WAVE PROPAGATION IN A MICROPOLAR TRANSVERSELY ISOTROPIC GENERALIZED THERMOELASTIC HALF-SPACE

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Rayleigh waves in a half-space exhibiting microplar transversely isotropic generalized thermoelastic properties based on the Lord-Shulman (L-S), Green and Lindsay (G-L) and Coupled thermoelasticty (C-T) theories are discussed. The phase velocity and attenuation coefficient in the previous three different theories have been obtained. A comparison is carried out of the phase velocity, attenuation coefficient and specific loss as calculated from the different theories of generalized thermoelasticity along with the comparison of anisotropy. The amplitudes of displacements, microrotation, stresses and temperature distribution were also obtained. The results obtained and the conclusions drawn are discussed numerically and illustrated graphically. Relevant results of previous investigations are deduced as special cases.

Key words: micropolar, transversely isotropic, generalized thermoelastic, amplitude ratios.

1. Introduction

Classical mechanics deals with the basic assumption that the effect of the microstructure of a material is not essential for describing its mechanical behavior. Such an approximation has been shown in many well-known cases. Often, however, discrepancies between the classical theory and experiments are observed, indicating that the microstructure might be important. For example, discrepancies have been found in the stress concentrations in the areas of holes, notches and cracks; elastic vibrations characterized by a high frequency and small wavelengths, particularly in granular composites consisting of stiff inclusions embedded in a weaker matrix, fibers or grains; and the mechanical behavior of complex fluids such as liquid crystals, polymeric suspensions, and animal blood. In general, granular composites, for example porous materials, are widely used in the area of passive noise control as sound absorbers and the effect of acoustical waves characterized by high frequencies and small wavelengths become significant.

To explain the fundamental departure of microcontinuum theories from the classical continuum theories, a continuum model with microstructures to describe the microscopic motion or a non local model to describe the long range material interaction is developed. This theory extends the application of the continuum model to microscopic space and short-time scales. The micromorphic theory (Suhubi and Eringen, 1964; Eringen, 1999) treats a material body as a continuous collection of a large number of deformable particles, with each particle possessing a finite size and inner structure. Using assumptions such as infinitesimal deformation and slow motion, the micromorphic theory can be reduced to Mindlin's microstructure theory (1964). When the microstructure of the material is considered rigid, it becomes the micropolar theory (Eringen, 1966).

Eringen's micropolar theory is more appropriate for geological materials like rocks, soils since, this theory takes into account the intrinsic rotation and predicts the behavior of a material with inner structure. Different researchers discussed different type of problems in a transversely isotropic elastic material. Abubakar (1962) discussed free vibrations of a transversely isotropic plate. Keck *et al*. (1971) derived the frequency equation for the propagation of train of a nontorsional axisymmetric harmonic wave in infinitely long shells, made of three concentric cylinders of different transversely isotropic materials. Shuvalov *et al*.

(1974) described the long wavelength onset of the fundamental branches for a free anisotropic plate with an arbitrary through plate variation of material properties. Payton in 1991 studied wave propagation in a restricted transversely isotropic elastic solid whose surface contains conical points. However, no attempt has been made to study the wave propagation in a micropolar transversely isotropic medium.

The aim of the present study is to improve our knowledge about the propagation of waves in a micropolar transversely isotopic layer. This study has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. After developing the solution, frequency equations connecting the phase velocity with the wave number, for symmetric and skew-symmetric wave modes are derived. The amplitude ratios of displacements and microrotation are also obtained. The dispersion curves, attenuation coefficients, amplitude ratio of displacements and microrotation for symmetric and skew-symmetric waves are presented and illustrated graphically to evince the effect of anisotropy.

2. Basic equations

Following Eringen (1999), the constitutive relations and balance laws in a general micropolar anisotropic medium possessing the center of symmetry, in the absence of body forces, body couples, are given by

Balance laws

$$t_{ji,j} = \rho \ddot{u}_i,$$

$$m_{ik,i} + \varepsilon_{imn} t_{mn} = \rho j \ddot{\varphi}_k. \tag{2.1}$$

Constitutive relations

$$t_{ij} = A_{ijkl} E_{kl} + G_{ijkl} \Psi_{kl} - \beta_{ij} \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T,$$

$$m_{ij} = G_{jikl} E_{kl} + B_{klji} \Psi_{kl}. \tag{2.2}$$

Heat conduction equation

$$K_{ij}^* T_{,ij} = \rho c^* \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \dot{T} + T_0 \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \beta_{ij} u_{i,j}. \tag{2.3}$$

The deformation and wryness tensor are defined by

$$E_{ji} = u_{i,j} + \varepsilon_{ijk} \varphi_k, \qquad \Psi_{ij} = \varphi_{i,j}. \tag{2.4}$$

The list of symbols is given in the nomenclature.

