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Load analysis of a patellofemoral joint by a quadriceps muscle

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Purpose: The aim of this paper is to develop a model of the patellofemoral joint by considering the linear displacement along axis of cylindrical joint and to use this model in the analysis of the femur spatial displacements caused by the quadriceps muscle force. *Method*: The linear displacement along the axis of cylindrical joint of the patellofemoral joint is computed using optimization methods – minimization of the difference between the modeled and measured spatial displacements of the femur with respect to the tibia over the full range of the knee flexion. Then, the instantaneous screw displacements of the femur with regard to the tibia and corresponding muscle forces are computed for the model developed. The moment of the force arm with respect to the vector of screw displacement is used to evaluate the effectiveness of the acting force. *Results*: The simulation results for the model developed show significant improvement of the modeled linear coordinates of the femur reference system with respect to tibia reference system. The displacement analysis of the femur loaded by quadriceps muscle force can be used to describe the patellofemoral dislocation problem. *Conclusions*: The model of the patella-femur joint where the linear displacement along axis of the cylindrical joint is considered can reproduce the actual patella displacements more accurately. It seems expedient to study elasto-statics problem of this mechanism. The model can be used to study some medical conditions such as patellofemoral dislocation.

Key words: *tibiofemoral joint, vector method, cylindrical joint*

1. Introduction

The patellofemoral joint is an important part of the knee joint as it transmits a muscle force that drives the knee. The knee joint allows the relative motion between three bones, i.e., the tibia, femur and patella. Mechanical model of this joint can be used by a surgeon in planning certain operations, by an orthopedist for diagnosis and rehabilitation purposes or for sport biomechanics. Using the identified model of the knee it could be possible to analyze the relative displacements and the loads of the joint elements during physical activities such as riding on the bicycle [14].

Selected models of the patellofemoral joint with two or three flexible point contact pairs have been introduced [2]. However, a set of several differential equations has to be solved in order to analyze the model. A different approach for the modeling of the knee joint has been shown in [10]–[12], where the knee is considered as a parallel platform mechanism. This method will be further explored in this paper.

The patellofemoral joint and the tibiofemoral joint cooperate as one complex, parallel mechanism – the knee joint. In this paper, the patellofemoral joint is assumed as an active leg added to the 5-5 parallel platform mechanism of the femur-tibia joint presented in [10] (Fig. 1). The considered patellofemoral joint is

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composed of three parts: a patella, a patellar tendon and a muscle system. Several years ago cylindrical model of the patellofemoral joint was presented (Fig. 1) [11]. However, a linear displacement along the cylindrical joint axis has not been considered. In the actual joint the linear displacement is allowed and coupled with angular displacement.

The aim of this work is to further complete the existing model of the patellofemoral joint presented in [11] and to analyze the load exerted on the femur by the quadriceps muscle.

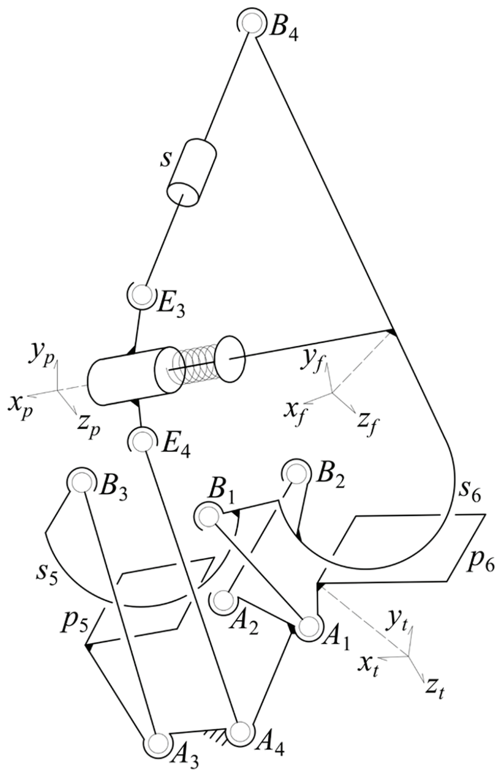

Fig. 1. The spatial 5-5 model of the tibiofemoral joint with the patellofemoral joint and the muscle, where: A_1B_1, A_2B_2 – the cruciate ligaments, A_3B_3 – the medial collateral ligament, A_4 (E_4) – the patellar tendon attachment to the tibia (the patella), s – the length of the substitute muscle, E_3 (B_4) – the substitute muscle attachment to the patella (the femur); $\{x_t\, y_t\, z_t\}$ – the tibia reference frame, $\{x_f\, y_f\, z_f\}$ – the femur reference frame, $\{x_p\, y_p\, z_p\}$ – the patella reference frame

