Analysis of Spectral Characteristics of Sound Waves Scattered from a Cracked Cylindrical Elastic Shell Filled with a Viscous Fluid

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The scattering of plane steady-state sound waves from a viscous fluid-filled thin cylindrical shell weakened by a long linear slit and submerged in an ideal fluid is studied. For the description of vibrations of elastic objects the Kirchhoff–Love shell-theory approximation is used. An exact solution of this problem is obtained in the form of series with cylindrical harmonics. The numerical analysis is carried out for a steel shell filled with oil and immersed in seawater. The modules and phases of the scattering amplitudes *versus* the dimensionless wavenumber of the incident sound wave as well as directivity patterns of the scattered field are investigated taking into consideration the orientation of the slit on the elastic shell surface. The plots obtained show a considerable influence of the slit and viscous fluid filler on the diffraction process.

Keywords: sound scattering, elastic shell, slit, scattering amplitude, pattern directivity, seawater, oil

1. Introduction

The acoustic diagnostics of pipelines with natural gas or oil is one of the important problems of the safety of transportation of these substances and of the preservation of the environment (MAKINO *et al.*, 2001). The methods of non-destructive testing of underwater pipelines are continuously developed (MOHAMED *et al.*, 2011). One of these methods is the long distance acoustic identification of the defects in underwater pipeline shells.

The echo-signals reflected from the thin circular cylindrical with a long linear slit elastic shell empty inside were investigated by GOLDSBERRY (1967), PIDDUBNIAK (1995), PIDDUBNIAK *et al.* (2009), POROKHOVSKI, (2008), and for case of the air-filled shell are described in the article of KERBRAT *et al.* (2002).

On the other hand, the problem of sound scattering by cylindrical viscous fluid-filled shells was considered in many works. And so KACHAENKO *et al.* (1990) had investigated the three-dimension problem of sound diffraction from orthotropic elastic cylindrical shells with a viscous fluid inside on the base of the Timoshenko thin elastic shell theory. The sound diffraction from the viscoelastic polymer hollow cylinders and spheres filled with viscous fluids was analyzed by HASHEMINEJAD, SAFARI (2003). KUBENKO *et al.* (1989) considered the diffraction of sound pulses from coaxial piezoceramic cylindrical shells also filled with a viscous fluid. The wave guided properties of elastic cylindrical shells containing the viscous fluids were investigated by VOLLMANN, DUAL (1997).

The aim of our paper is the study of the spectral structure of sound waves reflected from a thin elastic cylindrical shell surrounded by an ideal compressible fluid (sea water) and filled with a compressible viscous fluid (oil) in case when the shell is weakened by an infinitely long linear slit. We show that the slit present in the shell causes three additional components in the scattering amplitude connected with tangential, angular and radial vibrations of the shell edges on the crack. On the basis of the conclusions from this analysis we make an attempt to estimate the possibility of hydroacoustic diagnostics of oil-pipelines of this type.

2. Formulation of the problem and basic relations

Let us consider in the acoustic medium a thin cylindrical elastic shell of thickness h and a radius a of the

middle surface weakened along the $\theta = \theta_0$ line by an infinitely long slit. The plane harmonic sound wave incidents on the shell with the phase front parallel to the axis of the object (Fig. 1). The shell is filled with a viscous barotropic compressible fluid, in general moved in the axial direction.
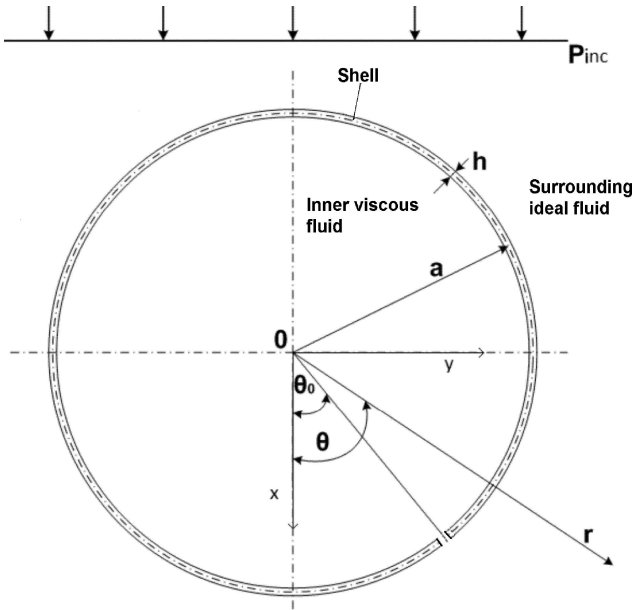

Fig. 1. Scheme of the interaction of sound wave and elastic shell filled with viscous fluid.

The sound waves reflected from the surface of the shell, as well as the waves re-radiated through the shell from the fluid filler, generate an acoustic scattered field. Therefore, it is necessary to study the spectral structure of the scattered sound waves, in particular, to investigate the influence of slit in shell and its orientation with respect to the direction of the incident wave, and also to take into consideration the presence of the viscous fluid filler.

For the description of the wave process we use the linear acoustic theories of ideal and viscous fluids and the linear Kirchhoff–Love theory of thin elastic shells.

From the first one, it is noted that the influence of the fluid motion in the tube must be neglected in the echo-signal because the front of the incident sound wave is parallel to the direction of this motion (YEH, 1968).

The pressure in the reflected wave is described by the Helmholtz equation (BREKHOVSKIKH, GODIN, 1989)

$$(\Delta + k^2)p_{sc}(r, \theta, \omega) = 0 \quad (r > a), \tag{1}$$

where k is the wave number of the ideal acoustical fluid, $k = \omega/c$, ω is the circular frequency, r and θ are the polar co-ordinates with an origin at the axis of the shell, c is the sound velocity in the outer fluid, Δ is the Laplace operator

$$\Delta \equiv \nabla \cdot \nabla = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{2}$$

$\nabla \equiv \mathrm{grad}$ is the Hamilton operator of gradient

$$\nabla \equiv \mathbf{i}_r \frac{\partial}{\partial r} + \mathbf{i}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}, \tag{3}$$

$\mathbf{i}_r$ and $\mathbf{i}_\theta$ are the unit vectors in the polar system of coordinates. The harmonic factor $\exp(-i\omega t)$ is neglected in this paper, $i = \sqrt{-1}$.

The particles displacement vector $\mathbf{u}_{sc}(r, \theta, \omega)$ in the outer acoustical medium is connected with the pressure $p_{sc}(r, \theta, \omega)$ as follows (BREKHOVSKIKH, GODIN, 1989):

$$\mathbf{u}_{sc}(r, \theta, \omega) = \frac{1}{\rho\omega^2}\nabla p_{sc}(r, \theta, \omega) \quad (r > a), \tag{4}$$

where ρ is the density of the outer fluid.

In particular, for the radial component of the displacement vector we obtain

$$w_{sc}(r, \theta, \omega) = \frac{1}{\rho\omega^2}\frac{\partial p_{sc}(r, \theta, \omega)}{\partial r}. \tag{5}$$

The vibrations of the thin elastic cylindrical shell are described by the Kirchhoff-Love equations (PIDSTRYHACH, SHVETZ, 1978)

$$N_2(\theta, \omega) - \frac{\partial Q_2(\theta, \omega)}{\partial \theta} = \rho_s \omega^2 ahw(\theta, \omega) \\ + aq_r(\theta, \omega), \tag{6}$$

$$\frac{\partial N_2(\theta, \omega)}{\partial \theta} + \frac{1}{a}\frac{\partial M_2(\theta, \omega)}{\partial \theta} = -\rho_s\omega^2 ahv(\theta, \omega) \\ - aq_\theta(\theta, \omega), \tag{7}$$

where $N_2(\theta, \omega)$ is the outer normal stress resultant in the shell, $M_2(\theta, \omega)$ is the stress couple, $Q_2(\theta, \omega)$ is the resultant shear stress, $v(\theta, \omega)$ is the angular component of the displacement vector in the shell, $w(\theta, \omega)$ is the radial component of this vector, $q_r(\theta, \omega)$ and $q_\theta(\theta, \omega)$ are the radial and angular components of the outer force vector, respectively, ρ_s is the material density of shell.

For these characteristics, the following physical and geometrical relations are accomplished (PIDSTRYHACH, SHVETZ, 1978):

$$N_2(\theta, \omega) = \frac{D_1}{a}\left[\frac{\partial v(\theta, \omega)}{\partial \theta} + w(\theta, \omega)\right], \tag{8}$$

$$M_2(\theta, \omega) = -\frac{D_2}{a}\frac{\partial \vartheta(\theta, \omega)}{\partial \theta}, \tag{9}$$

$$Q_2(\theta, \omega) = \frac{1}{a}\frac{\partial M_2(\theta, \omega)}{\partial \theta}, \tag{10}$$

$$\vartheta(\theta, \omega) = \frac{1}{a}\left[\frac{\partial w(\theta, \omega)}{\partial \theta} - v(\theta, \omega)\right], \tag{11}$$

where D_1 and D_2 are the tension and bending stiffness of the shell material, respectively

$$D_1 = \frac{Eh}{1 - \nu^2}, \qquad D_2 = \frac{Eh^3}{12(1 - \nu^2)}, \tag{12}$$

E is the Young modulus, ν is the Poisson number, $\vartheta(\theta, \omega)$ is the angle of the shell middle surface rotation.

For the description of sound propagation in the viscous barotropic fluid, we use the Navier-Stokes equations, which formally (in the harmonic regime of vibrations) coincide with equations of the theory of elasticity, but with the Lamé constants λ_f and μ_f depending on the frequencies of vibrations (LANDAU, LIFSITZ, 1987; BREKHOVSKIKH, GODIN, 1989):

$$\lambda_f(\omega) = \rho_f c_f^2 + i\omega\left(\frac{2}{3}\eta_f - \zeta_f\right),$$
$$\mu_f(\omega) = -i\omega\eta_f, \tag{13}$$

where ρ_f is the density of the viscous fluid, c_f is the sound velocity for the adiabatic process, η_s is the first (shear) viscosity, ζ_f is the second (volume) viscosity.

