Dawid Wójcik*
orcid.org/0000-0003-2453-8553

Krzysztof Raszczuk**
orcid.org/0000-0002-0121-0670

Jerzy Jasieńko***
orcid.org/0000-0002-8574-6121

Andrzej Kadłuczka****
orcid.org/0000-0001-7009-5330

Theoretical Analysis of Static Behavior of Masonry Pillars with Geometric Imperfections

Analiza teoretyczna pracy statycznej filarów murowych z imperfekcjami geometrycznymi

Keywords: masonry pillars, geometric imperfections, numerical analysis, static behaviour, historical structures

Słowa kluczowe: filary murowe, imperfekcje geometryczne, analiza numeryczna, praca statyczna, konstrukcje historyczne

Introduction

The assumption of ideal geometry for historical structures is a simplification which can result in incorrect conclusions related to failure modes of the main members of the load-bearing structure. Deformations of masonry structures may be caused by a number of factors, including, inter alia, changes in the subsoil (uneven settlement, as subsoil and groundwater conditions change) or structural errors (unfavorable static behavior).[1] One example of such deformations is the tilting and damaged bases of free-standing stone pillars of the Temple of Apollo in Bassae, which had been analyzed with reference to the effects of seismic impact. It was concluded that the negative impact of different imperfections on load-bearing capacity was additive.[2] The impact of the tilting of the pillars on their load-bearing capacity was also studied by Gurel et al.,[3] who presented an analytical solution for a deformed column, which involved the use of a model with an ideal geometry and equivalent horizontal loading. Results obtained from calculations indicated an immense susceptibility of columns with a tilting imperfection to seismic impact. Aside from deformations involving flexure, tilting or damage to the base of the column, there are also torsional and flexural—torsional imperfections. Examples of such deformations can be seen in some of the structural members of St. Mary's Church in Ostrow Tumski in Poznań. Rapp and Sielicki[4] performed a numerical analysis of a brick pillar with deformations in the form of reciprocal torsion of its cross-sections and flexure. The horizontal deflection in the extreme case reached $\Delta l=0.023$ and was linked to a concurrent torsion. This non-axial geometry resulted in 20% decrease of load-bearing capacity. This proved that analysis of a joint impact of these two types of deformation was reasonable. Consideration of the actual geometry in the case of masonry structures is particularly important due to their brittle response to tensile stresses. As previously mentioned, the issue of load-bearing capacity of pillars subject to seismic loading is already extensively covered in literature[5].

* *M.Sc. Eng., Faculty of Civil Engineering, Wrocław University of Science and Technology*
** *Ph.D. Eng., Faculty of Civil Engineering, Wrocław University of Science and Technology*
*** *Prof. D.Sc. Ph.D. Eng. Faculty of Civil Engineering, Wrocław University of Science and Technology*
**** *Prof. D.Sc. Ph.S. Eng. Arch., Faculty of Architecture, Cracow University of Technology*

* mgr inż., Wydział Budownictwa Lądowego i Wodnego Politechniki Wrocławskiej
** dr inż., Wydział Budownictwa Lądowego i Wodnego Politechniki Wrocławskiej
*** prof. dr hab. inż., Wydział Budownictwa Lądowego i Wodnego Politechniki Wrocławskiej
**** prof. dr hab. inż. arch., Wydział Architektury Politechniki Krakowskiej

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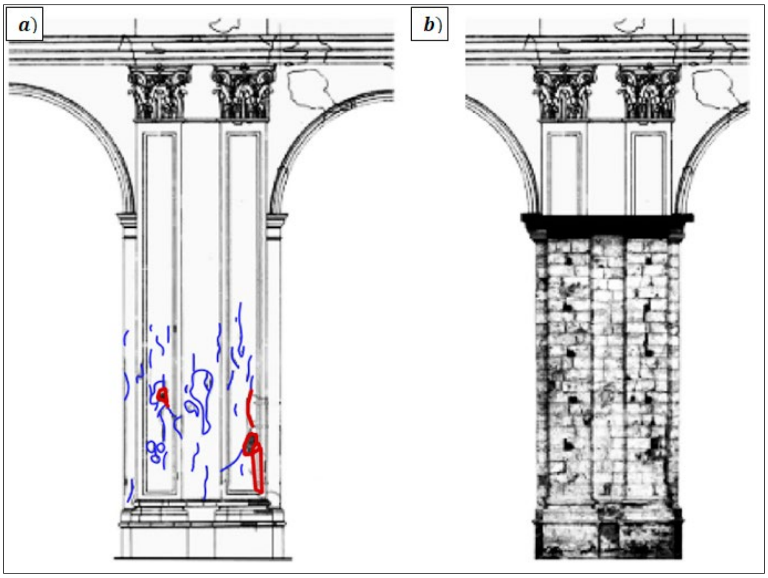

Fig. 1 Damage to the pillars in Noto Cathedral: a) image of the damage (blue—scratches, red - cracks), b) view of the damage; source: L. Binda et al., Non destructive testing techniques applied to masonry and timber structures of the Crocifisso Church in Noto, [in:] Retrofitting of Heritage Structures: Design and Evaluation of Strengthening Techniques, ed. S. Syngellakis, Ashurst 2013, p. 75–87.

Ryc. 1. Uszkodzenia kolumn w katedrze Noto: a) obraz uszkodzeń (niebieski – zarysowania, czerwony – pęknięcia), b) widok uszkodzeń; źródło: L. Binda et al., *Non destructive testing techniques applied to masonry and timber structures of the Crocifisso Church in Noto*, [w:] *Retrofitting of Heritage Structures: Design and Evaluation of Strengthening Techniques*, red. S. Syngellakis, Ashurst 2013, s. 75–87.

