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DEVELOPMENT OF THE AIRLINE BUSINESS MACROECONOMICS DYNAMICS MODELS

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Abstract

This paper proposes a solution to a certain macroeconomic model. A multi-alternative problem of aviation transportation optimal organisation in conditions of uncertainty of the subjective preference functions is considered. Conditional optimisation of the objective functional containing the entropy of the individuals' operational effectiveness functions preferences is carried out in the framework of the simplest macroeconomic problem. The principle of the Solow and Cobb–Douglas models, likewise for economic growth, is modified with the Subjective Entropy Maximum Principle. The advantages of the described optimisation approach are demonstrated in generalised terms of the operational effectiveness functions for aviation transportation organisation.

Keywords: aviation transportation; operational effectiveness; objective functional optimisation; simplest macroeconomic problem; entropy
Type of the work: research article

1. INTRODUCTION

Aviation industry and airlines are undergoing hard times due to the SARS COVID-19 pandemic period as well as the tragedy of the heroic rebuff of Ukraine to the fascist–russist full-scale warfare invasion.

The current circumstances require indispensable measures to be taken in the major macroeconomic airline industry components. The presented paper is dedicated to the simplest macroeconomic problem setting in the framework of the Solow [1–3] and Cobb–Douglas [4,5] models, likewise for economic growth [6], taking into account the individuals' subjective preferences functions of the available alternatives obtained based upon the Subjective Entropy Maximum Principle [7–10].

The Principle of the Subjective Entropy Maximum has been previously presented in the literature [7–10]. This principle was applied to the simplest problems of the macroeconomics dynamics. Those were the continuous models. They were in the type of the Leontief [11] ones.

Nevertheless, some important problems are neither included within nor converge into those classes of the simplest macroeconomics models: Solow [1–3], Mankiw et al. [6] and others.

A combination of macroeconomics models with the Principle of the Subjective Entropy Maximum is tried in the present work. The principle was developed during 1990–2010. Despite the fact that the principle formally hardly differs from that of Jaynes [12–14], the combination widens the area of the practical applications of the obtained results. This holds good especially in psychology [7–10], economics [7–10,15], theory of conflicts etc. [16–23]. And the approach could be recommended for implementation in various other spheres that share similar characteristics [24–32].

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2. PROBLEM STATEMENT

Let us consider development of the macroeconomics problem by Solow when there are some more elaborated members.

The production function is now given by the following expression:

$$Y_{t+1} = A\left(K_t\right)^{\alpha}\left(H_t\right)^{\beta}\left(L_t\right)^{1-(\alpha+\beta)}, \tag{1}$$

where Y_{t+1} is the production function Y at the time (step of iteration, recursion stage), $t+1$, the time following the previous time t; A is the coefficient; K_t is the Capital gained at the preceding point of time t; α is the power index for the Capital K_t; H_t is the Human potential required in order to gain Y_{t+1}; β is the power index for the Human component, H, analogously to α coefficient; and L_t is the Labour component L of the production process Y at the earlier point of time t.

Also, a Consumption member arising as an outcome of the splitting of the production process results should be distinguished as follows:

$$C_t = Y_t - \left(K_t + H_t\right), \tag{2}$$

where C_t is the Consumption.

3. PROPOSED SOLUTION

3.1. Subjective preferences functions distribution optimisation

In the presented problem setting, described with Eqs (1) and (2), determination of the objective functional for the individuals' subjective preferences π is given by

$$\Phi_{\pi} = -\sum_{i=1}^{N=3}\pi_t^i \ln \pi_t^i + \beta_{\pi}\left[\pi_t^K K_t + \pi_t^H H_t + \pi_t^C C_t\right] + \gamma\left[\sum_{i=1}^{N=3}\pi_t^i - 1\right], \tag{3}$$

where the uncertainty of the available alternatives is evaluated with the subjective entropy member

$$-\sum_{i=1}^{N=3}\pi_t^i \ln \pi_t^i; \tag{4}$$

β_{π} and γ are the corresponding cognitive coefficients.

