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Application of Smoothed Particle Hydrodynamics Method in Metal Processing: An Overview

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Abstract

Smoothed Particle Hydrodynamics (SPH) is a Lagrangian formula-based non-grid computational method for simulating fluid flows, solid deformation, and fluid structured systems. SPH is a method widely applied in many fields of science and engineering, especially in the field of materials science. It solves complex physical deformation and flow problems. This paper provides a basic overview of the application of the SPH method in metal processing. This is a very useful simulation method for reconstructing flow patterns, solidification, and predicting defects, limitations, or material destruction that occur during deformation. The main purpose of this review article is to give readers better understanding of the SPH method and show its strengths and weaknesses. Studying and promoting the advantages and overcoming the shortcomings of the SPH method will help making great strides in simulation modeling techniques. It can be effectively applied in training as well as for industrial purposes.

Keywords: Smooth particle hydrodynamic (SPH), Finite element method (FEM), Metal processing, Lagrangian

1. Introduction

The development of the SPH method began in the 1970s and it is the earliest particle method in computational mechanics, according to Monaghan [1] and Cleary et al [8]. It is a Lagrangian continuum method for solving partial differential equations and does not use any fixed meshes or meshes to monitor the material structure deformation and heat transfer. Material particles are modeled as blobs such as liquids or cohesive solids, which move around the bulk of the material and react with other particles. So far, this method has been widely developed and applied in many scientific fields. It has become a typical method for modeling and simulating the behavior of liquids, granular materials, or solids undergoing large deformations and a loss of material cohesion [1, 3, 6, 8-10, 12, 13, 20-22, 27, 29, 30, 31, 66, 93,102]. In this paper, the applicability of the SPH method in metal deformation will be highlighted through the synthesis, analysis, and evaluation of some previous research results.

Metal deformation includes a variety of processes for forming finished or semi-finished products. These processes use liquid metals such as casting, semi-solid metals such as extrusion, thixoforming, and thixoforging, or solid metals such as rolling, drawing, forging, extrusion, cutting, and stamping during machining [4, 18, 19, 25, 26, 30]. On the other hand, the large degree of strain in metallic materials will make numerical modeling a difficult task and a challenge for finite element methods (FEM). The FEM method is not capable of simulating the process of large deformation due to mesh distortion or mesh entanglement. Therefore, the simulation problem is easy to fail and requires re-meshing until it is suitable. However, the above problems can be overcome by combining or replacing FEM with SPH [4, 24].

2. The SPH method

Particle-based SPH is used for modeling coherent fluid flows, solid structure deformation, and heat transfer. In the computational domain of the SPH method, a given particle only interacts with its neighbors through a function multiplied by a constant radius. Interparticle interactions stop when the particles move out of each other's influence domains, according to Rushdie et al [24]. The particles represent blobs of liquid or an arbitrary solid that move around in response to liquid or solid stress generated by interaction with other particles. In SPH, a liquid is represented by a set of freely moving particles. Each particle has some basic physical properties such as mass, density, position, velocity, and other related properties. The detailed and comprehensive knowledge of SPH methods such as continuity equation, momentum equation, energy equation, equation of state, etc. has been fully presented in the references [1, 2, 5, 12, 13, 19, 34, 36, 44, 45, 62, 68, 86, 89]. Some basic theoretical knowledge related to metal deformation (heat transfer and phase transition model) of the SPH method is briefly presented in this section.

2.1. Heat transfer and phase transition model

Fluid flow and solid deformation of materials are often associated with important physical phenomena such as solidification, heat transfer, and phase change, according to Cleary et al. [5]. An SPH particle represents a specific material. Its properties and behavior can be altered to match the thermodynamic states occurring in the material. Changing the reference density of particles in SPH according to the equation of state leads to shrinkage during solidification. When the metal cools, the grain density increases, leading to a decrease in the mass of the grain. The process of heat exchange between material particles is characterized by heat conduction. The appropriate form of the heat conduction equation has been derived from Monaghan [1], Raden et al. [18] and is applied to many different heat conduction problems. The heat conduction equation in SPH is presented as:

$$\frac{dH}{dt} = \frac{1}{\rho}\nabla(k\nabla T) \tag{1}$$

where:

- H is enthalpy,
- k is thermal conductivity,
- T is temperature.

The SPH formulation of this equation is approximated by using the modified SPH approximation for the second derivative which is developed:

$$\frac{dH_i}{dt} = \sum_j \frac{m_j}{\rho_i \rho_j} \frac{4k_i k_j}{\left(k_i + k_j\right)} (T_i - T_j) \frac{r_{ij}.\nabla_i W_{ij}}{r_{ij}^2 + \eta^2} \tag{2}$$

where:

- t is time; i and j are neighboring particles,
- m, r, v, and ρ are the mass, position, velocity, and density of the particle; η is a small parameter to prevent singularity,
- W is an interpolation smoothing kernel.

This equation ensures that the automatic heat flow is continuous across different material surfaces. The temperature in equation (2) is calculated by considering the relationship between enthalpy and temperature. The solidification of liquid metal occurs within a certain temperature range and undergoes a paste state. Therefore, the non-isothermal phase change will depend on the relationship between enthalpy and temperature.

