Kołowrocki Krzysztof

Soszyńska Joanna
Maritime University, Gdynia, Poland

Safety and risk optimization of a ferry technical system

Keywords

safety function, risk function, operation process, optimization.

Abstract

The joint general model of safety of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system safety analysis and linear programming are applied in maritime transport to safety and risk optimization of a ferry technical system.

1. Introduction

Most real technical systems are very complex and it is difficult to analyze their safety. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their safety is complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' safety characteristics is often very difficult to fix and to analyze. A convenient tool for solving this problem is a semi-markov [3] modeling of the system operation processes linked with a multi-state approach for the system safety analysis [2], [12] and a linear programming for the system safety optimization [7]. This approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in [1], [4] and on semi-markov processes modeling discussed for instance in [10], [11].

2. System safety in variable operation conditions

We assume that the system during its operation process has v different operation states. Thus, we can define the system operation process $Z(t)$, $t \in < 0,+\infty >$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\}.$$

In practice, a convenient assumption is that $Z(t)$ is a semi-markov process [3] with its conditional sojourn times θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$. In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1xv}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{vxv}$, where $p_{bb}(t) = 0$ for $b = 1,2,...,v$,
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{vxv}$ of the process sojourn times θ_{bl}, $b \neq l$, in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,...,v$, $b \neq l$, and $H_{bb}(t) = 0$ for $b = 1,2,...,v$.

Under these assumptions, the sojourn times θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \quad b, l = 1,2,...,v, \quad (1)$$

$b \neq l$.

The unconditional distribution functions of the sojourn times θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,...,v,$ are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ b = 1,2,...,v.$$

The mean values $E[\theta_b]$ of the unconditional sojourn times θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,...,v, \quad (2)$$

where M_{bl} are defined by (1).
Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b), \ t \in <0,+\infty), \ b = 1,2,...,v,$$

are given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,...,v, \quad (3)$$

where the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (4)$$

In the case of a periodic operation process the limit transient probabilities p_b are long term proportions of sojourn times at the particular operation states z_b, $b = 1,2,...,v.$
We assume that the system is composed of n multistate components E_i, $i = 1,2,...,n,$ and that the changes of the operation process $Z(t)$ states have an influence on the system components E_i safety and on the system safety structure as well.
Consequently, we denote the component E_i lifetime in the safety states subset $\{u, u+1,...,z\}$ by $T_i^{(b)}(u)$ and by

$$s_i^{(b)}(t,\cdot) = [1, \ s_i^{(b)}(t,1), s_i^{(b)}(t,2), \ ..., \ s_i^{(b)}(t,z)],$$

where for $t \in <0,\infty)$, $b = 1,2,...,v$, $u = 1,2,...,z,$

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t | Z(t) = z_b),$$

its conditional safety function while the system is at the operational state z_b, $b = 1,2,...,v.$
Similarly, we denote the system lifetime in the safety states subset $\{u, u+1,...,z\}$ by $T^{(b)}(u)$ and by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\cdot) = [1, \ \boldsymbol{s}_{n_b}^{(b)}(t,1), \ \boldsymbol{s}_{n_b}^{(b)}(t,2), \ ..., \ \boldsymbol{s}_{n_b}^{(b)}(t,z)]$$

for $n_b \in \{1,2,...,n\}$, where n_b are the numbers of system components in the operation states z_b where for $t \in <0,\infty)$, $n_b \in \{1,2,...,n\}$, $b = 1,2,...,v$, $u = 1,2,...,z,$

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t | Z(t) = z_b),$$

is the conditional safety function of the system while the system is at the operational state z_b, $b = 1,2,...,v.$
Thus, the safety function $s_i^{(b)}(t,u)$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,...,z\}$ is not less than t, while the operation process $Z(t)$ is at the operation state z_b. Similarly, the safety function $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,...,z\}$ is not less than t, while the operation process $Z(t)$ is at the operation state z_b.
In the case when the system operation time is large enough, the unconditional safety function of the system is given by

$$\boldsymbol{s}_n(t,\cdot) = [1, \ \boldsymbol{s}_n(t,1), \ \boldsymbol{s}_n(t,2), \ ..., \ \boldsymbol{s}_n(t,z)], \ t \geq 0,$$

where by [6]

$$\boldsymbol{s}_n(t,u) = P(T(u) > t) \cong \sum_{b=1}^{v} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \quad (5)$$

for $t \geq 0$, $n_b \in \{1,2,...,n\}$, $u = 1,2,...,z,$ and $T(u)$ is the unconditional lifetime of the system in the safety state subset $\{u, u+1,...,z\}$.
The mean values of the system lifetimes in the safety state subset $\{u, u+1,...,z\}$ are

$$\mu(u) = E[T(u)] \cong \sum_{b=1}^{v} p_b \mu_b(u), \ u = 1,2,...,z, \quad (6)$$

where

$$\mu_b(u) = \int_0^{\infty} \boldsymbol{s}_{n_b}^{(b)}(t,u)dt, \ n_b \in \{1,2,...,n\}, \quad (7)$$

$u = 1,2,...,z.$

The mean values of the system lifetimes in the particular safety states u, are [4]

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 1,2,...,z-1,$$

$$\bar{\mu}(z) = \mu(z). \quad (8)$$

A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T^{(b)}(r) \le t),$$

$$t \in (-\infty, \infty),$$

that the system is in the subset of safety states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system or, in short, a risk [4].

Under this definition, from (6), we have

$$\boldsymbol{r}(t) = 1 - \boldsymbol{s}_n(t,r),\ t \in (-\infty, \infty). \quad (9)$$

and if τ is the moment when the risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (10)$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

3. Optimal transient probabilities maximizing system lifetimes

Considering the equation (5), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (6) for the mean values of the system unconditional lifetimes in the safety state subsets. From linear equation (6), we can see that the mean value of the system unconditional lifetime $\mu(u)$, $u = 1,2,...,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation states given by (3) and the mean values $\mu_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, of the system conditional lifetimes in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$, given by (7). Therefore, the system lifetime optimization approach based on the linear programming can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$ of the transient probabilities p_b in the system operation states to maximize the mean value $\mu(u)$ of the unconditional system lifetimes in the safety state subsets $\{u, u+1,...,z\}$ under the assumption that the mean values $\mu_b(u)$ of the system conditional lifetimes in the safety state subsets are fixed. As a special case of the above formulated system lifetime optimization problem, if r, $r = 1,2,...,z$, is a system critical safety state, then we want to find the optimal values $\dot{p}_b$ of the transient probabilities p_b in the system operation states to maximize the mean value $\mu(r)$ of the unconditional system lifetime in the safety state subset $\{r, r+1,...,z\}$ under the assumption that the mean values $\mu_b(r)$, $b = 1,2,...,\nu$, $r = 1,2,...,z$, of the system conditional lifetimes in this safety state subset are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following linear form

$$\mu(r) = \sum_{b=1}^{\nu} p_b \mu_b(r) \quad (11)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\sum_{b=1}^{\nu} p_b = 1,\ \ \breve{p}_b \le p_b \le \hat{p}_b,\ \ b = 1,2,...,\nu, \quad (12)$$

where $\mu_b(r)$, $\mu_b(r) \ge 0$, $b = 1,2,...,\nu$, are fixed mean values of the system conditional lifetimes in the safety state subset $\{r, r+1,...,z\}$ and

$$\breve{p}_b,\ 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b,\ 0 \le \hat{p}_b \le 1,$$

$$\breve{p}_b \le \hat{p}_b,\ b = 1,2,...,\nu, \quad (13)$$

are respectively the lower and upper bounds of the unknown transient probabilities p_b.