The normalising condition is expressed with the member of

$$\left[\sum_{i=1}^{N=3}\pi_t^i - 1\right]. \tag{5}$$

The necessary conditions for the objective functional in Eq. (3)'s extremum existence

$$\frac{\partial \Phi_{\pi}}{\partial \pi_t^i} = 0, \tag{6}$$

allow obtaining an optimal distribution of the individuals' subjective preferences π, as follows:

$$\pi_t^K,\ \pi_t^H \text{ and } \pi_t^C. \tag{7}$$

That is,

$$-\ln \pi_t^K - 1 + \beta_\pi K_t + \gamma = 0. \tag{8}$$

This yields

$$\ln \pi_t^K = \gamma - 1 + \beta_\pi K_t. \tag{9}$$

Thus,

$$\pi_t^K = e^{\gamma - 1 + \beta_\pi K_t}. \tag{10}$$

On the other hand

$$-\ln \pi_t^H - 1 + \beta_\pi H_t + \gamma = 0. \tag{11}$$

This yields

$$\ln \pi_t^H = \gamma - 1 + \beta_\pi H_t. \tag{12}$$

Thus,

$$\pi_t^H = e^{\gamma - 1 + \beta_\pi H_t}. \tag{13}$$

And,

$$-\ln \pi_t^C - 1 + \beta_\pi C_t + \gamma = 0. \tag{14}$$

This yields

$$\ln \pi_t^C = \gamma - 1 + \beta_\pi C_t. \tag{15}$$

Thus,

$$\pi_t^C = e^{\gamma - 1 + \beta_\pi C_t}. \tag{16}$$

The procedure of Eqs (3)–(16) leads to the following position:

$$\pi_t^K + \pi_t^H + \pi_t^C = e^{\gamma - 1 + \beta_\pi K_t} + e^{\gamma - 1 + \beta_\pi H_t} + e^{\gamma - 1 + \beta_\pi C_t} = 1. \tag{17}$$

The normalising condition, i.e. Eq. (17), means that

$$e^{\gamma - 1} = \frac{1}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}}. \tag{18}$$

Because of Eq. (18),

$$\pi_t^K = \frac{e^{\beta_\pi K_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}}. \tag{19}$$

In turn,

$$\pi_t^H = \frac{e^{\beta_\pi H_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}}. \tag{20}$$

And,

$$\pi_t^C = \frac{e^{\beta_\pi C_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}}. \tag{21}$$

Supposedly, the Labour depends upon the Consumption in the following way:

$$L_{t+1} = L_{\max} + \frac{L_{\min} - L_{\max}}{1 + \left(C_t\right)^{\mu}}, \tag{22}$$

where $L_{\max}$ is the maximal Labour contribution into the production, possible when

$$C_t \to \infty; \tag{23}$$

$L_{\min}$ is the minimal Labour contribution into the production function, possible when

$$C_t = 0; \tag{24}$$

μ is the coefficient.

3.2. Simulation

Using Eqs (1)–(24), the recursive system is ascertained as the following:

$$\begin{pmatrix} Y_{t+1} \\ K_{t+1} \\ H_{t+1} \\ L_{t+1} \\ C_{t+1} \\ \pi_{t+1}^K \\ \pi_{t+1}^H \end{pmatrix} = \begin{pmatrix} A\left(K_t\right)^{\alpha}\left(H_t\right)^{\beta}\left(L_t\right)^{1-(\alpha-\beta)} \\ \pi_t^K Y_{t+1} \\ \pi_t^H Y_{t+1} \\ L_{\max} + \dfrac{L_{\min} - L_{\max}}{1 + \left(C_t\right)^{\mu}} \\ Y_{t+1} - \left(K_{t+1} + H_{t+1}\right) \\ \dfrac{e^{\beta_\pi K_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}} \\ \dfrac{e^{\beta_\pi H_t}}{e^{\beta_\pi K_t} + e^{\beta_\pi H_t} + e^{\beta_\pi C_t}} \end{pmatrix}. \tag{25}$$

