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Malinowski Jacek
ORCID ID: 0000-0002-4413-1868
Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland

Modeling hazard-related interactions between the processes realized in and around the Baltic Sea Region ports

Keywords

probabilistic modeling, hazard-related events, processes interaction, cascading effect, risk calculation.

Abstract

This paper presents a probabilistic model of hazard-related interdependence between the operations carried out in the ports of the Baltic Sea region and in their neighborhoods. Each single operation, considered w.r.t. its hazardous aspect, will be defined as a point process consisting of undesired events (emergencies and/or accidents). Thus, the interdependence between these processes can be regarded as interaction between such events. The developed model will specify the impact of hazard related events occurring within one process on the risk of occurrence of such events in the other processes. This model will be a basis for the analysis of inter-process dependencies, including the feedback and cascading effects, as implied by the cause-effect relationships between the events occurring in different processes. Furthermore, it is envisaged to be used for assessing the potential effects of accidents or catastrophic events, and for developing the appropriate prevention measures. The procedures derived from the model will be applied to analyzing the mutual impacts between the processes realized in the oil and container terminals, forecasting negative effects of these impacts along with assessing their costs, and planning preventive actions aimed at avoiding such effects.

1. Notation and Definitions

1.1 The overall characteristic of processes and events

$p_1,\ldots,p_n$ – the individual processes as the constitutive elements of the considered environment; n – the number of these processes

$E_1^{(i)},\ldots,E_{m(i)}^{(i)}$ – different hazardous events that can occur in the process p_i; m(i) – the number of such events

$\lambda_1^{(i)},\ldots,\lambda_{m(i)}^{(i)}$ – the intensities with which $E_1^{(i)},\ldots,E_{m(i)}^{(i)}$ occur as primary events, i.e. not caused by another event in any process (given data)

$X_a^{(i)}$ – the strength of $E_a^{(i)}$; a random variable with values in a finite set S={1,…,s}

$\pi_a^{(i)}(x)$ – the probability that the strength of an primary $E_a^{(i)}$ is equal to x (given data)

$\pi_a^{(i)}(>x)$ – the probability that the strength of an primary $E_a^{(i)}$ exceeds x; note that $\pi_a^{(i)}(>x) = \sum_{y>x}\pi_a^{(i)}(y)$

$N_a^{(i)}(s,t)$ – the number of occurrences of a primary $E_a^{(i)}$ of any strength in the (s, t] time interval; a random variable

$N_a^{(i)}(s,t,x)$ – the number of occurrences of a primary $E_a^{(i)}$ of strength x in the (s, t] time interval; a random variable

$\coprod$ – the "inverted pi" operator used to compute the probabilities of sums of independent events, i.e.

$$\Pr(A_1 \cup \ldots \cup A_r) = \coprod_{i=1}^{r} \Pr(A_i) =$$

$$1 - \prod_{i=1}^{r} \Pr(\neg A_i)$$

where $A_1,\ldots,A_r$ are independent events

1.2 The cause-effect probabilities

$\pi_{b,a}^{(j,i)}(y, x)$ – the probability that $E_b^{(j)}$ of strength y directly causes $E_a^{(i)}$ of strength x (given data)

$\pi_{b,a}^{(j,i)}(y, >x)$ – Pr ($E_b^{(j)}$ of strength y directly causes $E_a^{(i)}$ of strength greater than x); note that $\pi_{b,a}^{(j,i)}(y, >x) = \sum_{z>x} \pi_{b,a}^{(j,i)}(y, x)$

$\pi_{b,a}^{(j,i)}(\cdot, x)$ – Pr ($E_b^{(j)}$ of any strength directly causes $E_a^{(i)}$ of strength x)

$\pi_{b,a}^{(j,i)}(\cdot, >x)$ – Pr ($E_b^{(j)}$ of any strength directly causes $E_a^{(i)}$ of strength greater than x)

$\pi_{b,a}^{(j,i)}(y, x; h)$ – the probability that $E_b^{(j)}$ of strength y causes $E_a^{(i)}$ of strength x as a result of h-step (but not less-than-h-step) cascading effect, $h \geq 2$

$\pi_{b,a}^{(j,i)}(y, >x; h)$, $\pi_{b,a}^{(j,i)}(\cdot, x; h)$, $\pi_{b,a}^{(j,i)}(\cdot, >x; h)$ – the probabilities defined analogously to $\pi_{b,a}^{(j,i)}(y, >x)$, $\pi_{b,a}^{(j,i)}(\cdot, >x)$, $\pi_{b,a}^{(j,i)}(\cdot, >x)$

The probabilities $\pi_{b,a}^{(j,i)}$ with various arguments will be called the cause-effect probabilities, as they quantify the cause-effect relations between the events occurring within the analyzed multi-process environment.

1.3 Various types of risks

$r_a^{(i)}(s, t, >x)$ – the risk that at least one event $E_a^{(i)}$ of strength >x occurs as a primary event in the (s, t] interval, $1 \leq a \leq m(i)$;

$r_{b,a}^{(j,i)}(s, t, >x)$ – the risk that $E_b^{(j)}$ (an event in p_j) directly causes at least one occurrence of $E_a^{(i)}$ (an event in p_i) of strength > x in the (s, t] interval, where $b \neq a$ for j=i;

$R_a^{(i)}(s, t, x, 1)$ – the total risk that at least one $E_a^{(i)}$ of strength x occurs (in p_i) in the (s, t] interval, as a direct effect of any event $E_b^{(j)}$ in any process p_j, $(b,j) \neq (a,i)$. The capital letter R indicates that all processes rather than one contribute to the risk.