Now, we can obtain the optimal solution of the formulated by (11)-(13) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the objective function given by (11). First, we arrange the system conditional lifetime mean values $\mu_b(r)$, $b = 1,2,...,\nu$, in non-increasing order

$$\mu_{b_1}(r) \ge \mu_{b_2}(r) \ge \ldots \ge \mu_{b_\nu}(r),$$

where $b_i \in \{1,2,...,\nu\}$ for $i = 1,2,...,\nu$.

Next, we substitute

$$x_i = p_{b_i}, \ \breve{x}_i = \breve{p}_{b_i}, \ \hat{x}_i = \hat{p}_{b_i} \ \text{for} \ i = 1,2,...,\nu \quad (14)$$

and we maximize with respect to x_i, $i = 1,2,...,\nu$, the linear form (11) that after this transformation takes the form

$$\mu(r) = \sum_{i=1}^{\nu} x_i \mu_{b_i}(r) \quad (15)$$

for a fixed $r \in \{1,2,...,z\}$ with the following bound constraints

$$\sum_{i=1}^{\nu} x_i = 1, \ \breve{x}_i \le x_i \le \hat{x}_i, \ i = 1,2,...,\nu, \quad (16)$$

where $\mu_{b_i}(r)$, $\mu_{b_i}(r) \ge 0$, $i = 1,2,...,\nu$, are fixed mean values of the system conditional lifetimes in the safety state subset $\{r, r+1,..., z\}$ arranged in non-increasing order and

$$\breve{x}_i, \ 0 \le \breve{x}_i \le 1 \ \text{and} \ \hat{x}_i, \ 0 \le \hat{x}_i \le 1,$$

$$\breve{x}_i \le \hat{x}_i, \ i = 1,2,...,\nu, \quad (17)$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,...,\nu$, respectively.
We define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i, \ \hat{y} = 1 - \breve{x} \quad (18)$$

and

$$\breve{x}^0 = 0, \ \hat{x}^0 = 0 \ \text{and} \ \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i, \ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \quad (19)$$

for $I = 1,2,...,\nu$.

Next, we find the largest value $I \in \{0,1,...,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \quad (20)$$

and we fix the optimal solution that maximize (15) in the following way:

i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \ \text{and} \ \dot{x}_i = \breve{x}_i \quad (21)$$

for $i = 2,3,...,\nu$;

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \ \text{for} \ i = 1,2,..., I,$$

$$\dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1} \ \text{and} \ \dot{x}_i = \breve{x}_i \quad (22)$$

for $i = I+2, I+3,...,\nu$;

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \ \text{for} \ i = 1,2,...,\nu. \quad (23)$$

Finally, after making the inverse to (14) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \ \text{for} \ i = 1,2,...,\nu, \quad (24)$$

that maximize the system mean lifetime $\mu(r)$ in the safety state subset $\{r, r+1,..., z\}$, defined by the linear form (11) giving its maximum value in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(r) \quad (25)$$

for a fixed $r \in \{1,2,..., z\}$.
From the above, replacing r by u, $u = 1,2,..., z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the safety state subsets $\{u, u+1,..., z\}$ of the form

$$\dot{\mu}(u) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(u) \ \text{for} \ u = 1,2,..., z. \quad (26)$$

Further, according to (5), the corresponding optimal unconditional multistate safety function of the system is

$$\dot{\boldsymbol{s}}_{\boldsymbol{n}}(t,\cdot) = [1, \ \dot{\boldsymbol{s}}_{\boldsymbol{n}}(t,1), ..., \ \dot{\boldsymbol{s}}_{\boldsymbol{n}}(t,z)], \quad (27)$$

where

$$\dot{\boldsymbol{s}}_n(t,u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{s}_n(t,u)]^{(b)} \ \text{for} \ t \ge 0, \quad (28)$$

$$u = 1,2,..., z$$

and by (8) the optimal solutions for the mean values of the system unconditional lifetimes in the particular safety states are of the form

$$\dot{\bar{\mu}}(u) = \dot{\mu}(u) - \dot{\mu}(u+1), \ u = 0,1,..., z-1,$$

$$\dot{\bar{\mu}}(z) = \dot{\mu}(z). \quad (29)$$

Moreover, considering (9) and (10), the corresponding optimal system risk function and the moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{s}}_n(t,r) \ \ t \in (-\infty, \infty), \quad (30)$$

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta), \quad (31)$$

where $\dot{\boldsymbol{r}}^{-1}(t)$, if it exists, is the inverse function of the risk function $\dot{\boldsymbol{r}}(t)$.

4. The ferry technical system safety and risk

We consider a passenger ro-ro ferry operating in Baltic Sea between the Gdynia port in Poland and the Karlskrona port in Sweden on regular everyday timetable.

Figure 1. The ferry on her voyage

We assume that the ferry is composed of seven subsystems S_1, S_2, S_3, S_4, S_5, S_6, S_7 having an essential influence on its safety [12]. These subsystems are:

S_1 - a navigational subsystem,

S_2 - a propulsion and controlling subsystem,

S_3 - a loading and unloading subsystem,

S_4 - a hull subsystem,

S_5 - an anchoring and mooring subsystem,

S_6 - a protection and rescue subsystem,

S_7 - a social subsystem.

In the ferry safety analysis we omit the protection and rescue subsystem S_6 and the social subsystem S_7 and we consider its strictly technical subsystems S_1, S_2, S_3, S_4 and S_5 only.

Additionally, we assume that subsystems S_i, $i = 1,2,3,4,5$ are composed of five-state components, i.e. $z = 4$, with the multi-state safety functions

$$s_i^{(b)}(t, \cdot)$$

$$= [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ s_i^{(b)}(t,3),\ s_i^{(b)}(t,4)\,],$$

$$t \in <0, \infty), \ b = 1,2,\ldots,18, \ u = 1,2,3,4,$$

with exponential co-ordinates different in various operation states z_b, $b = 1,2,\ldots,18$.

Further, assuming that the ferry is in the safety state subset $\{u, u+1, \ldots, 4\}$ if all its subsystems are in this subset of safety states, we conclude that the ferry is a series system [4] of subsystems S_1, S_2, S_3, S_4, S_5 with a scheme presented in *Figure 2*.