An example of damage to historical pillars is the partial catastrophe of the Noto Cathedral in 1996, which was preceded by and earthquake, which clearly damaged the main load-bearing structure. The research[6] showed that some of the damage was formed before the above-mentioned seismic impact, and the cracks were filled with gypsum mortar. Based on the non-destructive tests, it was estimated that the collapse could also occur due to the poor structure of the pillars (the interior was filled with rock rubble and mortar, and the outer layer was not carefully connected with the pillar core) and damage related to long-term effects. Images of cracks and scratches in one of the pillars are presented in Fig. 1. Among the earthquake-related damage, the collapse of the dome and the two main pillars in the Basilica Santa Maria di Collemaggio in L'Aquila should also be noted. The analyses of the load capacity[7] showed, inter alia, stress concentration in the base of the structure, related to the interaction of horizontal forces, which was confirmed by the observed cracks (Fig. 2). The analysis of the technical condition of the pillars in the Castle and Manor Farm

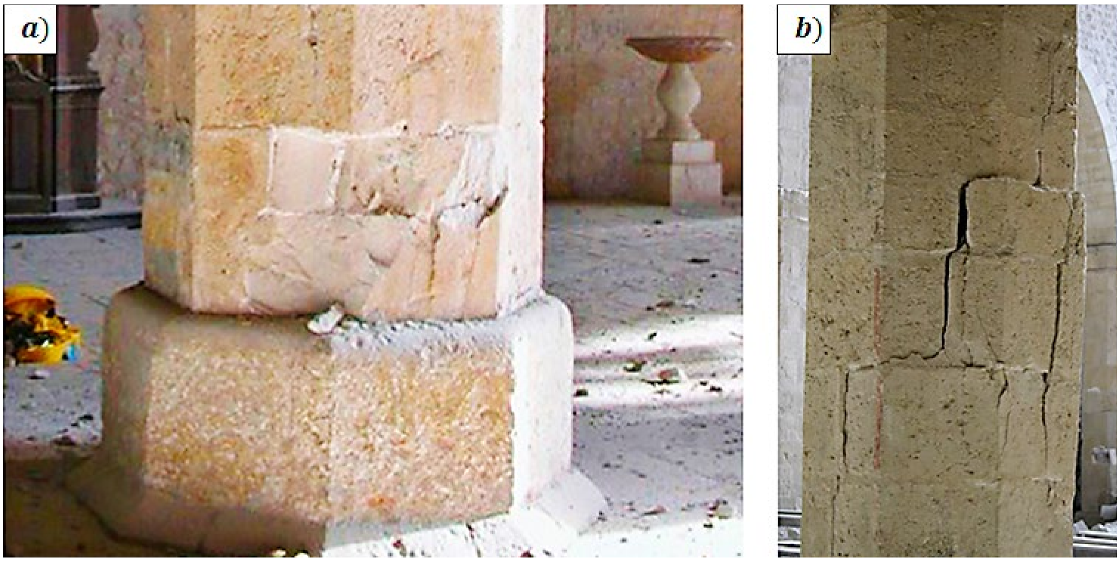

Fig. 2. Damage to the pillars in Basilica Santa Maria di Collemagio in L'Aquila: a) cracks at the base, b) vertical cracks; source: P. Crespi et al., Structural analysis of stone masonry columns of the Basilica p. Maria di Collemaggio, "Engineering Structures" 2016, No. 129, p. 81–90.

Ryc. 2. Uszkodzenia kolumn w bazylice Santa Maria di Collemagio w L'Aquili: a) pęknięcia u podstawy, b) pionowe pęknięcia; źródło: P. Crespi et al., *Structural analysis of stone masonry columns of the Basilica p. Maria di Collemaggio*, "Engineering Structures" 2016, nr 129, s. 81–90.

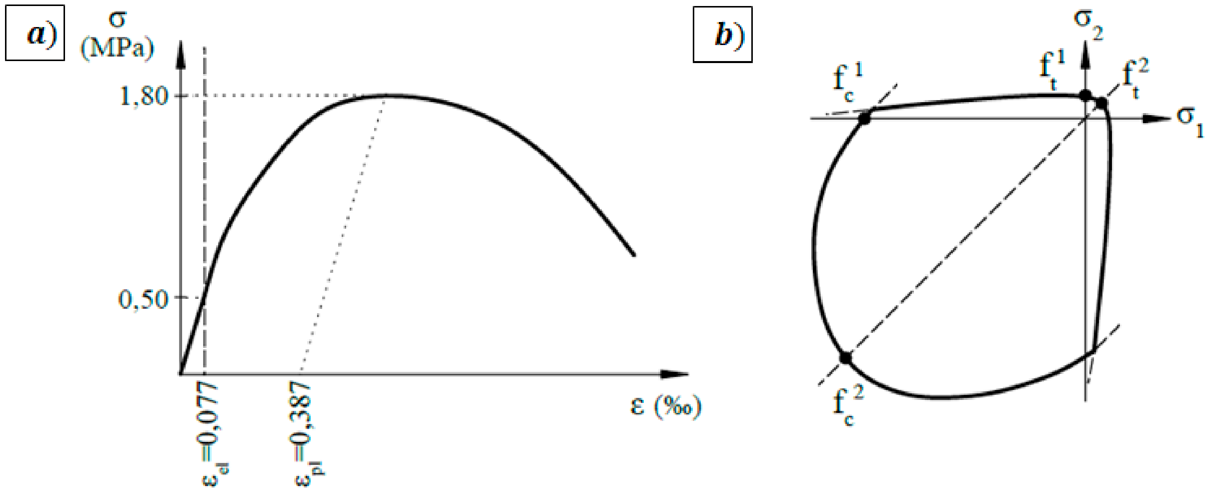

Fig. 3. Smeared cracking model definition: a—static equilibrium path, b—failure surfaces; by the authors.
Ryc. 3. Definicja modelu rys rozmazanych: a – ścieżka równowagi statycznej, b – powierzchnie uszkodzenia; oprac. autorzy.

in Krzyżowa in connection with the expansion planned at that time was performed by Jasieńko and Stawiski.[8] Based on ultrasonic measurements, the internal structure was reconstructed, and it was determined that the cracks that occur were probably caused by corrosion of steel elements that increased their volume causing internal damage. Particular attention should also be paid to the catastrophe at the Pomeranian Dukes' Castle in Szczecin in 2017, in which the pillars collapsed as a result of unfavorable processes occurring in the ground under the foundation.