The relationship between enthalpy and temperature, provided the specific heat is constant, can be written as [3]:

$$H = \begin{cases} c_s T, \quad T \le T_s \\ c_s T_s + c_{sl}(T - T_s), \quad T_s < T \le T_l \\ c_s T_s + c_{sl}(T_l - T_s) + c_l(T - T_l), \quad T > T_l \end{cases} \tag{3}$$

The temperature of each particle is then calculated by:

$$T_i = \begin{cases} \frac{H_i}{c_s}, H_i \le H_s = c_s T_s (Solid), \\ T_s + \frac{H_i - H_s}{c_{sl}}, H_s < H_i \le H_l = H_s + c_{sl}(T_l - T_s)(Mushy), \\ T_s + \frac{H_i - H_l}{c_l}, H_i > H_l\ (Liquid) \end{cases} \tag{4}$$

where:

- T_l and T_s are the solidus and liquidus temperatures, respectively,
- c_l and c_s are the specific heats in the solid and liquid phases,
- $c_{sl} = L/(T_l - T_s)$ with the latent heat (L) describes the energy released by a particle to change the phase from liquid to solid.

2.2. Boundary and interface conditions

The SPH particle method is very different from mesh-based numerical methods (FEM) in dealing with boundaries, and care must be taken in this regard. For the finite element method, the appropriate boundary conditions can be clearly defined and properly implemented without affecting the stability of the model. In FEM, the implementation of boundary conditions is generally straightforward according to the reference of Liu et al. [13]. In contrast, this problem in SPH is not as straightforward as in mesh-based numerical models. Although SPH has been developed for many years, boundary processing remains one of the main difficulties and is considered a weakness of particle methods. This difficulty is due to two reasons: First, complex boundaries cause technical problems (implementation). Second, for particles near the boundary, the nucleus will be truncated by the boundary, so the accuracy of the SPH approximation is reduced. In this case, appropriate and accurate boundary processing methods are always a concern for the correct and successful implementation of the SPH method [22]. In a finite element method (FEM) simulation, the temperature conditions for the boundary particles are specified and set before calculating and conducting the simulation. However, it is not necessary when using the SPH method because the SPH particles exchange heat and matter with neighboring particles whether they are in the same or different phase, as referenced by Monaghan [1] and Zhang [12]. If all boundaries were adiabatic, the particles would interact with each other and ensure that the system conserves thermal energy. Heat transfer from liquid or solid to boundary particles is calculated by considering the heat conduction process. This process takes place when the solution domain across the boundaries is cooled [19].

2.3. Liquid-solid interaction model

The phase transition of a particle from a liquid to a solid state occurs when the temperature of the particle is lower than the temperature of the solid. Solid particles behave like liquid ones, but they move under high viscosity. If a solid particle has more than two of its neighbors in the solid phase, then they act and move together as a solid group. The way to create solid groups has been described very specifically by Raden et al. [18]. Each solid particle is assigned an initial index. Then, each particle index is updated with the maximum index of its neighbor. It is repeated until each solid particle index converges to the maximum index in each group. This means the number of snake groups is determined by the number of remaining stats. Solid groups are considered incapable of being formatted. To prevent distortion, the motion of a solid particle must be converted into translational and rotational motion relative to the center of mass of that group of solids. The center of mass is calculated using the following formula [18]:

$$r_c = \frac{1}{N}\sum_j r_j \tag{5}$$

where: N is the number of solid particles in the solid group.

The moment of inertia (I) and angular velocity (ω) characteristic to translational and rotational motions are determined by the following formula:

$$I = \sum_i |q_i|^2 \tag{6}$$

$$\omega = \frac{1}{I}\sum_i q_i \cdot v_i \tag{7}$$

where: $q_i = r_i - r_c$ is the relative position of particle i to the center of mass. It is specifically identified in the references.

2.4. Historically dependent properties of metals

Each SPH particle represents a specific mass of metal and carries complete information about that material. This is an important property of Lagrangian methods. Thus, all information about the exact state of each metal can be found in the integrated history of the grain data. This shows the ability to track properties like [86]:

- metallic phase and microstructure,
- damage leading to predicted fault,
- surface oxide,
- cumulative plastic deformation,
- metal components and trapped gases.

3. A systematic literature reviews

As mentioned in the introduction, the SPH method is very commonly used in the study of metal deformation. Table 1 shows the system of metal deformation and machining processes and articles related to the application of the SPH simulation method. Each specific topic is identified in citations from the references. Among them, the SPH method is mostly applied by authors in two research topics: the casting process and the metal welding process.

Table 1. The list of articles related to the research theme

Process	Citations
Casting	[2, 5, 7, 15, 17, 33, 34, 61-65, 67-79, 107]
Cutting	[11, 14, 28, 32, 80-84, 94, 97, 104]
Forging	[16, 24, 35, 89, 90, 91, 103,105]
Extrusion	[15, 36, 85-88, 90]
Welding	[38, 39, 23, 37,40-60, 106]
Rolling	[92, 101, 102]
Drilling	[95, 96, 98, 99, 100]

3.1. Casting process

Casting is an important and very commonly used method for low-cost production of high-volume, complex-shaped parts. It is a method of creating cast ingots for other machining processes or finished products. The classification of casting methods is shown in the diagram in Figure 1. In this method, molten metal is poured into the cavity of a casting mold of the shape and size of the casting.

After the metal solidifies in the mold, casting with the same shape as the mold cavity is obtained. To obtain uniform molded components with minimal porosity, a proper and precise mold design is required. Therefore, the numerical simulation will effectively support the research, design, and manufacture of molds, reduce costs and improve machining quality and productivity [4].