3. Problem formulation and solution

We have used appropriate transformations following Slaughter (2002), on the set of Eq.(2.1) to derive equations for a micropolar generalized thermoelastic transversely isotopic medium and restricted our analysis to the two dimensional problem.

We consider a homogeneous, micropolar generalized thermoelastic transversely isotropic half-space initially in an undeformed state and at uniform temperature T_0. We take the origin of the coordinate system on the top plane surface and the x_3 axis pointing normally into the half-space, which is thus represented by $x_3 \geq 0$. We choose the x_1-axis along the direction of wave propagation so that all particles on a line parallel to the x_2-axis are evenly displaced. Therefore, all the field quantities will be independent of the x_2 coordinate. Further, the disturbance is assumed to be confined to the neighborhood of the free surface and hence vanishes as $x_3 \to \infty$. So, we assume the components of the displacement and microrotation vector of the form

$$\boldsymbol{u} = (u_1, 0, u_3), \qquad \boldsymbol{\varphi} = (0, \varphi_2, 0). \tag{3.1}$$

Thus, the field equations reduce to

$$A_{11} \frac{\partial^2 u_1}{\partial x_1^2} + A_{55} \frac{\partial^2 u_1}{\partial x_3^2} + (A_{13} + A_{56}) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + K_1 \frac{\partial \varphi_2}{\partial x_3} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{3.2}$$

$$A_{66} \frac{\partial^2 u_3}{\partial x_1^2} + A_{33} \frac{\partial^2 u_3}{\partial x_3^2} + (A_{13} + A_{56}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + K_2 \frac{\partial \varphi_2}{\partial x_1} - \beta_3 \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{3.3}$$

$$B_{77} \frac{\partial^2 \varphi_2}{\partial x_1^2} + B_{66} \frac{\partial^2 \varphi_2}{\partial x_3^2} - K_1 \frac{\partial u_1}{\partial x_3} + K_1 \frac{\partial \varphi_2}{\partial x_3} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{3.4}$$

$$K_1^* \frac{\partial^2 T}{\partial x_1^2} + K_3^* \frac{\partial^2 T}{\partial x_3^2} = \rho c^* \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + T_0 \left(1 + n_0 \tau_0 \frac{\partial}{\partial t} \right) \left(\beta_1 \frac{\partial \dot{u}_1}{\partial x_1} + \beta_3 \frac{\partial \dot{u}_3}{\partial x_3} \right), \tag{3.5}$$

$$t_{33} = A_{11} \frac{\partial u_1}{\partial x_1} + A_{33} \frac{\partial u_3}{\partial x_3} - \beta_3 \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T,$$

$$\tag{3.6}$$

$$t_{31} = A_{65} \frac{\partial u_3}{\partial x_1} - K_1 \varphi_2 + A_{55} \frac{\partial u_1}{\partial x_3}, \qquad m_{32} = B_{66} \frac{\partial \varphi_2}{\partial x_3}$$

where

$$K_1 = A_{56} - A_{55}, \qquad K_2 = A_{66} - A_{56}, \qquad X = K_2 - K_1,$$

and we have used the notations $11 \to 1, 33 \to 3, 12 \to 7, 13 \to 6, 23 \to 5$ for the material constants.

For the Lord and Shulman (L-S) theory we take $\tau_1 = 0$, $n_0 = 1$, for Green and Lindsay (G-L) theory we take $\tau_1 \geq \tau_0 \geq 0$, $n_0 = 0$, and for the coupled theory (CT) $\tau_1 = \tau_0 = 0$, $n_0 = 0$.

For further considerations, it is convenient to introduce the dimensionless variables defined by

$$(x'_1, x')_3 = \frac{\omega^*}{c_1}(x_1, x_3), \qquad (u'_1, u')_3 = \frac{\omega^*}{c_1}(u_1, u_3), \qquad t'_{ij} = \frac{t_{ij}}{A_{11}},$$

$$m'_{ij} = \frac{m_{ij}c_1}{B_{56}\omega^*}, \qquad\qquad \varphi'_2 = \frac{\varphi_2 A_{55}}{K_1}, \qquad\qquad T' = \frac{T}{T_0}, \qquad\qquad (3.7)$$

$$t' = \omega^* t, \qquad \tau'_0 = \omega^* \tau_0, \qquad \tau'_1 = \omega^* \tau_1, \qquad \omega^{*2} = \frac{X}{\rho j}, \qquad c_1^2 = \frac{A_{11}}{\rho}.$$

4. Normal mode analysis and solution of the problem

We assume the solution for representing propagating waves in the plane of the form

$$(u_1, u_3, \varphi_2, T) = (1, \bar{u}_3, \bar{\varphi}_2, \bar{T}) u_1 e^{i\xi(x_1 + mx_3 - ct)} \qquad\qquad (4.1)$$

where ξ is the wave number, $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave, m is the unknown parameter which signifies the penetration depth of the wave, $\bar{u}_3, \bar{\varphi}_2, \bar{T}$ are respectively, the amplitude ratios of the displacement u_3, microrotation φ_2 and temperature distribution to that of the displacement u_1.