$R_a^{(i)}(s, t, x, h)$ – the total risk that at least one $E_a^{(i)}$ of strength x occurs (in p_i) in the (s, t] interval, as a h-step (but not less-than-h-step) cascading effect of any event $E_b^{(j)}$ in any process p_j, $h \geq 2$.

$R_a^{(i)}(s, t, x, \geq 1)$ – the total risk that at least one $E_a^{(i)}$ of strength x occurs (in p_i) in the (s, t] interval, as a cascading effect of any step and any event $E_b^{(j)}$ in any process p_j.

$R_a^{(i)}(s, t, >x, 1)$, $R_a^{(i)}(s, t, >x, h)$, $R_a^{(i)}(s, t, >x, \geq 1)$ – the total risks defined as the three above ones, x being replaced by >x.

2. Introduction

The functioning of a port can be considered, according to the systems approach, as a set of processes which represent the operations carried out in various facilities, both within the port premises and in the surrounding areas. The aim of this work is to construct a model of hazard-related interdependence of these processes. This model should describe the impact of hazardous events originating in one process (caused by the operations carried out within a given process) on the risks of events adversely affecting the other processes.

The analytical part of the task, apart from defining the interactions between the processes, will include the analysis of feedback and cascading effects that can result from the mutual dependencies between the events occurring in different processes. The main analytical result consists in deriving the formulas which on the one hand express the risks of events of different types as triggered by other events in the same or other processes, and, on the other hand, quantify the consequences that a given event can entail, in the sense of adverse impact it can have on its own and other processes. Such formulas allow to assess the risk of the occurrence of a harmful event as a direct or indirect consequence of other events, as well as to assess the risk of a harmful impact that a given event has in the sense of causing other such events. They can be applied to the development and implementation of safeguards protecting against, or mitigating the effects of, hazard-related mutual impacts among the considered processes.

The events occurring in the individual processes are quantified by the random variables expressing the strength of each event. These variables can have "crisp" numerical values, or "non-crisp" descriptive values, i.e. fuzzy or linguistic ones, e.g. the strength of an event can be extreme, high, significant, considerable, medium, low, etc. The used quantification approach depends on the degree of accuracy of the intended risk analysis, and the amount and character of the available data. The applicable mathematical tools are the probability/possibility theory, evidence (Dempster-Schafer) theory, and simple arithmetic.

The considered harmful events are divided in four categories: primary (occurring by themselves), directly caused (by another event), indirectly caused by a cascading effect, and indirectly caused by a feedback effect. A cascading effect takes place when the events occur, on a cause-effect basis, in a series whose length exceeds 2; the first event is a primary one, and each other event in the series is directly caused by the preceding one. We will say that an event is a result of a h-step cascading effect if the event's number in the cause-effect series is h+1. A directly caused event can be regarded as a result of a 1-step cascading effect. A feedback effect is a special case of a cascading effect, where the last event in a series is an instance of the first event. More accurately, a feedback effect occurs if the event $E_a^{(i)}$ causes, by means of a h-step cascading effect ($h \geq 2$), another instance of $E_a^{(i)}$, possibly of different strength. A feedback effect cannot be a 1-step cascading effect, due to the natural assumption that an event cannot be directly caused by itself, i.e.

$$\pi_{a,a}^{(i,i)}(y,x) = 0,\ i\in\{1,\ldots,n\},\ a\in\{1,\ldots,m(i)\} \quad (1)$$

However, we admit the possibility of the internal impact which occurs if an event is directly caused by another event in the same process, i.e.

$$\pi_{b,a}^{(i,i)}(y,x) \geq 0,\ b\neq a \quad (2)$$

It is also assumed that the primary events are independent, both within one process, and among all the considered processes, i.e. the instances of $E_1^{(i)},\ldots,E_{m(i)}^{(i)}$, $i=1,\ldots,n$, as primary events, are mutually independent.

For the sake of computational tractability it is desirable that the sequences of cascading events caused by different primary event be mutually independent. If we assume that the events in a cascade follow each other in a quick succession, i.e. the time of the last event in a cascade triggered by a primary event always precedes the time of the next primary event (or, if there are delays between successive events in a cascade, that no two events can coincide), then this requirement is fulfilled by virtue of the following lemma.

Lemma 1

The sequences of cascading events caused by different triggering events are independent (Clearly, the events in one cascade are not independent).

Proof: the lemma follows directly from the assumption of the mutual independence of the instances of primary events, and the impossibility of causing one non-primary event by two primary ones, i.e. the impossibility of the occurrence of a common event in two cause-effect chains (clearly, chains with a common event would be mutually dependent). This impossibility is a consequence of the assumed instantaneousness of a cascading effect.