The SPH method has proven to be a useful and suitable method for modeling the shape and displacement rate of complex fluid flows during casting [2, 5, 7, 15, 17, 33, 34, 61-65, 67-79]. When discussing the application of the SPH method in casting simulation research, articles often refer to specific topics such as high pressure die casting [5, 33, 34, 62, 64, 67], ingot casting [7], continuous casting [17], low pressure die casting [61], gravity casting [63, 74], casting complex-shaped objects [69].

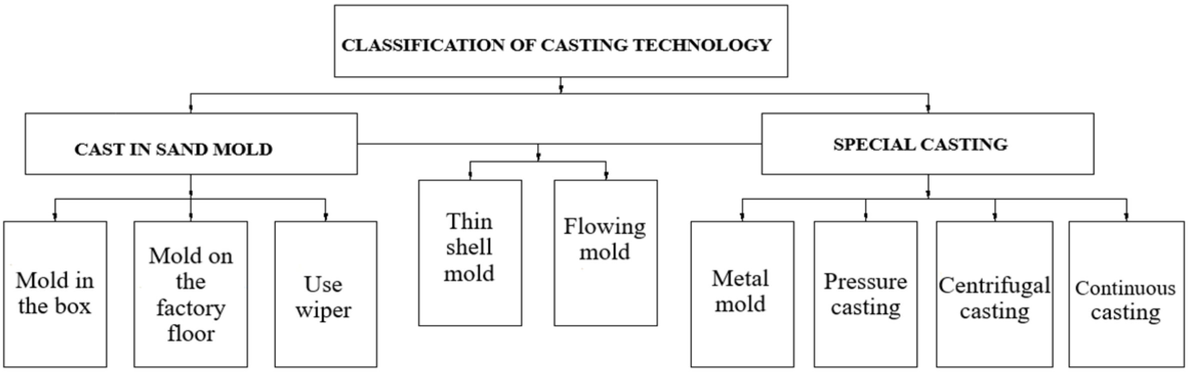


Fig. 1. Classification diagram of metal casting methods

3.2. Cutting process

Metalworking by cutting is a common processing method in the mechanical engineering industry. This method produces parts with high precision and surface smoothness. The essence of this technology is to remove part of the metal of the workpiece in the form of chips by cutting tools to create the part with the required shape and size. The FEM method has been widely used to model complex cutting processes. However, in the shearing process, a large strain occurs, leading to an increased element strain. FEM simulation problems encounter many difficulties. To evaluate the use of the SPH method in the framework of metal cutting, many researchers have built and developed an SPH model, demonstrating its ability to model chip formation, and accurately estimate shear force. Therefore, SPH is a method with many attractive features to solve the problem of large deformation. It was selected, applied and shown in the references [11, 14, 28, 32, 80-84, 94, 97].

3.3. Forging process

The basic principle of the forging method is to take advantage of the plasticity of metal and deform it in the solid state, using the action of external forces to create finished products and semi-finished products of a certain size, depending on the requirements. The large material deformation during forging makes numerical modeling of metals by FEM difficult due to mesh distortion and mesh entanglement. Therefore, the Lagrangian particle-based SPH meshless method is more suitable for simulating the shaping process. The occurrence of holes, cracks, and their interactions are modeled and simulated, becoming more intensive in terms of arithmetic and computation [16, 24, 35, 89, 90, 91, 103].

3.4. Extrusion Process

Stress, strain, and other phenomena experienced by materials during extrusion are predicted through numerical simulation modeling. These factors greatly affect the quality of the extruded product, as well as the speed of the production process. Large plastic deformations can lead to heavy re-meshing and digital diffusion. Here, the SPH method is used. Its meshless nature allows the large strain to be simulated without re-meshing and historically dependent properties such as plastic strain to be tracked, making it suitable for defect prediction [15, 36, 85-88, 90].

3.5. Welding process

Welding is a method of permanently joining two or more metal parts together. It is performed by heating pieces of metal to their melting point, and then either using pressure or not, pressing the welding parts against each other to stick together. There are many welding methods, and they are very commonly used in mechanical processing (Figure 2). Metal is melted, and then completely crystallizes to form the weld joint.