Then the radial w_f and angular v_f components of the displacement vector in the viscous cylinder can be represented in the following form (ACHENBACH, 1973):

$$w_f(r, \theta, \omega) = \frac{\partial \Phi(r, \theta, \omega)}{\partial r} + \frac{1}{r}\frac{\partial \Psi(r, \theta, \omega)}{\partial \theta}, \tag{14}$$

$$v_f(r, \theta, \omega) = \frac{1}{r}\frac{\partial \Phi(r, \theta, \omega)}{\partial \theta} - \frac{\partial \Psi(r, \theta, \omega)}{\partial r}, \tag{15}$$

where $\Phi(r, \theta, \omega)$ and $\Psi(r, \theta, \omega)$ are longitudinal and shear elastic field potential functions, which satisfy the following wave equations:

$$(\Delta + k_{fL}^2)\Phi(r, \theta, \omega) = 0 \quad (0 < r < a), \tag{16}$$

$$(\Delta + k_{fT}^2)\Psi(r, \theta, \omega) = 0 \quad (0 < r < a). \tag{17}$$

Here $k_{fA} = \omega/c_{fA}$ $(A = L, T)$ are the wave numbers in the viscous fluid, c_{fL} is the longitudinal wave velocity and c_{fT} is the shear wave velocity, which are defined from the relations:

$$c_{fL} \equiv c_{fL}(\omega) = \sqrt{\frac{\lambda_f(\omega) + 2\mu_f(\omega)}{\rho_f}},$$
$$c_{fT} \equiv c_{fT}(\omega) = \sqrt{\frac{\mu_f(\omega)}{\rho_f}}. \tag{18}$$

Using the connections between the components of the stress tensor, the vector of displacement (ACHENBACH, 1973), and the formulas (14), (15), we obtain the following necessary relations (PIDSTRYHACH, PIDDUBNIAK, 1986):

$$\sigma_{fr}(r, \theta, \omega) = 2\mu_f(\omega)$$
$$\cdot\left(-L_1\Phi(r, \theta, \omega) + L_2\frac{\partial \Psi(r, \theta, \omega)}{\partial \theta}\right), \tag{19}$$

$$\tau_{fr\theta}(r, \theta, \omega) = 2\mu_f(\omega)$$
$$\cdot\left(L_2\frac{\partial \Phi(r, \theta, \omega)}{\partial \theta} + L_1\Psi(r, \theta, \omega)\right), \tag{20}$$

where

$$L_1 \equiv \frac{1}{r}\left(\frac{\partial}{\partial r} + 2\frac{\partial^2}{\partial \theta^2}\right) + \frac{1}{2}k_{fT}^2,$$
$$L_2 \equiv \frac{1}{r}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right). \tag{21}$$

There, the following boundary conditions must be added to the Eqs. (1), (6), (7), (16), and (17):

$$q_r(\theta, \omega) = -[p_{\text{tot}}(a, \theta, \omega) + \sigma_{fr}(a, \theta, \omega)], \tag{22}$$

$$q_\theta(\theta, \omega) = -\tau_{fr}(a, \theta, \omega), \tag{23}$$

$$w(\theta, \omega) = w_{\text{tot}}(a, \theta, \omega) = w_f(a, \theta, \omega), \tag{24}$$

$$v(\theta, \omega) = v_f(a, \theta, \omega). \tag{25}$$

Here

$$p_{\text{tot}}(r, \theta, \omega) = p_{\text{inc}}(r, \theta, \omega) + p_{\text{sc}}(r, \theta, \omega),$$
$$w_{\text{tot}}(r, \theta, \omega) = w_{\text{inc}}(r, \theta, \omega) + w_{\text{sc}}(r, \theta, \omega) \tag{26}$$

are the total sound fields, which contain the sum of incident and scattered waves.

Next, we also assume that the shell edges on the slit $\theta = \theta_0$ are free from loadings, i.e.

$$N_2(\theta_0 \pm 0, \omega) = 0,$$
$$Q_2(\theta_0 \pm 0, \omega) = 0, \tag{27}$$
$$M_2(\theta_0 \pm 0, \omega) = 0.$$

3. The method of solution of the problem

Suppose the plane wave of acoustical pressure incidents on the cylindrical elastic shell

$$p_{\text{inc}}(x, \omega) \equiv p_{\text{inc}}(r, \theta, \omega) = p_0 e^{ikx} = p_0 e^{ikr\cos\theta}, \tag{28}$$

where p_0 is the constant value of the dimension of pressure.

The expression (28) can be represented as the Fourier-Bessel complex series (BATEMAN, ERDÉLYI, 1953; FELSEN, MARKUVITZ, 1973)

$$p_{\text{inc}}(r, \theta, \omega) = p_0 \sum_{m=-\infty}^{\infty} i^m J_m(kr)e^{-im\theta}$$
$$(r \geq a, 0 \leq \theta \leq 2\pi) \tag{29}$$

and similarly from Eq. (5)

$$w_{\text{inc}}(r, \theta, \omega) = \frac{1}{\rho\omega^2}\frac{\partial p_{\text{inc}}(r, \theta, \omega)}{\partial r}$$
$$= \frac{p_0}{\rho\omega^2} \sum_{m=-\infty}^{\infty} i^m J_m'(kr)e^{-im\theta}$$
$$(r \geq a, \ 0 \leq \theta \leq 2\pi), \tag{30}$$

$J_m(kr)$ and $J'_m(kr)$ are the Bessel functions of m-th order and its derivative over argument, respectively.

The solution of the Eq. (1) for the sound waves scattered from the shell may be represented in the form

$$p_{sc}(r,\theta,\omega) = \sum_{m=-\infty}^{\infty} p_{sm} H_m^{(1)}(kr) e^{-im\theta}$$
$$(r \geq a,\ 0 \leq \theta \leq 2\pi) \quad (31)$$

and the particle radial displacement in the scattered sound field is obtained from Eq. (5) as

$$w_{sc}(r,\theta,\omega) = \frac{1}{\rho\omega^2} \sum_{m=-\infty}^{\infty} p_{sm} H_m^{(1)'}(kr) e^{-im\theta}$$
$$(r \geq a,\ 0 \leq \theta \leq 2\pi), \quad (32)$$

where $H_m^{(1)}(kr)$ and $H_m^{(1)'}(kr)$ are the Hankel functions of second kind and m-th order and its derivate over argument, respectively; p_{sm} are the unknown quantities.

Similarly, we obtain the solutions of Eqs. (16) and (17) for the inner fluid viscous cylinder

$$\Phi(r,\theta,\omega) = \sum_{m=-\infty}^{\infty} \Phi_m J_m(k_{fL}r) e^{-im\theta}$$
$$(0 \leq r \leq a,\ 0 \leq \theta \leq 2\pi), \quad (33)$$

$$\Psi(r,\theta,\omega) = \sum_{m=-\infty}^{\infty} \Psi_m J_m(k_{fT}r) e^{-im\theta}$$
$$(0 \leq r \leq a,\ 0 \leq \theta \leq 2\pi), \quad (34)$$

where Φ_m and Ψ_m are the unknown quantities.

Substituting the relations (33), (34) to the Eqs. (14), (15), and (23)–(25) we obtain (PIDSTRYHACH, PIDDUBNIAK, 1986):

$$w_f(r,\theta,\omega) = \sum_{m=-\infty}^{\infty} [\Phi_m \chi_{1m}(k_{fL}r)$$
$$+ M\Psi_m \chi_{2m}(k_{fT}r)] e^{-im\theta}, \quad (35)$$

$$v_f(r,\theta,\omega) = -i \sum_{m=-\infty}^{\infty} m [\Phi_m \chi_{2m}(k_{fL}r)$$
$$+ \Psi_m \chi_{1m}(k_{fT}r)] e^{-im\theta}, \quad (36)$$

$$\sigma_{fr}(r,\theta,\omega) = 2\mu_f(\omega) \sum_{m=-\infty}^{\infty} [\Phi_m \chi_{3m}(k_{fL}r)$$
$$+ M\Psi_m \chi_{4m}(k_{fT}r)] e^{-im\theta}, \quad (37)$$

$$\tau_{fr\theta}(r,\theta,\omega) = -2\mu_f(\omega)i \sum_{m=-\infty}^{\infty} m [\Phi_m \chi_{4m}(k_{fL}r)$$
$$+ \Psi_m \chi_{3m}(k_{fT}r)] e^{-im\theta}, \quad (38)$$

with $0 \leq r \leq a$, $0 \leq \theta \leq 2\pi$ and

$$\chi_{1m}(k_{fA}r) = k_{fA} J'_m(k_{fA}r),$$

$$\chi_{2m}(k_{fA}r) = \frac{1}{r} J_m(k_{fA}r),$$

$$\chi_{3m}(k_{fA}r) = \left(-\frac{1}{2}k_{fT}^2 + \frac{M}{r^2}\right) J_m(k_{fA}r)$$
$$-\frac{1}{r}k_{fA} J'_m(k_{fA}r), \quad (39)$$

$$\chi_{4m}(k_{fA}r) = \frac{1}{r}\left[-\frac{1}{r}J_m(k_{fA}r) + k_{fA}J'_m(k_{fA}r)\right]$$
$$(A = L, T;\ M = m^2).$$

For the solution of the equations of the shell theory we use the distribution of unknown functions in the Fourier series over the angular variable θ. However, there must be taken into consideration that these functions and their first derivates are continuous, possibly except the discontinuities of the first kind at point $\theta = \theta_0$. Therefore,

$$f(\theta) = \sum_{m=-\infty}^{\infty} f_m e^{-im\theta} \quad (0 \leq \theta \leq 2\pi), \quad (40)$$

such that

$$f_m = \frac{1}{2\pi} \lim_{\delta \to 0} \left[\int_0^{\theta_0-\delta} f(\theta) e^{im\theta}\, d\theta + \int_{\theta_0+\delta}^{2\pi} f(\theta) e^{im\theta}\, d\theta \right]$$
$$(m = 0,\ \pm 1,\ \pm 2, ...). \quad (41)$$

For this reason, applying the operator $\int_0^{2\pi} (...) e^{im\theta}\, d\theta$ to the Eqs. (6) and (7) we obtain (PIDDUBNIAK, 1995)

$$2\pi N_{2m} - [Q_2] e^{im\theta_0} + 2\pi im Q_{2m}$$
$$= 2\pi(\rho_s \omega^2 a h w_m + a q_{rm}), \quad (42)$$

$$[N_2] e^{im\theta_0} - 2\pi im N_{2m} + \frac{1}{a}[M_2] e^{im\theta_0} - \frac{2\pi im}{a} M_{2m}$$
$$= -2\pi(\rho_s \omega^2 a h v_m + a q_{\theta m}), \quad (43)$$

where $[f] = f(\theta_0 - 0) - f(\theta_0 + 0)$ is the jump of function $f(\theta)$ resulting from transition through the slit in the shell.