With the help of Eqs (3.7) and (3.8), field Eqs (3.2)-(3.5) are reduced to (after suppressing primes)

$$\left[-m^2\xi^2 + a_1 - d_2 m\xi^2 \bar{u}_3 + i\xi m d_3 \bar{\varphi}_2 + a_2 \bar{T} \right] u_1 e^{i\xi(x_1 + mx_3 - ct)} = 0,$$

$$\left[-m\xi^2 d_5 + \left(-m^2\xi^2 + a_3 \right) \bar{u}_3 + i\xi d_6 \bar{\varphi}_2 + m a_4 \bar{T} \right] u_1 e^{i\xi(x_1 + mx_3 - ct)} = 0,$$

$$\left[-i\xi m d_9 - i\xi m d_{10} \bar{u}_3 + \left(-m^2\xi^2 + a_5 \right) \bar{\varphi}_2 \right] u_1 e^{i\xi(x_1 + mx_3 - ct)} = 0, \qquad\qquad (4.2)$$

$$a_6 + a_7 \bar{u}_3 + \left(m^2\xi^2 + a_8 \right) \bar{T} = 0$$

where

$$a_1 = -d_1\xi^2 + \omega^2, \qquad a_2 = -i\xi d_{13}\left(1 - i\omega\tau_1 \right), \qquad a_3 = -d_4\xi^2 + \omega^2 d_7,$$

$$a_4 = -i\xi d_{14}\left(1 - i\omega\tau_1 \right), \quad a_5 = -d_8\xi^2 - d_{11} + \omega^2 d_{12}, \quad a_6 = \varepsilon_2 \bar{\beta}\omega\xi\left(1 - i\omega n_0\tau_0 \right), \quad a_7 = a_6 / \bar{\beta},$$

$$d_1 = A_{11} / A_{55}, \qquad d_2 = \left(A_{13} / + A_{56} \right) / A_{55} = d_5, \qquad d_3 = K_1^2 / A_{55}^2,$$

$$d_4 = A_{66} / A_{33}, \qquad d_6 = K_1 K_2 / A_{33} A_{55}, \qquad d_7 = A_{55} / A_{33}, \qquad d_8 = B_{77} / B_{66},$$

$$d_9 = -A_{55} c_1^2 / B_{66} \omega^{*2}, \qquad\qquad d_{10} = -K_2 A_{55} c_1^2 / K_1 B_{66} \omega^{*2}, \qquad\qquad d_{11} = -X c_1^2 / B_{66} \omega^{*2},$$

$$d_{12} = A_{55} j / B_{66}, \qquad d_{13} = \beta_1 T_0 / A_{33}, \qquad d_{14} = \beta_3 T_0 / A_{33}, \qquad \varepsilon_1 = \rho c^* c_1^2 / K_3 \omega^*,$$

$$\varepsilon_2 = \beta_3 c_1^2 / K_3 \omega^*, \qquad \overline{K} = K_1 / K_3, \qquad \overline{\beta} = \beta_1 / \beta_3, \qquad d_{15} = K_1^2 / A_{55}^2, \qquad d_{16} = A_{65} / A_{55}.$$

The condition for the non trivial solution of the system of Eq.(4.2), yields a biquadratic equation in $q \left(= m^2 \right)$ as

$$Aq^4 + Bq^3 + Cq^2 + Dq + E = 0 \tag{4.3}$$

where

$$A = -\xi^8, \qquad B = -\xi^4 a_4 a_7 + \xi^6 \left(a_3 + a_5 + a_1 - a_8 + d_3 d_9 \right) + \xi^8 d_2 d_5,$$

$$\begin{aligned} C = \xi^4 & \left[a_2 \left(d_5 a_7 - a_6 \right) - a_3 a_5 + a_4 d_2 a_6 + d_6 d_{10} + \right. \\ & \left. - \left(a_1 - a_8 \right) \left(a_3 + a_5 \right) + a_1 a_8 - d_3 d_9 \left(a_7 - a_8 \right) \right] + \\ & + \xi^2 a_7 \left(a_4 a_1 + a_5 - d_3 d_9 \right) - \xi^6 \left(d_2 d_5 a_5 + d_2 d_6 d_9 + d_3 d_5 d_{10} \right), \end{aligned}$$

$$\begin{aligned} D = \xi^2 & \left[a_2 a_3 a_6 - a_2 a_5 \left(d_5 a_7 - a_6 \right) + a_7 d_6 d_9 a_2 + \right. \\ & \left. - a_4 a_6 \left(a_5 d_2 + d_3 d_{10} \right) + \left(a_1 - a_8 \right) a_3 a_5 - a_1 a_8 \left(a_3 + a_5 \right) - d_3 d_9 a_7 a_8 \right] + \\ & - a_4 a_5 a_1 a_7 - \xi^4 \left[\left(a_1 - a_8 \right) d_6 d_{10} + d_2 \left(d_5 a_5 a_8 + d_6 d_9 d_8 \right) + d_3 d_5 d_{10} d_8 \right], \end{aligned}$$

$$E = a_2 \left(\xi^2 d_6 a_6 d_{10} - a_5 a_3 a_6 \right) + a_1 a_8 \left(a_3 a_5 - \xi^2 d_6 d_{10} \right).$$