The accepted data are

$$A = 1.872, \quad \alpha = 0.3, \quad \beta = 0.3, \quad L_{\min} = 0.5, \quad L_{\max} = 2.5. \tag{26}$$

The initial conditions are

$$\begin{pmatrix} Y_0 \\ K_0 \\ H_0 \\ L_0 \\ C_0 \\ \pi_0^K \\ \pi_0^H \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 2.5 \\ 1.795 \\ 1.5 \\ 0.3 \\ 0.3 \end{pmatrix}; \quad (27)$$

and

$$\mu = 1.5, \quad \beta_\pi = 0.3; \quad (28)$$

The results of the computer simulation are shown below (Figs. 1–10):

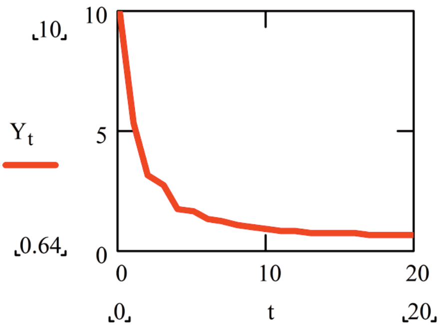

Figure 2. Capital function.

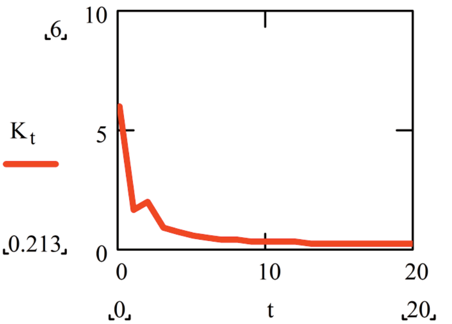

Figure 1. Production function.

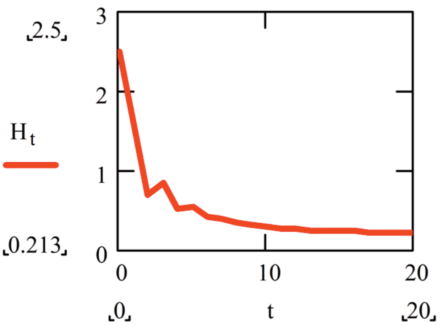

Figure 3. Human function.

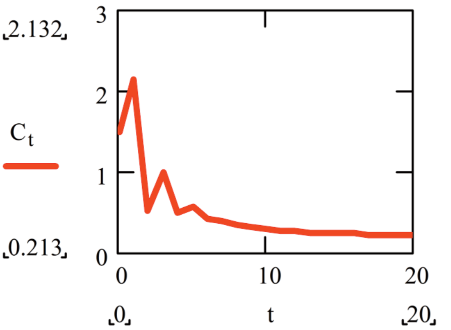

Figure 4. Consumption function.

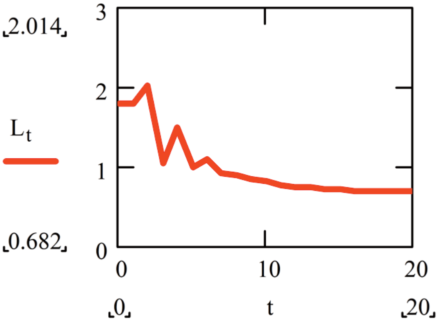

Figure 5. Labour function.

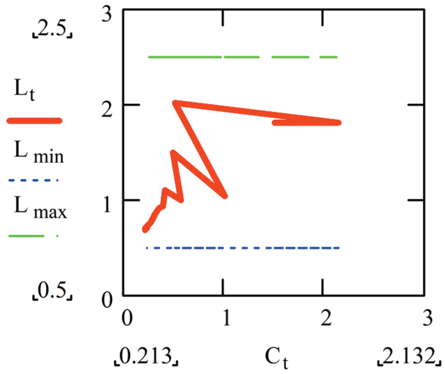

Figure 6. Labour–Consumption dependence.

