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MODELLING OF WEAR UNDER THE CONDITIONS OF HIGH SLIDING SPEEDS

MODELOWANIE PROCESÓW ZUŻYWANIA W WARUNKACH DUŻYCH PRĘDKOŚCI POŚLIZGU

Key words: wear, probabilistic model, sliding speed, Markov process, rate of change of friction coefficient.

Abstract The paper presents a probabilistic model of the wear process of machine elements under the conditions of high-speed friction using the thermo-kinetic theory of fracture. The definition of model parameters is based on the probabilistic-physical approach. The model is presented in a discrete form and adapted for using computer simulation methods that require spatio-temporal sampling of computational models. The analysis of calculation results shows that the dominant factor that affects the stress-strain state, and therefore the wear of the tribological system processes, is the rate of change of the friction coefficient from static to dynamic values. The rate of change in the friction coefficient was proposed as an effectiveness criterion of using methods for increasing the wear resistance of systems under the conditions of high-speed friction.

Słowa kluczowe: zużywanie, model probabilistyczny, prędkość poślizgu, proces Markowa, prędkość zmiany współczynnika tarcia.

Streszczenie W pracy omówiono probabilistyczny model procesu zużywania elementów maszyn w warunkach dużych prędkości tarcia z wykorzystaniem termokinetycznej teorii destrukcji. Określanie parametrów modelu opiera się na probabilistyczno-fizycznym podejściu. Model przedstawiono w dyskretnej formie i dostosowano do wykorzystania metod modelowania komputerowego, które potrzebują czasoprzestrzennej dyskretyzacji modeli obliczeniowych. Analiza wyników obliczeń wykazuje, że dominującym czynnikiem, który wpływa na stan naprężeń i odkształceń, a więc i na procesy zużywania badanego systemu tribologicznego, jest prędkość zmiany wartości współczynnika tarcia od wartości statycznej do wartości dynamicznej. Zaproponowano jako kryterium oceny skuteczności zastosowania metod zwiększenia odporności na ścieranie systemów pracujących w warunkach dużych prędkości tarcia, używać prędkość zmiany współczynnika tarcia.

INTRODUCTION

The calculation methods for analysis are the basic and effective tools used for the examination of tribological systems operating under extreme conditions. The difficulties that arise in the development of methods for the simulation of the wear of friction pairs operating under the conditions of high sliding speed are caused by incomplete knowledge of processes taking place on the tribo-contact surface of interaction. It means that the random nature of external impact factors and operating conditions of tribological systems should be taken into consideration.

In the development of the models of wear in friction pairs operating under the conditions of high-speed friction, a number of specific phenomena arising due to high speeds of relative displacement of the elements of friction pairs at sliding speeds exceeding 100 m/s should be considered. Above all, the authors stress **[L. 1, 2]** the phenomena affecting the course of significant reduction in the value of friction coefficient, which arises in relation to the pairs of friction for all materials. **[L. 2]** presents the results of experimental tests under the conditions of high sliding speed of the elements of steel pairs, which show that the value of friction coefficient in such cases may decrease to 0.0001. The deciding

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parameters affecting the volume of wear of the elements of friction pairs operating under the conditions of high sliding speed **[L. 3–6]** are the impact temperature and the state of stress, in particular, its dynamic nature.

Given the above, the properties of stress fields of the friction pairs should be determined with reference to the dynamic effects that occur under the conditions of high sliding speeds, and changes in the friction coefficient as a function of the sliding speed should be taken into account. In the development of analysis methods, the fact that the wear processes are non-stationary random processes was taken into consideration **[L. 7]**.

MODEL DEVELOPMENT

Wear as a result of the accumulation of tribological damages belongs to the class of accumulating damages. Based on the development of the experimental material, the authors **[L. 7]** substantiated that stochastic models of damage accumulation processes, built using random Markov processes with discrete time and states, describe the processes of accumulating damages with a high degree of probability.

