Development of mathematical model of duration of filling the finite-dimensional space with air at vacuum-gauge pressure

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Abstract. The article presents analytical dependences for simulating the process of filling the chambers of variable vacuum-gauge pressure of the system of pulser-teat cup with air. The simulation is carried out with an account of the law of mass saving for controlled volumes of gases by means of equation of mechanical energy of air transportation. The paper calculates the time of filling chambers of milking units with air to the conditions of atmospheric pressure.

Key words: milking unit, velocity coefficient, vacuum-gauge pressure, time of filling.

INTRODUCTION

Developments of new constructions of milking units pulsers require theoretical grounding of technical characteristics of their operation, i. e. air expenditure at preassigned geometrical dimensions of the construction which determine the very regime of the work of pulser, its energetical parameters and time characteristics of the derivative processes when coming from the cycle of sucking to the compression cycle. Therefore, in modeling constructive and technological parameters of the milking units pulser there exists a need in models of air expenditures caused by both separate elements and the whole milking units [22].

Air expenditures caused by a milking units is calculated under conditions of reaching normal atmospheric pressure taking into consideration the volumes of chambers of variable vacuum-gauge pressure, vacuum-gauge pressure, frequency of pulsations [1,2]. The given methods of calculation guarantee only average results and ignore specific properties of the pulser's construction. The character of pressure variation is accepted here by the exponencial law.

Experimental data on air expenditures caused by the milking unit and methods of research are presented in the works [3,4] which may be used in determining air expenditures caused by the existing now pulsers of conventional structures.

The average value of air expenditure may be calculated considering the air motion velocity in vacuum conductor of the milking unit, the lapse of vacuum-gauge pressure, constructive dimensions of vacuum drive and coefficient of pneumatic voltage of friction [5].

Model of calculating air expenditures with an account of the equation of the state of gas is closely approached to the solution of the predetermined problem [6].

To simulate air expenditures in displacement of condensed gas from the finite-dimensional space through the nozzle, the equation of Saint-Venant Wentzel is employed. The mentioned equation describes the process of filling space through a short attachment from the infinite-dimensional space [7].

When learning the impact of the pulser construction parameters on technological parameters of its functioning, the analysts are usually focused on the experimental study of the impact of pressure and the transmission hole diameter on the frequency of pulsation [8, 9].

In the dynamic analysis of pneumodrives thermodynamic processes are considered. The part of energy here is directed to the gas thermal extension taking into account initial and finite temperatures of extension [10-15].

The discussed above models get a number of drawbacks. They do not consider the regime of milking units operation and, as a result, the nature of variations of vacuum-gauge pressure. In addition, the aquation lacks the parameter of the losses of vacuum-gauge pressure and, therefore, it does not give the opportunity of calculating the amount of air for the preassigned time period.

Having developed the model of filling the finite-dimensional space with air, we shall further on consider the process of filling air considering only mechanical energy of air movement in the system of the teat cup – pulser [16, 19, 22].

OBJECTIVES

The article is aimed at the development of mathematical model for simulating time of filling the finite dimensional space with air depending upon the geometrical dimensions and technological parameters of the pulser functioning.

MAIN PRESENTATION

Let us consider the process of variation of pressure in the following system: interwall space of teat cup-pulser which formulates the volume of the space with variable vacuum-gauge pressure as a single system filled with air. Let the volume of the system equals *V*. Its vacuum gauge pressure is P_V. It is important to determine in the finite dimensional space the duration of pressure increase from P_V to P_A. At an arbitrary moment *t* pressure in the system will be P_i and, consequently, density of air stream ρ and its velocity υ.

We shall set up a differential equation for filling the space *V* with air within the system "teat cup-pulser" considering the fact that air mass *M* which will arrive in some time may be calculated by the equation [17]:

$$dM = m \cdot dt \,, \qquad (1)$$

where: *m* – mass of air expenditure according to the equation [22], kg/s,

$$m = \upsilon \cdot S_{cross} \cdot \rho_A \,, \qquad (2)$$

where: υ – velocity of air motion during filling of the volume *V*.

According to the equation [22] with corrections, the velocity of motion is calculated by the formula:

$$\upsilon = \sqrt{\frac{2g}{1+\xi} \cdot \frac{P_i}{\rho_i} \cdot \left(1 - \left(\frac{P_i}{P_A}\right)^{\frac{n-1}{n}}\right) \cdot dt} \,, \qquad (3)$$

where: P_i – running value of pressure in the space of vacuum-gauge pressure (P_i varies from P_V to P_A); P_A – atmospheric pressure; ρ_i – air density at the running value of pressure P_i.