The complex coefficients in Eq.(4.3) implies that four roots, q_j, $(j=1,2,3,4)$ of this equation may be complex. The complex phase velocities of the quasi-waves, will be varying with the direction of phase propagation. Therefore, the three waves propagating in such a medium are attenuating waves. These waves are called quasi-waves because polarisations may not be along the dynamic axis. Analogous to the propagation in a micropolar isotropic thermoelastic medium, these coupled waves may be called quasi-longitudinal displacement (QLD) waves, quasi-transverse microrotational (QCTM) waves, quasi- transverse displacement (QCTD) waves and quasi-thermal waves (QCT) that are propagating with the descending phase velocities respectively.

5. Boundary condition

The surface $x_3 = 0$ of the half-space is assumed to satisfy the following boundary conditions

$$t_{33} = 0, \qquad t_{31} = 0, \qquad m_{32} = 0, \qquad \frac{\partial T}{\partial x_3} + hT = 0$$

where $h \to 0$ is the surface heat transfer coefficient;
$h \to 0$ corresponds to thermally insulated boundaries and
$h \to \infty$ refers to isothermal boundaries. $\tag{5.1}$

Rayleigh wave equation

To obtain the solution for surface waves, it is essential that motion is confined to the free surface $x_3 = 0$ of the half-space, so that the characteristic roots (q_j) must satisfy the radiation conditions $(\text{Re}(q_j) \geq 0)$. So, we take the solution for the displacement, microrotation and temperature distribution of the form

$$(u_1, u_3, \varphi_2, T) = \sum_{j=1}^{4} A_j \left(1, r_j, s_j, t_j\right) e^{i\xi\left(x_1 - ct + ix_3 m_j\right)} \tag{5.2}$$

where

$$r_j = \frac{II}{I}, \qquad s_j = \frac{III}{I}, \qquad t_j = \frac{IV}{I},$$

$$I = \begin{vmatrix} -m_j^2\xi^2 + a_3 & i\xi d_6 & m_j a_4 \\ -i\xi d_{10} & -m_j^2\xi^2 + a_5 & 0 \\ m_j a_7 & 0 & m_j^2\xi^2 + a_8 \end{vmatrix}, \qquad II = \begin{vmatrix} -m_j\xi^2 d_5 & i\xi d_6 & m_j a_4 \\ -im_j\xi d_9 & -m_j^2\xi^2 + a_5 & 0 \\ a_6 & 0 & m_j^2\xi^2 + a_8 \end{vmatrix},$$

$$III = \begin{vmatrix} -m_j\xi^2 d_5 & -m_j^2\xi^2 + a_3 & m_j a_4 \\ -im_j\xi d_9 & -i\xi d_{10} & 0 \\ a_6 & m_j a_7 & m_j^2\xi^2 + a_8 \end{vmatrix}, \qquad IV = \begin{vmatrix} -m_j\xi^2 d_5 & -m_j^2\xi^2 + a_3 & i\xi d_6 \\ -im_j\xi d_9 & -i\xi d_{10} & -m_j^2\xi^2 + a_5 \\ a_6 & m_j a_7 & 0 \end{vmatrix}.$$

Using Eq.(5.2) in the boundary condition (5,1), we obtain the frequency equation of the Rayleigh wave in micropolar generalized thermoelastic transversely isotropic half-space of the form

$$m_1 m_2 AT1 + m_1 m_3 AT2 + m_1 m_4 AT3 + m_2 m_3 AT4 + m_2 m_4 AT5 + m_3 m_4 AT6 = 0 \tag{5.3}$$

where

$$AT1 = \left(s_1 t_2 - s_2 t_1\right)\left(a_3^* b_4^* - a_4^* b_3^*\right), \qquad AT2 = \left(s_1 t_3 - s_3 t_1\right)\left(a_4^* b_2^* - a_2^* b_4^*\right),$$

$$AT3 = \left(s_1 t_4 - s_4 t_1\right)\left(a_2^* b_3^* - a_3^* b_2^*\right), \qquad AT4 = \left(s_2 t_3 - s_3 t_2\right)\left(a_1^* b_4^* - a_4^* b_1^*\right),$$

$$AT5 = \left(s_2 t_4 - s_4 t_2\right)\left(a_3^* b_1^* - a_1^* b_3^*\right), \qquad AT6 = \left(s_3 t_4 - s_4 t_3\right)\left(a_1^* b_2^* - a_2^* b_1^*\right),$$

$$a_j^* = i\xi d_1 - d_7 r_j m_j \xi - d_{14} t_j / d_7, \qquad b_j^* = i\xi d_{16} r_j - m_j \xi + s_j d_{15}^2, \qquad j = 1,..4.$$