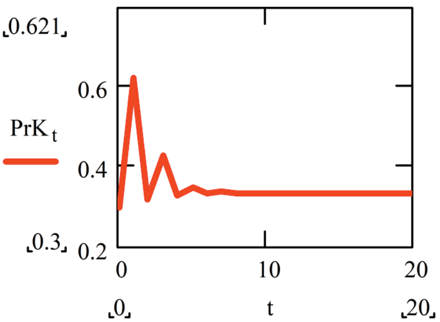

Figure 7. Capital preferences function.

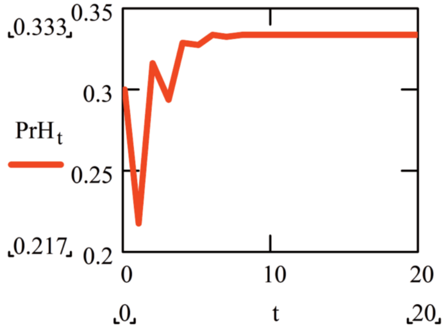

Figure 8. Human preferences function.

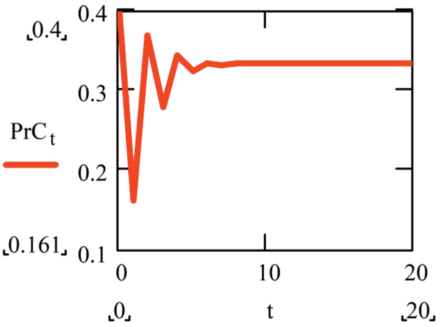

Figure 9. Consumption preferences function.

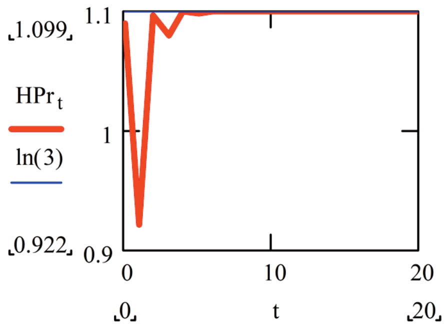

Figure 10. Subjective entropy.

4. ANALYSIS

Aviation industry problems could be considered through the prism of macroeconomics modelling. One of these models, expressed in Eqs (1)–(28), enables considerations to be arrived at based upon the multi-alternative approach.

The intellectual potential of Human brain work power is distinguished from the general Labour component. It is taken into account with the formula of the production function in Eq. (1).

In turn, the particular Labour component of the total production depends upon the Consumption fraction formed from the production function. This is described with the expression in Eq. (2). In fact, the model in Eq. (22) envisages logistic dependence between the Labour and Consumption components.

The objective functional (Eq. [3]) in the presented three-alternative problem setting helps to find the so-called canonical distribution for the individuals' subjective preferences functions (Eq. [7]) in the explicit view (Eqs [19]–[21]). The key point is the conditional optimisation of the subjective entropy function (Eq. [4]) in the framework of the objective functional (Eq. [3]) subject to the normalising conditions constraints (Eq. [5]) and the corresponding cognitive function construction. The simplest optimisation follows the procedure outlined via Eqs (6)–(21).

The elaborated model takes into consideration the Consumption limitations expressed in Eqs (23) and (24).

The recursive procedure provided with the use of the system of equations contained in Eq (25) is simulated with the accepted data indicated in Eqs (26)–(28). In the presented problem setting, the system of the individuals' subjective preferences functions (Eq. [7]), of the canonical view expressed in Eqs (19)–(21), converges to the maximally subjective entropy value, which ensures degrading conditions for the macroeconomic system (see Figs. 1–10).

5. CONCLUSIONS

Human intellect potential could be successfully taken into account through the described recursive procedures. The uncertainty of the macroeconomic system alternatives' subjective preferences functions (in the presented consideration, airline business alternatives' subjective preferences for Capital, Human and Labour) is evaluated with the subjective entropy measure.

A solution for the particular three-alternative problem formulated in the present study concerning the airline business is arrived at after evaluating the available alternatives among individuals' subjective preferences functions entropy maximum.

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