In comparison with parallel platform mechanisms (see Fig. 1) known from robotics, the model of the knee joint under consideration is characterized by two nonsymmetrical platforms (the femur and the tibia), connected with two point contact pairs (plane-sphere, where planes correspond to the tibial condyles and spheres correspond to the femoral condyles) and four legs: three passive legs S-S type (the cruciate ligaments and the collateral ligament with S – the spherical joints) and one active leg S-S-C type (the patellar tendon and the patella with C – the cylindrical joint). The mechanism considered operates in a limited workspace of one configuration, relatively distant from singular positions.

The substitute linear actuator represents the quadriceps muscle of the knee. Therefore, only the extension of the knee can be studied by using the present model. However, it is possible to add the hamstring muscles to the model and study the flexion but these muscles are not considered here. It is also assumed that the tibia, the femur and the patella are treated as rigid bodies and the lengths of ligaments are constant. The only flexible deformation is described as a linear displacement in the cylindrical joint. The knee joint is considered in the normal range of flexion (deep flexion is not taken into account).

The problem solved in this paper is formulated as follows: for a given set of tibiofemoral displacements and parameters of the cylindrical patellofemoral joint model find the linear displacement along the cylindrical joint axis that minimizes the difference between the model displacements and that of the actual joint. Then, determine the extension force moment arm and the angle between the muscle force and the instantaneous screw axis of the femur displacement with respect to the tibia.

2. Method

The displacement analysis of the patellofemoral joint requires a set of relative tibiofemoral positions that can be obtained from the experimental data or from numerical simulations using the tibiofemoral joint model.

In this paper, a 5-5 parallel tibiofemoral joint mechanism is analyzed (Fig. 1). The solution for the position problem of this mechanism is obtained using the constraint equation method that leads to a set of 5 nonlinear equations solved numerically, as in [10]. The relative tibiofemoral displacements obtained can be presented as functions: $\beta(\alpha)$, $\gamma(\alpha)$, $p_x(\alpha)$, $p_y(\alpha)$, $p_z(\alpha)$, where α, β, γ – angular displacements that correspond to the flexion(+)/extension(–), the adduction(+)/abduction(–) and the external(+)/internal(–) rotation of the femur with regard to the tibia and p_x, p_y, p_z – the translational displacements of the femur reference frame origin with regard to the tibia reference frame origin. A sequence of rotations (for the right knee) is assumed after [11] and presented earlier in [7]

$$\boldsymbol{R}_{ft} = \begin{bmatrix} c\alpha c\gamma + s\alpha s\beta s\gamma & -s\alpha c\gamma + c\alpha s\beta s\gamma & -c\beta s\gamma \\ s\alpha c\beta & c\alpha c\beta & s\beta \\ c\alpha s\gamma - s\alpha s\gamma - s\alpha s\beta c\gamma & -s\alpha s\gamma - c\alpha s\beta c\gamma & c\beta c\gamma \end{bmatrix}, \quad \boldsymbol{p}_{ft} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}, \tag{1}$$

where $\boldsymbol{R}_{ft}$ – the rotation matrix from the femur reference frame to the tibia reference frame, $\boldsymbol{p}_{ft}$ – the position vector of the femur reference frame with regard to the tibia reference frame, $s\alpha = \sin\alpha$, $c\alpha = \cos\alpha$. The flexion α is an angular displacement about the z_f axis of the femur reference frame (the z_f is orthogonal to the sagittal plane of the femur), the internal rotation γ occurs about the y_t axis of the tibia reference frame (the y_t is orthogonal to the transverse plane of the tibia), while the abduction β is an angular displacement about a floating axis that is perpendicular to the z_f and the y_t [11].