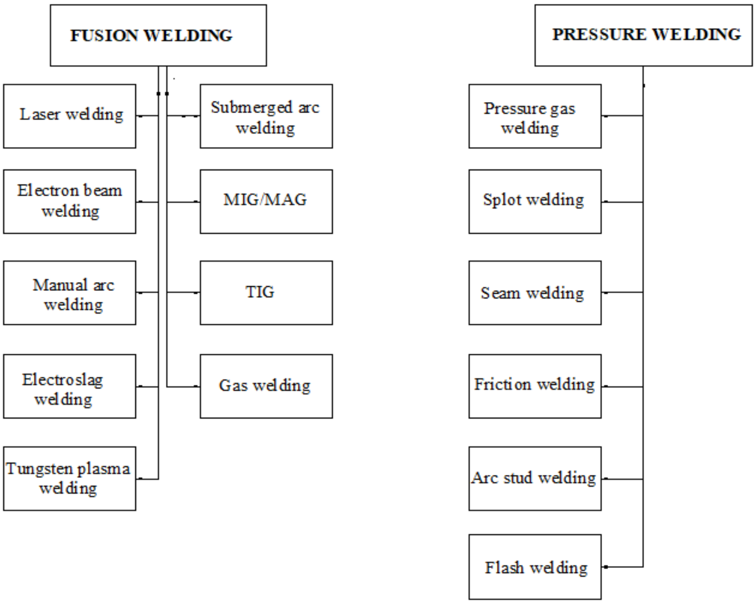


Fig. 2. Classification diagram of metal welding methods

Explosive welding involves processes like the detonation of explosives, the impact of metal structures, and strong fluid-structure interaction. The SPH method is applied to study the typical physical phenomena in the explosive welding process including blast wave, welding surface morphology, jet current, and plate acceleration [37, 38, 39, 57, 58, 59]. The simulation of the TIG welding process with the SPH method, takes into account the melting and solidification of the anodic metal, the free surface movement of the liquid, and the dominant current conduction force. Compared with simulation results obtained with the FEM method, SPH can give excellent results, especially based on the observed deviations between the investigated FEM methods, and thus proves the accuracy of the SPH method for industrially relevant engineering applications [41, 47, 55]. Besides, many other welding processes are also simulated with the SPH method, such as arc welding, friction stir welding, impact welding, laser welding, etc, [40, 41, 43-46, 48-56].

3.6. Rolling process

In the rolling process, metal is deformed between two rollers rotating in opposite directions with a gap smaller than the height of the workpiece, resulting in a decrease in the height of the workpiece and an increase in its length and width. The shape of the gap between the two rollers determines the shape of the product. The workpiece moves through the rolling clearance due to the friction between the two rollers and the workpiece. Rolling not only changes the shape and size of the workpiece but also improves the quality of the product. The SPH simulation approach was tested and applied to the cold rolling process. Thereby, it is possible to confirm the advantages of using SPH and reveal some improvements in simulation efficiency. From the research findings, it can be concluded that the SPH method can be a sufficient tool to achieve quick and simple simulation for complex deformation cases such as the metal rolling process [92, 101].

3.7. Drilling process

Drilling is one of the most common processes in metalworking. Its essence is the technique of making circular holes in different types of metal materials. It is widely used in many industries such as machine-building, mechanical engineering, and construction. The drill is fastened to the drilling machine with a chuck, and is clamped to prevent rotation. The tip of the drill bit cuts deep into the material, creating thin layers of chips. During the drilling process, the cutting force has a significant effect on the accuracy and quality of the hole. Uncompensated radial cutting forces lead to an increase in the diameter of the hole machined, reducing its accuracy. The SPH method has been applied to predict the cutting force and to study the influence of the chip separation criteria and the material model on the cutting force during the drilling process [100]. In addition, the SPH method is also used to investigate the mechanism of puddle removal during laser drilling of two different materials: aluminum and 316L stainless steel [98]. A simple numerical model was developed to predict the penetration depth of laser drilling [96] and simulate the three-phase laser micromachining process for manufacturing medical coronary stents [95] and simulate the heat transfer problem in the open state in the laser welding process [99].

4. Some applications of the SPH method in metal processing

Large plastic deformations in metals can lead to complex meshing requirements, which makes FEM difficult and leads to inaccurate results. Cleary et al. found that, when applying the SPH method, these problems can be handled more easily due to the meshless nature. The SPH can monitor large deformations beyond the capabilities of FEM methods, and specifically can monitor the deformation process of each part. Each position of the metal is directly controlled and deformations are predicted. The defect pattern is related to the flow and microstructure within the metal [4]. The following are some applications of the SPH method in the metal forming process in liquid, semi-solid and solid states, namely simulation of metal casting, welding, cutting, and forging processes. However, many of the problems involved are applicable to other metal forming technologies.

4.1. Gravity die casting process

Gravity Die Casting (GDC) technology is performed by pouring liquid metal from the cooking pot into the mold cavity under the action of gravity. GDC molds are usually made of sand, stone, or coal because these materials can be shaped easily. They do not break when subjected to sudden effects of high temperature, are not easily deformed, and are cheap. Figure 3 shows the schematic diagram of the structure of the mold and the products of the Gravity Die Casting process.

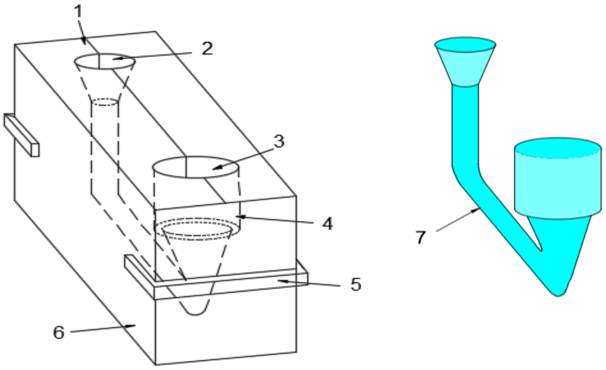


Fig. 3. Schematic diagram of the Gravity Die Casting process
1. Parting line; 2. Sprue; 3. Riser; 4. Casting
5. Clamps; 6. Permanent Mould; 7. Detail

The GDC process is capable of making complex and complete products, such as wheels, cylinder heads, engine blocks, and brake calipers, at a lower cost than most other casting methods. The cycle time for GDC is shorter than for other casting processes, resulting in a larger number of castings produced per time unit. Surface quality and internal quality (especially with regard to porosity) are also better, improving both the product quality and process productivity. These include the development of more efficient control of heat and mold filling performance [8].