From Eqs. (8)–(11), the following equations can be obtained in the same way:

$$2\pi N_{2m} = \frac{D_1}{a}\left([v]e^{im\theta_0} - 2\pi im v_m + 2\pi w_m\right), \quad (44)$$

$$2\pi M_{2m} = -\frac{D_2}{a}\left([\vartheta_2]e^{im\theta_0} - 2\pi im \vartheta_{2m}\right), \quad (45)$$

$$2\pi Q_{2m} = \frac{1}{a}\left([M_2]e^{im\theta_0} - 2\pi im M_{2m}\right), \quad (46)$$

$$2\pi \vartheta_{2m} = \frac{1}{a}\left([w]e^{im\theta_0} - 2\pi im w_m - 2\pi v_m\right). \quad (47)$$

The Eqs. (42), (43) and (46) contain the jumps of normal stress resultant $N_2(\theta,\omega)$, the stress couple $M_2(\theta,\omega)$ and shear stress resultant $Q_2(\theta,\omega)$, which in consequence of carrying out the boundary conditions (27), are zeros:

$$[N_2] = [M_2] = [Q_2] = 0. \tag{48}$$

Next, we introduce the definitions

$$
\begin{aligned}
W_m &= \frac{1}{2\pi}[w]e^{im\theta_0}, \\
V_m &= \frac{1}{2\pi}[v]e^{im\theta_0}, \\
T_m &= \frac{a}{2\pi}[\vartheta_2]e^{im\theta_0}, \\
\Theta_m &= T_m - imW_m.
\end{aligned}
\tag{49}
$$

Then taking into account the Eqs. (42)–(48), we obtain the following system of equations:

$$
\begin{aligned}
v_m &- im(1 + \varepsilon M)w_m \\
&= imV_m - im\varepsilon\Theta_m - \frac{a^2}{D_1}q_{\theta m}, \\
im(1 &+ \varepsilon M)v_m + (x_{10}^2 - 1 - \varepsilon M^2)w_m \\
&= V_m - \varepsilon M\Theta_m - \frac{a^2}{D_1}q_{rm},
\end{aligned}
\tag{50}
$$

where

$$
\begin{aligned}
\varepsilon &= \frac{h^2}{12a^2}, \qquad x_{10} = k_{10}a, \qquad k_{10} = \frac{\omega}{c_{10}}, \\
c_{10} &= \sqrt{\frac{E}{\rho_s(1 - v^2)}} = 2c_T\sqrt{1 - \frac{c_T^2}{c_L^2}},
\end{aligned}
\tag{51}
$$

c_{10} is the velocity of the shell elastic wave connected with the plate mode of shell vibrations (METSAVEER *et al.*, 1979), k_{10} is the wave number for this mode.

Now, we write the expressions for the quantities $q_{\theta m}$ and q_{rm}. On the basis of the boundary conditions (22), (23) and Eqs. (26₁), (29), (31), (37), (38), (40), (41), we find

$$
\begin{aligned}
q_{rm} = -\{&p_0 i^m J_m(x) + p_{sm}H_m^{(1)}(x) \\
&+ 2\mu_f[\Phi_m\chi_{3m}(x_{fL}) + M\Psi_m\chi_{4m}(x_{fT})]\},
\end{aligned}
\tag{52}
$$

$$q_{\theta m} = -2i\mu_{fm}[\Phi_m\chi_{4m}(x_{fL}) + \Psi_m\chi_{3m}(x_{fT})]\}, \tag{53}$$

where $x = ka$, $x_{fA} = k_{fA}a$ ($A = L, T$).

Next, from the conditions (24), (25) and Eqs. (30), (32), (35), (36) we have

$$w_{tot,m} = w_m = \frac{1}{\rho c\omega}[p_0 i^m J_m'(x) + p_{sm}H_m^{(1)'}(x)], \tag{54}$$

$$
\begin{aligned}
w_{fm} &= w_m \\
&= \frac{1}{a}[\Phi_m x_{fL}J_m'(x_{fL}) + M\Psi_m J_m(x_{fT})],
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
v_{fm} &= v_m \\
&= -\frac{im}{a}[\Phi_m J_m(x_{fL}) + \Psi_m x_{fT}J_m'(x_{fT})].
\end{aligned}
\tag{56}
$$

From the Eq. (54), the quantities p_{sm} are obtained as

$$p_{sm} = \frac{1}{H_m^{(1)'}(x)}[\rho c\omega w_m - p_0 i^m J_m'(x)] \tag{57}$$

and from the system of equations (55) and (56) the quantities Φ_{sm} and Ψ_{sm} may be written as

$$\Phi_m = \frac{1}{\Delta_m}[w_m\chi_{1m}(x_{fT}) - imv_m\chi_{2m}(x_{fL})], \tag{58}$$

$$\Psi_m = \frac{1}{\Delta_m}\left[-w_m\chi_{2m}(x_{fT}) + \frac{i}{m}v_m\chi_{1m}(x_{fL})\right]$$
$$(m \neq 0), \tag{59}$$

where

$$
\begin{aligned}
\Delta_m = a^2[&\chi_{1m}(x_{fL})\chi_{1m}(x_{fT}) \\
&- M\chi_{2m}(x_{fL})\chi_{2m}(x_{fT})].
\end{aligned}
\tag{60}
$$

Substituting Φ_{sm} and Ψ_{sm} into the expressions for the quantities q_{rm} and $q_{\theta m}$ (52), (53) we obtain after some transforms:

$$
\begin{aligned}
q_{rm} = -&[p_0 i^m J_m(x) + p_{sm}H_m^{(1)}(x)] \\
&- \frac{2\mu_f}{\Delta_m}[w_m f_{1m}(x_{fT}, x_{fL}) \\
&+ imv_m f_{2m}(x_{fT}, x_{fL})],
\end{aligned}
\tag{61}
$$

$$
\begin{aligned}
q_{\theta m} = -&\frac{2\mu_f}{\Delta_m}[imw_m f_{2m}(x_{fL}, x_{fT}) \\
&- v_m f_{1m}(x_{fL}, x_{fT})]
\end{aligned}
\tag{62}
$$

with

$$
\begin{aligned}
f_{1m}(x_{fA}, x_{fB}) &= a^3[\chi_{1m}(x_{fA})\chi_{3m}(x_{fB}) \\
&\qquad - M\chi_{2m}(x_{fA})\chi_{4m}(x_{fB})], \\
f_{2m}(x_{fA}, x_{fB}) &= a^3[-\chi_{2m}(x_{fA})\chi_{3m}(x_{fB}) \\
&\qquad + \chi_{1m}(x_{fA})\chi_{4m}(x_{fB})] \\
&\qquad (A, B = L, T; \ A \neq B).
\end{aligned}
\tag{63}
$$

Now, using the expressions (57) for p_{sm} and the expressions (61), (62) for q_{rm} and $q_{\theta m}$, the equations (50) can be reduced to the systems of equations in v_m and w_m:

$$
\begin{aligned}
L_{11m}v_m + L_{12m}w_m &= imV_m - im\varepsilon\Theta_m, \\
L_{21m}v_m + L_{22m}^0 w_m &= V_m - \varepsilon M\Theta_m + g_m,
\end{aligned}
\tag{64}
$$

where

$$L_{11m} = x_{10}^2 - (1+\varepsilon)M + \frac{2a\xi_{f0}}{\Delta_m} f_{1m}(x_{fL}, x_{fT}),$$

$$L_{12m} = -im \left[1 + \varepsilon M + \frac{2a\xi_{f0}}{\Delta_m} f_{2m}(x_{fL}, x_{fT}) \right],$$

$$L_{21m} = im \left[1 + \varepsilon M - \frac{2a\xi_{f0}}{\Delta_m} f_{2m}(x_{fT}, x_{fL}) \right],$$

$$L_{22m} = x_{10}^2 - 1 - \varepsilon M^2 - \frac{2a\xi_{f0}}{\Delta_m} f_{1m}(x_{fT}, x_{fL}),$$

$$L_{22m}^0 = L_{22m} + z_m, \qquad z_m = -x\xi_0 \frac{H_m^{(1)}(x)}{H_m^{(1)'}(x)}, \quad (65)$$

$$g_m = \kappa_0 \widetilde{g}_m, \qquad \widetilde{g}_m = \frac{i^m}{H_m^{(1)'}(x)},$$

$$\kappa_0 = \frac{2ip_0 a^2}{\pi D_1 x}, \qquad \xi_0 = \frac{a}{h} N_s \left(\frac{c}{c_{10}} \right)^2,$$

$$\xi_{f0} = \frac{a}{h} N_{fs} \left(\frac{c_{fT}}{c_{10}} \right)^2,$$

$$N_s = \frac{\rho}{\rho_s}, \qquad N_{fs} = \frac{\rho_f}{\rho_s}.$$

Similarly as the expression for g_m, the unknown characteristics can be written as

$$\{v_m, w_m, \vartheta_m, V_m, \Theta_m\} = \kappa_0 \{\widetilde{v}_m, \widetilde{w}_m, \widetilde{\vartheta}_m, \widetilde{V}_m, \widetilde{\Theta}_m\}. \quad (66)$$

