega} + \frac{2D_m}{\omega_m} + \frac{T}{2}\right) \cdot s^2 + s + Kv} \tag{10}$$

To check if it is correct to substitute 6-th order with second order system, we will simulate the system transfer function response on step function with simulation program MATLAB. Numerical values of the parameters of the examined system are:
$\omega = 1000\,s^{-1}$, D=0.7, $\omega_m = 663\,s^{-1}$, $D_m = 0.17$, T=0.006 s and Kv=$100\,s^{-1}$.

Following transfer function will be compared:

-6-th order system transfer function

$$\frac{Xo(s)}{Xi(s)} = \frac{100}{6.82 \cdot 10^{-15} \cdot s^6 + 1.33 \cdot 10^{-11} \cdot s^5 + 1.57 \cdot 10^{-8} \cdot s^4 + 9.72 \cdot 10^{-6} \cdot s^3 + 4.91 \cdot 10^{-3} \cdot s^2 + s + 100}$$

-second order system transfer function

$$\frac{Xo(s)}{Xi(s)} = \frac{100}{4.91 \cdot 10^{-3} \cdot s^2 + s + 100}$$

Fig. 4 gives responses of the position control loop transfer function of 6-th and 2-nd order on step function.

From Fig. 4 it is obvious that the differences caused by substitution are minimal. It makes substitution completely acceptable. For the second order system it is possible very easy and fast to calculate Kv-factor for necessary position control loop damping.

We can write the second order system transfer function in the following form:

$$\frac{Xo(s)}{Xi(s)} = \frac{1}{\frac{1}{\omega_n^2} \cdot s^2 + \frac{2\zeta}{\omega_n} \cdot s + 1} \tag{11}$$

where ζ is position control loop damping ($0 < \zeta < 1$), and ω_n is nominal angular frequency of the position control loop. We will transform equation (10) in the form of equation (11).

$$\frac{Xo(s)}{Xi(s)} = \frac{Kv}{\left(\frac{2D}{\omega} + \frac{2D_m}{\omega_m} + \frac{T}{2}\right) \cdot s^2 + s + Kv} = \frac{1}{\frac{\left(\frac{2D}{\omega} + \frac{2D_m}{\omega_m} + \frac{T}{2}\right)}{Kv} \cdot s^2 + \frac{1}{Kv} \cdot s + 1} \tag{12}$$

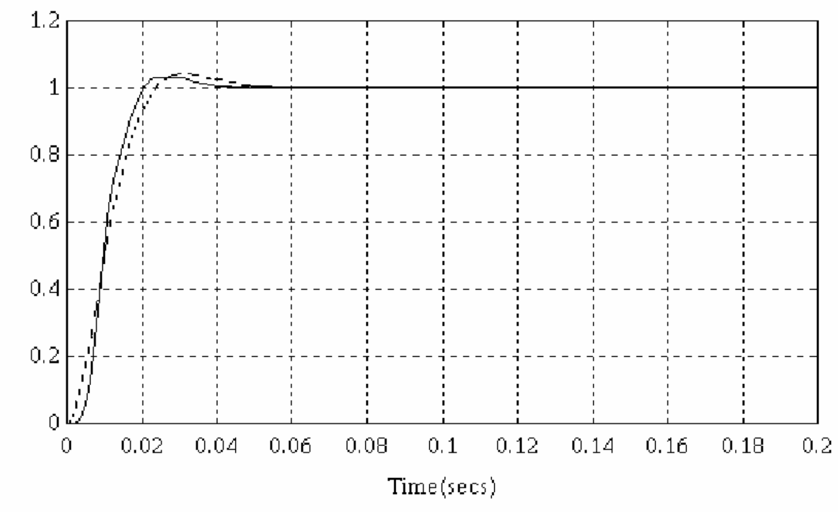

Fig. 4. (--------) Time response of the 6-th order system
(- - - - -) Time response of the 2-nd order system

Comparing (11) and (12) we can obtain:

$$\omega_n = \sqrt{\frac{Kv}{\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}}} \quad \text{and} \quad \zeta = \frac{1}{2}\cdot\sqrt{\frac{1}{Kv\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)}} \tag{13}$$

In order to have required position control loop damping ζ, Kv-factor should be calculated with the following equation:

$$Kv = \frac{1}{4\zeta^2\left(\frac{2D}{\omega}+\frac{2D_m}{\omega_m}+\frac{T}{2}\right)} \tag{14}$$

Equation (14) gives direct analytical relationship between Kv-factor and ω, D, ω_m, D_m, T and ζ, which are already known variables, or can be calculated very easy.

With the equation (14) it is possible to estimate CNC machine tool servo drive position loop gain Kv without performing experiments.

We will check correctness of the equation (14) on real servo drive position control loop of CNC milling machine FGS 32-CNC. Position loop damping ζ=0.7 is preferable according [2,6,8]. That is the value, which gives minimal contouring errors. Other numerical values of the examined system are: ω=1000 s^{-1}, D=0.7, ω_m=663 s^{-1}, D_m=0.17 and T=0.006 s. With the substitution in the equation (14) the position loop gain value Kv=103.85 s^{-1} is calculated. Experimentally tuned value of Kv-factor on examined machine tool axis was Kv=100 s^{-1}. The difference between analytically calculated and experimentally obtained value of Kv-factor is around 4%, which is completely acceptable.

4. CONCLUSION

The equation (14) enables very fast, simple and precise analytical calculation of position loop gain Kv as a function of already known position control loop parameters (ω-nominal angular frequency of the servo drive electrical parts, D-damping of the servo drive electrical parts, ω_m-nominal angular frequency of the mechanical transmission elements, D_m-damping of the mechanical transmission elements and T-sampling time). In that way we can avoid long-time experimental tuning of the Kv-factor on machine tool. And of course analytical calculation of the Kv factor gives possibility to estimate the accuracy of the system in the design phase.

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