Cleary et al. [2] have modeled the mold filling process in GDC for a complex mold by applying SPH and MAGMAsoft simulation methods. Both simulation methods can predict the overall structure of the liquid metal filling process. However, when comparing the simulation results of the above two methods, combined with the experimental results, they confirmed that the SPH simulation can capture the surface wave behavior very well and is close to the behavior of free surface waves and fine details of flow inside the mold cavity. The technology of GDC was used by authors [107] to analyze thermal effects during the solidification process of jet engine blades (Figure 4).

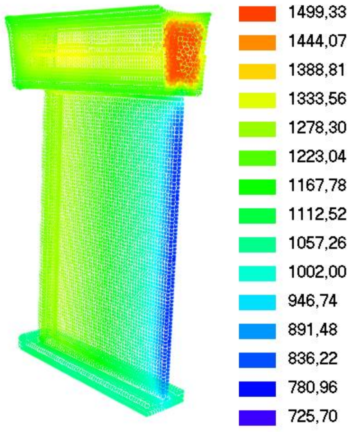


Fig. 4. Temperature distribution during solidification process (jet engine blade) [107]

4.2. High pressure die casting process

High-Pressure Die Casting (HPDC) is a process of filling the mold with molten metal at a high speed and pressure. It is a popular method for producing large-sized and high-yield parts. The molded products have high precision, good surface quality, uniformity, and high mechanical properties. For the product to achieve the above advantages, it is necessary to design a mold with high accuracy, ensuring technical factors in the machining process [8]. An example of simulation with the SPH method is provided by Cleary et al. [5,8]. It is applied to HPDC through a detailed form such as a servo piston to steering column components and a full engine rocker cover. The SPH simulation results are compared with the simulation result of the MAGMAsoft method and the water analog test results. MAGMAsoft's predictions are mostly qualitative, while SPH predictions show better compliance with experimental results, especially in predicting the flow of liquid into different parts of the mold, the relative intensity of the jets, and the time of filling back to the trapped airflow. In effect, SPH is capable of capturing finer details of fluid motion and splashing, especially the relative velocity of flow around sharp bends and through thin passages [8, 21]. Cleary et al. firmly confirmed the accuracy of SPH in predicting mold filling in the case with or without heat transfer and solidification. A four-part servo piston modeled to the size of the shot sleeve leads through a converging right-angle elbow runner into a chisel gate with a height of 3 mm. The base is 0.4 to 0.7 cm thick and has a diameter of 11 cm. The diameter of the inner cylindrical structure is 6.4 cm, and its height is 3.2 cm. The particle size used for the simulation is 1.4 mm. The piston head is roughly axially symmetrical, and there are three vents on the distal side of each piston. The simulation process of filling the mold as well as the speed of the liquid metal flow is shown through the color system. The results from the SPH simulation problem that Cleary achieved provided information on the order of filling, predicting potential locations of defects (porosity, etc....). They can be used as the basis for adjusting, improving, and optimizing technological parameters, supporting the development, and manufacturing of molds.

4.3. Ingot casting process

Some performance improvements of the new filling system for aluminum ingot casting from a combination of the SPH model with pilot-scale testing were performed by Prakash and Cleary [7]. They used the SPH methodology to design and simulate a new cast aluminum wheel, combined with testing, through several tweaks to optimize the technological parameters. Simulation results showed that for the flow of liquid metal from the wheel into the nozzles with the absence of the central flat part between the nozzles, the splashing phenomenon was significantly reduced. The fluid followed the contours of the nozzle, reducing the creation of new surfaces in contact with the air. There were no puddles in the new wheel design reducing flow complexity. A vortex flow pattern appears in the nozzles as the liquid fills the faucet. Such a vertical flow pattern has not been seen in previous designs. Prakash and Cleary compared the oxide content for the new optimized wheel design, the two previous best cases from the first stage optimization, and the original wheel design head [7]. The results of this process confirm that the new wheel design now offers several significant benefits: the new cast aluminum ingot wheel system can operate at 50% higher throughput (30 t/h) compared to the industry standard (20 t/h), with the oxide content expected to be approximately 12% less than the previous best design and approximately 53% lower than the original design, improving quality surface leads to more ingots being marketed. A similar method was used by authors [107] to simulate flow and optimize technological parameters of critical parts of aircraft engines.

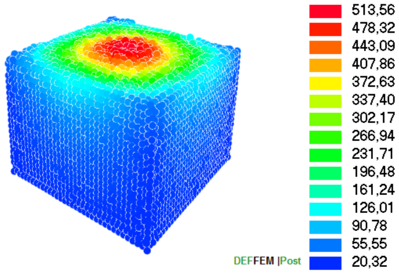


Fig. 5. Temperature distribution during ingot solidification (Inconel 718) [107]