After transformation, the system of Eqs. (64) leads to

$$\begin{aligned}
L_{11m}\widetilde{v}_m + L_{12m}\widetilde{w}_m &= im\widetilde{V}_m - im\varepsilon\widetilde{\Theta}_m, \\
L_{21m}\widetilde{v}_m + L_{22m}\widetilde{w}_m &= \widetilde{V}_m - \varepsilon M\widetilde{\Theta}_m + \widetilde{g}_m.
\end{aligned} \quad (67)$$

Solving these equations we obtain the connections between the quantities $\widetilde{v}_m$ and $\widetilde{w}_m$, on the one hand, and the characteristics $\widetilde{V}_m$ and $\widetilde{\Theta}_m$, on the other hand

$$\widetilde{v}_m = \frac{1}{D_m}(D_{vm} - L_{12m}\widetilde{g}_m), \quad (68)$$

$$\widetilde{w}_m = \frac{1}{D_m}(D_{wm} + L_{11m}\widetilde{g}_m), \quad (69)$$

where

$$\begin{aligned}
D_m &= -\frac{1}{H_m^{(1)'}(x)} [x\xi_0 L_{11m} H_m^{(1)}(x) \\
&\quad - D_m^0 H_m^{(1)'}(x)], \\
D_{vm} &= (imL_{22m}^0 - L_{12m})\widetilde{V}_m \\
&\quad - (imL_{22m}^0 - ML_{12m})\varepsilon\widetilde{\Theta}_m, \quad (70) \\
D_{wm} &= (L_{11m} - imL_{21m})\widetilde{V}_m \\
&\quad - (ML_{11m} - imL_{21m})\varepsilon\widetilde{\Theta}_m, \\
D_m^0 &= L_{11m}L_{22m} - L_{12m}L_{21m},
\end{aligned}$$

The substitution of w_m into the expression for p_{sm} (57) yields

$$p_{sm} = p_{sm}^0 + \frac{2ip_0\xi_0}{\pi H_m^{(1)'}(x)} \frac{D_{wm}}{D_m}, \quad (71)$$

where the quantities

$$p_{sm}^0 = -p_0 i^m \frac{\xi_0 x L_{11m} J_m(x) - D_m^0 J_m'(x)}{\xi_0 x L_{11m} H_m^{(1)}(x) - D_m^0 H_m^{(1)'}(x)} \quad (72)$$

represent the case of the shell without slit.

For the estimation the influence of the slit on the spectral structure of the acoustical echo-signal, it is necessary to find the quantities $\widetilde{V}_m$ and $\widetilde{\Theta}_m$ ($\widetilde{W}_m$, $\widetilde{T}_m$), which are contained in the expressions for D_{vm} and D_{wm}. The conditions, with the help of which these quantities may be determined, are obtained from Eq. (27).

First, let us calculate the quantities N_{2m}, M_{2m} and Q_{2m} from Eqs. (44)–(46), (49) and next from the Eqs. (66), (68)–(70) and (40). The system of linear algebraic equations with unknown quantities $\widetilde{V}_m$, $\widetilde{\Theta}_m$ ($\widetilde{W}_m$, $\widetilde{T}_m$) is obtained:

$$\sum_{m=-\infty}^{\infty} [A_{NVm}\widetilde{V}_m + A_{N\Theta m}\widetilde{\Theta}_m + A_{Npm}\widetilde{g}_m]e^{-im\theta_0} = 0,$$

$$\sum_{m=-\infty}^{\infty} [A_{MVm}\widetilde{V}_m + A_{M\Theta m}\widetilde{\Theta}_m + A_{Mpm}\widetilde{g}_m]e^{-im\theta_0} = 0, \quad (73)$$

$$\sum_{m=-\infty}^{\infty} m[A_{MVm}\widetilde{V}_m + A_{M\Theta m}\widetilde{\Theta}_m + A_{Mpm}\widetilde{g}_m]e^{-im\theta_0} = 0,$$

where

$$\begin{aligned}
A_{NVm} &= 1 + \frac{1}{D_m}[ML_{22m}^0 + im \\
&\quad \cdot (L_{12m} - L_{21m}) + L_{11m}], \\
A_{N\Theta m} &= \varepsilon A_{MVm}, \\
A_{MVm} &= -\frac{1}{D_m}[ML_{22m}^0 + im \\
&\quad \cdot (ML_{12m} - L_{21m}) + ML_{11m}], \quad (74) \\
A_{Npm} &= \frac{1}{D_m}(imL_{12m} + L_{11m}), \\
A_{Mpm} &= -\frac{1}{D_m}(imL_{12m} + ML_{11m}) \\
A_{M\Theta m} &= 1 + \frac{\varepsilon M}{D_m}[L_{22m}^0 + im \\
&\quad \cdot (L_{12m} - L_{21m}) + ML_{11m}].
\end{aligned}$$

From the definitions (49) and (66), it follows that

$$\{\widetilde{V}_m, \widetilde{W}_m, \widetilde{T}_m\}e^{-im\theta_0} = \frac{1}{2\pi}[\widetilde{v}, \widetilde{w}, \widetilde{\vartheta}_2] \\ = \{\widetilde{V}_0, \widetilde{W}_0, \widetilde{T}_0\}. \tag{75}$$

Then, from the Eqs. (73) we obtain the system of equations with the quantities $\widetilde{V}_0, \widetilde{T}_0, \widetilde{W}_0$:

$$A_{11}\widetilde{V}_0 + A_{12}\widetilde{T}_0 = B_1,$$
$$A_{21}\widetilde{V}_0 + A_{22}\widetilde{T}_0 = B_2, \tag{76}$$
$$A_{33}\widetilde{W}_0 = B_3,$$

where

$$A_{11} = \sum_{m=0}^{\infty} \varepsilon_m A_{MVm},$$

$$A_{12} = \sum_{m=1}^{\infty} A_{N\Theta m} = \varepsilon A_{21},$$

$$A_{22} = \sum_{m=0}^{\infty} \varepsilon_m A_{M\Theta m},$$

$$A_{33} = \sum_{m=1}^{\infty} M A_{M\Theta m}, \tag{77}$$

$$B_1 = -\sum_{m=0}^{\infty} \varepsilon_m A_{Npm}\widetilde{g}_m \cos(m\theta_0),$$

$$B_2 = -\sum_{m=1}^{\infty} A_{Mpm}\widetilde{g}_m \cos(m\theta_0),$$

$$B_3 = -\sum_{m=1}^{\infty} m A_{Mpm}\widetilde{g}_m \sin(m\theta_0).$$

Here $\varepsilon_m = 1-0.5\delta_{m0}$, δ_{m0} is the Kronecker symbol, and following properties are used as well:

$$A_{NV,-m} = A_{NVm}, \qquad A_{MV,-m} = A_{MVm},$$
$$A_{N\Theta,-m} = A_{N\Theta m}, \qquad A_{M\Theta,-m} = A_{M\Theta m},$$
$$A_{Np,-m} = A_{Npm}, \qquad A_{Mp,-m} = A_{Mpm}, \tag{78}$$
$$A_{MV0} = 0, \qquad A_{N\Theta 0} = 0,$$
$$A_{Mp0} = 0.$$

Solving the Eqs. (76) and (77) we obtain

$$\widetilde{V}_0 = \frac{B_1 A_{22} - \varepsilon B_2 A_{21}}{A_{11}A_{22} - \varepsilon A_{21}^2},$$

$$\widetilde{T}_0 = -\frac{B_1 A_{21} - B_2 A_{11}}{A_{11}A_{22} - \varepsilon A_{21}^2}, \tag{79}$$

$$\widetilde{W}_0 = \frac{B_3}{A_{33}}.$$

Having the expressions for quantities $\widetilde{V}_0, \widetilde{W}_0$ and $\widetilde{T}_0$, from the Eqs. (31), (70)–(72), (75), (79), we can determine the complex amplitude of the acoustic pressure in the scattered sound wave:

$$p_{\text{sc}}(r, \theta, \omega) = 2\sum_{m=0}^{\infty} \varepsilon_m p_{sm}^0 \cos(m\theta)$$

$$+ \frac{2}{\pi}ip_0\xi_0 \sum_{m=0}^{\infty} \frac{H_m^{(1)}(kr)}{H_m^{(1)\prime}(x)D_m}$$

$$\cdot\{[(\varepsilon_m L_{11m} - imL_{21m})\widetilde{V}_0$$

$$- \varepsilon(ML_{11m} - imL_{21m})\widetilde{T}_0]\cos[m(\theta - \theta_0)]$$

$$+ \varepsilon m(ML_{11m} - imL_{21m})\widetilde{W}_0 \sin[m(\theta - \theta_0)]\}. \tag{80}$$

This formula is obtained taking into consideration the Bessel function property in the Eq. (72) and the equality $p_{s,-m}^0 = (-1)^m p_{sm}^0$.