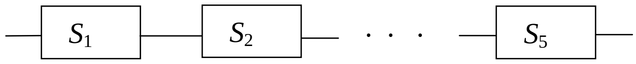

Figure 2. The scheme of a ferry technical system safety structure

We assume that the changes of the ferry operation states have an influence on its technical system safety structure and on its subsystems S_i safety functions as well [12]. Taking into account the operation process of the considered ferry we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia port,
- an operation state z_2 – unmooring operations at Gdynia port,
- an operation state z_3 – leaving Gdynia port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoring” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoring” buoy to “Verko” Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona port,
- an operation state z_8 – unloading at Karlskrona port,
- an operation state z_9 – loading at Karlskrona port,
- an operation state z_{10} – unmooring operations at Karlskrona port,
- an operation state z_{11} – ship turning at Karlskrona port,
- an operation state z_{12} – leaving Karlskrona port and navigation at restricted waters to “Angoring” buoy,
- an operation state z_{13} – navigation at open waters from “Angoring” buoy to the entering Traffic Separation Scheme,

- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia port,
- an operation state z_{17} – mooring operations at Gdynia port,
- an operation state z_{18} – unloading at Gdynia port.

The ferry operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,...,17$, to the neighboring state z_{b+1}, $b = 1,2,...,17$, and from z_{18} to z_1 only. Therefore, the probabilities of transitions between the operation states are given by

$$[p_{bl}] = \begin{bmatrix} 0\,1\,0 \ldots 0\,0 \\ 0\,0\,1 \ldots 0\,0 \\ \ldots \\ 0\,0\,0 \ldots 0\,1 \\ 1\,0\,0 \ldots 0\,0 \end{bmatrix}.$$

On the basis of statistical data coming from experts [12], the mean values of the conditional sojourn times in the operation states are (in minutes):

$$M_{12} = 54.33,\ M_{23} = 2.57,\ M_{34} = 36.57,$$

$$M_{45} = 52.5,\ M_{56} = 525.95,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 21.43,\ M_{910} = 53.69,$$

$$M_{1011} = 2.93,\ M_{1112} = 4.38,\ M_{1213} = 23.86,$$

$$M_{1314} = 509.69,\ M_{1415} = 50.14,\ M_{1516} = 34.28,$$

$$M_{1617} = 4.52,\ M_{1718} = 5.62,\ M_{181} = 18.74.$$

Hence, by (2), the unconditional mean sojourn times in the operation states are (in minutes):

$$M_1 = 54.33,\ M_2 = 2.57,\ M_3 = 36.57,$$

$$M_4 = 52.5,\ M_5 = 525.95,\ M_6 = 37.16,$$

$$M_7 = 7.02,\ M_8 = 21.43,\ M_9 = 53.69,$$

$$M_{10} = 2.93,\ M_{11} = 4.38,\ M_{12} = 23.86,$$

$$M_{13} = 509.69,\ M_{14} = 50.14,\ M_{15} = 34.28,$$

$$M_{16} = 4.52,\ M_{17} = 5.62,\ M_{18} = 18.74.$$

Since from the system of equations (4) we get

$$\pi_i = 0.056 \text{ for } i = 1,2,...,18,$$

then the long term proportion of transients p_b at the operational states z_b, according to (3), are given by

$$p_1 = 0.037,\ p_2 = 0.002,\ p_3 = 0.025,\ p_4 = 0.036,$$

$$p_5 = 0.364,\ p_6 = 0.025,\ p_7 = 0.005,\ p_8 = 0.014,$$

$$p_9 = 0.037,\ p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.017,$$

$$p_{13} = 0.354,\ p_{14} = 0.035,\ p_{15} = 0.024,\ p_{16} = 0.003,$$

$$p_{17} = 0.004,\ p_{18} = 0.013. \qquad (32)$$

Under the assumption that the changes of the ferry operation states have an influence on the subsystem S_i safety and on the ferry safety structures as well, on the basis of expert opinions and statistical data given in [12], the ferry technical system safety structures and their components safety functions at different operation states can be determined.

For instance, at the operation state z_1, i.e. at the loading state the ferry built of $n_1 = 2$ subsystems S_3 and S_4 forming a series structure [4] shown in *Figure 3*.

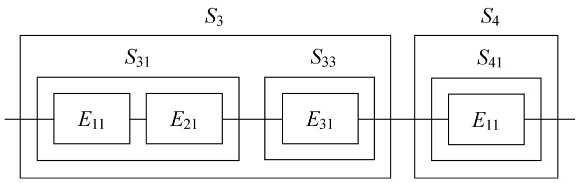

Figure 3. The scheme of the ferry structure at the operation state z_1

Considering that the ferry is in the safety state subsets $\{u, u+1,...,4\}$, $u = 1,2,3,4$, if all its subsystems are in this safety state subset, the considered system is a five-state series system [4] and the conditional safety function of the ferry while the ferry is at the operational state z_1 is given by

$$\bar{\mathbf{s}}_2^{(1)}(t,\ \cdot)$$

$= [1, \bar{\boldsymbol{s}}_2^{(1)}(t,\ 1), \bar{\boldsymbol{s}}_2^{(1)}(t,\ 2), \bar{\boldsymbol{s}}_2^{(1)}(t,\ 3), \bar{\boldsymbol{s}}_2^{(1)}(t,\ 4)\ ]$, (33)

where

$$\bar{\boldsymbol{s}}_2^{(1)}(t,u) = \boldsymbol{s}_{3;1,1,1}^{(1)}(t,\ u)\ \boldsymbol{s}_{1,1}^{(1)}(t,\ u) \quad (34)$$

for $t \in < 0, \infty)$, $u = 1,2,3,4$,

i.e.

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 1) = \exp[-0.433t]\ \exp[-0.05t] = \exp[-0.483t], \quad (35)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 2) = \exp[-0.59t]\ \exp[-0.06t] = \exp[-0.65t] \quad (36)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 3) = \exp[-0.695t]\ \exp[-0.065t] = \exp[-0.76t], \quad (37)$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 4) = \exp[-0.85t]\ \exp[-0.07t] = \exp[-0.92t]. \quad (38)$$

The expected values and standard deviations of the ferry conditional lifetimes in the safety state subsets calculated from the above result given by (33)-(38), according to (7), at the operational state z_1 are:

$\mu_1(1) \cong 2.07$, $\mu_1(2) \cong 1.54$

$\mu_1(3) \cong 1.32$, $\mu_1(4) \cong 1.09$ years, (39)

$\sigma_1(1) \cong 2.07$, $\sigma_1(2) \cong 1.54$,

$\sigma_1(3) \cong 1.32$, $\sigma_1(4) \cong 1.09$ years, (40)

and further, using (8), it follows that the ferry conditional lifetimes in the particular safety states at the operational state z_1 are:

$\bar{\mu}_1(1) \cong 0.53$, $\bar{\mu}_1(2) \cong 0.22$,

$\bar{\mu}_1(3) \cong 0.23$, $\bar{\mu}_1(4) \cong 1.09$ years. (41)

At the operation states z_2, i.e. at the unmooring operations state the ferry is built of $n_2 = 3$ subsystems S_1, S_2 and S_5 forming a series structure [4] shown in *Figure 4*.