4.4. High-speed and orthogonal cutting process

High-speed cutting (HSCP) is a machining method that focuses on ultra-fast, light, low-pressure cuts, which reduce excessive heat loss and transfer, reduce energy consumption, and increase machining productivity. A numerical model of the high-speed cutting process is approached, based on the SPH method. This process is simulated by Limido et al [11, 14] using Ls-Dyna software. Modeling based on two-dimensional SPH was performed to demonstrate continuous local chip formation for cutting as well as accurate estimation of shear forces. His research group has studied the possibility of the SPH shear model compared with the classical FEM approach and summarized the main features of the SPH shear model approach compared with the classical Lagrangian FE models. The results of the simulation study with the SPH method in Limido's research team were compared with experimental results. The validation criteria were defined as the chip morphology and the cutting forces. Thereby, seeing the appropriateness of the selected tool and the SPH model can predict continuous and shear localized chips and all the steps of its formation. The model also correctly estimates the shear force components (approximately 10% and 30% errors on, respectively, tangential and normal components). The SPH model has the advantage of the total transparency of the assumptions and does not use adjustment parameters such as coefficient of friction, and fault control parameters [11]. The 2D SPH cutting model has also been implemented as a useful tool to understand chip formation and describing tool wear. In this part, the authors only stop at the 2D SPH model. Therefore, it is hoped that in the future the 3D SPH model will be focused on, researched, and applied to specifically solve problems in HSCP (free surface, friction, heat exchange). Besides, Afrasiabi et al. [104] also used the SPH method to numerically simulate the orthogonal metal cutting process with an advanced thermal model. The numerical analysis includes both thermal and mechanical effects to solve thermal equations and examine the elastic properties of materials. SPH simulation results with different thermal models were used by the research team to compare the rake face temperature. The bar graph comparing the predicted temperature in the simulation with the experimentally measured data is shown in Figure 6 and Figure 7. The red and blue bars are the average temperatures of the particles inside black frames indicated on the rake face, corresponding to 905 K and 744 K. Using the SPH method, the authors have shown that using advanced thermal modeling reduces the over-prediction error of T_{RF} from 33% to 9%. In addition, they showed that the agreement between the experimentally measured forces and those predicted by numerical simulations was satisfactory, with the average error between them being less than 14% and the temperature values obtained from the SPH simulation being consistent with the experimental results.

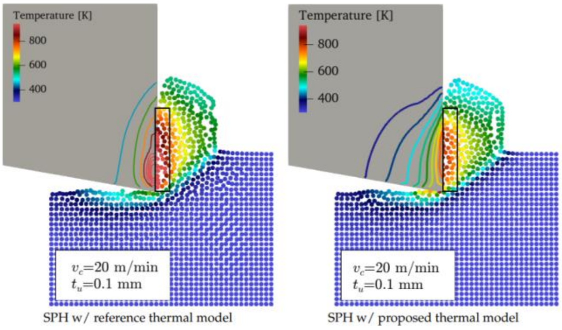


Fig. 6. Temperature distribution for the reference model (a), and proposed thermal model (b) [104]

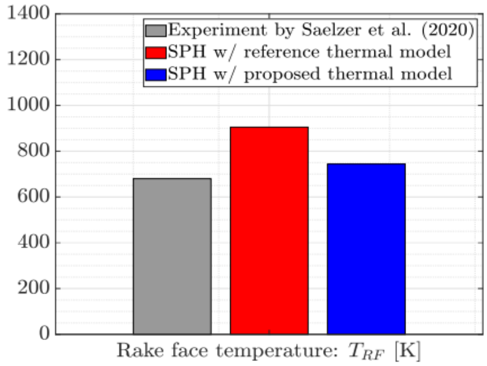
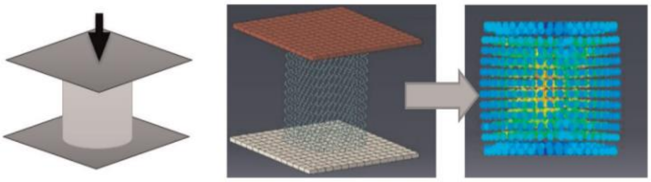


Fig. 7. The average value of rake surface temperature between SPH simulations and experiment results [104]

4.5. Forging and high-compression processes

Forging is a processing process in which metal is compressed under a great deal of pressure to form parts with superior material properties, uniform structure, and high strength compared to other technologies. In this process, metal is usually heated to the appropriate temperature, and then specialized equipment is used to process and shape the product. The deformability of workpiece metals and forging tools is based on ductility theory. Stress-strain and microstructural changes in materials can be predicted using numerical simulation methods [4, 24, 16]. Cleary and Prakash [16] have shown that SPH is a useful simulation method for insights into material deformation and flow patterns during forging. The influence of process parameters and material properties on the quality of the forging component was evaluated through SPH simulation. To accomplish that process, they have solved several key problems such as: determining the required forging force to fill the mold; controlled material hardness by the degree of heat treatment. SPH is used to predict forging defects such as under-filling, asymmetry in forging components, etc. At the same time, SPH can also be used to evaluate the quality of products and optimize the forging system.