In *Figure 1* a diagram illustrating the interaction of three processes is presented. $E_2^{(1)}$ is a primary event in p_1, causing an occurrence of $E_3^{(2)}$ which, in turn, causes an occurrence of $E_4^{(1)}$. The above three events form a cascade chain of step 2. As shown in the picture, a certain time elapses between $E_2^{(1)}$ and $E_3^{(2)}$, and then between $E_3^{(2)}$ and $E_4^{(1)}$, thus $E_2^{(1)}$ and $E_3^{(2)}$ do not have instantaneous effect. However, we assume that such an effect is possible, as in the case of $E_4^{(2)}$ and $E_3^{(3)}$. Furthermore, one event can directly cause two or more secondary events, as $E_4^{(2)}$ does. A harmful event may cause a temporary break in a process, $E_1^{(2)}$ and $E_3^{(3)}$ being examples of such events, which is illustrated by discontinuities in the time axes for p_1 and p_2.

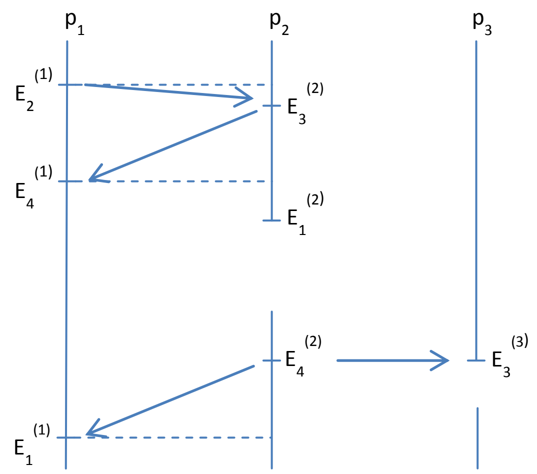

Figure 1. A diagram of three processes' interaction

Practical implementation of the developed model should consist of four phases:

1. Identifying hazards involved in the individual processes, and evaluating/estimating/assessing the intrinsic risks $r_a^{(i)}(s, t, >x)$, $1\leq i\leq n$, $1\leq a\leq m(i)$.
2. Identifying hazard-related interactions between the processes, and evaluating/estimating/assessing the extrinsic risks $r_{a,b}^{(i,j)}(s, t, >x)$, $1\leq a\leq m(i)$, $1\leq b\leq m(j)$, $j\neq i$; $i,j\in\{1,\ldots,n\}$.
3. Calculating all the risks defined in Section 1.
4. Developing procedures aimed at mitigation, minimization or elimination of possible harmful consequences of the identified hazards, using the risks calculated in step 3.

3. Examples of real-life scenarios described by the processes interaction model

Let the processes and harmful events be defined as follows:

p_1 – ship traffic to and from the oil and container terminals
$E_1^{(1)}$ – collision with a pier/breakwater
$E_2^{(1)}$ – vessel on fire
$E_3^{(1)}$ – damage on vessel not caused by $E_1^{(1)}$

p_2 – truck traffic to and from the container terminal
$E_1^{(2)}$ – truck collision
$E_2^{(2)}$ – truck accident other than collision

p_3 – cargo storage and handling in the container terminal
$E_1^{(3)}$ – crane accident
$E_2^{(3)}$ – fire in the storage yard

p_4 – oil storage (onshore tanks) and transport (pipeline)
$E_1^{(4)}$ – pipeline breakage and oil spill
$E_2^{(4)}$ – onshore tank fire

p_5 – weather extremes
$E_1^{(5)}$ – lightning
$E_2^{(5)}$ – violent storm
$E_3^{(5)}$ – arrival of extreme heat (>35°C)
$E_4^{(5)}$ – arrival of heavy frost (<–20°C)

Catastrophic scenario 1:
$E_2^{(2)}$ – truck accident other than collision (tire burst and skidding)
↓
$E_1^{(4)}$ – pipeline breakage and oil spill (truck skids into the pipeline)
│ ↘
│ $E_2^{(4)}$ – onshore tank fire (due to crash-related ignition of the spreading oil spill)
↓
$E_2^{(1)}$ – vessel on fire (due to flow of burning oil into the port basin)

$E_2^{(1)}$ and $E_2^{(4)}$ occur as a result of a cascading effect of step 2. The basic reason of these events is the truck route passing too close to the pipeline.

Catastrophic scenario 2:
$E_2^{(5)}$ – violent storm
↓
$E_1^{(3)}$ – crane accident (collapse of the crane boom)
↓
$E_3^{(1)}$ – damage on vessel (caused by the collapsing boom)

$E_3^{(1)}$ occurs as a result of a cascading effect of step 2. The basic reason of this event is the mooring of the damaged vessel within the range of the collapsing boom.

4. Formulas for the cause-effect probabilities defined in section 1.2

In this section, we will derive formulas for the cause-effect probabilities defined in 1.2. Let us point out that in order to use the obtained formulas we only need the quantities marked in section 1 as "given data". First, the "direct effect" probabilities $\pi_{b,a}^{(j,i)}(\cdot, x)$ and $\pi_{b,a}^{(j,i)}(\cdot, >x)$, where $b \neq a$ if $j=i$, will be calculated. It holds that:

$$\pi_{b,a}^{(j,i)}(\cdot,\ x) =$$

$$\Pr\begin{pmatrix} E_b^{(j)} \textit{of any strength} \\ \textit{directly causes } E_a^{(i)} \\ \textit{of strength } x \end{pmatrix} =$$