Given the details of operation of friction pairs under the conditions of high sliding speeds, in most cases, the computer modelling methods use spatio-temporal sampling of computational models to analyse and determine the impact of dominant factors on the wear processes. Respectively, at the first stage of model development, a solution to an issue requires the wear process to be discretised.

Using the model proposed in **[L. 7]**, the wear process can be described in a discrete form, making a number of assumptions:

1. The process of interaction between the elements of friction pairs consists of repeated load cycles. A load cycle is the basic period of the operation of a friction pair during which tribological defects may accumulate resulting in wear. Load cycles are used to measure the time, which is discrete in this case. The assumption that the occurrence of wear is possible only during the load cycle facilitates developing computational models.
2. A transition of a tribological system from one state of wear to another state corresponds to the wear of the surface layer by a certain value. The wear states of a tribological system are discrete and transient. The assumption of discontinuity of wear states correlates well with experimental data on a discrete nature of the course of damages diffused in micro-volumes.
3. The parameters that determine the load cycle remain unchanged within this cycle. This assumption postulates that wear of a system is determined at the beginning and the end of the load cycle. It is a condition of the random Markov process according to which the volume of wear depends only on the load cycle and state of wear at the beginning of the cycle.
4. In order to determine the probabilistic parameters of the wear processes, we will use the w_{ii} symbol for the probability that no wear will happen during the load cycle. Then the probability of wear occurrence during this cycle is equal to $1 - w_{ii}$, because these events make up a complete group of random events. If the volume of wear in a specified number of load cycles exceeds a certain critical value, the system will go into the absorbing state, with a zero probability to leave it.

In this way, a model of the wear process was obtained, and it can be described by using random Markov processes with discrete states and time.

Determination of model parameters

The Markov chain parameters are considered defined if the vector of initial states and the matrix of transition probabilities are given. In most cases, the vector of initial states $\pi_j(t=0)$ is determined on the assumption that, at the initial moment of time of system operation, there is no wear so the system is in state 1:

$$\left[\pi_j(t=0)\right] = [1, 0, 0, \ldots, 0] \tag{1}$$

The probabilities of system states at the moment of time $t > 1$ are determined by the product $\left[\pi_j(t-1)\right]$ of the vector of unconditional probabilities at the moment $(t-1)$ and the matrix of transition probabilities, which describes the behaviour of a tribological system at the moment of time t:

$$\left[\pi_j(t)\right] = \left[\pi_j(t-1)\right]\left[\boldsymbol{W}_{ij}\right], i, j = 1, 2, \ldots, K_s \tag{2}$$

where $\left[\pi_j(t-1)\right]$ – vector of unconditional probabilities that the system is in the i^{th} state $i = 1,2,..., K_s$ at the moment of time $(t-1)$;
$[\boldsymbol{W}_{ij}]$ – matrix of transition probabilities;
K_S – number of system states.

The matrix of transition probabilities is determined on the basis of the matrix type and the value of matrix components w_{ij}.

The authors **[L. 7]** proved that the matrix of transition probabilities with a single increase in jumps and the presence of absorbing state best reflect the essence of wear.

The issues related to the determination of values of the transition probability matrix components were investigated from a position of the determination of the correlation between parameters of the mathematical model and physical characteristics of the wear process under the conditions of high sliding speeds.

Due to the physics of the wear process, it is believed that transitions of the tribological system from one state to the next take place under the action of the stream of wear that occurs. During the events arising from the stream of wear, the system moves from one state to the next.

A wear stream event is assumed to be the wear of certain value h. In accordance with the basic limit theorem of streams, a stream of wear is the Poisson process, which means that it shows the properties of regularity, stationarity, and the lack of sequence, which is without prejudice to the requirements of the random Markov process.

Given the intensity of stream $\lambda(t)$ as the average number of events per unit of time for the elementary period of Δt, compatible to t **[L. 9]**, the wear stream intensity $\lambda_I(t)$ at the moment of time t can be calculated as the rate of wear $V_I(t)$ divided by the value of h:

$$\lambda_I(t) = \frac{V_I(t)}{h}, \quad [1/\text{czas}] \tag{3}$$

where $V_I(t)$ – rate of wear at the moment of time t (length/time, volume/time, mass/time);
h – a value, which is determined from the condition of stream regularity and has a dimension of length, mass or volume, respectively, depending on the rate of wear used, i.e. linear, mass or volumetric. The value of wear h is determined from the condition that the probability of the occurrence of wear higher than h during one load cycle is negligibly low.