Let us substitute the equation (3) into the equation (2) and get:

$$m = S_{cross} \cdot \rho_A \cdot \sqrt{\frac{2g}{1+\xi} \cdot \frac{P_i}{\rho_i} \cdot \left(1 - \frac{P_i}{P_A}\right)^{\frac{n-1}{n}}} \,. \qquad (4)$$

We shall accept:

$$x_i = \frac{P_A}{P_i} \,. \qquad (5)$$

Then the value of mass of air expenditure during filling the chamber of variable vacuum-gauge pressure and its getting the conditions of atmospheric pressure will be:

$$m_\Delta = S_{cross} \cdot \sqrt{\frac{2g}{1+\xi} \cdot P_i \rho_i \cdot \left(x_i^{\frac{2}{n}} - x_i^{\frac{3-n}{n}}\right)} \,. \qquad (6)$$

Variation of air mass in chambers of variable vacuum-gauge pressure will equal:

$$dM_A = S_{cross} \cdot \psi_i \cdot \sqrt{P_i \rho_i} \cdot dt \,, \qquad (7)$$

where:

$$\psi_i = \sqrt{\frac{2g}{1+\xi} \cdot \left(x_i^{\frac{2}{n}} - x_i^{\frac{3-n}{n}}\right)} \cdot \frac{м^{\frac{1}{2}}}{s} \,, \qquad (8)$$

- coefficient of velocity.

Then considering the equations (6), (8) the equation (7) will get the from:

$$dM_A = S_{cross} \psi_i \cdot \sqrt{P_i \rho_A \left(\frac{P_i}{P_A}\right)^{\frac{1}{n}}} \cdot dt = $$
$$= S_{cross} \cdot \psi_i \sqrt{P_A \rho_A \cdot \frac{P_i}{P_A} \cdot \left(\frac{P_i}{P_A}\right)^{\frac{1}{n}}} \cdot dt = . \qquad (9)$$
$$= f_{cross} \cdot \psi_i \sqrt{P_A \rho_A} \cdot \sqrt{\left(\frac{P_i}{P_A}\right)^{\frac{n+1}{n}}} \cdot dt$$

When filling the volume *V*, with air, its mass will vary with simultaneous variation of its density. Variation of the air mass may be described in the following way:

$$dM = V \cdot d\rho_i \,. \qquad (10)$$

We shall insert expression for ρ_i (4), to the equation (10) and get:

$$dM = V \cdot \rho_A \cdot d\left(\left(\frac{P_i}{P_A}\right)^{\frac{1}{n}}\right) = \,. \qquad (11)$$
$$= \frac{V}{n} \cdot \rho_A \cdot \left(\frac{P_i}{P_A}\right)^{\frac{1-n}{n}} \cdot d\left(\frac{P_i}{P_A}\right)$$

Let us match the equations (11) and (9):

$$\rho_A \cdot \frac{V}{n}\left[\frac{P_i}{P_A}\right]^{\frac{1-n}{n}} \cdot d\left(\frac{P_i}{P_A}\right) = \qquad (12)$$
$$= S_{cross} \psi_i \sqrt{P_A \rho_A} \cdot \sqrt{\left(\frac{P_i}{P_A}\right)^{\frac{n+1}{n}}} dt \,.$$

let us shorten the equation (12) to the expression $\left(\frac{P_i}{P_A}\right)^{\frac{1}{n}}$.

$$\rho_A \cdot \frac{V}{n}\left(\frac{P_i}{P_A}\right)^{-1} \cdot d\left(\frac{P_i}{P_A}\right) = \,. \qquad (13)$$
$$= S_{cross} \psi_i \sqrt{P_A \rho_A} \cdot \sqrt{\left(\frac{P_i}{P_A}\right)^{\frac{n-1}{n}}} dt \,.$$

Let us make transformation in the equation (13):

$$\frac{1}{n} \cdot \frac{1}{\frac{P_i}{P_A} \cdot \left(\frac{P_i}{P_A}\right)^{\frac{n-1}{2n}}} \cdot d\left(\frac{P_i}{P_A}\right) = \frac{S_{cross} \psi_i}{V} \cdot \sqrt{\frac{P_A}{\rho_A}} \cdot dt \,.$$

or:

$$\frac{1}{n}\cdot\left(\frac{P_i}{P_A}\right)^{\frac{1-3n}{2n}}\cdot d\left(\frac{P_i}{P_A}\right)=\frac{S_{cross}\psi_i}{V}\cdot\sqrt{\frac{P_A}{\rho_A}}\cdot dt\ . \quad (14)$$

Let us integrate the equation (14) having set the limits of integration from $\frac{P_i}{P_A}$ to *1* and from *0* to *t*:

$$\frac{1}{n}\int_{\frac{P_i}{P_A}}^{1}\left(\frac{P_i}{P_A}\right)^{\frac{1-3n}{2n}}\cdot d\left(\frac{P_i}{P_A}\right)=\frac{S_{cross}\psi_i}{V}\cdot\sqrt{\frac{P_A}{\rho_A}}\int_0^t dt\,.$$

$$\frac{1}{n}\cdot\frac{\left(\frac{P_i}{P_A}\right)^{\frac{1-n}{2n}}\Big|_{P_i/P_A}^{1}}{\frac{1-n}{2n}}=\frac{S_{nep}\psi_i}{V}\cdot\sqrt{\frac{P_A}{\rho_A}}\cdot t\Big|_0^t\ . \quad (15)$$