$$\Pr\left(\bigcup_{y\in S} \begin{Bmatrix} E_b^{(j)} \textit{of strength } y \\ \textit{directly causes } E_a^{(i)} \\ \textit{of strength } x \end{Bmatrix} \right) =$$

$$\sum_{y\in S} \Pr\left(X_a^{(i)} = x | X_b^{(j)} = y\right) \Pr\left(X_b^{(j)} = y\right) =$$

$$\sum_{y\in S} \pi_{b,a}^{(j,i)}(y, x)\pi_b^{(j)}(y) \tag{3}$$

and

$$\pi_{b,a}^{(j,i)}(\cdot,\ > x) =$$

$$\Pr\begin{pmatrix} E_b^{(j)} \textit{of any strength} \\ \textit{directly causes } E_a^{(i)} \\ \textit{of strength } > x \end{pmatrix} =$$

$$\Pr\left(\bigcup_{y\in S} \begin{Bmatrix} E_b^{(j)} \textit{of strength } y \\ \textit{directly causes } E_a^{(i)} \\ \textit{of strength } > x \end{Bmatrix} \right) =$$

$$\sum_{y\in S} \Pr\left(X_a^{(i)} > x | X_b^{(j)} = y\right) \Pr\left(X_b^{(j)} = y\right) =$$

$$\sum_{z>x} \sum_{y\in S} \Pr\left(X_a^{(i)} = z | X_b^{(j)} = y\right) \times$$

$$\times \Pr\left(X_b^{(j)} = y\right) =$$

$$\sum_{z>x} \sum_{y\in S} \pi_{b,a}^{(j,i)}(y, z)\pi_b^{(j)}(y) \tag{4}$$

Now we pass to the calculation of the probabilities related to the cascading effect of degree h≥2. To make the analysis more detailed, different formulas will be obtained depending on whether the internal impact and/or the feedback effect are taken into consideration or not.

Lemma 2

If both the internal impact and feedback effect are taken into consideration, then $\pi_{b,a}^{(j,i)}(y, >x, h)$, where h≥2, is given by the following recursive formula:

$$\pi_{b,a}^{(j,i)}(y,\ >x,\ h)=$$

$$\coprod_{\substack{k=1,\dots,n\\ c=1,\dots,m(k)}}\ \sum_{\substack{z\in S\\ z\le x\ if\ (k,c)=(i,a)}} \pi_{b,c}^{(j,k)}(y,z)\times$$

$$\times\,\pi_{c,a}^{(k,i)}(z,>x,h-1) \tag{5}$$

where

$$\pi_{c,a}^{(k,i)}(z,\ >x,\ 1)=\pi_{c,a}^{(k,i)}(z,\ >x)=$$

$$\sum_{u>x}\pi_{c,a}^{(k,i)}(z,u) \tag{6}$$

Remark 1:
Under the adopted assumptions, (5) also holds for (j,b)=(i,a), and (k,c)=(i,a) is in the range of the "inverted pi" operator (feedback effect). However, it should be remembered that $\pi_{b,c}^{(j,k)}(y,z)=0$ for (k,c)=(j,b) and $\pi_{c,a}^{(k,i)}(z,>x,1)=0$ for (k,c)=(i,a) – see (1).

Remark 2:
If (k,c)=(i,a), then z>x are not in the range of the summation operator, because taking such z into consideration would amount to admitting the possibility that $E_b^{(j)}$ directly causes $E_a^{(i)}$ of strength >x. This would contradict the requirement that $E_a^{(i)}$ of strength >x cannot be a less-than-h-step cascading effect of $E_b^{(j)}$.

Proof of (5):

$$\pi_{b,a}^{(j,i)}(y,\ >x,\ h)=$$

$$\Pr\begin{pmatrix} E_a^{(i)}\ of\ strength>x\\ occurs\ in\ h\ steps\\ as\ a\ cascading\ effect\\ of\ E_b^{(j)}\ of\ strength\ y,\\ but\ not\ in\ 1,\dots,or\ (h-1)\ steps\end{pmatrix}=$$

$$\Pr\left(\bigcup_{\substack{k=1,\dots,n\\ c=1,\dots,m(k)}}\ \bigcup_{\substack{z\in S\\ z\le x\ if\ (k,c)=(i,a)}} A_{b,c,a}^{(j,k,i)}(y,z,>x)\right)$$

where

$$A_{b,c,a}^{(j,k,i)}(y,z,x)=$$

$$\begin{Bmatrix} E_b^{(j)}\ of\ strength\ y\\ directly\ causes\\ E_c^{(k)}\ of\ strength\ z,\\ and\ E_a^{(i)}\ of\ strength\ >x\\ occurs\ in\ (h-1)\ steps\\ as\ a\ cascading\ effect\\ of\ E_c^{(k)}\ of\ strength\ z,\\ but\ not\ in\ 1,\dots,or\ (h-2)\ steps\end{Bmatrix}$$

The latter probability is equal to

$$\coprod_{\substack{k=1,\dots,n\\ c=1,\dots,m(k)}}\ \sum_{\substack{z\in S\\ z\le x\ for\ (k,c)=(i,a)}} \pi_{b,c}^{(j,k)}(y,z)\times$$

$$\times\,\pi_{c,a}^{(k,i)}(z,\ >x,\ h-1)$$

which fact follows from Lemma 1 stating that the sequences of cascading events caused by different triggering events (namely, the events $E_c^{(k)}$) are independent. The proof is thus completed.