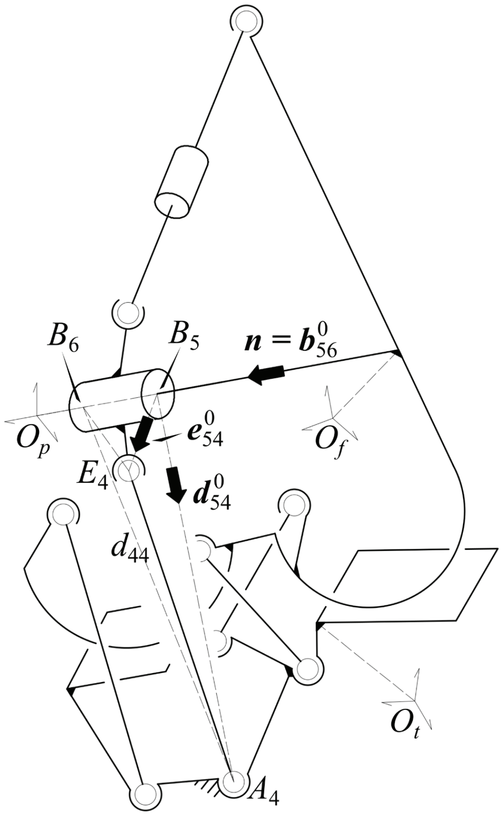

Fig. 2. The spatial 5-5 model of the tibiofemoral joint with the patellofemoral joint and the muscle, the vector tetrahedron describing the patellofemoral joint

Then, the position problem of the patellofemoral joint model with fixed linear displacement is solved using the vector method (yielding a closed-form solution). The mechanism considered can be described with one vector tetrahedron as shown in Fig. 2. In order to obtain the position of the patella reference frame with respect to the tibia reference frame a unit vector $\boldsymbol{e}_{54}^o$ describing the patella orientation can be determined using the formula for finding one of three unit vectors, when two unit vectors and two dot products of each of these vectors with the unknown unit vector are known [3]. The unknown unit vector $\boldsymbol{e}_{54}^o$ is determined using two known unit vectors $\boldsymbol{b}_{56}^o$, $\boldsymbol{d}_{54}^o$ and three known dot products $c_1 = \boldsymbol{b}_{56}^o \cdot \boldsymbol{d}_{54}^o$, $c_2 = \boldsymbol{b}_{56}^o \cdot \boldsymbol{e}_{54}^o$, $c_3 = \boldsymbol{d}_{56}^o \cdot \boldsymbol{e}_{54}^o$

$$\boldsymbol{e}_{54}^o = [(c_2 - c_1 c_3)\boldsymbol{b}_{56}^o + (c_1 - c_1 c_3)\boldsymbol{d}_{54}^o \pm \sqrt{D}(\boldsymbol{b}_{56}^o \times \boldsymbol{d}_{54}^o)]/(1 - c_1^2), \tag{2}$$

where $D = 1 - c_1^2 - c_2^2 - c_3^2 + 2c_1c_2c_3$.

In the next step, Horn's method [8] is used to obtain the transformation matrix from the patella reference frame to the femur reference frame. Then, the obtained results are presented as functions: $\alpha^p(\alpha)$, $\beta^p(\alpha)$, $\gamma^p(\alpha)$, $p_x^p(\alpha)$, $p_y^p(\alpha)$, $p_z^p(\alpha)$, where: α^p, β^p, γ^p – angular displacements that correspond to the flexion, the abduction and the external rotation of the patella with regard to the femur (the aforementioned convention is used [7], [11]) and p_x^p, p_y^p, p_z^p are components of the translational displacements of the patella reference frame origin with regard to the femur reference frame origin.

Now, if the relative positions of the tibia and the femur and a solution for the respective positions of the simplified patellofemoral joint are known, then linear displacement along the cylindrical joint axis can be considered.

2.1. The linear displacement along the cylindrical joint axis

The additional linear displacement along the cylindrical joint axis is considered as a passive degree of freedom. This additional degree of freedom results from elastic deformation of the flexible elements (bursae, ligaments). One way to determine the value of this displacement is to solve the elasto-statics problem of the mechanism. However, it is also possible to search for this displacement using optimization methods – minimization of the difference between the model and the actual joint spatial displacements over the full range of the knee flexion. In this case, the linear displacement will be presented as a function of the flexion angle of the knee α – the independent geometrical variable in the joint. The latter method is considered in this paper.

The linear displacement x along the cylindrical joint axis of the patellofemoral joint can be considered by assuming that the position vector of the point B_5 is dependent on x, as follows:

$$\boldsymbol{b}_5^f = \boldsymbol{b}_{5s}^f + x\boldsymbol{n}^f, \tag{3}$$

where $\boldsymbol{b}_{5s}^f$ – the starting position vector of the point B_5 in the femur reference frame (obtained from the previous parameter estimation), $\boldsymbol{b}_5^f$ – the current position vector of the point B_5 in the femur reference frame, x – the linear displacement along the axis of the cylindrical joint, $\boldsymbol{n}^f$ – the unit vector of the cylindrical axis described in the femur reference frame.