Amplitudes of stresses and temperature distribution

The amplitudes of stresses and temperature distribution for plane waves can be obtained as

$$t_{33} = \sum_{j=1}^{4} A_i a_j^* e^{i\xi(x_1 - ct + im_i x_2)}, \qquad\qquad t_{31} = \sum_{j=1}^{4} A_i b_j^* e^{i\xi(x_1 - ct + im_i x_2)},$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (5.4)$$

$$m_{32} = \sum_{j=1}^{4} A_i s_j m_j e^{i\xi(x_1 - ct + im_i x_2)}, \qquad\qquad t_{33} = \sum_{j=1}^{4} A_i a_j^* e^{i\xi(x_1 - ct + im_i x_2)}.$$

Particular case:

(i) The resulting expressions of stresses and temperature distribution for coupled theory (C-T) can be obtained by substituting $\tau_1 = \tau_0 = 0$, $n_0 = 0$ in Eq.(5,4).

(ii) Taking, $\tau_1 = 0$, $n_0 = 1$ in Eq.(5.4), we obtain the expressions of stresses and temperature distribution corresponding to the Lord and Shulman (L-S) theory.

(iii) To obtain the expressions of stresses and temperature distribution corresponding to the Green and Lindsay (G-L) theory we substitute $\tau_1 \geq \tau_0 \geq 0$, $n_0 = 0$ in Eqs (5.4).

6. Numerical results and discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computations, we take the values for relevant parameters for the micropolar transversely isotropic thermoelastic solid as

$$A_{11} = 13.8 \times 10^{10} \, Nm^{-2}, \qquad\qquad A_{33} = 14.43 \times 10^{10} \, Nm^{-2},$$

$$A_{55} = 3.7 \times 10^{10} \, Nm^{-2}, \qquad\qquad A_{66} = 4.2 \times 10^{10} \, Nm^{-2},$$

$$A_{13} = 8.85 \times 10^{10} \, Nm^{-2}, \qquad\qquad A_{56} = 2.977 \times 10^{10} \, Nm^{-2},$$

$$B_{77} = 3.71 \times 10^{9} \, N, \qquad\qquad B_{66} = 3.9 \times 10^{9} \, N,$$

$$\rho = 1.74 \, Kg/m^3, \qquad\qquad j = 0.2 \times 10^{-15} \, m^2,$$

$$K^* = 0.6 \times 10^{-2} \, J/m\sec\,{}^oC, \qquad C^* = 0.23 \, J/Kg\,{}^oC, \qquad T = 298 \, K.$$

Following Gauthier (1982) we take the non dimensional values for aluminium epoxy like composite as

$$\rho = 2.19 \times 10^3\, Kgm^3, \qquad \lambda = 9.4 \times 10^{10}\, Nm^{-2}, \qquad \mu = 4 \times 10^{10}\, Nm^{-2}, \qquad K = 1 \times 10^{10}\, Nm^{-2},$$

$$C^* = 1.04\, CalK^{-1}, \qquad \gamma = 0.779 \times 10^5\, N, \qquad j = 0.2 \times 10^{-4}\, m^{-2}.$$

All numerical computations are carried out for a single fixed value of frequency $\omega = .75$ and for two given values of the wave number $\xi = .25, .35$. The computations were carried out within the range $0 \leq x_1 \leq 10$.

The solid lines represent the case of the micropolar thermoelastic transversely isotropic solid (MTIS) and the broken lines represent the case of the micropolar thermoelastic isotropic solid (MIS). A comparison of the three theories of generalized thermoelasticity, namely, coupled thermoelasticity (C-T), Lord Shulman (L-S) and Green Lindsay (G-L) is shown in all the graphs. The solid and dotted line without the center symbol corresponds to the C-T theory, solid and dotted line with the center symbol $(-o-o-)$ corresponds to the L-S theory and the solid and dotted line with the center symbol $(-\times-\times-)$ corresponds to the case of the G-L theory.

Figures 1-2 show the variation of phase velocity and specific loss with the wave number and Figs 3-9 show the variation of stresses, displacements, microrotation and temperature distribution with respect to distance.