Analysis of the static behavior of masonry pillars is a complex issue, even if one assumes the ideal geometry of members analyzed. It is necessary also to take into account the stability, susceptibility to second-order effects and nonlinear, non-homogeneous and anisotropic parameters of the material used. This has been extensively discussed in literature but a number of issues still remain to be resolved. Fossetti et al.[9] suggested analysis of pillars exposed to eccentric loading using a numerical procedure which took into account the second-order effects. The procedure correlated well with numerical calculations obtained in the ATENA3D environment. The authors pointed to the need to verify the method against the results of experimental testing. Mura[10] investigated cantilever pillars subject to axial and eccentric loading, which included second-order effects and cracking and made use of the Finite Difference Method. La Mendola and Papia[11] proposed a model for analysis of prismatic cantilever pillars subject to eccentric loading, which assumed the material was characterized by a zero tensile strength and linear-elastic response to compression. The model was subsequently adapted for the purpose of analyzing walls exposed to transverse loading[12] and extended with a nonlinear constitutive relation under compressive loading.[13] The analysis of these issues were later developed for members with a circular cross-section.[14] Broseghini et al.[15] analyzed the stability of masonry pillars with a circular cross-section using two methods: semi-analytical (based on the assumption that the masonry does not have tensile strength) and numerical using macro-modeling in the LUSAS environment. Libecajtova[16] carried out numerical simulations of the load-bearing capacity of the pillars, using micromodeling, consisting in assigning a constitutive plastic model with a damage plasticity to the masonry elements and mortar. The results of the analyzes were compared with the experimental studies, obtaining a satisfactory convergence. The model based on plasticity with failure, however, for the whole homogenized medium, was also used by Ombres and Verre,[17] analyzing the pillars before and after strengthening with composite materials. The above examples and many others prove the topicality of the problem of static behavior of pillars, and a summary of the state of art on strengthening pillars is also presented in Jasieńko et al.[18]

The analysis conducted by the authors of this paper aimed at assessing the impact of torsional, flexural and flexure-torsional imperfections on the static behavior of pillars, which are subjected to loading by a force concentrated at the center of gravity of the upper cross-section.

Methods of analysis

Numerical analyses of masonry structures use different strategies to model the material. In general terms, these can be categorized as micro-modelling and macro-modelling. In the case of existing structures, where the mechanical parameters of masonry components are known, it is possible to determine the global characteristic features of the continuum using homogenization methods.[19] This is why a uniform and homogenized masonry model was assumed appropriate for analytical purposes. Anisotropy of the material was not factored in and the material parameters were adopted arbitrarily based on the literature.[20]

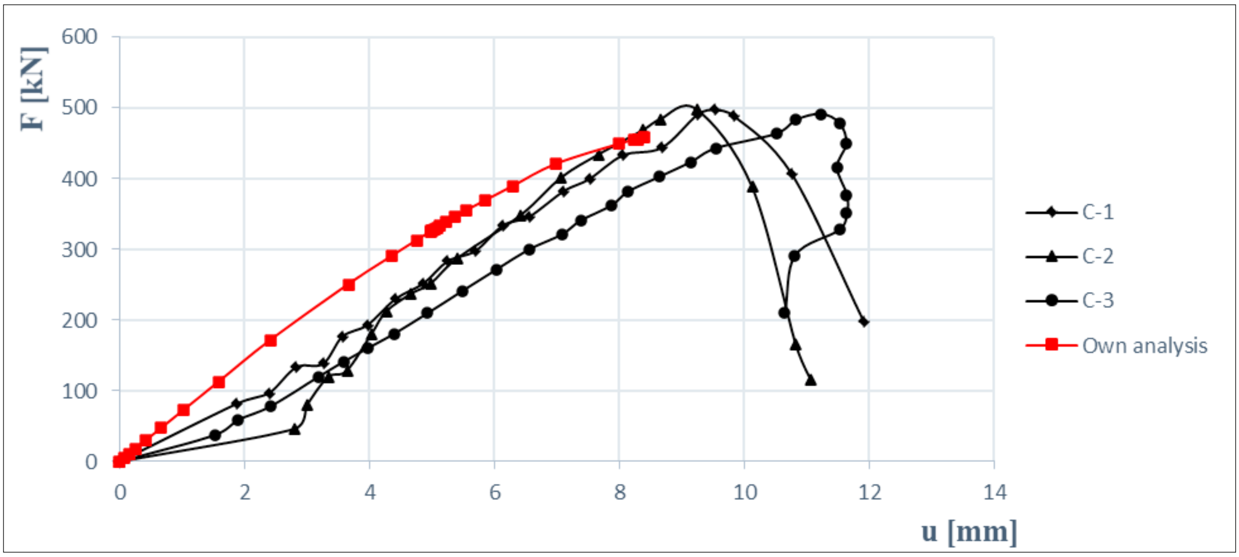

Fig. 4. Comparison of static equilibrium paths for pillars C-1, C-2, C-3 with numerical analysis; by the authors.
Ryc. 4. Porównanie ścieżek równowagi statycznej dla słupów C-1, C-2, C-3 z analizą numeryczną; oprac. autorzy.