Besides, the SPH method was also used by Ba [105] to test the axial compression of a cylindrical specimen (25.4 mm in diameter; 25.4 mm in length) at a constant velocity (2.54 mm s^{-1}) and high temperature (400 °C) (Figure 8). This research aimed to demonstrate the effectiveness of the proposed overall formula Lagrangian SPH. The author compared the results of stress-strain curves obtained from numerical simulations with experiments to verify the accuracy and stability of the code (Figure 9). The results of this research showed that the SPH method was accurate compared to the experiment. Less than 5% error was noted between curves.



a) b) c)

Fig. 8. Example simulation results of the axial compression process of a cylindrical sample with the 3D model (a), particle model (b), and SPH results (c) [105]

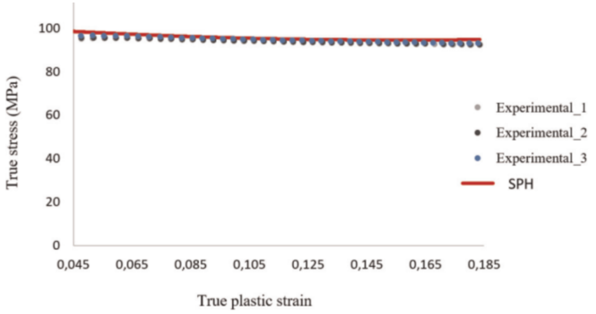


Fig. 9. Stress-strain curve comparison between numerical simulation and experimental results [105]

4.6. Aluminum alloy extrusion process

The plastic deformation process of aluminum alloy in the 3D cylindrical geometry model with extrusion ratio and mold angle is studied and evaluated specifically by Prakash and Cleary in reference [36]. The extrusion process was found to consist of three distinct stages: the initial stage where the extrusion force increased sharply, the second stage where the force was applied stably, and the final stage when the extrusion force dropped sharply, and the metal was fully extruded. The authors showed that the SPH method can simulate strain for a wide range of die sections ranging from a simple circular section to a remarkably complex industrial part with a mold angle of 180◦. The metal flow is easily modeled as it passes through a conical die, and it is stable to different extrusion ratios and die angles. In addition, they also showed that a complex section simulation with the SPH method has the advantage of following very large strains (beyond the capabilities of FEM methods) and a good predictability of the distribution.

4.7.Welding process

Laser welding is a state-of-the-art welding process that allows metals and their alloys to be bonded. This technology has many outstanding features compared to previous traditional welding methods. It creates a durable weld without additional metal. The schematic diagram of the laser welding process is shown in Figure 10.

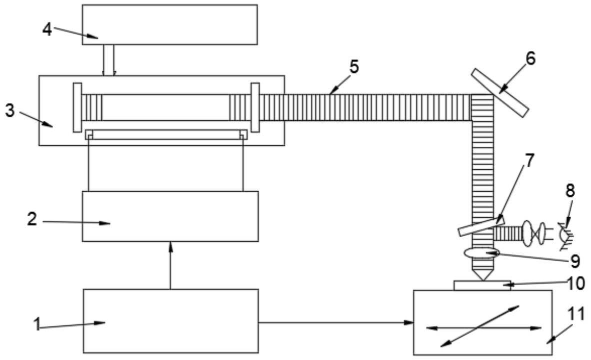


Fig. 10. Schematic diagram of the laser welding process
1. Control cabinet; 2. Power supply; 3. Laser head;
4. Cooling system; 5. Laser beam; 6. Reflector;
7. Filter mirror; 8. Observation system; 9. Converging lens; 10. Detail; 11. Workpiece placement table

The conduction mode laser welding process was simulated by Hu et al. using the SPH method [23]. The solid phase is modeled based on the tuning equations in thermal elasticity. For the liquid phase, surface tension effects are taken into account to simulate the hot flow of metal in the weld pool, using the standard formulation system of weakly compressed SPH proposed by Monaghan [1]. The fundamental heat transfer phenomena including the solid-liquid interface and the occurring phase transitions, melting and freezing, are modeled [3]. Hu's research team has simulated the process of spot welding and laser welding of aluminum materials with dimensions of 2 x 2 x 1.15 mm, respectively (divided into 41 x 41 x 24 = 40,344 particles) and 2 x respectively, 2.6 x 1.3 mm (divided into 21 x 27 x 14 = 7938 seeds). The simulation results have a laser power magnitude of 3600 W. Thereby, the influence of the Gaussian distribution of laser radiation on the temperature field, and clearly shows the Marangoni force acting in the flow due to laser radiation. The surface tension coefficient depends on the temperature. The numerical results obtained with the model are presented for laser spot welding and aluminum and iron seam welding. The variation of process parameters such as welding speed and laser power and their influence on weld size is studied. Therefore, the application of SPH that simulates the welding process can determine the optimal process parameters such as welding speed, laser power for each welding application, and maximum temperature during the entire process. In addition, the authors analyzed scalability to develop an efficient, large-scale SPH code with many cores, and a large number of particles. The process implemented in Pasimodo is run in a shared memory environment. Scalability analysis shows that the computation time increases linearly with the increasing number of particles. The simulation runs most efficiently when using as many threads as the cores in the CPU. Fraser et al. [106] applied the Lagrangian method without mesh (SPH) to simulate the FSW (friction stir-welding) process. This approach allows determining the temperature evolution, elastic and plastic deformation, defect formation, residual stress, and material flow in the same model. By using the SPH method, the large plastic deformation and material mixing common to FSW are well captured and easier to calculate. The authors calculated the simulation torque by taking the cross-product of the contact force and the distance vector between the tool axis and the SPH element. Research results have shown a good correlation of force and torque parameters between experiment and simulation shown in Figure 10. A flash height of 4.2 mm was measured experimentally; case 1 predicts a flash height of 4.5 mm, 3.9 mm for case 2, and less than 1 mm for case 3. On the other hand, the change of process parameters affects the force and torque. This can be verified by investigating the plastic strain contours in the weld zone as shown in Figure 11. We can see that the tool torque and forge force for case 2 are lower than those for cases 1 and 3.