It follows from Fig.1 that the variation of phase velocity for both MTIS and MIS decreases sharply and then attains a constant value with an increase in the wave number. Figure 2 presents that the value of specific loss initially oscillates within the interval $(0, 2)$ and then becomes constant with the wave number for both MTIS and MIS. Figures 3 and 4 show that the value of normal stress and tangential stress increases sharply within the interval $(0, 2)$ and then decreases to become constant at the end. The variations for all the 3 theories are similar with a slight difference in their amplitude. It follows from Fig.5 that the value of tangential couple stress initially goes on increasing with an increase in the distance up to a value 6 of distance, but after that it decreases with a further increase in the distance, when the wave number is $.25$. However, as the value of the wave number gets increased, the value of tangential couple stress initially oscillates and then decreases.
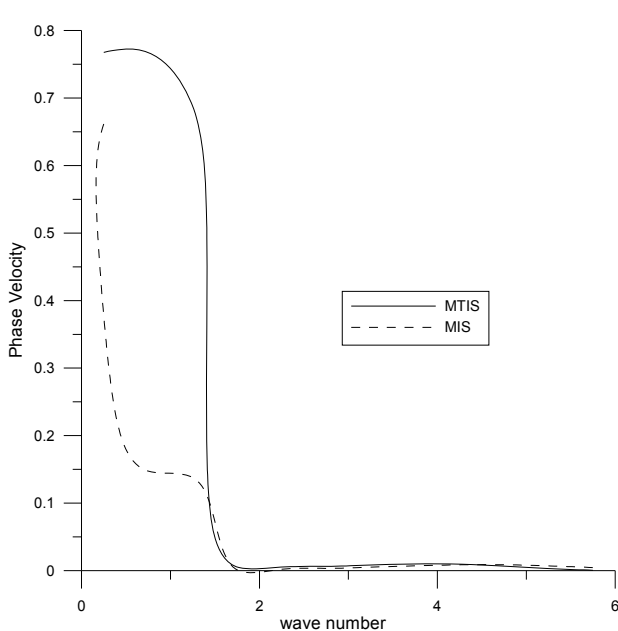

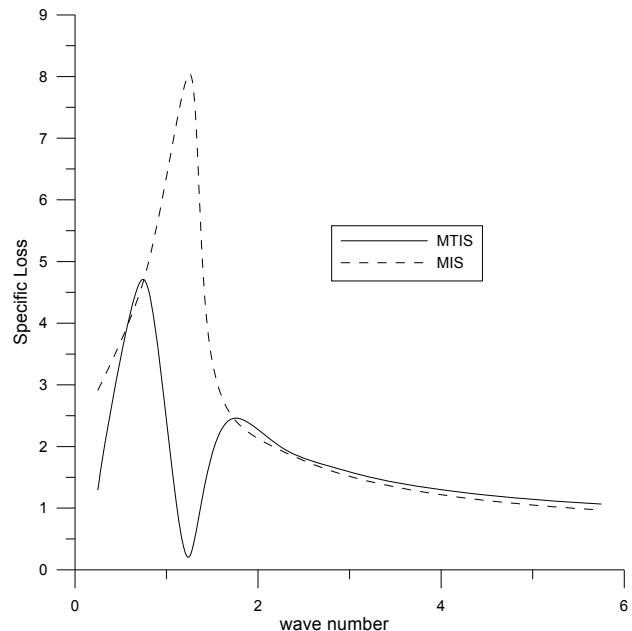Fig.1. Variation of phase velocity with wave number.   Fig.2. Variation of specific loss with wave number.
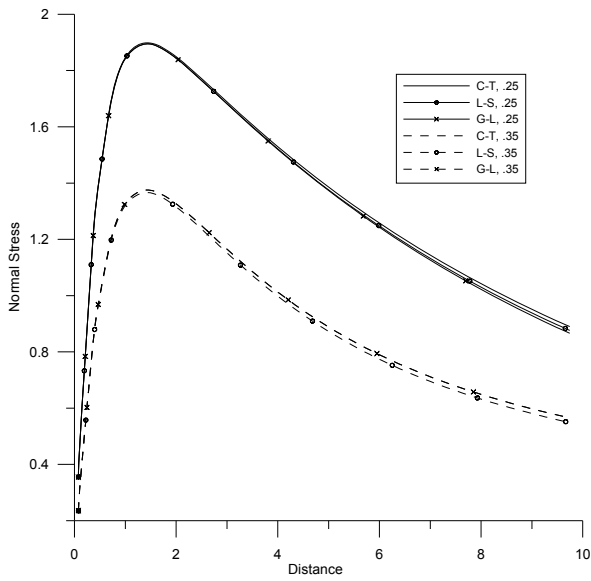
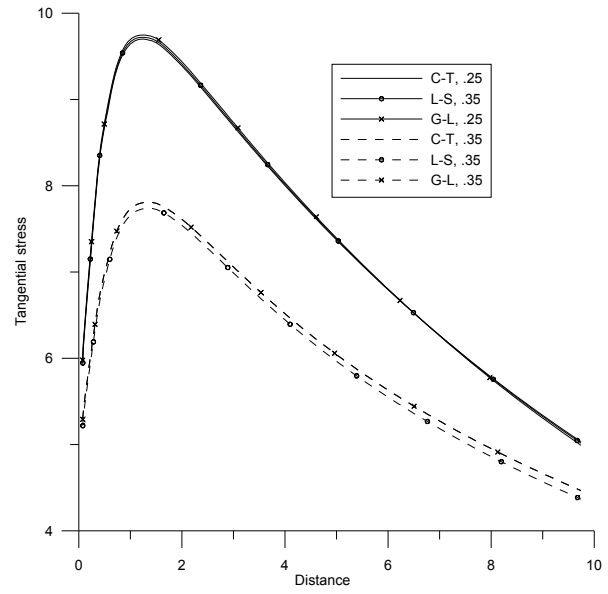Fig.3. Variations of normal stress with distance.