Nonlinear behavior of masonry, resulting from masonry cracking, can be factored in by describing discrete and smeared cracking. The use of the latter method is more favorable in situations where the global behavior of the structure is being investigated and there is no need to track cracking. For this reason, the smeared cracking model[21] was used, which is available in the ABAQUS environment. The model assumes an initial linear-elastic behavior of the material, based on preselected values of Young's modulus and Poisson's ratio. The Rankine criterion was used to detect cracking formations. When cracking appears, the stiffness is reduced, which leads to a nonlinear behavior consistent with the defined curve. In the analysis, the boundary conditions are described by four failure ratios listed in Table 1. The effects of tension stiffening and shear retention were not included in the analysis.

The parameters of historical material modelled using the smeared cracking method, were defined in accordance with Fig. 3 and Table 1.

Table 1. Model parameters.

f_k (MPa)	1.8
E (GPa)	6.5
v (-)	0.167
Failure ratios (-)	1.12; 0.08; 1.33; 0.28

In order to verify the assumptions adopted, numerical analysis was carried out for the pillars which had also been investigated in experimental tests by Chunyi et al.[22] Next, the convergence of the results obtained in both analyses was compared. The static equilibrium path in the analysis was defined using the following equation:

$$\sigma/f_k = 2(\varepsilon/\varepsilon_0) - (\varepsilon/\varepsilon_0)^2 \qquad (1)$$

with an assumed value of =0.002. The compressive strength of masonry was determined based on the tests results using the following formula:[23]

$$f_k = 0.45 f_b^{0,7} f_m^{0,3} \qquad (2)$$

A short term Young's modulus, consistent with records, was adopted as follows:

$$E = 600 f_k \qquad (3)$$

The values of the parameters obtained for the material model are listed in Table 2. The pillar investigated was assumed to have the geometry of a cuboid with the height of h=4365 mm and a rectangular cross-section with dimensions 360 x 240 mm. The results obtained from the authors' analysis were compared to the results of experimental testing of the three elements: C-1, C-2, C-3. The static equilibrium path obtained indicated a satisfactory level of accuracy of the approximation method, which was selected to conduct further numerical analyses (Fig. 4).

Table 2. Parameters of the masonry material model.

f_k (MPa)	6
E (GPa)	3.6
v (-)	0.167
Failure ratios (-)	1.12; 0.08; 1.33; 0.28

Assumptions for authors' analysis

A cantilever static diagram of a 14 m high pillar was adopted for analytical purposes. The pillar was subject to loading with a vertical force F. Vertical displacements *u* (Fig. 5a) were observed. Numerical models were con-

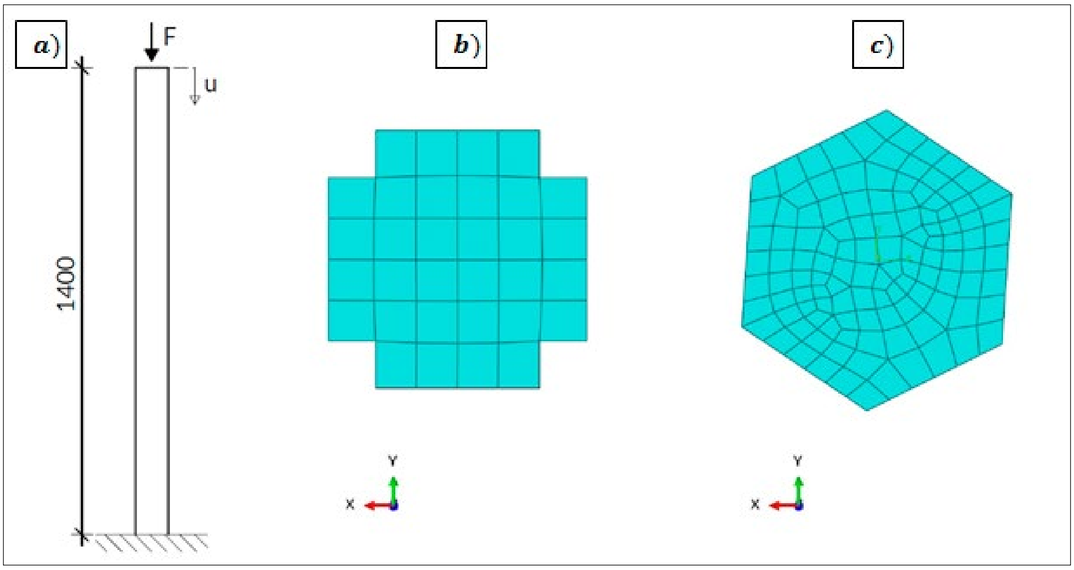

Fig. 5. Calculation models: a—static diagram, b—numerical model (hexagonal cross-section), c—numerical model (cruciform cross-section); by the authors.
Ryc. 5. Modele obliczeniowe: a – schemat statyczny, b – model numeryczny (przekrój heksagonalny), c – model numeryczny (przekrój krzyżowy); oprac. autorzy.