To determine the rate of wear $V_I(t)$, the thermo-kinetic theory of fracture **[L. 8]**, which allows conducting the investigations on the wear of a tribological system taking into account the stress and impact temperature, can be used.

The durability of the material under load σ according to the following **[L. 8]**:

$$\tau = \tau_0 \exp\left(\frac{\mathrm{U}_0 - \gamma\sigma}{\mathrm{kT}}\right) \tag{4}$$

Taking into account the proposals of the authors [**L. 10**] regarding taking into account the impact of the external environment to overcome the energy barrier, we will obtain the following:

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma \pm \Delta G_d}{\mathrm{kT}}\right) \tag{5}$$

where τ_0 – a time constant equal to the period of atomic fluctuations in a body of $10^{-13}...10^{-12}$ s;
k – Boltzmann constant, J/K;
T – absolute temperature, K;
U_0 – energy of activation of the leading tribological damage mechanism, J/mol;
γ – structural factor;
σ – load, Pa;
ΔG – factor which takes into account the impact of the external environment (ΔG < 0 – the external environment reduces the wear resistance, ΔG > 0 – the external environment increases the wear resistance, and ΔG = 0 – the external environment has a neutral effect), J/mol.

By changing the Boltzmann constant k into the gas constant R, we convert (5) into

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma \pm \Delta G}{R\mathrm{T}}\right) \tag{6}$$

Where R – gas constant, J(mol · K).
Given the cyclical nature of load process, we express the durability τ using the number of load cycles N and duration of a single load cycle t_c and distinguish the product by the following:

$$\sigma\gamma = U_0 - RT\, ln\left(\frac{N \cdot t_{\mathrm{A}}}{\tau_0}\right) \pm \Delta G \tag{7}$$

where N – the number of load cycles;
t_c – duration of a single load cycle.

σγ reflects the part of the work performed by the external load in the layer destruction process.

Assuming that the work of friction forces A_f is connected with the operation of external forces, we can write:

$$\sigma\gamma = \frac{A_f}{V_w} \tag{8}$$

where V_w – volume of the layer being worn, mol;
A_f – work of friction forces, J.

Given that $A_f = F_f L_S = f \cdot F_N \cdot L_S$ and that, $V_w = (A_n \cdot h_w)/\mathrm{M}$, we transform (8) to the form of:

$$\sigma\gamma = \frac{f \cdot F_N \cdot L_S \cdot M}{A_n \cdot h_w} \tag{9}$$

where f – friction coefficient;
F_N – pressure force, N;
L_S – sliding friction path, m;
M – molar volume, m^3/mol;
A_n – nominal contact area, m^2;
h_w – thickness of wear layer, m.

Assuming that F_N/A_N are contact pressures, σ_N, it can be written as

$$\sigma\gamma = \frac{f \cdot \sigma_N \cdot L_S \cdot M}{h_w} \tag{10}$$

where σ_N – contact pressures, Pa.

If the generalised Amontons-Coulomb law is assumed as the model of friction, taking into account the Stribek effect, $f = f_d + (f_s - f_d) \cdot \exp(-\beta v_{sk})$, which considers changes in the friction coefficient depending on the sliding speed, then the expression (10) can be presented as:

$$\sigma\gamma = \frac{\sigma_N \cdot M \cdot L_S \cdot \left(f_d + \left(f_s - f_d\right) \cdot \exp\left(-\beta v_{sk}\right)\right)}{h_w} \quad (11)$$

where f_d – dynamic friction coefficient;
f_s – statc friction coefficient;
v_{sk} – relative sliding speed at the contact point, m/c;
β – coefficient determined experimentally.