In Fig. 1, the patella is presented as a rigid body with the coordinate system assigned. If the position and the orientation of this coordinate system are computed then a difference between the actual patella displacements and that of the model can be obtained. In order to calculate this difference, at a selected flexion angle α, six relative indicators are computed. Each indicator can be calculated as follows

$$\Delta k = \frac{|k_s(\alpha_i) - k_r(\alpha_i)|}{z_k}, \tag{4}$$

where Δk – indicator of $k \in \{\alpha^p, \beta^p, \gamma^p, p_x^p, p_y^p, p_z^p\}$; $k_s(\alpha_i)$ – the value of k obtained using the 5-5 tibiofemoral joint model at α_i (α is the flexion angle; here: $\alpha_i \in \langle 6.00°; 102.00° \rangle$, $\Delta\alpha_i = 12.00°$); $k_r(\alpha_i)$– the measured value of k at α_i, z_k – the range of k obtained from the experimental data.

The objective function that computes the difference between the corresponding displacements of the model and that of the actual joint at a selected value of the flexion angle α, can be written as follows

$$h = (w_1\Delta\alpha^p + w_2\Delta\beta^p + w_3\Delta\gamma^p$$

$$+ w_4\Delta p_x^p + w_5\Delta p_y^p + w_6\Delta p_z^p) + g, \tag{5}$$

where w_i – the weight factor of the respective indicator (here, $w_i = 1$ [$i = 1..6$]), g – the penalty function (assumes a large value if the solution is not real).

The next step involves a numerical search for a value of the linear displacement x (see equation (3)) at a selected value of the flexion angle α using the objective function defined in equation (5). This numerical search has to be performed several times (here, 9) in order to obtain the values of the linear displacement x over the assumed range of the flexion angle α (here, $\alpha_i \in \langle 6.00°; 102.00° \rangle$, $\Delta\alpha_i = 12.00°$). In this paper, the estimation of x has been performed using the Nedler–Mead method.

The result of this procedure is the linear displacement x as a discrete function of the flexion angle α that enables the cylindrical model to reproduce the actual patella displacements more accurately.

2.2. The quadriceps muscle force

It is notable that in the actual patellofemoral joint there are few forces acting on it. The analysis of reaction forces seems expedient but very difficult, due to the complex nature of the mechanism. On the other hand, analysis of the quadriceps muscle force considered as the only external force acting on the mechanism is simple and can result in interesting conclusions. In this case, two parameters are studied. The

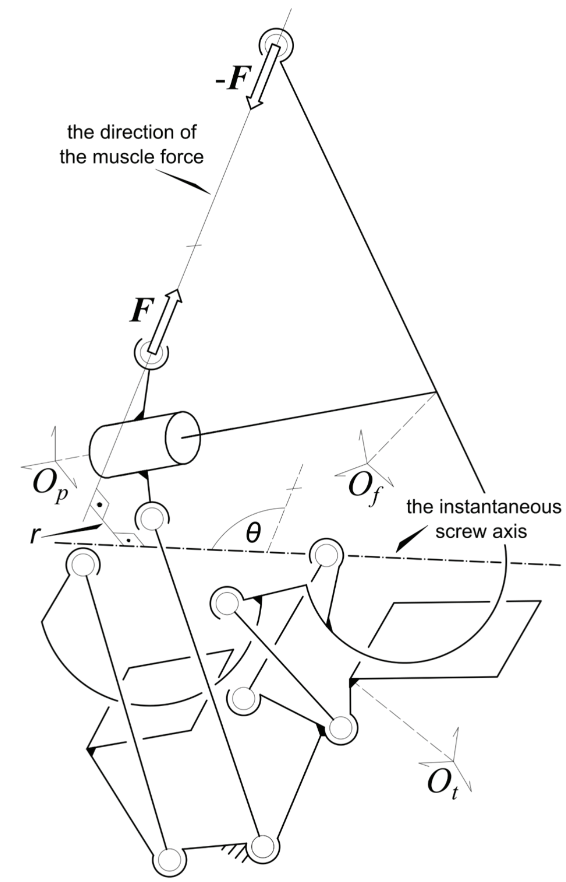

Fig. 3. The spatial 5-5 model of the tibiofemoral joint with the patellofemoral joint and the muscle, where $\boldsymbol{F}$ – the quadriceps muscle force, θ – the angle between the direction of the muscle force and the instantaneous screw axis, r – the arm the of quadriceps force moment

first one is the arm r of the muscle force moment computed as a distance between the vector of muscle force and the axis of instantaneous screw displacement of the femur with respect to the tibia using a procedure described in [16] (see Fig. 3). The arm r of the muscle force moment is computed for selected values of the flexion angle α. The other parameter is the angle θ between the vector of the muscle force and the axis of the vector of the screw displacement (Fig. 3) that can be computed from the dot product of these two vectors.