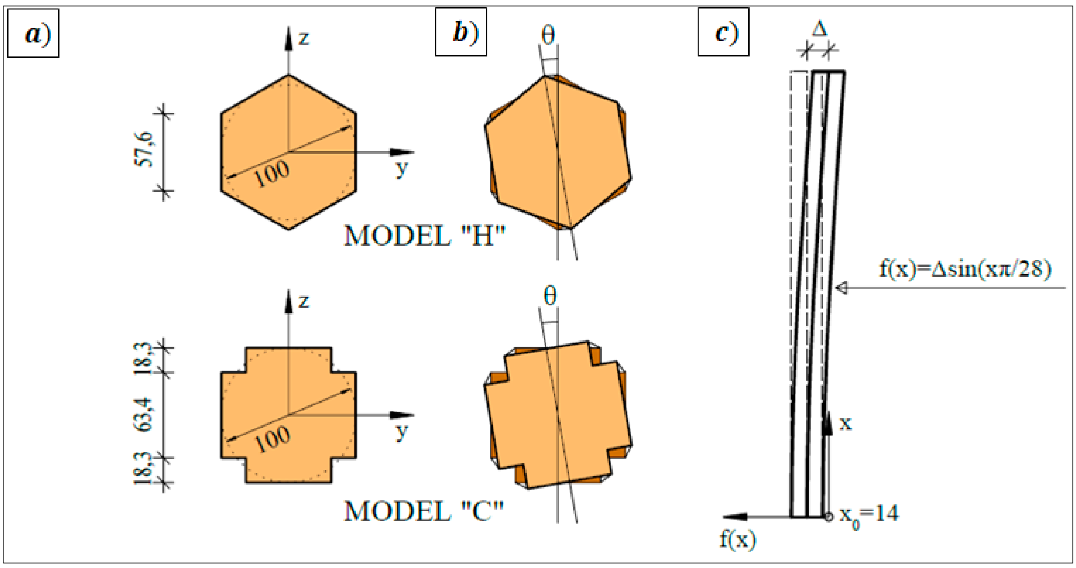

Fig. 6. Models adopted for analysis: a—cross-sections, b—definition of torsional imperfection θ, c—definition of flexural imperfection Δ; by the authors.
Ryc. 6. Modele przyjęte do analizy: a – przekrój, b – definicja odkształcenia od skręcania θ, c – definicja odkształcenia od zginania Δ; oprac. autorzy.

structed using homogeneous *Solid* type blocks, divided into a hexagonal grid of node finite elements of C3D8R type (Continuum, 3-D, 8-node, Reduced integration) (Fig. 5b, 5c). Two types of cross-sections most commonly found in historical structures were used—hexagonal cross-sections and cruciform ones, circumscribed around a circle with the diameter of 1 m (Fig. 6a). Aside from analyses of straight pillars, the calculations were also made for models of pillars with geometric imperfections. A torsional imperfection was introduced using a reciprocal rotation by the angle of the two extreme cross-sections (Fig. 6b). The shape of a member with a flexural deformation was adopted as consistent with the form of the buckling of the cantilever schema, that is it constitutes a quarter of the period of sine function (Fig. 6c). The slenderness of pillars was defined as follows:

$$\lambda = l_w / i_{min} \qquad (4)$$

The effective buckling length was adopted as $l_w = 2l_o$. Geometric parameters of the pillars are listed in Table 3.

Table 3. Geometric parameters of the pillars.

Cross-section	A [m²]	[m³]	[m]	[-]
H	0.865	0.0446	0.227	123.13
C	0.866	0.0470	0.233	120.02

Numerical analyses were conducted for 14 models, seven with a hexagonal cross-section and seven with a cruciform one. The symbol of each model indicates the type of the model's cross-section and the value of its imperfection, e.g., C20/10 describes a pillar with a cruciform cross-section and with a torsional imperfection of and a flexural imperfection of =10 cm. Parameters for all the pillars are listed in Table 4. The following members were analyses:

- straight pillars (θ=0, Δ=0),
- pillars with torsional imperfections (θ>0, Δ=0),
- pillars with flexural imperfections (θ=0, Δ>0),
- pillars with flexure-torsional imperfections (θ>0, Δ>0).

Table 4. Models used for analysis.

Item	Model symbol	θ (º)	Δ (cm)	Cross section
1	H 0/0	0	0	hexagonal
2	H 10/0	10	0	hexagonal
3	H 20/0	20	0	hexagonal
4	H 0/10	0	10	hexagonal
5	H 0/20	0	20	hexagonal
6	H 20/10	20	10	hexagonal
7	H 20/20	20	20	hexagonal
8	C 0/0	0	0	cruciform
9	C 10/0	10	0	cruciform
10	C 20/0	20	0	cruciform
11	C 0/10	0	10	cruciform
12	C 0/20	0	20	cruciform
13	C 20/10	20	10	cruciform
14	C 20/20	20	20	cruciform

Results

Six numerical models, three with hexagonal and three with cruciform cross-sections, were prepared to assess the impact of the preliminary torsion of the pillar's cross-section on its static behavior. Geometric imperfections of these pillars were graded every 10° (H 0/0, H10/0, H20/0, C 0/0, C 10/0, C 20/0). Behavior of pillars with ideal geometry (H 0/0, C 0/0) is universally acknowledged—the distribution of stresses is almost completely uniform along the height of the member, whereas abnormalities and concentration of stresses occur in the support zone. A pillar with a 20° torsion is characterized by a similar static behavior, but some irregularities occur especially in the area where the loading is applied and in the support zone. First cracking zones of pillars with torsion (Fig. 10c, 10d) are nearly identical as in the case of straight pillars (Fig. 10a, 10b).