Fig.4. Variations of tangential stress with distance.

Figures 6-9 depict the variations of amplitudes and displacement, microrotation and temperature distribution with distance. It is illustrated in these figures that the value of displacements and microrotation go on decreasing as the distance from the surface increases. This variation pattern is physically admissible since the characteristics of Rayleigh waves are that the amplitude of the wave decreases rapidly with the depth. The rate of the decrease depends on the wavelength. Figure 9 shows that the value of temperature distribution initially increases at a depth of 5 units from the surface and then goes on decreasing with an increase in the distance. The value of displacements and microrotation gets decreased with an increase in the wave number. The variation pattern for all the three theories of thermoelasticity varies in the same manner with a slight difference in magnitude.
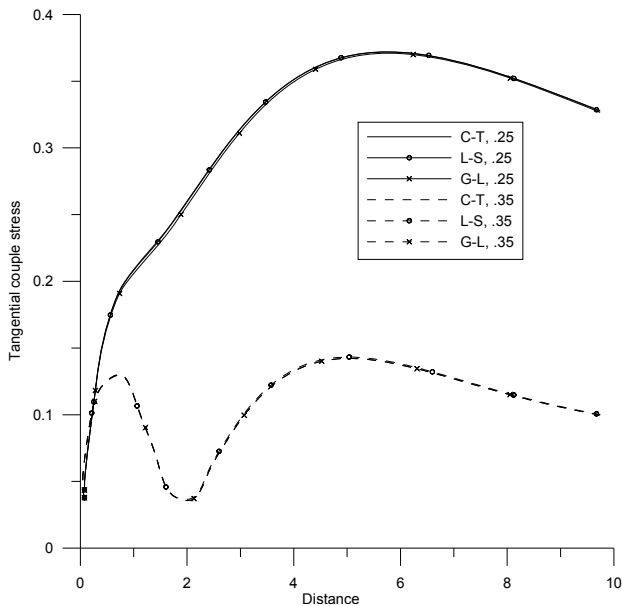

Fig.5. Variations of tangential couple stress with distance.
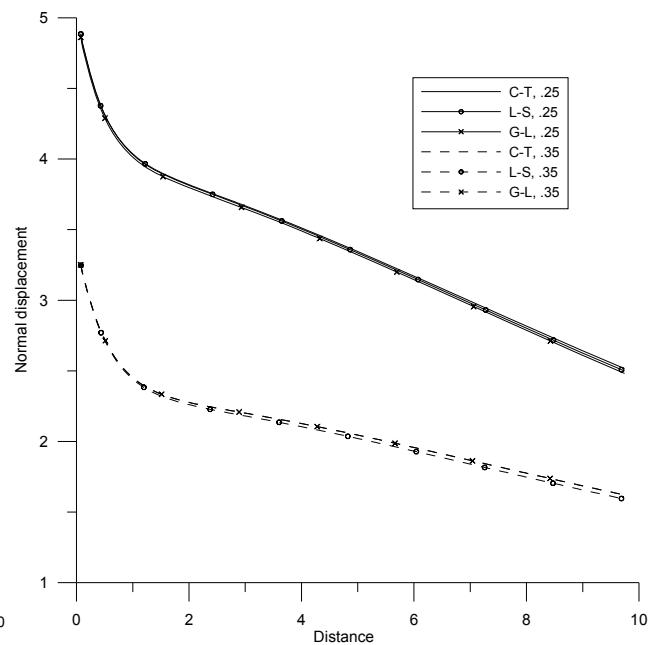

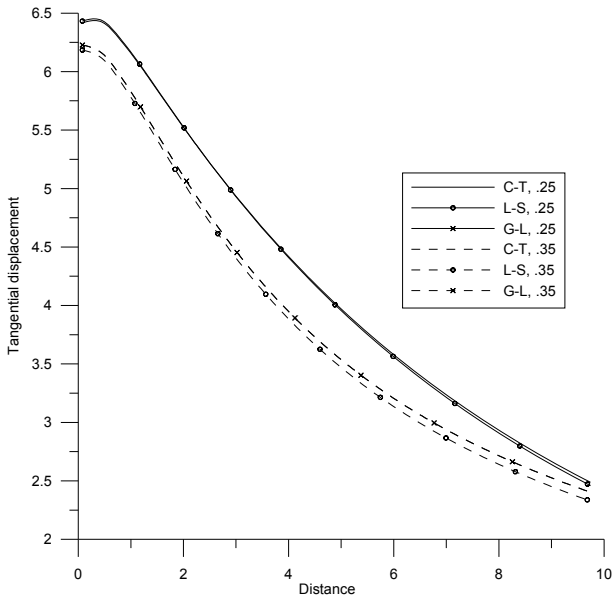Fig.6. Variation of normal displacement with distance.