A spatial displacement of a rigid body (a bone) can be completely specified by four screw parameters: $\boldsymbol{p}_s$ – a position vector of a point located on a screw axis, $\boldsymbol{s}^o$ – a unit vector that defines the screw axis direction, d – a linear displacement along the screw axis, ϕ – an angular displacement about the screw axis. These parameters can be computed if two consecutive locations of a rigid body are known. In this paper, a method by Bottema and Roth is used [5].

For a given set of three initial and final position vectors of points B_1, B_2 and B_3 located on the femur and described in the tibia reference frame ($\boldsymbol{b}_{1i}$, $\mathbf{b}_{2i}$, $\boldsymbol{b}_{3i}$ and $\boldsymbol{b}_{1f}$, $\mathbf{b}_{2f}$, $\boldsymbol{b}_{3f}$), the angular displacement ϕ and the unit vector $\boldsymbol{s}^0$ can be obtained from [5]

$$\tan\left(\frac{\phi}{2}\right)s^0 = \frac{[(\boldsymbol{b}_{3f}-\boldsymbol{b}_{2f})-(\boldsymbol{b}_{3i}-\boldsymbol{b}_{2i})]\times[(\boldsymbol{b}_{1f}-\boldsymbol{b}_{2f})-(\boldsymbol{b}_{1i}-\boldsymbol{b}_{2i})]}{[(\boldsymbol{b}_{3f}-\boldsymbol{b}_{2f})-(\boldsymbol{b}_{3i}-\boldsymbol{b}_{2i})]\cdot[(\boldsymbol{b}_{1f}-\boldsymbol{b}_{2f})+(\boldsymbol{b}_{1i}-\boldsymbol{b}_{2i})]}. \quad (6)$$

Then, the position vector $\boldsymbol{p}_s$ and the linear displacement d are computed as follows [5]

$$\boldsymbol{p}_s = \frac{1}{2}\left[\boldsymbol{b}_{1i}+\boldsymbol{b}_{1f} + \left(\boldsymbol{s}^o\times(\boldsymbol{b}_{1f}-\boldsymbol{b}_{1i})/\tan\left(\frac{\phi}{2}\right)\right) - \boldsymbol{s}^o\cdot(\boldsymbol{b}_{1f}+\boldsymbol{b}_{1i})\boldsymbol{s}^o\right], \quad (7)$$

$$d = \boldsymbol{s}^o\cdot(\boldsymbol{b}_{1f}-\boldsymbol{b}_{1i}).$$

3. Results

Input data set for the position analysis contains: $\boldsymbol{b}_5^f$ ($\boldsymbol{b}_5^p$), $\boldsymbol{b}_6^f$ ($\boldsymbol{b}_6^p$) – the position vectors of the points B_5 and B_6 on the cylindrical joint axis in the femur (the patella) reference frame, $\boldsymbol{a}_4$ – the position vector of the point A_4 – the center of the patellar tendon attachment to the tibia given in the tibia reference frame, $\boldsymbol{e}_4^p$ – the position vector of the point E_4 – the center of the patellar tendon attachment to the patella described in the patella reference frame, d_{44} – the length of the patellar tendon. The assumed 19 parameters are listed below (the coordinates of position vectors and d_{44} are expressed in [mm])

$$\boldsymbol{a}_4 = \begin{bmatrix} 24.88 \\ -25.10 \\ 10.20 \end{bmatrix}, \quad \boldsymbol{b}_5^f = \begin{bmatrix} 7.63 \\ 17.38 \\ 0.00 \end{bmatrix}, \quad \boldsymbol{b}_5^p = \begin{bmatrix} -44.99 \\ 9.77 \\ 1.60 \end{bmatrix},$$

$$\boldsymbol{b}_6^f = \begin{bmatrix} 7.53 \\ 17.60 \\ -1.70 \end{bmatrix}, \quad \boldsymbol{b}_6^p = \begin{bmatrix} -45.48 \\ 10.18 \\ 0.00 \end{bmatrix},$$

$$\boldsymbol{e}_4^p = \begin{bmatrix} 2.10 \\ -23.35 \\ -5.15 \end{bmatrix}, \quad d_{44} = 39.87.$$

The parameter set has been estimated using the experimental data and the initial estimates presented in [11]. The input data set also contains a set of the rotation matrices $\boldsymbol{R}_{ft}(\alpha, \beta, \gamma)$, the translation vectors $p_{ft} = [p_x, p_y, p_z]^T$ from the femur reference frame $\{x_f\, y_f\, z_f\}$ to the tibia reference frame $\{x_t\, y_t\, z_t\}$ obtained from the tibiofemoral joint model.