Using (5) we obtain the following "aggregate" probabilities:

$$\pi_{b,a}^{(j,i)}(\cdot,\ >x,\ h)=$$

$$\sum_{y\in S}\pi_b^{(j)}(y)\pi_{b,a}^{(j,i)}(y,\ >x,\ h),\ \ h\ge 2 \tag{7}$$

and

$$\pi_{b,a}^{(j,i)}(\cdot,\ \cdot,\ h)=\pi_{b,a}^{(j,i)}(\cdot,\ >0,\ h),\ \ h\ge 2 \tag{8}$$

Lemma 3

If the internal impact is not taken into consideration, but the feedback effect is, we have:

$$\pi_{b,a}^{(j,i)}(y,\ >x,\ h)=$$

$$\coprod_{\substack{k=1,\dots,n;\ k\ne j\\ c=1,\dots,m(k)}}\ \sum_{\substack{z\in S\\ z\le x\ for\ (k,c)=(i,a)}} \pi_{b,c}^{(j,k)}(y,z)\times$$

$$\times\,\pi_{c,a}^{(k,i)}(z,\ >x,\ h-1),\ \ h\ge 3 \tag{9}$$

Note that k=j is not in the range of the "inverted pi", otherwise an internal impact within p_j would be taken into account. For h=2 (9) changes to:

$$\pi_{b,a}^{(j,i)}(y,\ >x,\ 2)=$$

$$\coprod_{\substack{k=1,\dots,n;\ k\ne j,k\ne i\\ c=1,\dots,m(k)}}\sum_{z\in S}\pi_{b,c}^{(j,k)}(y,z)\times$$

$$\times \pi_{c,a}^{(k,i)}(z, > x) \quad (10)$$

Note that now k=i and k=j are not in the range of the "inverted pi", otherwise an internal impact in p_i or p_j would be taken into account. Also note that the summation over $z \in S$ is not limited to $z \leq x$ for (k,c)=(i,a), because k=i is not in the range of the "inverted pi".

If the feedback is not taken into consideration, then the following two lemmas hold:

Lemma 4

If the feedback effect <u>is not</u> taken into consideration, but the internal impact is, then we have:

$$\pi_{b,a}^{(j,i)}(y, > x, h) =$$

$$\coprod_{\substack{k=1,\dots,n \\ c=1,\dots,m(k) \\ (k,c)\neq(i,a)}} \sum_{z \in S} \pi_{b,c}^{(j,k)}(y,z) \times$$

$$\times \pi_{c,a}^{(k,i)}(z, > x, h-1), \ h \geq 2 \quad (11)$$

where (j,b)≠(i,a) – otherwise $E_a^{(i)}$ would be a h-step feedback effect of itself. For a similar reason (k,c)=(i,a) is not in the range of the "inverted pi" – otherwise $E_a^{(i)}$ would be a (h–1)-step feedback effect of itself. In consequence, the summation over $z \in S$ is not limited to $z \leq x$ for (k,c)=(i,a) as in (5) or (9).

Lemma 5

If both the feedback effect and internal impact <u>are not</u> taken into consideration, then it holds that

$$\pi_{b,a}^{(j,i)}(y, > x, h) =$$

$$\coprod_{\substack{k=1,\dots,n;\ k\neq j \\ c=1,\dots,m(k) \\ (k,c)\neq(i,a)}} \sum_{z \in S} \pi_{b,c}^{(j,k)}(y,z) \times$$

$$\times \pi_{c,a}^{(k,i)}(z, > x, h-1), \ h \geq 3 \quad (12)$$

where (j,b)≠(i,a). Note that (k,c)=(i,a) is not in the range of the "inverted pi" – otherwise $E_a^{(i)}$ would be a (h–1)-step feedback effect of itself. Also, k=j is not in that range – otherwise an internal impact in p_j would be taken into account. For h=2 (12) changes to:

$$\pi_{b,a}^{(j,i)}(y, > x, 2) =$$

$$\coprod_{\substack{k=1,\dots,n;\ k\neq j,k\neq i \\ c=1,\dots,m(k)}} \sum_{z \in S} \pi_{b,c}^{(j,k)}(y,z) \times$$

$$\times \pi_{c,a}^{(k,i)}(z, > x) \quad (13)$$

where (j,b)≠(i,a). Note that k=j and k=i are not in the range of the "inverted pi" – otherwise an internal impact in p_j or p_i would be taken into account.

The proofs of Lemmas 3–5 are similar to that of Lemma 2. As to the formulas for "aggregate" probabilities, i.e. (7) and (8), they hold for all the cases considered in Lemmas 3–5, where the probabilities $\pi_{b,a}^{(j,i)}(y, >x; h)$ are given by (9)-(13).

5. Formulas for various types of risks defined in section 1.3

In this section we will present several theorems stating that the occurrences of $E_a^{(i)}$, whether as a primary or a secondary event, constitute a Poisson process with the appropriate intensity. The formulas for risks defined in sections 1.3 are given as corollaries to the respective theorems. As in the previous section, the obtained formulas use only the quantities marked in section 1 as "given data".