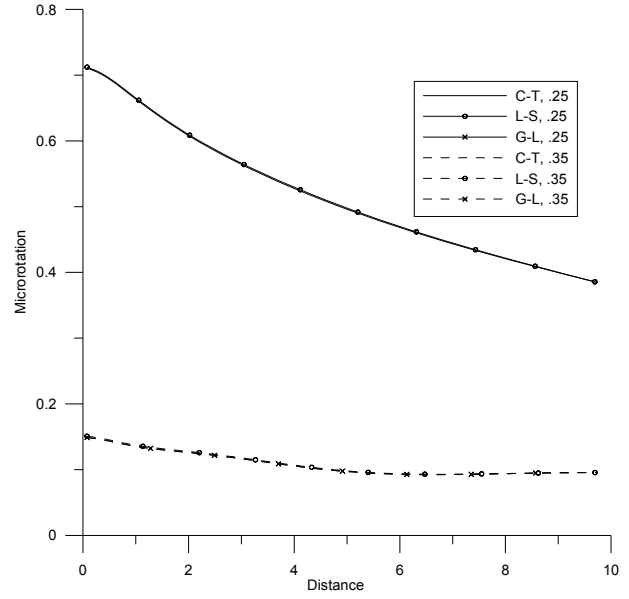Fig.7. Variations of tangential displacement with distance.

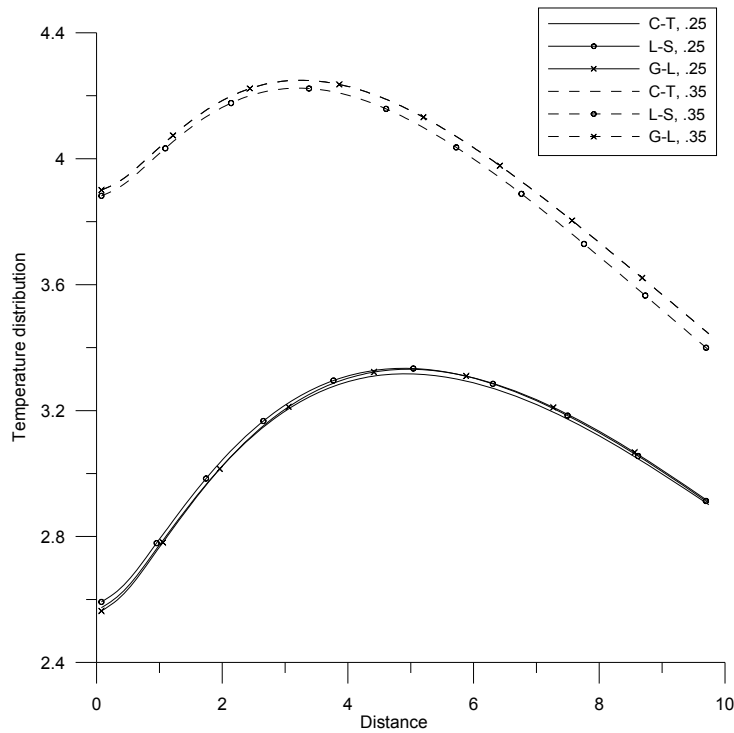Fig.8. Variations of microrotation with distance.

Fig.9. Variations of temperature distribution with distance.

5. Conclusions

The propagation of waves in a micropolar transversely isotropic medium possessing thermoelastic properties based on the Lord-Shulman (L-S), Green and Lindsay (G-L) and coupled thermoelasticty (C-T) theories are discussed. After developing the solution, the frequency equation for the propagation of Rayleigh waves has been derived. The expressions for amplitudes of stresses, displacements, microratation and

temperature distribution have been derived and computed numerically. The numerical results have been plotted graphically. The results computed numerically are found to be in close agreement with the theoretical results.

Nomenclature

$A_{ijkl}, G_{ijkl}, B_{ijkl}$ — characteristic constants of the material following the symmetry properties given by Eringen (1999)

c^* — specific heat at constant strain

j — microinertia

$K_{ij}^* = K_i \delta_{ij}$ — thermal conductivity

m_{ij} — components of couple stress

T — temperature change

T_0 — uniform reference temperature

t_{ij} — components of stress

u_i — components of displacement

β_{ij} — thermal elastic coupling tensor

δ_{ij} — Kronecker delta

ρ — density

τ_0, τ_1 — thermal relaxation times

φ — microrotation vector

φ_i — components of microrotation

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