3.1. The linear displacement along the cylindrical joint axis

It is worth noting that the difference between the experimental data and the values obtained from the

Table 1. Simulation results: MaxDiff – maximum; MeanDiff – mean difference between results obtained from patellofemoral joint model where the linear displacement$x$ is not considered and the experimental data presented in [8]

	α^p [deg]	β^p [deg]	γ^p [deg]	p_x^p [mm]	p_y^p [mm]	p_z^p [mm]
MaxDiff	1.91	3.09	4.93	1.88	5.54	1.39
MeanDiff	0.44	0.63	1.28	0.01	0.15	0.27
Range	76.10	5.54	26.58	45.08	29.53	1.94

Table 2. Simulation results: MaxDiff – maximum; MeanDiff – mean difference
between results obtained from patellofemoral joint model
where the linear displacement x is considered and the experimental data presented in [8]

	α^p [deg]	β^p [deg]	γ^p [deg]	p_x^p [mm]	p_y^p [mm]	p_z^p [mm]
MaxDiff	1.92	3.09	4.91	1.93	5.54	0.00
MeanDiff	0.44	0.64	1.28	0.01	0.13	0.00
Range	76.31	5.57	26.64	45.25	29.46	3.53

patellofemoral joint model where the linear displacement is fixed can be up to 80% on p_z^p (see Table 1). However, if linear displacement x along the cylindrical joint axis is allowed, the results can be improved.

It is worth noting that the effect of the additional linear displacement x on the angular displacements is negligible (see Table 2). On the other hand, significant improvement can be seen in the course of the translational displacement p_z^p. The model can now accurately reproduce the translational displacements of the patella with respect to the femur.

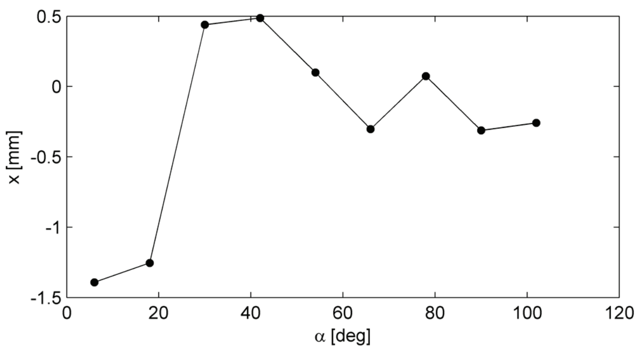

Fig 4. The linear displacement x along the cylindrical axis
as a function of the knee flexion angle α

The linear displacement along the cylindrical axis x as a function of the flexion angle of the knee α is presented in Fig. 4. The greatest linear displacement can be observed for α between 6 and 18 degrees.

3.2.The quadriceps muscle force

As the flexion angle of the knee α increases, the value of the arm of the quadriceps force moment significantly decreases (Fig. 5). This result corresponds quantitatively to the one presented in literature [6]. On the other hand, only for the flexion angle $\alpha = 68 \pm 3$ deg, the angle between the muscle force and the instantaneous screw axis θ is close to 90 ± 3 deg. These results can also be presented in the form of normalized components of the moment arm as shown in Fig. 6. If the angle between the force and the screw axis θ is not equal to 90 deg, it is possible for the patella to be pushed away from the patellofemoral groove. This condition is known as patellofemoral dislocation. The results of this study show that the patellofemoral dislocation is most likely to occur at knee flexion $\alpha = 25 \pm 3$ deg, where the torque arm is relatively high and the angle $\theta = 102 \pm 3$ deg.

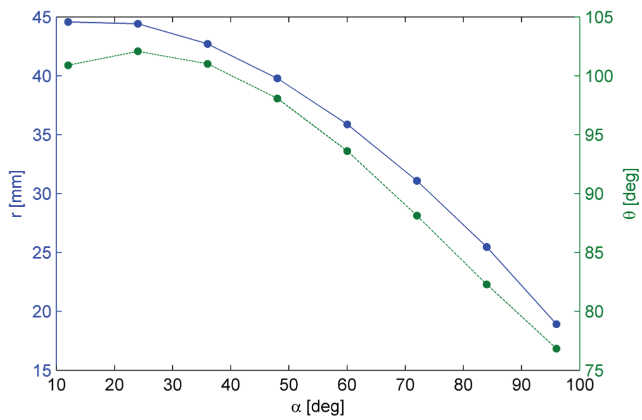

Fig. 5. The arm of the quadriceps force moment r, the angle θ
between the muscle force and the instantaneous screw axis
as functions of the knee flexion angle α