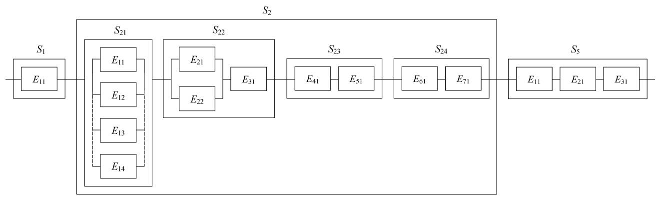

Figure 4. The scheme of the ferry structure at the operation state z_2

Considering that the ferry is in the safety state subsets $\{u, u+1,...,4\}$, $u = 1,2,3,4,$ if all its subsystems are in this safety state subset, the considered system is a five-state series system [4] and the conditional safety function of the ferry while the ferry is at the operational state z_2 is given by

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ \cdot)$$

$$=[1, \bar{\boldsymbol{s}}_3^{(2)}(t,\ 1), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 2), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 3), \bar{\boldsymbol{s}}_3^{(2)}(t,\ 4)\ ], \quad (42)$$

where

$$\bar{\boldsymbol{s}}_3^{(2)}(t,u)$$

$$= \boldsymbol{s}_{1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{7;4,2,1,1,1,1,1}^{(2)}(t,\ u)\ \boldsymbol{s}_{3;1,1,1}^{(2)}(t,\ u) \quad (43)$$

for $t \in < 0, \infty)$, $u = 1,2,3,4$,

i.e.

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 1) = 12\exp[-0.462t] + 8\exp[-0.561t] -16\exp[-0.495t]- 3\exp[-0.594t] \quad (44)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 2) = 12\exp[-0.54t] + 8\exp[-0.65t] + 6\exp[-0.62t] - 16\exp[-0.58t] - 6\exp[-0.61t] - 3\exp[-0.69t], \quad (45)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 3) = 12\exp[-0.62t] + 8\exp[-0.745t] + 6\exp[-0.72t] - 16\exp[-0.67t] - 6\exp[-0.695t] - 3\exp[-0.795t], \quad (46)$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 4) = 12\exp[-0.685t] + 8\exp[-0.82t] + 6\exp[-0.795t] - 16\exp[-0.74t] - 6\exp[-0.765t] - 3\exp[-0.875t]. \quad (47)$$

The expected values and standard deviations of the ferry conditional lifetimes in the safety state subsets calculated from the above result given by (42)-(47), according to (7), at the operational state z_2 are:

$$\mu_2(1) \cong 2.86, \;\; \mu_2(2) \cong 0.43$$

$$\mu_2(3) \cong 2.14, \;\; \mu_2(4) \cong 1.93 \text{ years}, \qquad (48)$$

$$\sigma_2(2) \cong 2.74, \;\; \sigma_2(2) \cong 2.35,$$

$$\sigma_2(3) \cong 2.05, \;\; \sigma_2(4) \cong 1.85 \text{ years}, \qquad (49)$$

and further, using (8), it follows that the ferry conditional lifetimes in the particular safety states at the operational state z_2 are:

$$\bar{\mu}_2(1) \cong 0.43, \;\; \bar{\mu}_2(2) \cong 0.29,$$

$$\bar{\mu}_2(3) \cong 0.21, \;\; \bar{\mu}_2(4) \cong 1.93 \text{ years}. \qquad (50)$$

At the remaining operation states z_b, $b = 3,..,18$ we proceed in an analogous way. We determined the system conditional safety functions in particular operation states and the expected values and standard deviations of the ferry conditional lifetimes.
In the case when the system operation time is large enough, the unconditional safety function of the ferry is given by the vector

$$\boldsymbol{s}_5(t, \cdot)$$

$$= [1, \; \boldsymbol{s}_5(t,1), \; \boldsymbol{s}_5(t,2), \; \boldsymbol{s}_5(t,3), \; \boldsymbol{s}_5(t,4)], \; t \geq 0, (51)$$

where, according to (5) and after considering the values of p_b, $b = 1,2,...,18$, given by (32), its co-ordinates are as follows:

$$\boldsymbol{s}_5(t,u) = 0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,u) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,u)$$

$$+ 0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,u) + 0.036 \cdot \bar{\boldsymbol{s}}_3^{(4)}(t,u)$$

$$+ 0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,u) + 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,u)$$

$$+ 0.005 \cdot \bar{\boldsymbol{s}}_3^{(7)}(t,u) + 0.014 \cdot \bar{\boldsymbol{s}}_2^{(8)}(t,u)$$

$$+ 0.037 \cdot \bar{\boldsymbol{s}}_2^{(9)}(t,u) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(10)}(t,u)$$

$$+ 0.003 \cdot \bar{\boldsymbol{s}}_2^{(11)}(t,u) + 0.017 \cdot \bar{\boldsymbol{s}}_3^{(12)}(t,u)$$

$$+ 0.354 \cdot \bar{\boldsymbol{s}}_3^{(13)}(t,u) + 0.035 \cdot \bar{\boldsymbol{s}}_3^{(14)}(t,u)$$

$$+ 0.024 \cdot \bar{\boldsymbol{s}}_2^{(15)}(t,u) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(16)}(t,u)$$

$$+ 0.004 \cdot \bar{\boldsymbol{s}}_3^{(17)}(t,u) + 0.013 \cdot \bar{\boldsymbol{s}}_2^{(18)}(t,u), \qquad (52)$$

for $t \geq 0$, $u = 1,2,3,4$, where $\boldsymbol{s}_2^{(1)}(t,u)$ and $\boldsymbol{s}_3^{(2)}(t,u)$ are respectively given by (35)-(38) and (44)-(47) and $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ for $b = 3,4,...,18$, are given in [12].