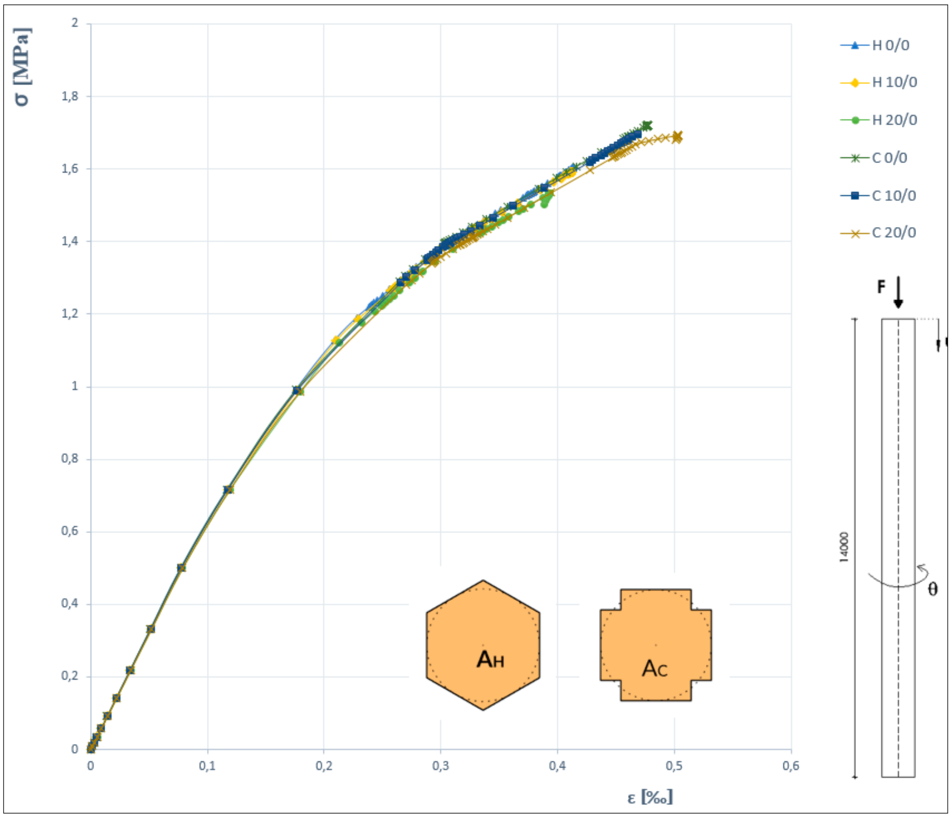

Fig. 7. Static equilibrium paths for models: H 0/0, H 10/0, H 20/0, C 0/0, C 10/0, C 20/0; by the authors.
Ryc. 7. Ścieżki równowagi statycznej dla modeli: H 0/0, H 10/0, H 20/0, C 0/0, C 10/0, C 20/0; oprac. autorzy.

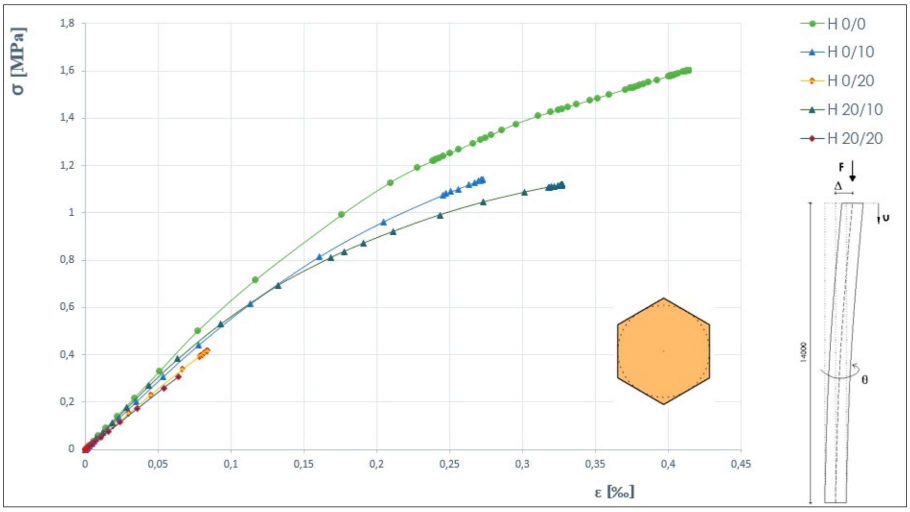

Fig. 8. Static equilibrium paths for models: H 0/0, H 0/10, H 0/20, H 20/10, H 20/20; by the authors.
Ryc. 8. Ścieżki równowagi statycznej dla modeli: H 0/0, H 0/10, H 0/20, H 20/10, H 20/20; oprac. autorzy.

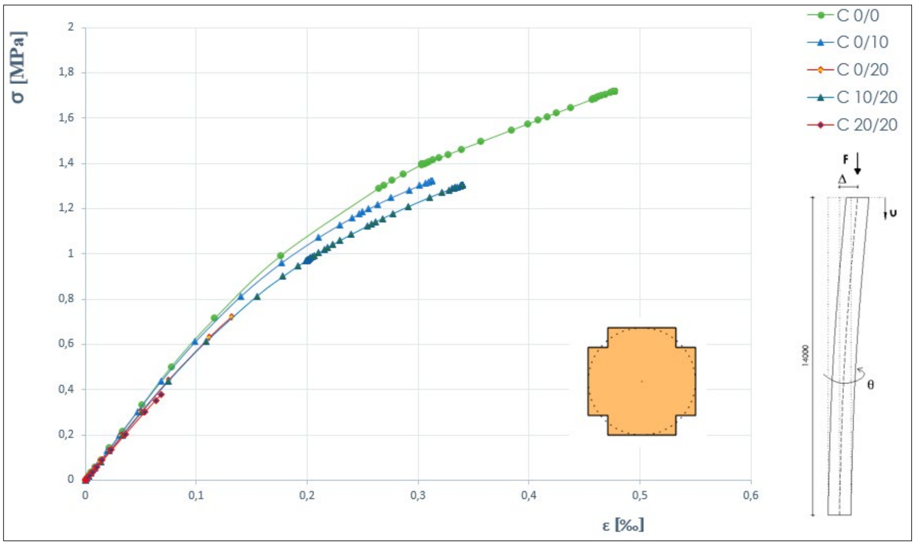

Fig. 9. Static equilibrium paths for models: H 0/0, H 0/10, H 0/20, H 20/10, H 20/20; by the authors.
Ryc. 9. Ścieżki równowagi statycznej dla modeli: H 0/0, H 0/10, H 0/20, H 20/10, H 20/20; oprac. autorzy.