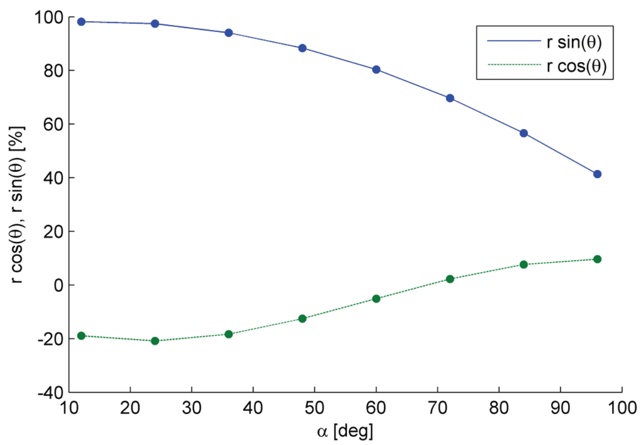

Fig. 6. The normalized (by the maximum value) components
of the moment arm of the quadriceps muscle force moment
acting on the femur, where $r \sin(\theta)$ – the arm component
of the moment responsible for the knee extension,
$r \cos(\theta)$ – the arm component of the moment responsible
for pushing the patella from the patellofemoral groove

The results can also be presented in a 3D space by using the actual muscle force vectors and screw displacement axes. A set of three pairs of these vectors is shown in Fig. 7.

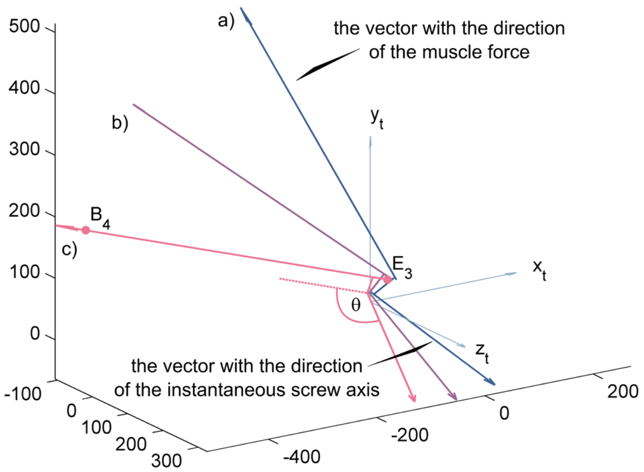

Fig 7. The set of three pairs of the vectors with the direction of the muscle force and the vectors of the instantaneous screw axes computed for three consecutive positions of the mechanism, where E_3 (B_4) – center of the muscle attachment to the femur (the patella). The vectors are described in the tibia reference frame

The rotation matrices and position vectors defining the displacement from which the screw axis a (Fig. 7) is computed are presented below

$$\boldsymbol{R}_{ft}(\alpha = 18.00°) = \begin{bmatrix} 0.9511 & -0.3081 & -0.0231 \\ 0.3088 & 0.9503 & 0.0394 \\ 0.0098 & -0.0446 & 0.9990 \end{bmatrix},$$

$$\boldsymbol{p}_{ft}(\alpha = 18.00°) = \begin{bmatrix} -0.13 \\ 20.92 \\ -3.04 \end{bmatrix} \text{mm},$$

$$\boldsymbol{R}_{ft}(\alpha = 42.00°) = \begin{bmatrix} 0.7394 & -0.6669 & -0.0919 \\ 0.6691 & 0.7431 & 0.0091 \\ 0.0744 & -0.0548 & 0.9957 \end{bmatrix},$$

$$\boldsymbol{p}_{ft}(\alpha = 42.00°) = \begin{bmatrix} -1.13 \\ 21.78 \\ -2.86 \end{bmatrix} \text{mm}.$$

The four screw parameters representing this displacement are as follows

$$\phi = 24.41°, \quad d = 0.00 \text{ mm},$$

$$\boldsymbol{s}^0 = \begin{bmatrix} 0.0555 \\ -0.1445 \\ 0.9879 \end{bmatrix}, \quad \boldsymbol{p}_s = \begin{bmatrix} -2.30 \\ 18.16 \\ 2.79 \end{bmatrix} \text{mm}.$$

It can be seen that the displacement from α = 18.00° to α = 42.00° of the tibiofemoral joint model is nearly spherical. The value of the angular displacement about the screw axis ϕ = 24.43°, is comparable to the change in the value of the flexion angle $\Delta\alpha$ = 24.00°.