Taking into account the (11), the expression (6) takes the following form:

$$\tau = \tau_0 \exp\left(\frac{U_0 - \left[\frac{\sigma_N \cdot M \cdot L_S \cdot \left(f_d + \left(f_s - f_d\right) \cdot e^{\left(-\beta v_{sk}\right)}\right)}{h_w}\right] \pm \Delta G}{RT}\right) \quad (12)$$

From the point of view of the kinetic concept of destruction, durability is one of the basic characteristics of mechanical strength of a material. It can be treated as an inversely proportional value of the averaged rate of the destruction process:

$$V_I\left(x,y,z,t\right) = \frac{1}{\tau} = \frac{1}{\tau_0} \exp\left(-\frac{U_0 - \left[\frac{\sigma_N\left(x,y,z,t\right) \cdot M \cdot L_S \cdot \left(f_d + \left(f_s - f_d\right) \cdot e^{\left(-\beta v_{sk}\right)}\right)}{h_w}\right] \pm \Delta G}{RT\left(x,y,z,t\right)}\right) \quad (13)$$

where $V_I\left(x,y,z,t\right)$, $\sigma_N\left(x,y,z,t\right)$, $T\left(x,y,z,t\right)$ – rate of wear, contact pressure and temperature, respectively, at the point with coordinates (*x, y, z*) at the moment of time *t*.

Then the expression (3) will take the following form:

$$\lambda_I\left(t\right) = \frac{1}{h\tau_0} \exp\left(-\frac{U_0 - \left[\frac{\sigma_N\left(x,y,z,t\right) \cdot M \cdot L_S \cdot \left(f_d + \left(f_s - f_d\right) \cdot e^{\left(-\beta v_{sk}\right)}\right)}{h_w}\right] \pm \Delta G}{RT\left(x,y,z,t\right)}\right) \quad (14)$$

It means that, by using the wear rate function, the basic characteristics of the stream of wear can be specified, i.e. the intensity of the stream of wear $\lambda_I\left(t\right)$, which determines the transitions of a tribological system from one state to another.

The probability of transition $w_{ij}\left(t\right)$ from a state in which the system was at the moment of time *t* to the state *j* during the elementary time of Δt in the Markov chain can be calculated as follows:

$$w_{ij}\left(t\right) \approx \lambda_I\left(t\right)\Delta t, \quad i \neq j \quad (15)$$

Moreover, $0 \le w_{ij}\left(t\right) \le 1$, $\lambda_I\left(t\right)\Delta t \le 1$, which gives $0 \le \Delta t \le 1/\lambda_I$.

It follows from the above that, the lower Δt, the more accurately the probability of system transition from one state to the next can be determined.

Although an increase in the sliding speed, in accordance with (13), results in a reduction in the friction coefficient in the contact area, the degree of this effect actually depends on the value of coefficient β. Taking into consideration that an increase in the sliding speed results in an increase in temperature and dynamic loads in the contact area, and thus in a reduction in the activation energy and increase in the destructive processes in the layer, it can be concluded that the temperature and contact pressures have a dominant effect on the wear process.

In the assessment of the rate of wear of the surface layer, an important role is played by the coefficient ΔG, which takes into account the impact of the external environment. In the wear process, the coefficient ΔG can be interpreted as a factor that takes into account the surface layer shaping technology and its impact on the activation energy of destruction processes.

In this way, by using the probabilistic-physical approach, the parameters of the wear model for surface layers of the elements of friction pairs under the conditions of high speeds were obtained.

The proposed phenomenological model, described using Markov processes with discrete states and time, allows one to select and specify the basic factors, distinguish their relationships, and analyse the degree of their impact on the wear processes of friction pairs operating under the conditions of high-speed friction.

The use of the model allowed a number of tests to be performed in order to assess the degree of impact of the basic factors on the wear processes of friction pairs under the conditions of high sliding speeds. As part of the tests, the analysis of the stress-strain state of the elements of friction pairs taking into account the dynamic nature of interactions and the analysis of the effect of temperature on the stress state and the effect of change in the friction coefficient, depending on the sliding speed, on the wear processes under the conditions of dynamic load and high sliding speeds were carried out.