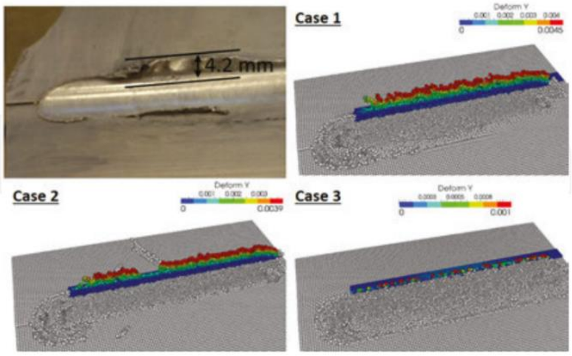


Fig. 11. Flash height comparison at the end of the advancing phase [106]

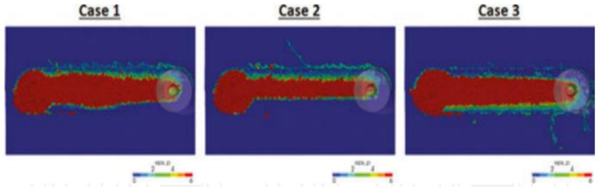


Fig. 12. Plastic strain at the end of the advancing phase shows the effective weld zone [106]

4.8. Cold rolling process

Regarding a series of studies on the simplicity and computational efficiency of the SPH method, Hoseinpour et al. [92] applied this method to the simulation of the cold rolling process, to confirm the advantages of the SPH method and suggest some improvements to make the simulation more efficient. Rolling tests were performed on a strip of Al 6061 aluminum. Several simulation results such as stress distribution patterns were compared with results from a FEM study. The comparison shows a good agreement between the two calculated results. Hoseinpour's research group tested the effects of decreasing diameter, thickness, and angular velocity on pressure. They showed that a decreasing angular velocity had no effect on pressure, but decreasing diameter and thickness reduced pressure. In addition, a decreasing angular velocity and diameter cause plastic deformation after a short time, and a decreasing thickness increases the gradient of elastic strain and stress. The work of the authors has once again confirmed that the SPH simulation method takes less time than the FEM mesh method. The lamination process with SPH was not conditional and controlees like the mesh method. Thereby, we can see the superiority of the SPH method, from which to research, improve and apply it on a large scale.

4.9. Twist drilling process

FEM cutting simulation modeling methods have been developed considerably for a long time, allowing to predict the shape and size of the chip, machined surface, stress-strain state, and temperature field. However, the simulation results are more consistent with the experiment in terms of quality than quantity. The main causes for this situation are imperfect algorithms for fault and friction modeling, along with inaccuracies in input data preparation. Boldyrev et al. proposed the SPH method for calculating the shear force when drilling uniform and isotropic 6061-T6 aluminum alloy [100]. Numerical simulation results were compared with calculations using empirical formulas and experimental results of other authors. These results showed that the SPH method could predict the axial shear force when drilling aluminum alloy with 20% accuracy, for torque the results were more satisfying. The team showed that this was a useful method for predicting the accuracy of boreholes, as well as evaluating the quality of the machined surface when machining composites. The numerical simulation results are presented in Figure 13 and Figure 14. Figure 13 shows the equivalent stress distribution in the workpiece, and Figure 14 depicts the values of thrust and moment in the drilling process. The computation time for a 6-core processor was 2.5 hours.

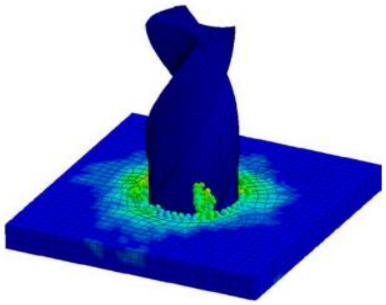


Fig. 13. Distribution of Von Mises stress in the workpiece (final stage) [100]

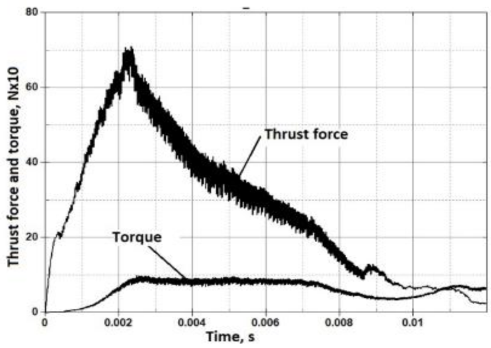


Fig. 14. Thrust force and torque prediction [100]

5. Summary

The SPH method has been known for a long time. We have witnessed its great success in methodology and application. SPH method has great potential in many technical and scientific problems. It is a promising alternative to mesh-based techniques, specifically involving multiphysics phenomena, in a wide range of industrial fields. It is also recognized as a powerful tool and superior to traditional meshing methods. However, there are still many tasks and challenges that need to be done, researched, improved, and developed to achieve a more reliable solution and improve calculation accuracy, consistency, efficiency, and stability. This paper shows that the SPH method has the advantage of being able to perform simulations of large strain processes (beyond the capabilities of FEM methods) and track the specific history of each metal part to control the model. Some specific research studies on the metal forming process using the SPH method to simulate and verify experimental results have been synthesized and analyzed here. Thereby, it is found that SPH is a very useful tool for the reconstruction of the flow model and the solidification process, prediction of possible defects in the deformation process, finding out reasonable machining conditions, and optimizing process parameters.

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