4. Discussion

The modelling of the patellofemoral joint has been explored by many researchers over the years. In the literature, a model where the patella and the femur connection is described using point contact pairs (usually two) can often be found [2]. Such a model, while providing a very accurate description of the patellofemoral joint, is not easy to analyze and use as a basis in knee orthotics design. On the other hand, the simplified model of the patellofemoral joint where the patella and the femur connection is described using a cylindrical joint has been presented in [11]. The model is very easy to solve and can reproduce the relative displacements of the patella with regard to the femur rather accurately. The authors [11] have not considered the linear displacement x along the cylindrical joint's axis. In this paper, such displacement is considered using the estimation procedure. It can be seen that the consideration of this linear displacement x can vastly improve the relative translational displacements of the patella with regard to the femur. The maximum difference between the p_z^p obtained from the model (where the displacement x along the cylindrical joint axis is fixed) and experimental results [11] is 80%, while there is no difference for the model with the displacement x considered – the model accurately reproduces the changes in p_z^p. The mean difference of p_z^p can be reduced from 14% to 0%. The effect of the additional displacement x on the other five components of displacements is negligible.

The ability of the quadriceps muscle to generate torque that drives the knee has been studied by many research groups over the years. These studies have mostly involved an experiment where the relative positions of the patella, the tibia and the femur have been registered and then the quadriceps muscle or the patellar tendon moment arm has been computed. The moment arm can be obtained as a shortest distance between an instantaneous center of rotation and the patellar tendon [15]. The three-dimensional moment arm can be expressed as a shortest distance between the instantaneous screw axis and the patellar tendon as in [1]. Some research groups also compute the moment arm using the principle of the virtual work instead of geometry [13]. These approaches seem very useful when analyzing the load on the tibia. However, it seems to be more general to study the moment arm of the quadriceps muscle. Such approach has been undertaken in [6], where two-dimensional muscle moment arm is computed using the principle of the virtual work. While being very well suited for analyzing planar loading conditions of the patella and the femur, it cannot provide insight into the effectiveness of the quadriceps muscle force. An interesting approach has been presented in [9], where an indicator is proposed that takes into account both the patellar tendon and the quadriceps muscle.

It is important to note that the moment arm of the quadriceps muscle with regard to the instantaneous screw axis is not the only factor that determines the effectiveness of the muscle force. The angle between the force and the instantaneous screw axis is similarly important. In this paper, these two separate indicators (the moment arm r and the angle of the force and the screw axis θ) are presented in a much easier to analyze form as the normalized components of the moment arm. Such methodology is used frequently in robotics [4]. It is shown that the quadriceps muscle force not only generates the moment that drives the extension of the knee, where the vector of this moment is parallel to the instantaneous screw axis, but also a smaller, perpendicular moment that pushes the patella from the patellofemoral groove. It can be seen that the muscle force is most effectively transferred into moment that drives the knee for the flexion angle α between 10 and 20 degrees, where the effective moment arm $r\ \sin(\theta)$ (when the angle of the muscle is considered) can be up to 44 mm. In this flexion range, the perpendicular moment's arm value ($r\cos(\theta)$) is also the highest (up to 10 mm). Hence, the risk of the patella being pushed from the patellofemoral groove is significant. It is notable that the comparison of the obtained results with literature is very hard to perform, as the moment arm over the knee flexion is highly dependent on the knee dimensions (which differ from specimen to specimen) and the method of acquiring the moment arm as shown in [15]. However, the obtained results for the moment arm are in agreement with the ones presented in [6].

The model presented in [11] is further completed by considering the linear displacement of the patella with respect to the femur along the cylindrical joint axis. It is shown that this additional linear displacement can give better approximation of the measured results. The model can be used to analyze the ability of the quadriceps muscle to drive the knee joint during extension. The results of this analysis can provide some insights into medical condition known as the patellofemoral dislocation.

5. Conclusion

The model of the patella-femur joint where the linear displacement along axis of the cylindrical joint is considered can reproduce the actual patella displacements more accurately. Significant improvement can be seen in the linear coordinate p_z^p of the patella reference system with respect to tibia reference system. It seems expedient to study elasto-statics problem of this mechanism.

The relative displacements of the femur with respect to the tibia under the quadriceps muscle load are studied using instantaneous screw axes. The effects of the muscle force are described by using the arm of the force moment and the skew angle between the force vector and the axis of screw displacement. The effectiveness is dependent on the flexion angle of the knee joint.

The model can be used in the research regarding the knee joint ability to bear loads. The conclusions obtained from such analysis can be potentially useful in sport biomechanics. It is possible to predict which parts of the knee motion are most vital when considering the patellofemoral dislocation.

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