Theorem 1

Primary events $E_a^{(i)}$ of strength x constitute a Poisson process with the intensity $\lambda_a^{(i)} \cdot \pi_a^{(i)}(x)$, while those of strength greater than x – a P. process with the intensity $\lambda_a^{(i)} \cdot \pi_a^{(i)}(>x)$.

Proof: It follows from the adopted assumptions that the primary events $E_a^{(i)}$ of any strength constitute a Poisson process with the intensity $\lambda_a^{(i)}$, hence we have:

$$\Pr\left[N_a^{(i)}(s,t,x) = r\right] =$$

$$\sum_{q=r}^{\infty} \Pr\left[\begin{matrix} X_a^{(i)} = x \ for\ r\ out\ of\ the\ q \\ instances\ of\ E_a^{(i)} \mid N_a^{(i)}(s,t) = q \end{matrix}\right] \times$$

$$\times \Pr\left[N_a^{(i)}(s,t) = q\right] =$$

$$\sum_{q=r}^{\infty} \binom{q}{r} \left[\pi_a^{(i)}(x)\right]^r \left[1 - \pi_a^{(i)}(x)\right]^{q-r} \times$$

$$\times \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^q}{q!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] =$$

$$\left[\lambda_a^{(i)} \cdot \pi_a^{(i)}(x) \cdot (t-s)\right]^r \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \sum_{q=r}^{\infty} \frac{q!}{r!(q-r)!} \left[1-\pi_a^{(i)}(x)\right]^{q-r} \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^{q-r}}{q!} =$$

$$\frac{\left[\lambda_a^{(i)} \cdot \pi_a^{(i)}(x) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \sum_{q=r}^{\infty} \left[1-\pi_a^{(i)}(x)\right]^{q-r} \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^{q-r}}{(q-r)!} =$$

$$\frac{\left[\lambda_a^{(i)} \cdot \pi_a^{(i)}(x) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \exp\left(\left[1-\pi_a^{(i)}(x)\right]\left[\lambda_a^{(i)} \cdot (t-s)\right]\right) =$$

$$\frac{\left[\lambda_a^{(i)} \cdot \pi_a^{(i)}(x) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot \pi_a^{(i)}(x) \cdot (t-s)\right]$$

The above derivation indicates that the first part of the thesis is true.

In the same way it is proved that the occurrences of $E_a^{(i)}$ of strength >x constitute a Poisson process with the intensity $\lambda_a^{(i)} \cdot \pi_a^{(i)}(>x)$.

Corollary 1:
The "primary" risks $r_a^{(i)}(s, t, x)$ and $r_a^{(i)}(s, t, >x)$ are given by the following formulas:

$$r_a^{(i)}(s,t,x) =$$

$$1-\exp\left[-\pi_a^{(i)}(x) \cdot \lambda_a^{(i)} \cdot (t-s)\right] \quad (14)$$

and

$$r_a^{(i)}(s,t,>x) =$$

$$1-\exp\left[-\pi_a^{(i)}(>x) \cdot \lambda_a^{(i)} \cdot (t-s)\right] \quad (15)$$

Now we pass to the calculation of the total risk that one or more events $E_a^{(i)}$ of strength x or exceeding x occur on the (s, t] interval, provided that all the processes can contribute to $E_a^{(i)}$.

Theorem 2 (direct impact, no cascading effect)

The events $E_a^{(i)}$ of strength x or greater than x, directly caused by primary events $E_b^{(j)}$ of any strength, constitute a Poisson process with the intensity $\lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot, x)$ or $\lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot, >x)$ respectively, where the probabilities $\pi_{b,a}^{(j,i)}(\cdot, x)$ and $\pi_{b,a}^{(j,i)}(\cdot, >x)$ are given by (3) and (4).

Further, the occurrences of $E_a^{(i)}$ of strength x or greater than x, directly caused by any primary event in any process (including p_i), constitute a Poisson process with the intensities given by the following formulas:

$$\Lambda_a^{(i)}(x,1) =$$

$$\sum_{\substack{j=1,\dots,n \\ b=1,\dots,m(j);\ b\neq a \text{ for } j=i}} \lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot,\ x) \quad (16)$$

and

$$\Lambda_a^{(i)}(>x,1) =$$

$$\sum_{\substack{j=1,\dots,n \\ b=1,\dots,m(j);\ b\neq a \text{ for } j=i}} \lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot,\ >x) \quad (17)$$

Proof: Let $N_{b,a}^{(j,i)}(s,t,x)$ be the number of the events $E_a^{(i)}$ of strength x, directly caused by primary events $E_b^{(j)}$ of any strength. We have:

$$\Pr\left[N_{b,a}^{(j,i)}(s,t,x) = r\right] =$$

$$\sum_{q=r}^{\infty} Pr\left(\begin{matrix} \textit{the events } E_b^{(j)} \\ \textit{directly cause} \\ r \textit{ events } E_a^{(i)} \\ \textit{of strength } x \end{matrix} \middle| N_b^{(j)}(s,t) = q\right) \times$$

$$\times \Pr\left(N_b^{(j)}(s,t) = q\right) =$$

$$\sum_{q=r}^{\infty} \binom{q}{r} \left[\pi_{b,a}^{(j,i)}(\cdot,\ x)\right]^r \left[1-\pi_{b,a}^{(j,i)}(\cdot,\ x)\right]^{q-r} \times$$

$$\times \frac{\left[\lambda_b^{(j)} \cdot (t-s)\right]^q}{q!} \exp\left[-\lambda_b^{(j)} \cdot (t-s)\right]$$