For the farfield case ($r \to \infty$) the following asymptote for the Hankel function of first kind must be used (BATEMAN, ERDÉLYI, 1953):

$$H_m^{(1)}(kr) \approx \frac{2(-i)^m}{\sqrt{\pi x i}}\sqrt{\frac{a}{2r}}e^{ikr}. \tag{81}$$

Then, for the acoustic pressure in the scattered field we have

$$p_{\text{sc}}(r, \theta, \omega) \approx p_0\sqrt{\frac{a}{2r}}f(\theta, k)e^{ikr}, \tag{82}$$

where $f(\theta, k)$ is the scattering amplitude:

$$f(\theta, k) = f_0(\theta, k) + f_v(\theta, k) + f_\vartheta(\theta, k) + f_w(\theta, k). \tag{83}$$

Here $f_0(\theta, k)$ is the scattering amplitude for the case of the shell without slit:

$$f_0(\theta, k) = -\frac{4}{\sqrt{\pi i x}}\sum_{m=0}^{\infty} \varepsilon_m$$

$$\cdot\frac{\xi_0 x L_{11m}J_m(x) - D_m^0 J_m'(x)}{\xi_0 x L_{11m}H_m^{(1)}(x) - D_m^0 H_m^{(1)\prime}(x)}\cos(m\theta) \tag{84}$$

and $f_v(\theta, k)$, $f_\vartheta(\theta, k)$, $f_w(\theta, k)$ are the disturbances in the scattering amplitude caused by the tangential, v, angular, ϑ, and radial, w, displacements of the shell edges on the slit, respectively:

$$f_v(\theta, k) = -\frac{8i\xi_0}{\pi\sqrt{\pi i x}}\widetilde{V}_0 \sum_{m=0}^{\infty} (-i)^m$$

$$\cdot\frac{(\varepsilon_m L_{11m} - imL_{21m})\cos[m(\theta - \theta_0)]}{\xi_0 x L_{11m}H_m^{(1)}(x) - D_m^0 H_m^{(1)\prime}(x)}, \tag{85}$$

$$f_\vartheta(\theta, k) = \frac{8i\xi_0\varepsilon}{\pi\sqrt{\pi i x}}\widetilde{T}_0 \sum_{m=1}^\infty (-i)^m$$
$$\cdot \frac{(ML_{11m}-imL_{21m})\cos[m(\theta-\theta_0)]}{\xi_0 x L_{11m}H_m^{(1)}(x)-D_m^0 H_m^{(1)\prime}(x)}, \quad (86)$$

$$f_w(\theta, k) = -\frac{8i\xi_0\varepsilon}{\pi\sqrt{\pi i x}}\widetilde{W}_0 \sum_{m=1}^\infty (-i)^m$$
$$\cdot \frac{m(ML_{11m}-imL_{21m})\sin[m(\theta-\theta_0)]}{\xi_0 x L_{11m}H_m^{(1)}(x)-D_m^0 H_m^{(1)\prime}(x)}. \quad (87)$$

It should be noted that here

$$\xi_0\varepsilon = \frac{h}{12a}N_s\frac{c^2}{c_{10}^2}.$$

4. Numerical calculations and analysis of results

The numerical calculations have been performed for the case of the steel shell with radius $a = 1$ m and wall thickness $h = 0.025$ m, for which $\rho_s = 7900$ kg/m³, $c_L = 5240$ m/s, $c_T = 2978$ m/s, $c_{10} = 4901$ m/s (ANSON, CHIVERS, 1981).

The shell is immersed in seawater of parameters: $\rho = 1000$ kg/m³, $c = 1410$ m/c (HICKLING, MEANS, 1968; ARTOBOLEVSKI, 1976).

The shell has been filled with an oil. In the general case, the density of the viscous fluid and the sound velocity in this substance depend on temperature, pressure and frequency of vibrations (LITOVITZ, DAVIS, 1965; MENG *et al.*, 2006; DUKHIN, GOETZ, 2009; TITTMANN, 2011). The density of this fluid had varied in the limits from 650 to 1040 kg/m³. We select the middle value from this range $\rho_f = 870$ kg/m³ corresponding to the oil of the 30WeightOil type for room temperature 25°C[1]. For this temperature, the dynamical viscosity equals $\eta_f = 0.110$ Pa·s. In addition, $c_f = 1300$ m/s (ÅSENG, 2006).

In respect to the volume viscosity, it the approximate equality $\zeta_f \approx 4\eta_f/3$ may be used for oil (TAŞKÖPRÜLÜ *et al.*, 1961). Then, from Eqs. (13) we obtain

$$\lambda_f(\omega) = \rho_f\left(c_f^2 - \frac{2}{3}i\omega\nu_f\right),$$
$$\mu_f(\omega) = -i\omega\rho_f\nu_f, \quad (88)$$

where $\nu_f = \eta_f/\rho_f$ is the kinematic viscosity: $\nu_f = 1.26\cdot 10^{-4}$ m²/s.

Next, from the Eqs. (18) the wave velocities in a viscous fluid may be written as

$$c_{fL} = \sqrt{c_f^2 - \frac{8}{3}i\omega\nu_f},$$
$$c_{fT} = \sqrt{-i\omega\nu_f}. \quad (89)$$

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[1]Oil Viscosity/Zplus™ Tech Brief #13, 2008, June 29, 1–11, www.zddplus.com/TechBrief13%20-%20Oil%20Viscosity.pdf.

Next, we obtain the dimensionless wave numbers for the viscous fluid in the complex form (DWIGHT, 1957):

$$x_{fL} = \frac{\omega a}{c_{fL}} = \frac{x_f}{\sqrt{1-(8/3)ix_f n_f}} = \frac{x_f}{\sqrt{2}\sqrt{1+(8x_f n_f/3)^2}}$$
$$\times \left[\sqrt{\sqrt{1+(8x_f n_f/3)^2}+1}\right.$$
$$\left. + i\sqrt{\sqrt{1+(8x_f n_f/3)^2}-1}\right], \quad (90)$$

$$x_{fT} = \sqrt{\frac{x_f}{2n_f}}(1+i), \qquad x_f = \frac{\omega a}{c_f}, \quad (91)$$

where $n_f = \nu_f/(c_f a)$ is the dimensionless kinematic viscosity.

It should be noted that performing the calculations we have applied the technique of elimination of the numerical instabilities of the Bessel functions with complex arguments in the same manner as it was done by ANSON, CHIVERS (1993).

Figures 2 a and b present the dependence of the real and imaginary parts of the dimensionless longitudinal wave numbers x_{fL} for the oil *versus* the dimensionless wave numbers for seawater. As seen the imaginary part
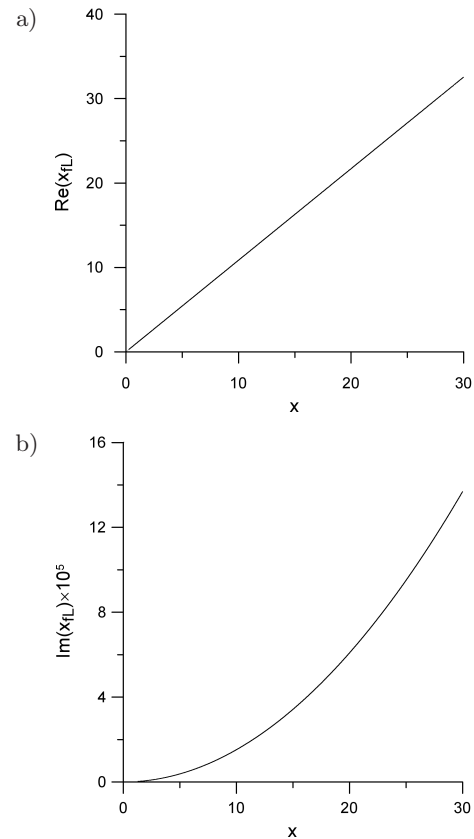

Fig. 2. The dependences of dimensionless complex longitudinal wave number x_{fL} in the oil on the dimensionless wave number in seawater x: a) $\mathrm{Re}(x_{fL})$; b) $\mathrm{Im}(x_{fL})$.

is very small in the considered frequency range, i.e. the attenuation of longitudinal waves is slight. Another type of behavior are observe in Fig. 3, where a similar dependence is presented for the dimensionless shear wave numbers. In this case, the real and imaginary parts of the wave numbers have the same values and increase quickly as the sound frequency increases (proportional to the square root of the frequency). Thus, the shear waves are characterized by great attenuation.
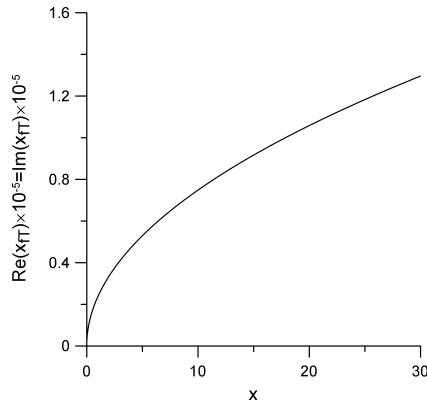

Fig. 3. The dependence of dimensionless complex shear wave number x_{fT} in the oil on the dimensionless wave number in seawater x: $\mathrm{Re}(x_{fT}) = \mathrm{Im}(x_{fT})$.

The module of the scattering amplitude $|f(\theta, k)|$ (it is known as "form-function") calculated in the "back" direction, i.e. for $\theta = 180°$ *versus* dimensionless frequency in the range $0 < x < 30$ for the case of shell without a slit, is shown in Fig. 4. It is seen that this amplitude contains many oscillations caused by superposition of the scattered waves arriving to the observation point with the difference phases. Another feature characterizing the sound scattering is the large quality of sharp peaks and dips aroused by the contribution of vibrations of the shell with a viscous liquid filler to the eigen frequencies. Thus, the figure shows the effects of the resonance acoustic scattering (GAUNAURD, WERBY, 1990; VEKSLER, 1993; KAPLUNOV *et al.*, 1994, 1998; BELOV *et al.*, 1999).
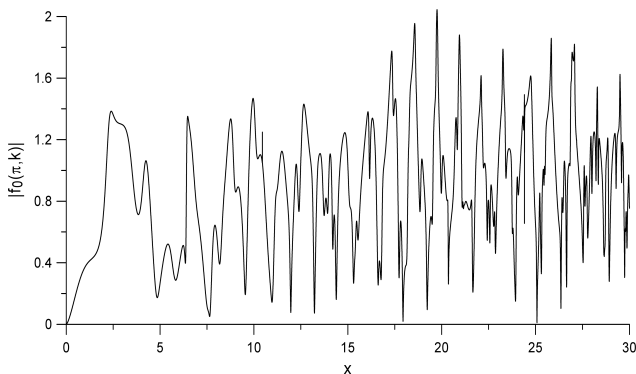
Fig. 4. Form-function $|f_0(\pi, k)|$ of non-defected steel shell filled with oil and surrounded in seawater vs $x = ka$.