The mean values and standard deviations of the system unconditional lifetimes in the safety state subsets, according to (6)-(7) respectively are:

$$\mu(1) \cong 0.037 \cdot 2.07 + 0.002 \cdot 2.86 + 0.025 \cdot 4.94$$

$$+ 0.036 \cdot 4.2 + 0.364 \cdot 4.2 + 0.025 \cdot 4.01$$

$$+ 0.005 \cdot 2.86 + 0.014 \cdot 3.53 + 0.037 \cdot 3.53$$

$$+ 0.002 \cdot 2.86 + 0.003 \cdot 3.91 + 0.017 \cdot 4.2$$

$$+ 0.354 \cdot 4.2 + 0.035 \cdot 4.2 + 0.024 \cdot 4.94$$

$$+ 0.003 \cdot 3.91 + 0.004 \cdot 2.86 + 0.013 \cdot 2.07$$

$$\cong 4.07, \qquad (53)$$

$$\sigma(1) \cong 4.1,$$

$$\mu(2) \cong 0.037 \cdot 1.54 + 0.002 \cdot 2.43 + 0.025 \cdot 3.9$$

$$+ 0.036 \cdot 3.80 + 0.364 \cdot 3.80 + 0.025 \cdot 3.24$$

$$+ 0.005 \cdot 2.43 + 0.014 \cdot 2.50 + 0.037 \cdot 2.50$$

$$+ 0.002 \cdot 2.43 + 0.003 \cdot 3.37 + 0.017 \cdot 3.80$$

$$+ 0.354 \cdot 3.80 + 0.035 \cdot 3.80 + 0.024 \cdot 3.90$$

$$+ 0.003 \cdot 3.37 + 0.004 \cdot 2.43 \cong 3.59, \qquad (54)$$

$$\sigma(2) \cong 3.34,$$

$$\mu(3) \cong 0.037 \cdot 1.32 + 0.002 \cdot 2.14 + 0.025 \cdot 3.44$$

$$+ 0.036 \cdot 3.38 + 0.364 \cdot 3.38 + 0.025 \cdot 2.88$$

$$+ 0.005 \cdot 2.14 + 0.014 \cdot 2.17 + 0.037 \cdot 2.17$$

$$+ 0.002 \cdot 2.14 + 0.003 \cdot 3.07 + 0.017 \cdot 3.38$$

$+0.354 \cdot 3.38 + 0.035 \cdot 3.38 + 0.024 \cdot 3.44$

$+0.003 \cdot 3.07 + 0.004 \cdot 2.14 + 0.013 \cdot 1.32$

$\cong 3.19,$ (55)

$\sigma(3) \cong 3.65,$

$\mu(4) \cong 0.037 \cdot 1.09 + 0.002 \cdot 1.93 + 0.025 \cdot 3.1$

$+0.036 \cdot 3.05 + 0.364 \cdot 3.05 + 0.025 \cdot 2.61$

$+0.005 \cdot 1.93 + 0.014 \cdot 1.92 + 0.037 \cdot 1.92$

$+0.002 \cdot 1.93 + 0.003 \cdot 2.76 + 0.017 \cdot 3.05$

$+0.354 \cdot 3.05 + 0.035 \cdot 3.05 + 0.024 \cdot 3.10$

$+0.003 \cdot 2.76 + 0.004 \cdot 1.93 + 0.013 \cdot 1.09$

$\cong 2.87,$ (56)

$\sigma(4) \cong 2.75.$

The mean values of the system lifetimes in the particular safety states, by (8), are

$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.48,$

$\bar{\mu}(2) = \mu(2) - \mu(3) = 0.4,$

$\bar{\mu}(3) = \mu(3) - \mu(4) = 0.32,$

$\bar{\mu}(4) = \mu(4) = 2.87.$ (57)

If the critical safety state is $r = 2$, then the system risk function, according to (9) [6] , is given by

$\boldsymbol{r}(t) = 1 - \boldsymbol{s}_5(t, 2)$

$= 1 - [0.037 \cdot \bar{\boldsymbol{s}}_2^{(1)}(t,2) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(2)}(t,2)$

$+0.025 \cdot \bar{\boldsymbol{s}}_2^{(3)}(t,2) + 0.036 \cdot \bar{\boldsymbol{s}}_3^{(4)}(t,2)$

$+0.364 \cdot \bar{\boldsymbol{s}}_3^{(5)}(t,2) + 0.025 \cdot \bar{\boldsymbol{s}}_3^{(6)}(t,2)$

$+0.005 \cdot \bar{\boldsymbol{s}}_3^{(7)}(t,2) + 0.014 \cdot \bar{\boldsymbol{s}}_2^{(8)}(t,2)$

$+0.037 \cdot \bar{\boldsymbol{s}}_2^{(9)}(t,2) + 0.002 \cdot \bar{\boldsymbol{s}}_3^{(10)}(t,2)$

$+0.003 \cdot \bar{\boldsymbol{s}}_2^{(11)}(t,2) + 0.017 \cdot \bar{\boldsymbol{s}}_3^{(12)}(t,2)$

$+0.354 \cdot \bar{\boldsymbol{s}}_3^{(13)}(t,2) + 0.035 \cdot \bar{\boldsymbol{s}}_3^{(14)}(t,2)$

$+0.024 \cdot \bar{\boldsymbol{s}}_2^{(15)}(t,2) + 0.003 \cdot \bar{\boldsymbol{s}}_2^{(16)}(t,2)$

$+0.004 \cdot \bar{\boldsymbol{s}}_3^{(17)}(t,2) + 0.013 \cdot \bar{\boldsymbol{s}}_2^{(18)}(t,2)]$ (58)

for $t \geq 0$.

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (10), is

$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.19$ years. (59)

5. Optimization of the ferry technical system operation process

In this case, as the critical state is $r = 2$, then considering the expression for $\mu(2)$ in (54), the objective function (11), takes the form

$\mu(2) = p_1 \cdot 1.54 + p_2 \cdot 2.43 + p_3 \cdot 3.90$

$+ p_4 \cdot 3.80 + p_5 \cdot 3.80 + p_6 \cdot 3.24$

$+ p_7 \cdot 2.43 + p_8 \cdot 2.50 + p_9 \cdot 2.50$

$+ p_{10} \cdot 2.43 + p_{11} \cdot 3.37 + p_{12} \cdot 3.80$

$+ p_{13} \cdot 3.80 + p_{14} \cdot 3.80 + p_{15} \cdot 3.90$

$+ p_{16} \cdot 3.37 + p_{17} \cdot 2.43 + p_{18} \cdot 1.54.$ (60)

The lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,18$, coming from experts, respectively are [7]:

$\breve{p}_1 = 0.0006$, $\breve{p}_2 = 0.001$, $\breve{p}_3 = 0.018$, ,

$\breve{p}_4 = 0.027$, $\breve{p}_5 = 0.286$, $\breve{p}_6 = 0.018$,

$\breve{p}_7 = 0.002$, $\breve{p}_8 = 0.001$, $\breve{p}_9 = 0.001$,

$\breve{p}_{10} = 0.001$, $\breve{p}_{11} = 0.002$, $\breve{p}_{12} = 0.013$,

$\breve{p}_{13} = 0.286$, $\breve{p}_{14} = 0.025$, $\breve{p}_{15} = 0.018$,

$\breve{p}_{16} = 0.002$, $\breve{p}_{17} = 0.002$, $\breve{p}_{18} = 0.001$,

$\hat{p}_1 = 0.056,\ \hat{p}_2 = 0.002,\ \hat{p}_3 = 0.027,$

$\hat{p}_4 = 0.056,\ \hat{p}_5 = 0.780,\ \hat{p}_6 = 0.024,$

$\hat{p}_7 = 0.018,\ \hat{p}_8 = 0.018,\ \hat{p}_9 = 0.056,$

$\hat{p}_{10} = 0.003,\ \hat{p}_{11} = 0.004,\ \hat{p}_{12} = 0.024,$

$\hat{p}_{13} = 0.780,\ \hat{p}_{14} = 0.043,\ \hat{p}_{15} = 0.024,$

$\hat{p}_{16} = 0.004,\ \hat{p}_{17} = 0.007,\ \hat{p}_{18} = 0.018.$

Therefore, according to (12)-(13), we assume the following bound constraints

$$\sum_{b=1}^{18} p_b = 1, \qquad (61)$$

$0.0006 \le p_1 \le 0.056,\ 0.001 \le p_2 \le 0.002,$

$0.018 \le p_3 \le 0.027,\ 0.027 \le p_4 \le 0.056,$

$0.286 \le p_5 \le 0.780,\ 0.018 \le p_6 \le 0.024,$

$0.002 \le p_7 \le 0.018,\ 0.001 \le p_8 \le 0.018,$

$0.001 \le p_9 \le 0.056,\ 0.001 \le p_{10} \le 0.003,$

$0.002 \le p_{11} \le 0.004,\ 0.013 \le p_{12} \le 0.024,$

$0.286 \le p_{13} \le 0.780,\ 0.025 \le p_{14} \le 0.043,$

$0.018 \le p_{15} \le 0.024,\ 0.002 \le p_{16} \le 0.004,$

$0.002 \le p_{17} \le 0.007,\ 0.001 \le p_{18} \le 0.018. \qquad (62)$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,18,$ that maximize the objective function (60), we arrange the system conditional lifetimes mean values $\mu_b(2)$, $b = 1,2,...,18,$ in non-increasing order

$\mu_3(2) \ge \mu_{15}(2) \ge \mu_4(2) \ge \mu_5(2) \ge \mu_{12}(2) \ge$

$\mu_{13}(2) \ge \mu_{14}(2) \ge \mu_{11}(2) \ge \mu_{16}(2) \ge \mu_6(2) \ge$

$\mu_8(2) \ge \mu_9(2) \ge \mu_2(2) \ge \mu_7(2) \ge \mu_{10}(2) \ge$

$\mu_{17}(2) \ge \mu_1(2) \ge \mu_{18}(2).$

Next, according to (14), we substitute

$x_1 = p_3 = 0.025,\ x_2 = p_{15} = 0.024,$

$x_3 = p_4 = 0.036,\ x_4 = p_5 = 0.364,$

$x_5 = p_{12} = 0.017,\ x_6 = p_{13} = 0.354,$

$x_7 = p_{14} = 0.035,\ x_8 = p_{11} = 0.00,$

$x_9 = p_{16} = 0.003,\ x_{10} = p_6 = 0.025,$

$x_{11} = p_8 = 0.014,\ x_{12} = p_9 = 0.037,$

$x_{13} = p_2 = 0.002,\ x_{14} = p_7 = 0.005,$

$x_{15} = p_{10} = 0.002,\ x_{16} = p_{17} = 0.004,$

$x_{17} = p_1 = 0.037,\ x_{18} = p_{18} = 0.013, \qquad (63)$

and

$\breve{x}_1 = 0.018,\ \breve{x}_2 = 0.018,\ \breve{x}_3 = 0.027,$

$\breve{x}_4 = 0.286,\ \breve{x}_5 = 0.013,\ \breve{x}_6 = 0.286,$

$\breve{x}_7 = 0.025,\ \breve{x}_8 = 0.002,\ \breve{x}_9 = 0.002,$

$\breve{x}_{10} = 0.018,\ \breve{x}_{11} = 0.001,\ \breve{x}_{12} = 0.001,$

$\breve{x}_{13} = 0.001,\ \breve{x}_{14} = 0.002,\ \breve{x}_{15} = 0.001,$

$\breve{x}_{16} = 0.002,\ \breve{x}_{17} = 0.0006,\ \breve{x}_{18} = 0.001,$

$\hat{x}_1 = 0.027,\ \hat{x}_2 = 0.024,\ \hat{x}_3 = 0.056,$

$\hat{x}_4 = 0.780,\ \hat{x}_5 = 0.024,\ \hat{x}_6 = 0.780,$

$\hat{x}_7 = 0.043,\ \hat{x}_8 = 0.004,\ \hat{x}_9 = 0.004,$

$\hat{x}_{10} = 0.024,\ \hat{x}_{11} = 0.018,\ \hat{x}_{12} = 0.056,$

$\hat{x}_{13} = 0.002,\ \hat{x}_{14} = 0.018,\ \hat{x}_{15} = 0.003,$

$\hat{x}_{16} = 0.007,\ \hat{x}_{17} = 0.056,\ \hat{x}_{18} = 0.018, \qquad (64)$

where $\breve{x}_i$ and $\hat{x}_i$ are lower and upper bounds of the unknown limit transient probabilities x_i, $i = 1,2,...,18,$ respectively and we maximize with respect to x_i, $i = 1,2,...,18,$ the linear form (60) that according to (15) takes the form

$$\mu(2) = x_1 \cdot 3.90 + x_2 \cdot 3.90 + x_3 \cdot 3.80$$
$$+ x_4 \cdot 3.80 + x_5 \cdot 3.80 + x_6 \cdot 3.80$$
$$+ x_7 \cdot 3.80 + x_8 \cdot 3.37 + x_9 \cdot 3.37$$
$$+ x_{10} \cdot 3.24 + x_{11} \cdot 2.50 + x_{12} \cdot 2.50$$
$$+ x_{13} \cdot 2.43 + x_{14} \cdot 2.43 + x_{15} \cdot 2.43$$
$$+ x_{16} \cdot 2.43 + x_{17} \cdot 1.54 + x_{18} \cdot 1.54, \quad (65)$$

with the following bound constraints

$$\sum_{i=1}^{18} x_i = 1, \quad (66)$$

$$0.018 \le x_1 \le 0.027,\ 0.018 \le x_2 \le 0.024,$$
$$0.027 \le x_3 \le 0.056,\ 0.286 \le x_4 \le 0.780,$$
$$0.013 \le x_5 \le 0.024,\ 0.286 \le x_6 \le 0.780,$$
$$0.025 \le x_7 \le 0.043,\ 0.002 \le x_8 \le 0.004,$$
$$0.002 \le x_9 \le 0.004,\ 0.018 \le x_{10} \le 0.024,$$
$$0.001 \le x_{11} \le 0.018,\ 0.001 \le x_{12} \le 0.056,$$
$$0.001 \le x_{13} \le 0.002,\ 0.002 \le x_{14} \le 0.018,$$
$$0.001 \le x_{15} \le 0.003,\ 0.002 \le x_{16} \le 0.007,$$
$$0.0006 \le x_{17} \le 0.056,\ 0.001 \le x_{18} \le 0.018, \quad (67)$$