Proceeding further as in the proof of Theorem 1 we obtain the following formula:

$$\Pr\left[N_{b,a}^{(j,i)}(s,t,x) = r\right] = \frac{\left[\lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot,\ x) \cdot (t-s)\right]^r}{r!} \times$$

$$\times \exp\left[-\lambda_b^{(j)} \cdot \pi_{b,a}^{(j,i)}(\cdot,\ x) \cdot (t-s)\right]$$

In the same way we obtain

$$\Pr\left[N_{b,a}^{(j,i)}(s,t,>x)=r\right]=$$

$$\frac{\left[\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot\,,\,>x)\cdot(t-s)\right]^r}{r!}\times$$

$$\times\exp\left[-\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot\,,\,>x)\cdot(t-s)\right]$$

The first part of the thesis is thus proved. For the proof of the second part let us note that the primary events $E_b^{(j)}$, $j\in\{1,\ldots,n\}$, $b\in\{1,\ldots,m(j)\}$, $b\neq a$ for $j=i$, occur independently, thus it follows from Lemma 1 that the occurrences of $E_a^{(i)}$ directly caused by these $E_b^{(j)}$-s can be regarded as a superposition of independent Poisson processes, hence their intensity is equal to the sum of the intensities of the individual processes.

Corollary 2:
The "secondary" risks $r_{b,a}^{(j,i)}(s, t, x)$ and $r_{b,a}^{(j,i)}(s, t, >x)$ are given by the following formulas:

$$r_{b,a}^{(j,i)}(s,t,x)=$$

$$1-\exp\left[-\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot,x)\cdot(t-s)\right] \tag{18}$$

and

$$r_{b,a}^{(j,i)}(s,t,>x)=$$

$$1-\exp\left[-\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot,>x)\cdot(t-s)\right] \tag{19}$$

Corollary 3:
The total risks $R_a^{(i)}(s, t, x, 1)$ and $R_a^{(i)}(s, t, >x, 1)$ are given by the following formulas:

$$R_a^{(i)}(s,t,x,1)=$$

$$1-\exp\left[-\Lambda_a^{(i)}(x,1)\cdot(t-s)\right] \tag{20}$$

and

$$R_a^{(i)}(s,t,>x,1)=$$

$$1-\exp\left[-\Lambda_a^{(i)}(>x,1)\cdot(t-s)\right] \tag{21}$$

where $\Lambda_a^{(i)}(x,1)$ and $\Lambda_a^{(i)}(>x,1)$ are given by (16) and (17).

Theorem 3 (cascading effect of step h≥2)

The events $E_a^{(i)}$ of strength x or greater than x, each of which is a h-step (but not less-than-h-step) cascading effect of a primary event $E_b^{(j)}$ of any strength, constitute a Poisson process with the intensity $\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot\,, x, h)$ or $\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot\,, >x, h)$ respectively, where the probabilities $\pi_{b,a}^{(j,i)}(\cdot\,, x, h)$ and $\pi_{b,a}^{(j,i)}(\cdot\,, >x, h)$ are given by the formulas in Lemmas 2 – 5. We recall that these formulas differ depending on whether the internal impact and/or feedback effect are taken into consideration.

Further, the events $E_a^{(i)}$ of strength x or greater than x, each of which is a h-step (but not less-than-h-step) cascading effect of any primary event in any process (including p_i), constitute a Poisson process with the intensities given by the following formulas:

$$\Lambda_a^{(i)}(x,h)=$$

$$\sum_{\substack{j=1,\ldots,n\\ b=1,\ldots,m(j)}}\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot,\ x,h) \tag{22}$$

and

$$\Lambda_a^{(i)}(>x,h)=$$

$$\sum_{\substack{j=1,\ldots,n\\ b=1,\ldots,m(j)}}\lambda_b^{(j)}\cdot\pi_{b,a}^{(j,i)}(\cdot,\ >x,h) \tag{23}$$

If the feedback effect is not taken into consideration, then (j,b)=(i,a) is excluded from the range of the summation operator in (22) and (23) – see Lemmas 4 and 5.

Proof: the proof is similar to that of Theorem 2.

Corollary 4:
The total risks $R_a^{(i)}(s, t, x, h)$ and $R_a^{(i)}(s, t, >x, h)$, h≥2, are computed from the following formulas:

$$R_a^{(i)}(s,t,x,h)=$$

$$1-\exp\left[-\Lambda_a^{(i)}(x,h)\cdot(t-s)\right] \tag{24}$$

and

$$R_a^{(i)}(s,t,>x,h)=$$

$$1-\exp\left[-\Lambda_a^{(i)}(>x,h)\cdot(t-s)\right] \tag{25}$$

where $\Lambda_a^{(i)}(x, h)$ and $\Lambda_a^{(i)}(>x, h)$ are given by (22) and (23).