The same characteristics, but for the case of the shell weakened by a slit, are given in Figs. 5–9 for $\theta_0 = 0°$, $45°$, $90°$, $135°$, and $180°$, respectively. The comparison of the plots in these figures and Fig. 4 demonstrates the resonance nature of the sound wave diffraction from objects of both types. However, large differences must be noted in the resonance amplitudes excited in consequence of the present slit and the vibrations of the elastic shell edges. Moreover, additional resonances arise that are caused by the surface wave propagation on the elastic shell. These waves, running on the shell periphery from one slit boundary to the other one, radiate sound energy into the surroundings including the "back" direction. The amplitudes of these resonances depend on the slit location.
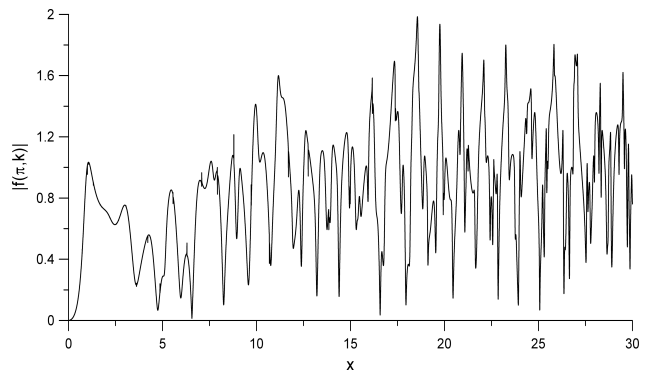

Fig. 5. Form-function $|f(\pi, k)|$ of defected steel shell filled with oil and surrounded in seawater vs $x = ka$ ($\theta_0 = 0°$).
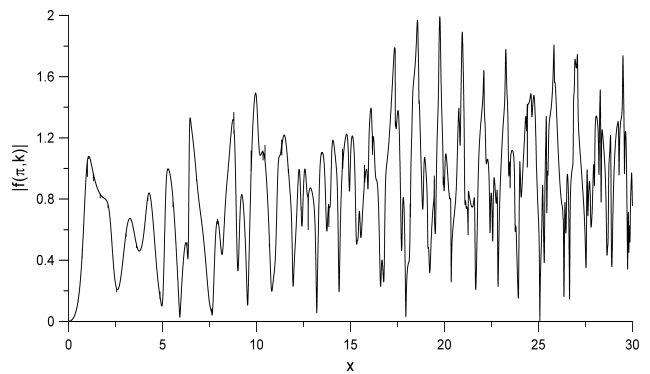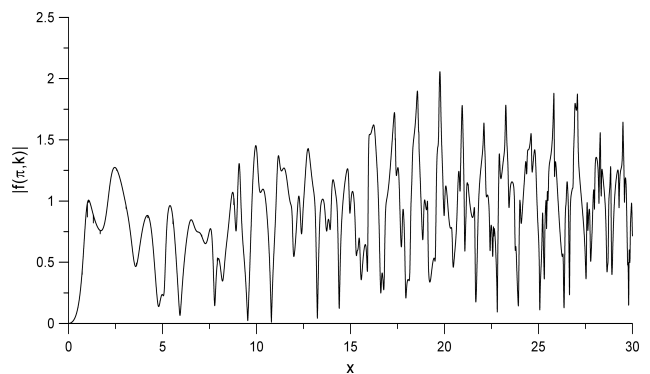

Fig. 6. Form-function $|f(\pi, k)|$ for $\theta_0 = 45°$.
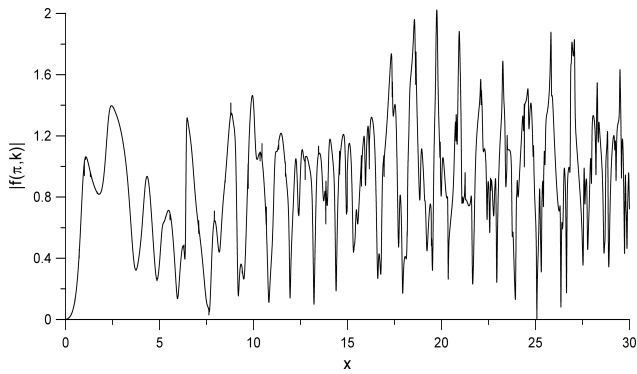

Fig. 7. Form-function $|f(\pi, k)|$ for $\theta_0 = 90°$.
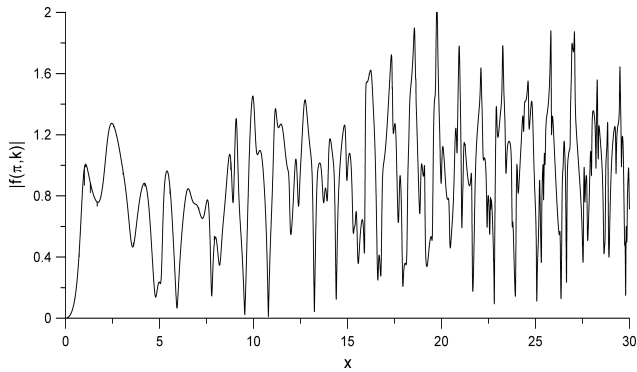
Fig. 8. Form-function $|f(\pi, k)|$ for $\theta_0 = 135°$.

Fig. 9. Form-function $|f(\pi, k)|$ for $\theta_0 = 180°$.

Figures 10–15 show the phase of the scattering amplitude $\phi(\theta, k) = \arctan[\mathrm{Im}f(\theta, k)/\mathrm{Re}f(\theta, k)]$, from which visually can be noted the resonance character of the sound diffraction and crack influence on the scattering process.

The influence of slit orientation on the spectral wave structure is illustrated in more detail in Fig. 16 for the scattering function phase portraits calculated for the "back" direction and the frequency range $0.001 \leq x \leq 30$. From these plots it is seen how the localization of the defect is reflected in the phase of echo-signal. The small loops in the displayed plots (in the form of small "circulars") for the case of shell with defect correspond to the signals with small amplitude and are caused by the particular vibrations of the shell edges.

Figures 17 represent the space distribution of the form-function $|f(\theta, k))|$ (in dB) at one of resonance frequency $x = 5.459$ for the case of shell without and with defect for some slit positions: $\theta_0 = 0°$, $45°$, $90°$, $135°$, $180°$. For the case of the cracked shell this characteristic is non-symmetrical with respect to the direction $\theta = 180°$ (or for $\theta = 0°$) and the asymmetry is most noted in "light" side of the object.

The directivity patterns $D(\theta, k) = |f(\theta, k)|/|f(\pi, k)|$ are presented in Figs. 18. The numerical calculations have been performed using the same parameters as above. We see here that the main part of the scattered acoustic energy is located in the "forward"
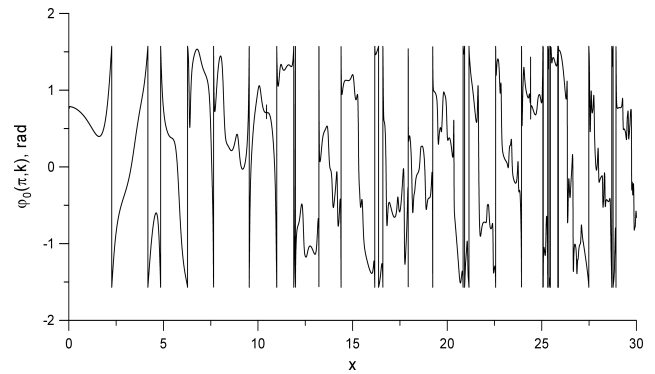

Fig. 10. Phase of backscattering far-field amplitude $\phi_0(\pi, k) = \mathrm{arctg}[\mathrm{Im}f_0(\pi, k)/\mathrm{Re}f_0(\pi, k)]|$ of non-defected shell vs $x = ka$.
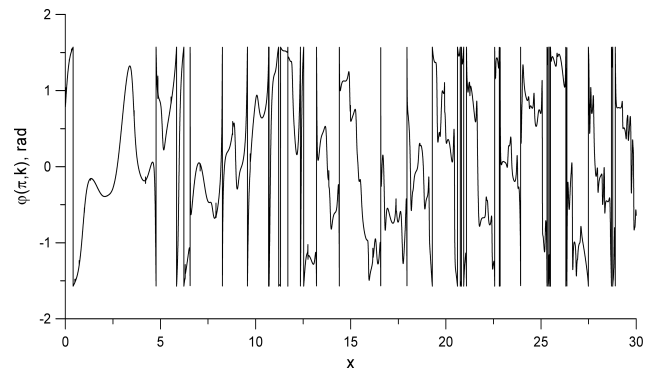

Fig. 11. Phase $\phi(\pi, k) = \mathrm{arctg}[\mathrm{Im}f(\pi, k)/\mathrm{Re}f(\pi, k)]|$ of defected shell vs $x = ka$ ($\theta_0 = 0°$).
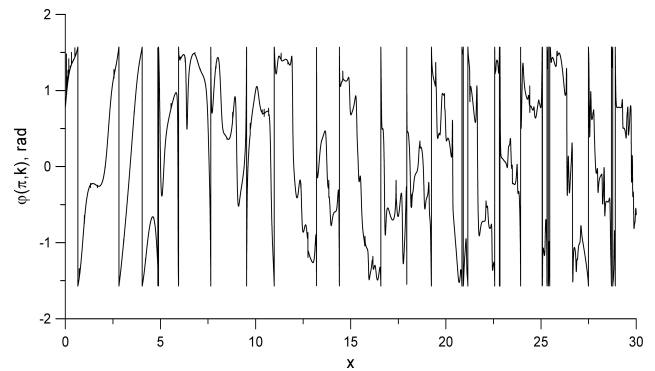

Fig. 12. Phase $\phi(\pi, k) = \mathrm{arctg}[\mathrm{Im}f(\pi, k)/\mathrm{Re}f(\pi, k)]|$ for $\theta_\circ = 45°$.
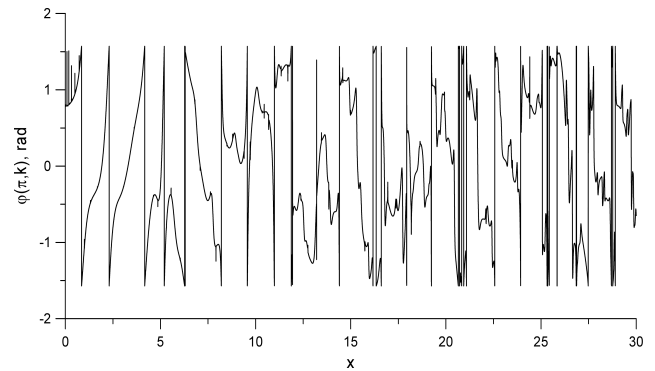