According to (18), we find

$$\breve{x} = \sum_{i=1}^{18} \breve{x}_i = 0.705,\ \hat{y} = 1 - \breve{x} = 1 - 0.705 = 0.295 \quad (68)$$

and according to (19), we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.018\ \ \hat{x}^1 = 0.027,\ \hat{x}^1 - \breve{x}^1 = 0.009$$
$$\breve{x}^2 = 0.018\ \ \hat{x}^2 = 0.024,\ \hat{x}^2 - \breve{x}^2 = 0.006,$$
$$\breve{x}^3 = 0.027\ \ \hat{x}^3 = 0.056,\ \hat{x}^3 - \breve{x}^3 = 0.029,$$
$$\breve{x}^4 = 0.286\ \ \hat{x}^4 = 0.78,\ \hat{x}^4 - \breve{x}^4 = 0.494,$$
.
.
.
$$\breve{x}^{18} = 0.001\ \ \hat{x}^{18} = 0.018,\ \hat{x}^{18} - \breve{x}^{18} = 0.017. \quad (69)$$

From the above, as according to (68), the inequality (20) takes the form

$$\hat{x}^I - \breve{x}^I < 0.295, \quad (70)$$

then it follows that the largest value $I \in \{0,1,...,18\}$ such that this inequality holds is $I = 4$.
Therefore, we fix the optimal solution that maximize linear function (65) according to the rule (22). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.027,$$
$$\dot{x}_2 = \hat{x}_2 = 0.024,$$
$$\dot{x}_3 = \hat{x}_3 = 0.056,$$
$$\dot{x}_4 = \hat{y} - \hat{x}^3 + \breve{x}^3 + \breve{x}_4$$
$$= 0.295 - 0.056 + 0.027 + 0.286 = 0.552, \quad (71)$$
$$\dot{x}_5 = \breve{x}_5 = 0.013,\ \dot{x}_6 = \breve{x}_6 = 0.286,$$
$$\dot{x}_7 = \breve{x}_7 = 0.025,\ \dot{x}_8 = \breve{x}_8 = 0.002,$$
$$\dot{x}_9 = \breve{x}_9 = 0.002,\ \dot{x}_{10} = \breve{x}_{10} = 0.018,$$
$$\dot{x}_{11} = \breve{x}_{11} = 0.001,\ \dot{x}_{12} = \breve{x}_{12} = 0.001,$$
$$\dot{x}_{13} = \breve{x}_{13} = 0.001,\ \dot{x}_{14} = \breve{x}_{14} = 0.002,$$
$$\dot{x}_{15} = \breve{x}_{15} = 0.001,\ \dot{x}_{16} = \breve{x}_{16} = 0.002,$$
$$\dot{x}_{17} = \breve{x}_{17} = 0.0006,\ \dot{x}_{18} = \breve{x}_{18} = 0.001. \quad (72)$$

Finally, according to (24) after making the inverse to (63) substitution, we get the optimal transient probabilities

$$\dot{p}_3 = \dot{x}_1 = 0.027,\ \dot{p}_{15} = \dot{x}_2 = 0.024,$$

$\dot{p}_4 = \dot{x}_3 = 0.056$, $\dot{p}_5 = \dot{x}_4 = 0.552$,

$\dot{p}_{12} = \dot{x}_5 = 0.013$, $\dot{p}_{13} = \dot{x}_6 = 0.286$,

$\dot{p}_{14} = \dot{x}_7 = 0.025$, $\dot{p}_{11} = \dot{x}_8 = 0.002$,

$\dot{p}_{16} = \dot{x}_9 = 0.002$, $\dot{p}_6 = \dot{x}_{10} = 0.018$,

$\dot{p}_8 = \dot{x}_{11} = 0.001$, $\dot{p}_9 = \dot{x}_{12} = 0.001$,

$\dot{p}_2 = \dot{x}_{13} = 0.001$, $\dot{p}_7 = \dot{x}_{14} = 0.002$,

$\dot{p}_{10} = \dot{x}_{15} = 0.001$, $\dot{p}_{17} = \dot{x}_{16} = 0.002$,

$\dot{p}_1 = \dot{x}_{17} = 0.0006$, $\dot{p}_{18} = \dot{x}_{18} = 0.001$, (73)

that maximize the system mean lifetime in the safety state subset $\{2,3,4\}$ expressed by the linear form (60) giving, according to (15) and (73), its optimal value

$$\dot{\mu}(2) \cong \dot{p}_1 \cdot 1.54 + \dot{p}_2 \cdot 2.43 + \dot{p}_3 \cdot 3.90$$
$$+ \dot{p}_4 \cdot 3.80 + \dot{p}_5 \cdot 3.80 + \dot{p}_6 \cdot 3.24$$
$$+ \dot{p}_7 \cdot 2.43 + \dot{p}_8 \cdot 2.50 + \dot{p}_9 \cdot 2.50$$
$$+ \dot{p}_{10} \cdot 2.43 + \dot{p}_{11} \cdot 3.37 + \dot{p}_{12} \cdot 3.80$$
$$+ \dot{p}_{13} \cdot 3.80 + \dot{p}_{14} \cdot 3.80 + \dot{p}_{15} \cdot 3.90$$
$$+ \dot{p}_{16} \cdot 3.37 + \dot{p}_{17} \cdot 2.43 + \dot{p}_{18} \cdot 1.54$$
$$= 0.0006 \cdot 1.54 + 0.001 \cdot 2.43$$
$$+ 0.027 \cdot 3.90 + 0.056 \cdot 3.80$$
$$+ 0.552 \cdot 3.80 + 0.018 \cdot 3.24$$
$$+ 0.002 \cdot 2.43 + 0.001 \cdot 2.50$$
$$+ 0.001 \cdot 2.50 + 0.001 \cdot 2.43$$
$$+ 0.002 \cdot 3.37 + 0.013 \cdot 3.80$$
$$+ 0.286 \cdot 3.80 + 0.025 \cdot 3.80$$
$$+ 0.024 \cdot 3.90 + 0.002 \cdot 3.37$$
$$+ 0.002 \cdot 2.43 + 0.001 \cdot 1.54 = 3.83 \quad (74)$$