Let us substitute the limits of integration into the equation (15):

$$\frac{1}{n}\cdot\frac{1-\left(\frac{P_i}{P_A}\right)^{\frac{1-n}{2n}}}{\frac{1-n}{2n}}=\frac{S_{cross}\psi_i}{V}\cdot\sqrt{\frac{P_A}{\rho_A}}\cdot t\ . \quad (16)$$

Coming from the equation (16) we determine duration of filling chambers of variable vacuum gauge pressure of the system “teat cup-pulser” with air to the conditions of atmospheric pressure:

$$t=\frac{1}{n}\cdot\frac{2n}{1-n}\left(1-\sqrt{\left(\frac{P_i}{P_A}\right)^{\frac{1-n}{n}}}\right)\cdot\frac{V}{S_{cross}\psi_i}\cdot\sqrt{\frac{\rho_A}{P_A}}\ ,$$

or:

$$t=\frac{2}{1-n}\cdot\frac{V}{S_{cross}\psi_i}\cdot\sqrt{\frac{\rho_A}{P_A}}\cdot\left[1-\sqrt{\left(\frac{P_i}{P_A}\right)^{\frac{1-n}{n}}}\right]. \quad (17)$$

where *V*– volume of chambers of variable vacuum-gauge pressure of the system “teat cup-pulser”, м^3; S_{cross} – area of crossing the transmission channel “pulser – teat cup”, м^2; ψ_i – coefficient of velocity characterizing pressures relations, м$^{1/2}$/s; ρ_A – density of air at atmospheric pressure, kg/м^3; P_F – atmospheric pressure of air, kg/м^2; *n* – parameter of polytropy.

In approbating the received equation we calculated the time of filling the system “teat cup-pulser” with air using the following initial data: *V* – 0,00005; 0,0001; 0,00015; 0,0002 м^3; d_{cross} – diameter of the permitting channel of the pulser, d_{cross} – 0,002; 0,003; 0,004 м; ρ_V = 1,25 kg/м^3; P_V = 10000 kg/м^2; *n* = 1,4.

The calculation results are presented in fig. 1-2.

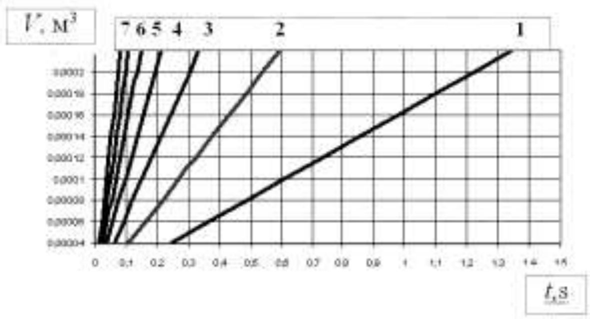

Fig. 1. Dependence of time filling with air *t* on the volume of space V “teat cup-pulser” at preassigned diameter of the transmission chanel of the pulser: 1,2,3,4,5,6,7 - d_{cross} = 0,001; 0,0015; 0,002; 0,0025; 0,003; 0,0035; 0,004 м

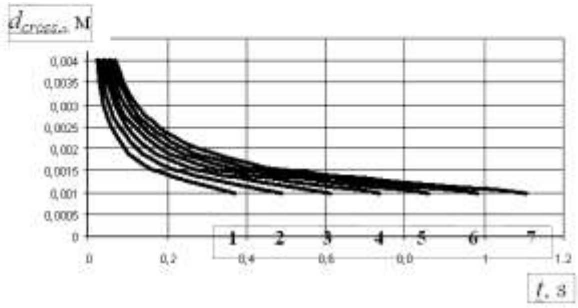

Fig. 2. Dependence of time filling with air *t* on the diameter of transmission hole d_{cross} of the pulser at the preassigned volume of space “teat cup-pulser”: 1,2,3,4,5,6,7 - *V* = 0,00006; 0,00008; 0,0001; 0,00012; 0,00014; 0,00016; 0,00018 м^3

CONCLUSIONS

The suggested mathematical dependences allow to simulate velocity and expenditure of air, time of filling depending on the volumes of chambers of variable vacuum-gauge pressure of the milking unit, constructive dimensions of air-conductors, regime of variation of pressure which allow to determine theoretically constructive and dynamical characteristics of the system “pulser – milking unit”.

Calculations proved the position that optimal diameters of permissing hole of the pulser are within 0,002…0,003 м at the volumes of variable pressures from 0,00005 м^3 to 0,0002 м^3.

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