Fig. 13. Phase $\phi(\pi, k) = \mathrm{arctg}[\mathrm{Im}f(\pi, k)/\mathrm{Re}f(\pi, k)]|$ for $\theta_0 = 90°$.
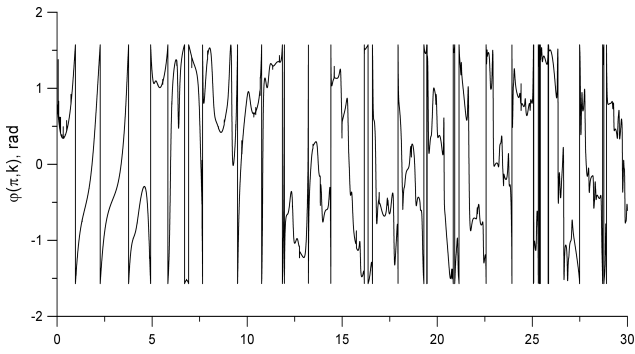
Fig. 14. Phase $\phi(\pi, k) = \mathrm{arctg}[\mathrm{Im}f(\pi,k)/\mathrm{Re}f(\pi,k)]|$ for $\theta_0 = 135°$.
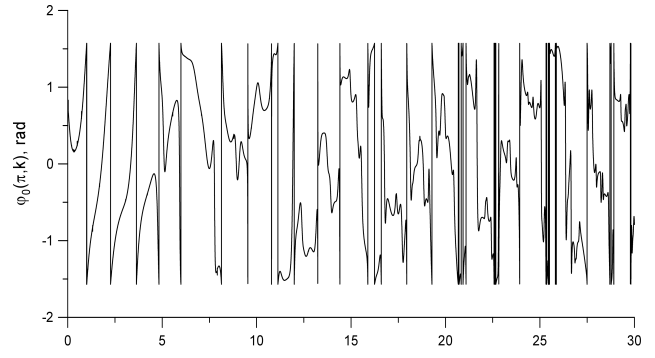
Fig. 15. Phase $\phi(\pi, k) = \mathrm{arctg}[\mathrm{Im}f(\pi,k)/\mathrm{Re}f(\pi,k)]|$ for $\theta_0 = 180°$.
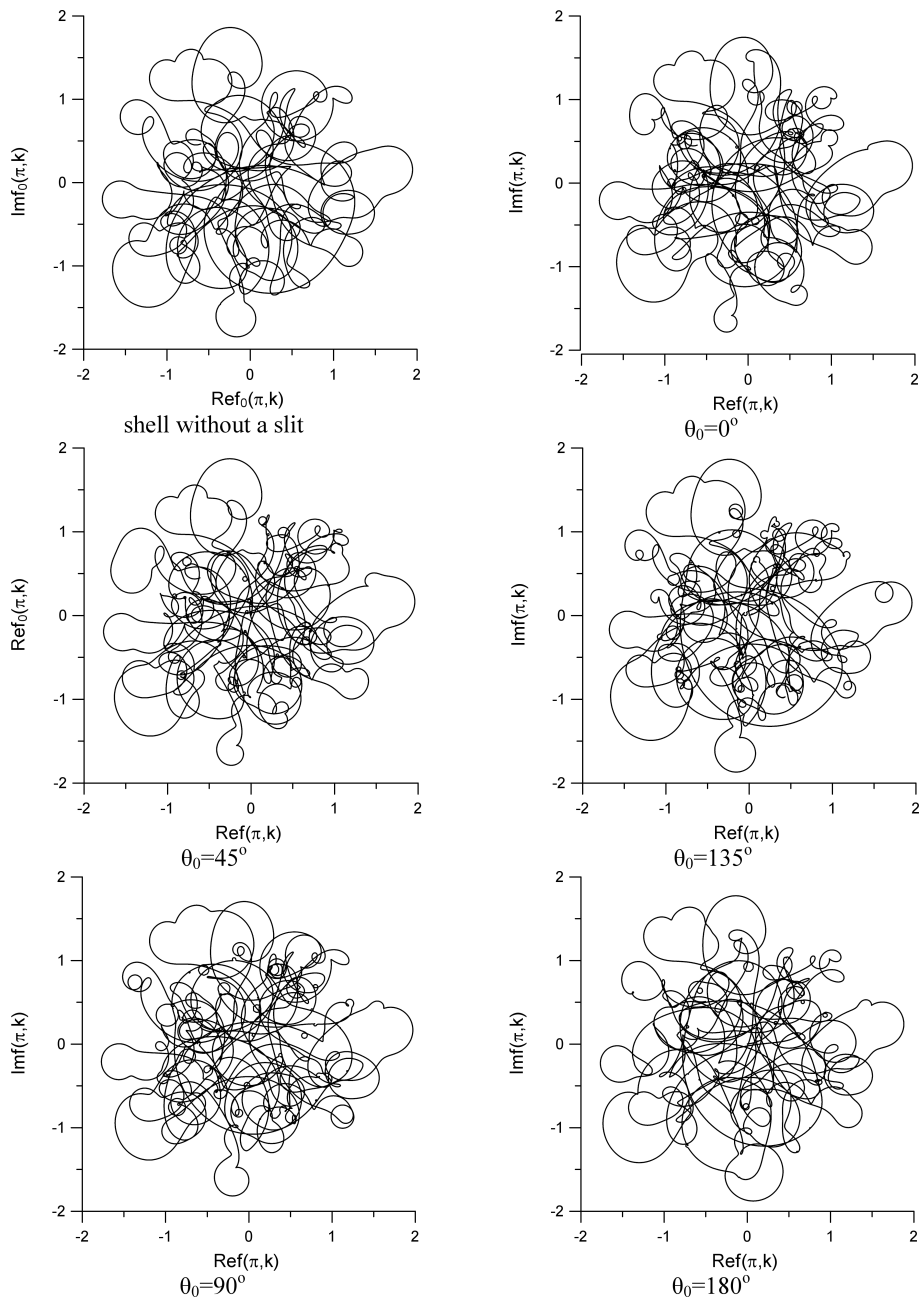
Fig. 16. Phase portraits of backscattering amplitude for $0.001 \leq x = ka \leq 30$.