NUMERICAL ANALYSIS AND DISCUSSION OF RESULTS

A barrel of a firearm was used as an object for the assessment of the impact of the basic factors on the wear processes under the conditions of high sliding speeds. Firearm barrels refer to systems operating under particularly difficult conditions of high dynamic pressures, high temperatures, and the mechanical operation of a bullet, which moves through the barrel at high sliding speeds of above 300 m/s.

To perform the analysis, the computational dynamic analysis package ANSYS Autodyn was used, where a model of bullet movement through the barrel under the influence of gunpowder gases was built. The "Gunpowder gas pressure – Time" curve **(Fig. 1)** was taken from the internal ballistics. The adequacy of the computational model was assessed by comparing the velocity of the bullet at the moment of its departure from the barrel, obtained based on the results of experimental and simulation tests **(Fig. 2)**. The discrepancy between the velocities obtained experimentally and by simulations was less than 1%.

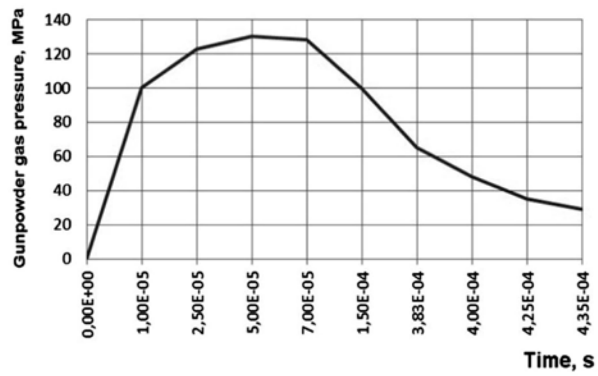

Fig. 1. "Gunpowder gas pressure – Time" curve
Rys. 1. Krzywa „Ciśnienie gazów prochowych – czas"

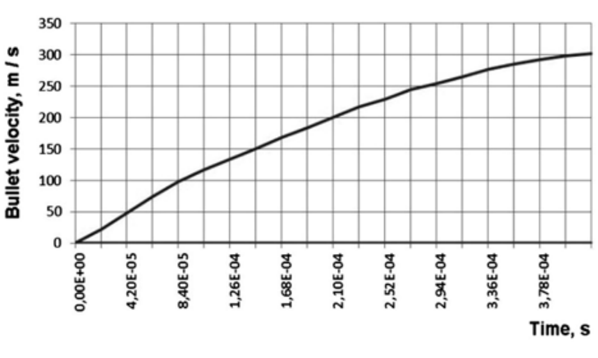

Fig. 2. The calculated velocity of a bullet in the barrel
Rys. 2. Obliczeniowa prędkość pocisku w lufie

In the course of the tests, the analysis of the stress-strain state at different moments of time during the bullet's movement over the barrel was performed (**Fig. 3**). The analysis of the impact of the dynamic nature of load on the tribological damage accumulation process was made. It was proven by calculations that the size of stress that occurred in the barrel was of damped dynamic vibrations in nature (**Fig. 4**) and affected the unevenness of wear of the barrel.