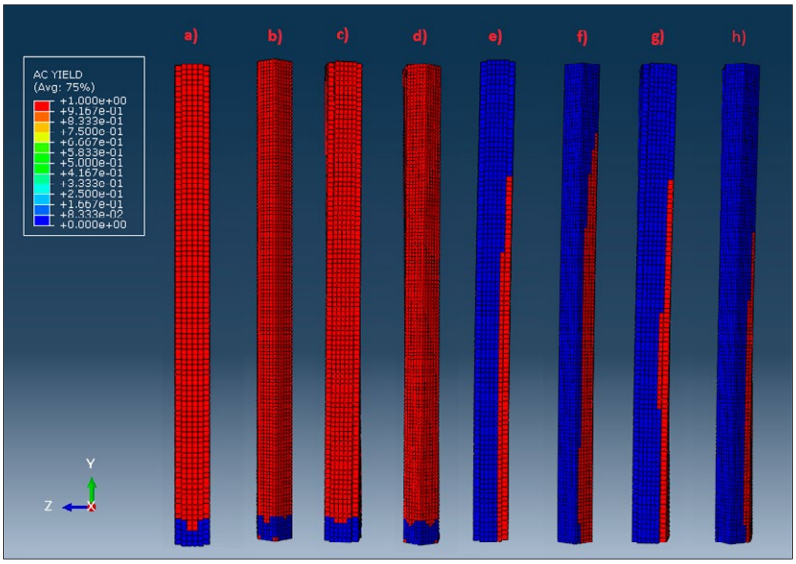

Fig. 10. First reduction of stiffness pictured for models: a—C 0/0, b—H 0/0, c—C 20/0, d—H 20/0, e—C 0/20, f—H 0/20, g—C 20/20, h—H 20/20, the stiffness reduction area marked in red; by the authors.
Ryc. 10. Pierwsza redukcja sztywności zobrazowana dla modeli: a – C 0/0, b – H 0/0, c – C 20/0, d – H 20/0, e – C 0/20, f – H 0/20, g – C 20/20, h – H 20/20, obniżenie sztywności oznaczono kolorem czerwonym; oprac. autorzy.

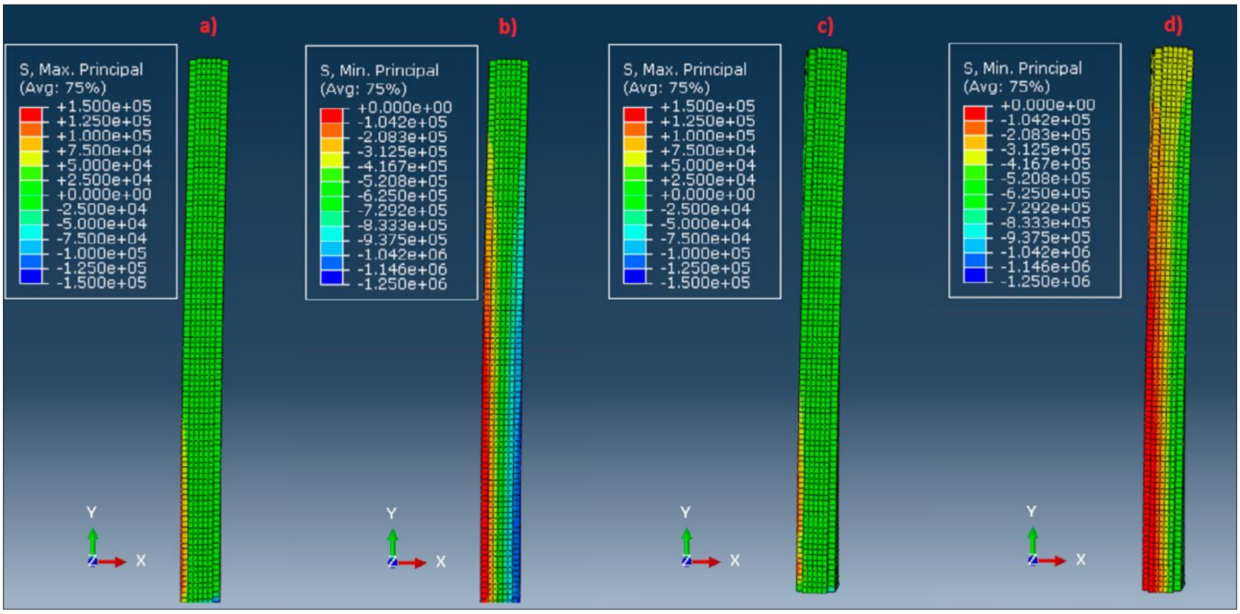

Fig. 11. Distribution of the main stresses: a—tensile stresses C 0/20, b—compressive stresses C 0/20, c—tensile stresses C 20/20, d—compressive stresses C 20/20; by the authors.
Ryc. 11. Rozkład głównych obciążeń: a – obciążenia zginające C 0/20, b – obciążenia ściskające C 0/20, c – obciążenia zginające C 20/20, d – obciążenia ściskające C 20/20; oprac. autorzy.