non-defected shell

$\theta_0=90^{\mathrm{o}}$

$\theta_0=0^{\mathrm{o}}$

$\theta_0=135^{\mathrm{o}}$

$\theta_0=45^{\mathrm{o}}$

$\theta_0=180^{\mathrm{o}}$
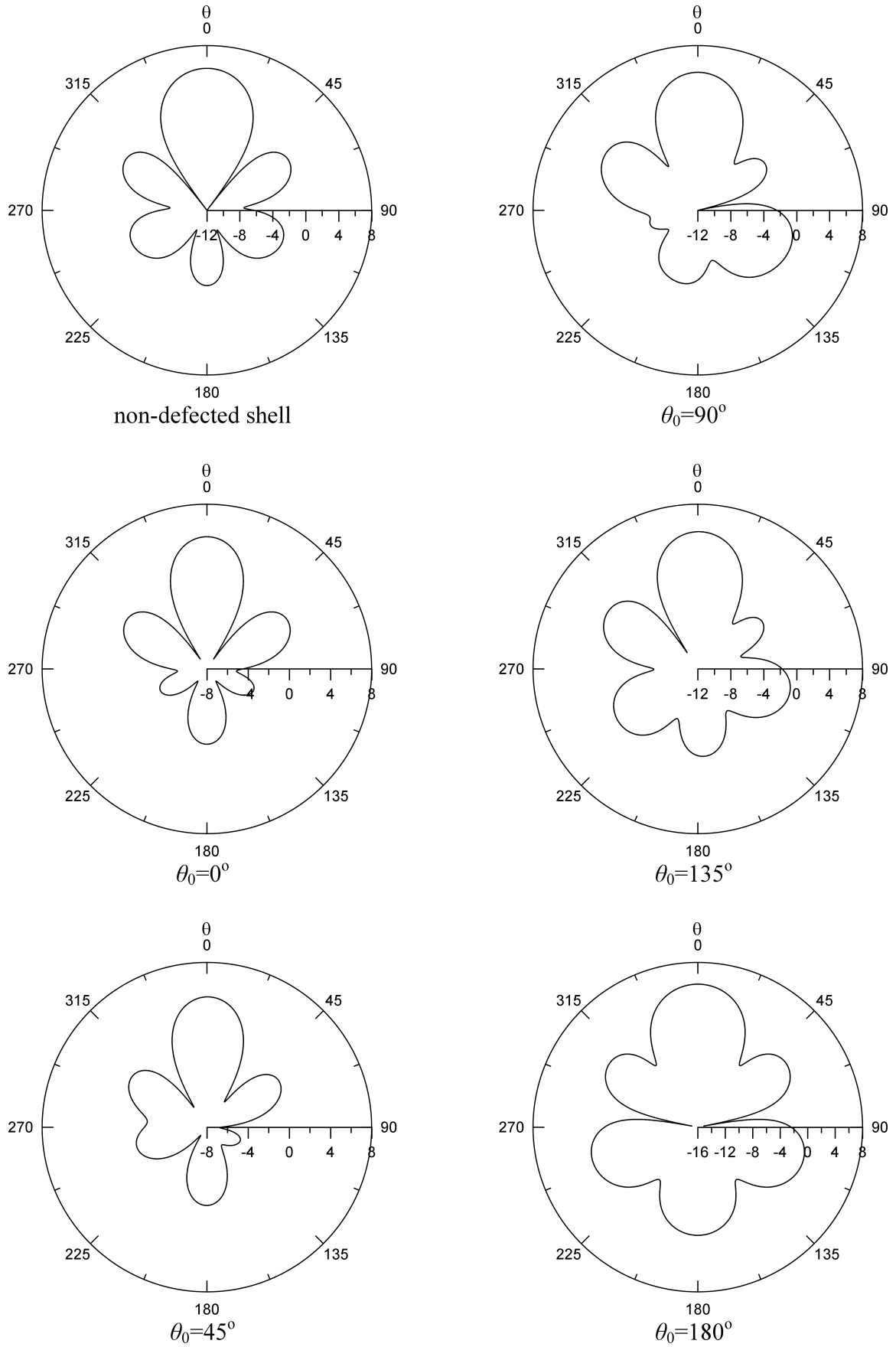
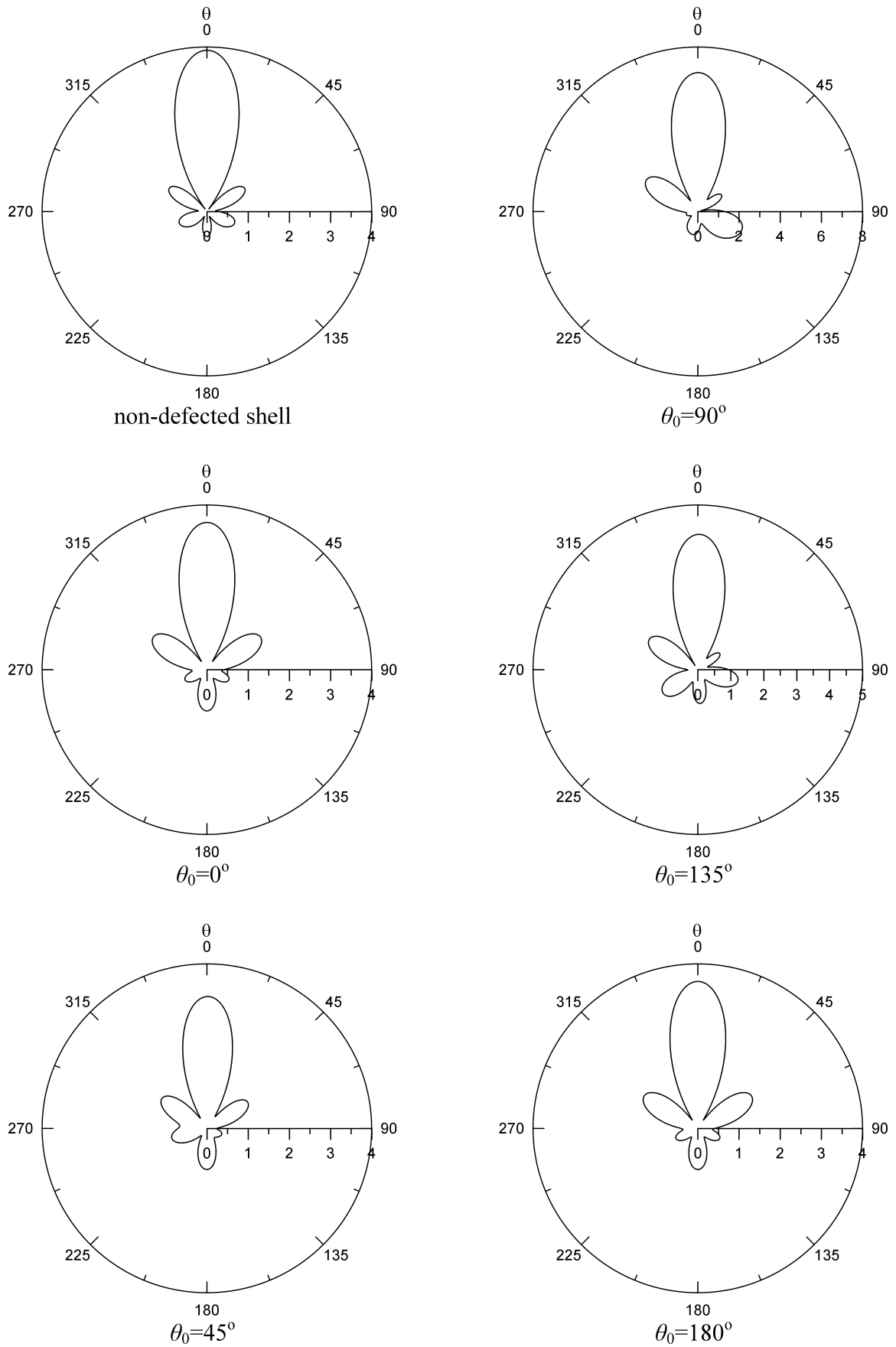Fig. 17. Space distribution of module of scattering amplitude $|f(\theta,k)|$ (in dB) for $x=5.459$.

Fig. 18. Directivity pattern $D(\theta, k) = |f(\theta, k)|/|f(\pi, k)|$ for x=5.459 for different position of slit.
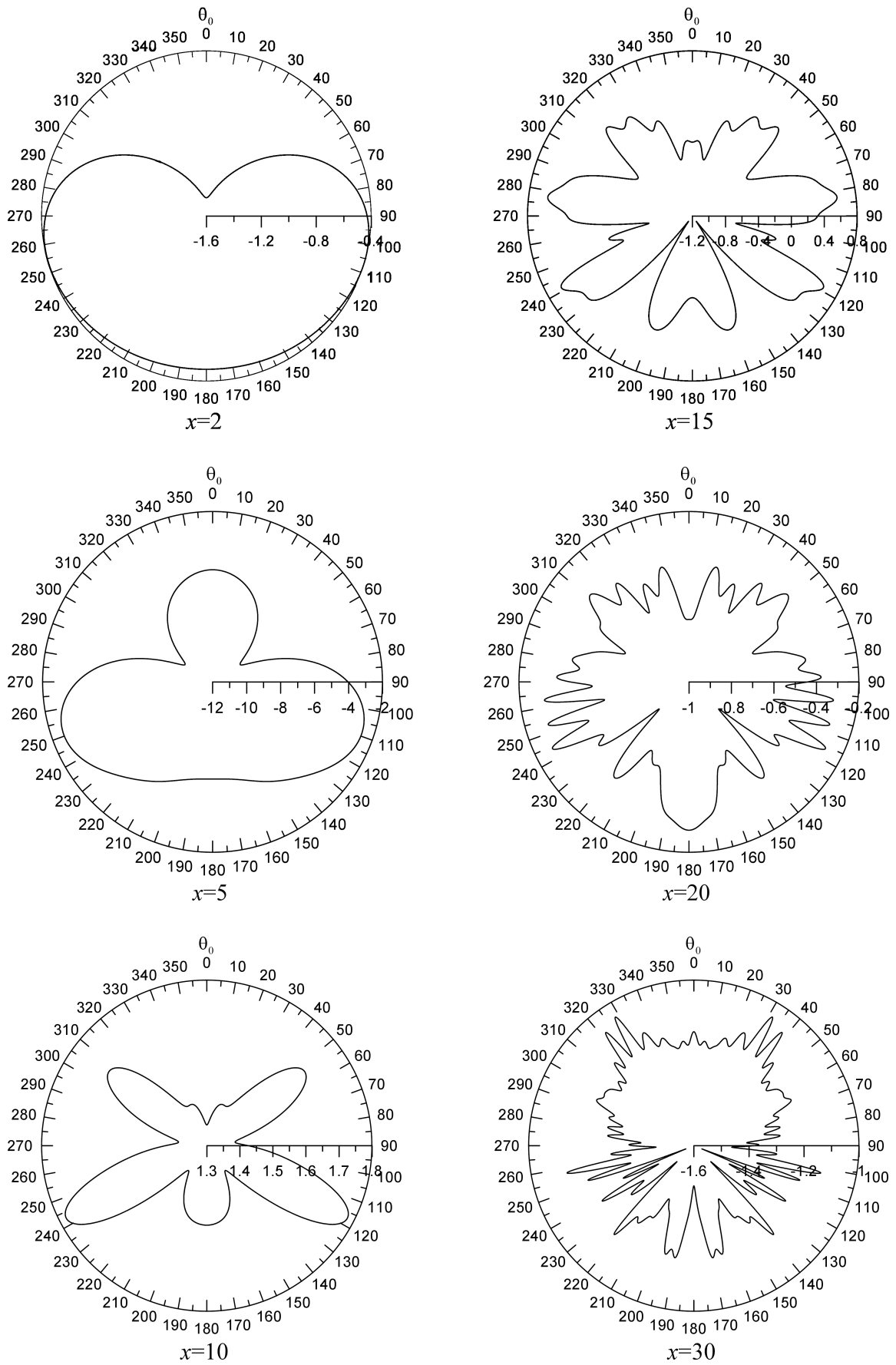
Fig. 19. Form-function $|f(\pi, k)|$ (in dB) vs angular variable of slit position θ_0 for different $x = ka$.

direction. The calculation also indicates that directivity pattern reaches its maximum for $\theta_0 = 90°$. In this case the side lobes of this characteristic are the most ones as well.

One further approach, which have been applied consists in the calculation of the form-function $|f(\theta, k)|$ (in dB) in the "back" direction *versus* the slit location θ_0. From the plots displayed in Figs. 19 for different frequencies $x = 2, 5, 10, 15, 20, 30$, it follows that having the value of the echo-signal amplitude on the transducer the slit location on the shell may be determined with very precision.

5. Conclusions

In this article we have presented the mathematical model of the scattering of plane harmonic sound waves from thin elastic cylindrical shells filled with a compressible viscous fluid and weakened by a long linear crack. The exact solution, i.e. the complex amplitude of the acoustic pressure, is found in the form of infinite Fourier-Bessel series.

On the basis of the solution obtained and the numerical calculations carried out for the case of the steel shell filled with oil and surrounded by seawater, it is shown that the spectrum of the scattered acoustic pressure is formed as the result of the resonance wave reflection from the object. These resonances are caused by shell vibrations on the eigen frequencies. The new additional effects are contributions to the resonance scattering caused by tangential, angular and radial vibrations of the free shell edges of the slit.

The numerical calculations show also that the presence of the slit is most visible in the low dimensionless frequency range, i.e. approximately in the range $0 < x < 10$.

The analysis of amplitudes and phases of the scattered sound pressure, the pattern directivity and influence of slit location on the scattering characteristics show the possibility of distance acoustic identification of cracked shells of the oil pipeline.

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