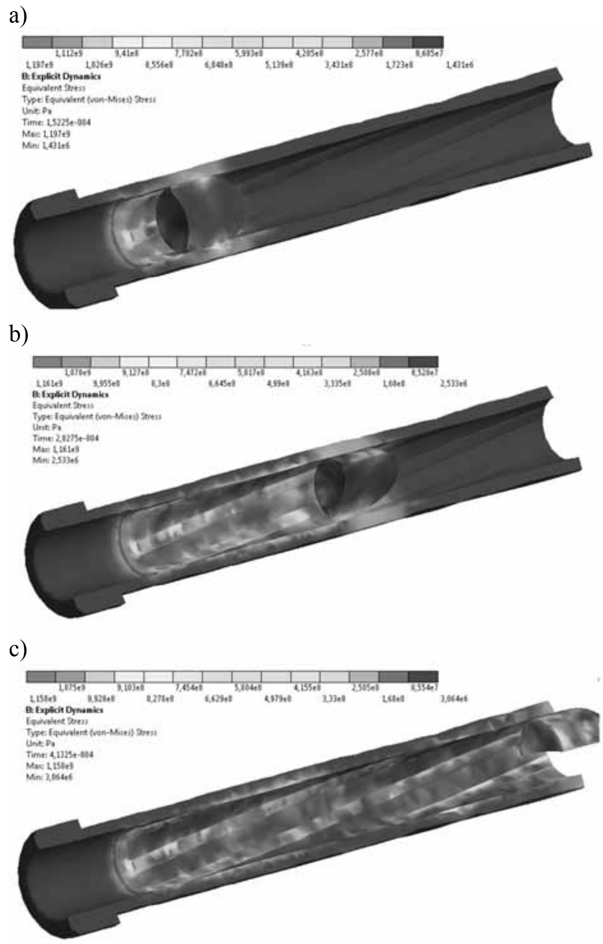

Fig. 3. Equivalent stresses in time: a) 1.52 × 10^{-4}; b) 2.82 × 10^{-4}; *c* – 4.13 × 10^{-4} s
Rys. 3. Naprężenia równoważne w czas: a) 1.52 × 10^{-4}; b) 2.82 × 10^{-4}; c) 4,13 × 10^{-4} s

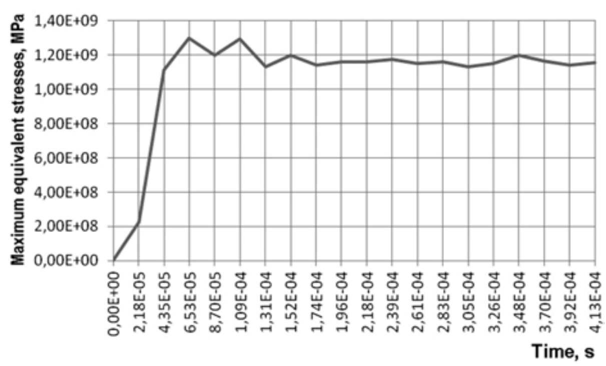

Fig. 4. Maximum equivalent stresses
Rys. 4. Maksymalne naprężenia równoważne

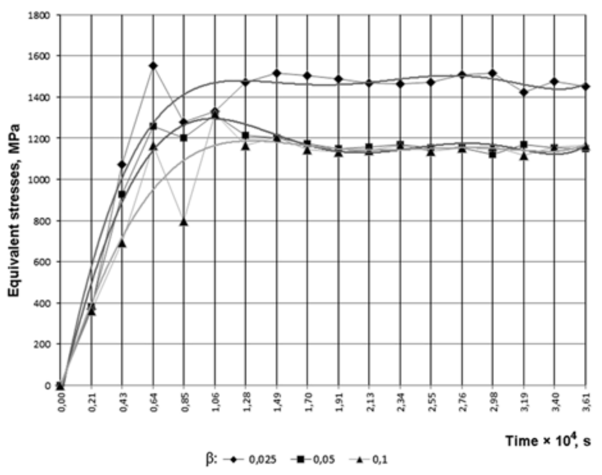

Fig. 5. Dependences of maximum equivalent stresses in the barrel on time for f_s = 0.4; f_d = 0.025

Rys. 5. Zależności maksymalnych naprężeń zredukowanych wg Misesa w lufie od czasu dla f_s = 0,4; f_d = 0,025

The investigations on the impact of friction coefficient on the stress-strain state in a system under the conditions of high sliding speeds were carried out. In order to perform the analysis of the impact of the friction coefficient on the stress-strain state, the dependences of the maximum reduced stresses in the barrel were defined in accordance with the von Mises hypothesis for different values of the dynamic f_d and static f_s friction coefficient and degree indicator β of the generalised model of friction.