Models for assessing the impact of flexural deformations on the load-bearing capacity and static behavior of pillars were also prepared. These were similar to those used for the numerical testing described above. In contrast to the case of torsional imperfections, the introduction of a preliminary flexural imperfection had a significant impact on the stress distribution in a pillar. The main compressive stresses and cracking (Fig. 10e, 10f) concentrate along the plane consistent with the direction of the displacement applied. The largest values are attained in the support zone and decrease along the height of the member in the direction of the loading plane. Compressive stresses on the opposite side of the member reach ten times smaller values, which makes them negligible in relation to the compressive strength of masonry (Fig. 11b). The main tensile stresses are also generated in this zone (Fig. 11a), leading to the destruction of the member. This means that the failure mode in the case of pillars with a flexural imperfection is strictly related to the propagation of cracking along the plane consistent with the direction of the preliminary deformation.

Based on the analysis of pillars H 0/10, H 0/20, C 0/10, H 0/20 it was concluded that the introduction of a flexural imperfection results in a concentration of stresses in the plane consistent with the direction of the deflection of the member. A similar effect can be observed in the distribution of main compressive stresses in a pillar with a flexure-torsional imperfection (Fig. 11d). The difference is that in the latter case the effect is of an irregular character. The impact of torsional imperfections is clearly demonstrated in the distribution of the main tensile stresses (Fig. 11c). The comparison of stress distribution in pillars with a flexural imperfection and pillars with a flexure-torsional imperfection indicates that the failure mode remains unchanged, but a significant flexure-torsional imperfection considerably accelerates its occurrence. The first cracking mode remains similar to the models with a flexural imperfection but the irregularity of cracking distribution is much higher (Fig. 10g, 10h). The failure mode for pillars with a flexure-torsional imperfection is based on the advancing process of cracking along the plane consistent with the direction of the preliminary deformation, but the impact of tensile stresses increases together with the increase of the imperfection, leading to brittle cracking in the support zone.

Conclusion

Based on the analysis conducted, the following conclusions were drawn:

- the introduction of a torsional imperfection into a model of a pillar with a hexagonal cross-section leads to increased stress concentrations but does not change failure mode;
- cruciform pillars with a torsional imperfection are exposed to the effect of propagating cracking along the internal corners, in which stresses concentrate;
- static equilibrium paths (Fig. 7) indicate a negligible influence of torsional imperfections (reduction in load-bearing capacity by less than 5%) on the load-bearing capacity of pillars with hexagonal as well as cruciform cross-sections;
- static equilibrium paths (Fig. 8, 9) indicate a high susceptibility of pillars to flexural imperfections (reduction in load-bearing capacity of up to more than 70%), which is also confirmed by the analysis carried out by Rapp and Sielicki[24] (decrease by 20% with flexural imperfection equal 2.3 cm);

- cumulative impact of torsional and flexural imperfections is significant when compared to straight pillars. The effect of reduced load-bearing capacity due to the torsional deformation depends on the size of the flexural imperfection (Fig. 8, 9);
- the presented analyzes of the impact of imperfection of masonry pillars on the state of stresses and crack propagation are the basis for the assessment of the effort of structural elements in real objects and the adoption of appropriate strengthening techniques, among which the methods that meet the conservation requirements, based on the use of FRCM[25] materials and cords,[26] deserve special attention.

Symbols:

Δl	horizontal displacement to member heights ratio
θ	the rotation angle between the top and the bottom cross-section of a model
Δ	pillar deflection
u	vertical displacement
h	height
F	loading
f_c^1	uniaxial compressive strength
f_c^2	biaxial compressive strength
f_t^1	uniaxial tensile strength
f_c^2	biaxial tensile strength
E	Young's modulus
ν	Poisson's ratio
l_w	effective buckling length of a member
i_{min}	the minimum radius of the section inertia
l_0	theoretical length of a member
ε_0	strain equivalent to compressive strength of masonry
f_k	compressive strength of masonry
f_b	compressive strength of a masonry unit
f_m	compressive strength of mortar

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Abstract

The paper discusses issues related to the static behavior of masonry pillars with geometric imperfections on the basis of numerical analyses, which were carried out. The main types of deformation and damage of historical masonry structures are described along with the impact of such deformations on the load-bearing capacity of a structure, as well as on the failure mode of the main load-bearing members. The authors underline the necessity of including geometric imperfections in the analysis of historical masonry pillars, with a special emphasis on flexural, torsional and flexure-torsional imperfections. The homogenized and isotropic continuum model was used for numerical testing. Physical nonlinearity of masonry members was obtained using the smeared cracking model. Cracking modes linked to particular types of geometric nonlinearity were also described as they could prove useful in selecting appropriate methods for strengthening historical masonry pillars with geometric imperfections.

Streszczenie

W artykule, na podstawie przeprowadzonych analiz numerycznych, omówiono zagadnienia związane z pracą statyczną filarów murowych z imperfekcjami geometrycznymi. Opisano uszkodzenia filarów oraz podstawowe typy deformacji, które występują w historycznych konstrukcjach murowych, a następnie zwrócono uwagę na znaczący wpływ tych odkształceń na nośność i mechanizm zniszczenia głównych elementów nośnych. Podkreślono konieczność uwzględniania imperfekcji geometrycznych w analizie historycznych filarów murowych, zwłaszcza deformacji giętnych, skrętnych oraz giętno-skrętnych. Do przeprowadzonych doświadczeń numerycznych wykorzystano zhomogenizowany i izotropowy model ośrodka ciągłego, a nieliniowość fizyczną muru uzyskano przy użyciu modelu rys rozmytych. Przedstawiono także obrazy zarysowań towarzyszące poszczególnym typom nieliniowości geometrycznych, które mogą być przydatne przy doborze metody wzmacniania historycznych filarów murowych z imperfekcjami geometrycznymi.