6. Optimal safety characteristics of the ferry technical system

Further, substituting the optimal solution (73) into the formulae (26), we obtain the optimal solution for the mean value of the system unconditional lifetime in the safety state subset $\{1,2,3,4\}$, $\{3,4\}$ and $\{4\}$ that respectively amounts:

$$\dot{\mu}(1) \cong 4.28, \quad \dot{\mu}(3) \cong 3.41, \quad \dot{\mu}(4) \cong 3.08, \quad (75)$$

and according to (29), the optimal solutions for the mean values of the system unconditional lifetimes in the particular safety states

$$\dot{\bar{\mu}}(1) \cong 0.45, \quad \dot{\bar{\mu}}(2) \cong 0.42,$$
$$\dot{\bar{\mu}}(3) \cong 0.33, \quad \dot{\bar{\mu}}(1) \cong 3.08. \quad (76)$$

Moreover, according to (27)-(28) and (51)-(52), the corresponding optimal unconditional multistate safety function of the system is of the form

$$\dot{\boldsymbol{s}}_5(t, \cdot) =$$
$$[1, \ \dot{\boldsymbol{s}}_5(t,1), \dot{\boldsymbol{s}}_5(t,2), \dot{\boldsymbol{s}}_5(t,3), \dot{\boldsymbol{s}}_5(t,4)] \quad (77)$$

for $t \geq 0$,

where according to (5) and after considering the values of p_b given by (73), its co-ordinates are as follows:

$$\dot{\boldsymbol{s}}_5(t,u) \cong 0.0006 \cdot \boldsymbol{s}_2^{(1)}(t,u) + 0.001 \cdot \boldsymbol{s}_3^{(2)}(t,u)$$
$$+ 0.027 \cdot \boldsymbol{s}_2^{(3)}(t,u) + 0.056 \cdot \boldsymbol{s}_3^{(4)}(t,u)$$
$$+ 0.552 \cdot \boldsymbol{s}_3^{(5)}(t,u) + 0.018 \cdot \boldsymbol{s}_3^{(6)}(t,u)$$
$$+ 0.002 \cdot \boldsymbol{s}_3^{(7)}(t,u) + 0.001 \cdot \boldsymbol{s}_2^{(8)}(t,u)$$
$$+ 0.001 \cdot \boldsymbol{s}_2^{(9)}(t,u) + 0.001 \cdot \boldsymbol{s}_3^{(10)}(t,u)$$
$$+ 0.001 \cdot \boldsymbol{s}_2^{(11)}(t,u) + 0.013 \cdot \boldsymbol{s}_3^{(12)}(t,u)$$
$$+ 0.286 \cdot \boldsymbol{s}_3^{(13)}(t,u) + 0.025 \cdot \boldsymbol{s}_3^{(14)}(t,u)$$
$$+ 0.024 \cdot \boldsymbol{s}_2^{(15)}(t,u) + 0.002 \cdot \boldsymbol{s}_2^{(16)}(t,u)$$
$$+ 0.002 \cdot \boldsymbol{s}_3^{(17)}(t,u) + 0.001 \cdot \boldsymbol{s}_2^{(18)}(t,u) \quad (78)$$

for $t \geq 0$, $u = 1,2,3,4$, where $\boldsymbol{s}_2^{(1)}(t,u)$ and $\boldsymbol{s}_3^{(2)}(t,u)$ are respectively given by (35)-(38) and (44)-(47) and $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ for $b = 3,4,...,18$, are given in [12].

If the critical safety state is r = 2, then the system risk function, according to (30), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{s}}_5(t,2) \text{ for } t \geq 0, \quad (79)$$

where $\dot{\boldsymbol{s}}_5(t,2)$ is given by (78) for $u = 2$.

Hence, considering (31), the moment when the optimal system risk function exceeds a permitted level, for instance δ = 0.05, is

$$\dot{\boldsymbol{\tau}} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.25 \text{ years.} \quad (80)$$

Comparing the ferry safety characteristics after its operation process optimization given by (74)-(79) with the corresponding characteristics before this optimization determined by (52)-(59) justifies the sensibility of this action.

7. Conclusion

The joint model of safety of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to system safety analysis was constructed. Next, the final results obtained from this joint model and a linear programming were used to build the model of complex technical systems safety optimization. These tools can be useful in safety evaluation and optimization of a very wide class of real technical systems operating in varying conditions that have an influence on changing their safety structures and their components safety characteristics. These tools practical application to safety and risk evaluation and optimization of a technical system of a ferry operating in variable operation conditions at the Baltic Sea waters and the results achieved are interesting for safety practitioners from maritime transport industry and from other industrial sectors as well.

Acknowledgements

The paper describes the work in the Poland-Singapore Joint Research Project titled “Safety and Reliability of Complex Industrial Systems and Processes” supported by grants from the Poland’s Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

References

[1] Aven, T. (1985). Reliability evaluation of multi-state systems with multi-state components. *IEEE Transactions on reliability* 34, pp 473-479.

[2] Dziula, P., Jurdzinski, M., Kolowrocki, K. & Soszynska, J. (2007). On multi-state approach to ship systems safety analysis. *Proc. 12th International Congress of the International Maritime Association of the Mediterranean, IMAM 2007.* A. A. Balkema Publishers: Leiden - London - New York - Philadelphia - Singapore.

[3] Grabski, F. (2002). *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Science.

[4] Kolowrocki, K. (2004). *Reliability of Large Systems*. Elsevier, Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney - Tokyo.

[5] Kolowrocki, K. & Soszyńska, J. (2006). Reliability and availability of complex systems. *Quality and Reliability Engineering International* Vol. 22, Issue 1, J. Wiley & Sons Ltd. 79-99.

[6] Kolowrocki, K. & Soszynska, J. (2008). A general model of ship technical systems safety. *Proc. International Conference on Industrial Engineering and Engineering Management, IEEM 2008*, Singapore, 1346-1350, 2008.

[7] Kolowrocki, K., Soszynska, J., Xie, M. & Blokus-Roszkowska, A. (2009). Safety and reliability optimisation of complex industrial systems and processes. WP5. Reliability, risk and availability based optimization of complex technical systems operation processes with applications in port, shipyard and maritime transport. Task 5.3 – English – 30.11.2009. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.

[8] Lisnianski, A., Levitin, G. (2003). *Multi-state System Reliability. Assessment, Optimisation and Applications.* World Scientific Publishing Co., New Jersey, London, Singapore , Hong Kong.

[9] Meng, F. (1993). Component- relevancy and characterisation in multi-state systems. *IEEE Transactions on reliability* 42, pp 478-483.

[10] Soszynska, J. (2006). Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping* Vol. 83, Issue 4, 304-310.

[11] Soszynska, J. (2007). Systems reliability analysis in variable operation conditions. *International Journal of Reliability, Quality and Safety Engineering. System Reliability and Safety* Vol. 14, No 6, 1-19.

[12] Soszyńska, J., Kołowrocki, K., Kamiński, P., Judziński, M. & Milczek, B. (2009). Data mining for identification and prediction of safety and reliability characteristics of complex industrial systems and processes WP6. Safety and risk analysis and evaluation of a Stena Baltica ferry in variable operation conditions. WP6 - Sub-Task 6.2.5 – English – 30.11.2009. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0.