Theorem 4 (cascading effect of any step)
The events $E_a^{(i)}$ of strength x or greater than x, each of which is a cascading effect of any step h≥1 of a primary event $E_b^{(j)}$ of any strength, constitute a Poisson process with the intensity $\lambda_b^{(j)} \cdot \sum_{h\geq 1} \pi_{b,a}^{(j,i)}(\cdot, x, h)$ or $\lambda_b^{(j)} \cdot \sum_{h\geq 1} \pi_{b,a}^{(j,i)}(\cdot, >x, h)$ respectively.

Further, the events $E_a^{(i)}$ of strength x or greater than x, each of which is a cascading effect of any step h≥1 of any primary event in any process (including p_i), constitute a Poisson process with the intensities given by the following formulas:

$$\Lambda_a^{(i)}(x, \geq 1) = \sum_{h\geq 1} \Lambda_a^{(i)}(x, h) \qquad (26)$$

and

$$\Lambda_a^{(i)}(>x, \geq 1) = \sum_{h\geq 1} \Lambda_a^{(i)}(>x, h) \qquad (27)$$

where $\Lambda_a^{(i)}(x, h)$ and $\Lambda_a^{(i)}(>x, h)$ are given by (22) and (23).

Proof: The secondary events $E_a^{(i)}$, each of which occurs as a cascading effect of any step triggered by a primary event $E_b^{(j)}$, constitute a superposition of Poisson processes X_h, h≥1, where the process X_h is a sequence of $E_a^{(i)}$-s, each of which occurs as a cascading effect of step h (but not less-than-h) triggered by a primary event $E_b^{(j)}$. These processes are independent, because, by one of the basic assumptions, the triggering events of the events $E_a^{(i)}$ in the compound process are independent. The first part of the thesis is thus a consequence of the first part of Theorem 3.

The secondary events $E_a^{(i)}$, each of which is a result of a cascading effect of any step triggered by any primary event, constitute a superposition of Poisson processes X_h, h≥1, where the process X_h is a sequence of $E_a^{(i)}$-s, each of which is a result of a h-step (but not less-than-h-step) cascading effect triggered by any primary event. These processes are independent, by the same argument as used in the first part of the proof. Thus, the second part of the thesis is a consequence of the second part of Theorem 3.

Corollary 5:
The total risks $R_a^{(i)}(s, t, x, \geq 1)$ and $R_a^{(i)}(s, t, >x, \geq 1)$ can be found from the following formulas:

$$R_a^{(i)}(s, t, x, \geq 1) =$$
$$1 - \exp\left[-\Lambda_a^{(i)}(x, \geq 1) \cdot (t - s)\right] \qquad (28)$$

and

$$R_a^{(i)}(s, t, >x, \geq 1) =$$
$$1 - \exp\left[-\Lambda_a^{(i)}(>x, \geq 1) \cdot (t - s)\right] \qquad (29)$$

where $\Lambda_a^{(i)}(x, \geq 1)$ and $\Lambda_a^{(i)}(>x, \geq 1)$ are given by (26) and (27).

To end this section, we point out a problem that can be encountered when attempting to calculate the overall risk that the event $E_a^{(i)}$ of strength x occurs in the time interval (s,t], as a primary event or as a result of a cascading effect of step h, h≥1. Let this risk be denoted as $R_a^{(i)}(s,t,x,\geq 0)$. Let us define, in addition to $N_a^{(i)}(s,t)$ defined in section 1, the following (Poisson) processes:

$N_{b,a}^{(j,i)}(s,t,x)$ – number of occurrences of $E_a^{(i)}$ of strength x, as directly caused by $E_b^{(j)}$

$N_{b,a}^{(j,i)}(s,t,x,h)$ – number of occurrences of $E_a^{(i)}$ of strength x, as a result of a cascading effect of step h triggered by $E_b^{(j)}$, h≥2.

We have:

$R_a^{(i)}(s,t,x,\geq 0) =$

$$\Pr[\, N_a^{(i)}(s,t,x) + \sum_{(j,b) \neq (i,a),\, b\in\{1,\dots,m(j)\}} N_{b,a}^{(j,i)}(s,t,x) +$$
$$+ \sum_{h\geq 2} \sum_{j\in\{1,\dots,n\},\, b\in\{1,\dots,m(j)\}} N_{b,a}^{(j,i)}(s,t,x,h) \geq 1 \,]$$

It would be possible to easily compute the above probability if the three underlying processes were independent, because the superposition of independent Poisson processes is also a Poisson process. However, $N_{b,a}^{(j,i)}(s,t,x)$ and $N_{b,a}^{(j,i)}(s,t,x,h)$ are not independent of $N_a^{(i)}(s,t)$, as their composing events are triggered by the events in $N_a^{(i)}(s,t)$. A way to tackle the above problem will be a topic of further research.

6. Conclusion

The main purpose of this paper is to define and calculate the risks of various unwanted or harmful events that can occur during the operations carried out in or around a port area, where the hazard-related aspects of those operations are modeled by a set of mutually dependent stochastic point processes. The dependence between the processes follows from the fact that events occurring in one process can cause events in the other processes, thus, apart from the primary events (assumed to occur independently), there also occur secondary events as a result of cascading or feedback effect. The derived formulas are effect-oriented in the sense that they express the total probabilities of the considered events without specifying the degree to which individual primary events contribute to these probabilities. However, the obtained formulas can be modified to the cause-oriented ones, i.e. quantifying the possible effects of individual primary events. This will be the subject of future work.

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