The maximum equivalent stresses according to **(Figs. 5, 6, 7)** take the nature of oscillatory changes quickly damped in time. It should be noted that a decrease in the degree indicator β to 0.025 results in an increase in the value of maximum amplitudes in the initial period. At β = 0.025, all stresses show the highest values and, respectively, a pair of friction is loaded to the maximum in this case. Such a behaviour remains for different values of the dynamic and static friction coefficient.

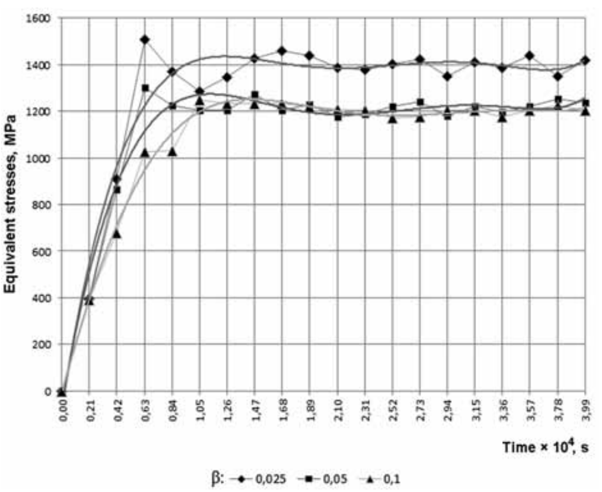

Fig. 6. Dependences of maximum equivalent stresses in the barrel on time for f_s = 0.4; f_d = 0.05

Rys. 6. Zależności maksymalnych naprężeń zredukowanych wg Misesa w lufie od czasu dla f_s = 0,4; f_d = 0,05

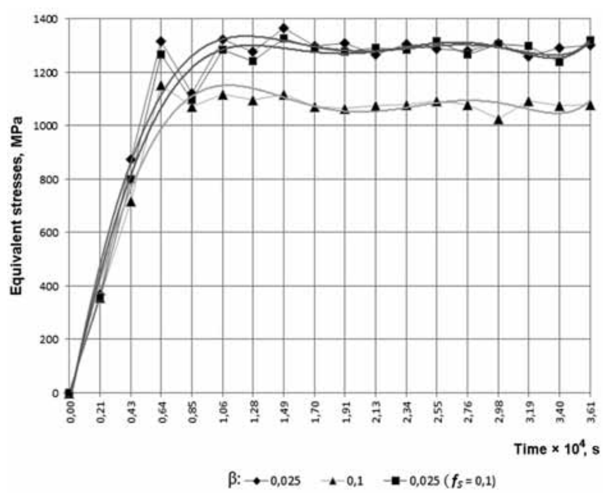

Fig. 7. Dependences of maximum equivalent stresses in the barrel on time for f_s= 0,2; f_d = 0.025

Rys. 7. Zależności maksymalnych naprężeń zredůkowanych wg Misesa w lufie od czasu dla f_s= 0,2; f_d = 0,025

CONCLUSIONS

1. The computational analysis has shown that the values of dynamic friction coefficient should not exceed 0.055...0.06 to ensure stable operation of the test tribological system. When values of the dynamic friction coefficient exceed the indicated ones, there occurs a significant increase in stress, and thus quickly slowing down of the bullet in the barrel and a large reduction in the bullet velocity at the moment of leaving the barrel's channel.
2. The dynamic friction coefficient f_d in the examined range of values 0.025...0.05 has no significant impact on absolute values of the maximum equivalent stress amplitudes.
3. The value of β indicates the rate of the reduction of the friction coefficient from the value of the static friction coefficient f_s to the value of the dynamic friction coefficient f_d, which means that the value of β indicates the speed of sliding of the elements of friction pairs during which the actual friction coefficient approaches the constant value.
4. The obtained results show that the rate of changes of the friction coefficient can be used as an effectiveness criterion of using materials, coating modification processes, and other methods for increasing the wear resistance of the elements of tribological systems operating under the conditions of high sliding speeds by adopting the coefficient β as the quantitative assessment.

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