Design and modeling of a uni-directional piston expander, a case study

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Abstract
A design of a reciprocating piston expander based on the uni-directional flow principle is proposed. The conversion of low value heat into mechanical work and electrical energy is a basic problem of small co-generation power plants. It is postulated that the proposed expander is appropriat for such applications. A methodology for engine design and modeling is brought forward and outlined. The design principle is based on the outlines proposed for steam engines by Stumpf [1]. The proposed design is of a horizontal, low speed unit designed with ease of manufacture in mind. Calculations based on the model show isentropic efficiencies around 70%, with the nominal power of 1,2 kW, for a machine working on 7 bar, mildly superheated steam, with the outlet pressure of 0,3 bar and condenser coolant used for residential heating. Furthermore, the most crucial mechanical and stress calculations are outlined.

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1. Introduction

An interest can be seen in utilizing piston expanders in small Rankine cycle systems [2]. Volumetric expanders are considered to be more efficient at small sizes than turbomachinery [3]. Expanders utilizing pistons are more commonly and historically known as steam engines.
A conventional steam engine despite its simplicity shows several thermodynamic losses:
- clearance losses
- port throttling losses
- incomplete expansion losses
- wall losses (irreversible heat transfer losses) [4]

The first three are symptomatic of most volumetric machinery. The wall losses are however a steam expansion related issue. In short, wall losses are created by the cooling of cylinder walls by the flow of the expanded (cooler) steam through the cylinder and ports which cools the walls down during the exhaust stroke (the problem is magnified if exhaust and admission are done through the same ports and valve chambers). On the admission stroke, the incoming steam meeting the cold walls can condense (if the wall temperature is below saturation temperature for a given pressure), increasing the steam consumption even several times [5]. In condensing engines, where exhaust temperature can reach values as low as 30°C this is especially problematic. Even when condensation does not occur, wall losses exist as the specific volume of the incoming steam diminishes. Ochęduszko notes several ways of diminishing wall losses [4]:
- the use of superheated steam
- steam jacketing of cylinders
- increasing the engine's cut-off parameter (ε)
- increasing the rotational speed of the engine
- avoiding working the engine on a deep vacuum (which decreases the cycle efficiency)
- compounding (dividing expansion into stages in separate cylinders)
- uni-directional expansion as proposed by Stumpf

The cut-off "ε" is defined as a portion of the stroke (or volume) during which the inlet valve is open (filling volume divided by expansion volume). Increasing the cut-off has the problem of also increasing the loss of incomplete expansion. Working the engine on lower vacuums decreases the thermal efficiency of the cycle. However the conclusion can be derived that the internal efficiency of the engine will be better therefore during counter-pressure working (or with shallow vacuum), which entails that engines are only fit to work when exhaust heat is to be utilized.

Historically, superheat was considered to be a sufficient precaution. In fact, historical engineers not yet understanding the fundamentals of engine cycles have perfected superheating as a way of decreasing steam engine wall losses [6]. The original works of Schmidt on superheating also concentrate on this issue [4]. It has to be noted that superheat does not cancel the wall effects completely – as it only diminishes them by the fact that the convection heat transfer coefficient is much lower for gasses such as superheated steam than it is for saturated vapors [7]. Also, it must be noted, that water droplets or film of condensation can co-exist with superheated steam in meta-stable conditions [7]. Superheating is of course desirable as it increases the efficiency of the Rankine cycle, in addition to the minimization of wall losses. It is imperative that a modern piston expander needs to be designed in such a way as to be able to take a high superheat temperature, but due to the fact that it might often be utilized to work on saturated vapors its design must also minimize wall losses working on saturated vapors.

Compounding decreases the differential between admission and inlet temperature in a given stage which decreases wall losses considerably. Experimental data as quoted by Ochęduszko [4], shows that wall losses will however still be significant in the last, coldest, stage of expansion. This is not to be misunderstood, as this still increases the isentropic (internal) efficiency of the engine overall. However it prompts, that the efficiency could still be increased by looking into the processes occurring in the last stage cylinder.

Steam jacketing is experimentally shown to be beneficial [4], but in a conventional engine the minor disadvantage of the jackets heating up exhaust steam and therefore imparting the engine with a higher outlet enthalpy has to be noted. Stumpf proposed the decrease of this effect by jacketing only the cylinder head [1].

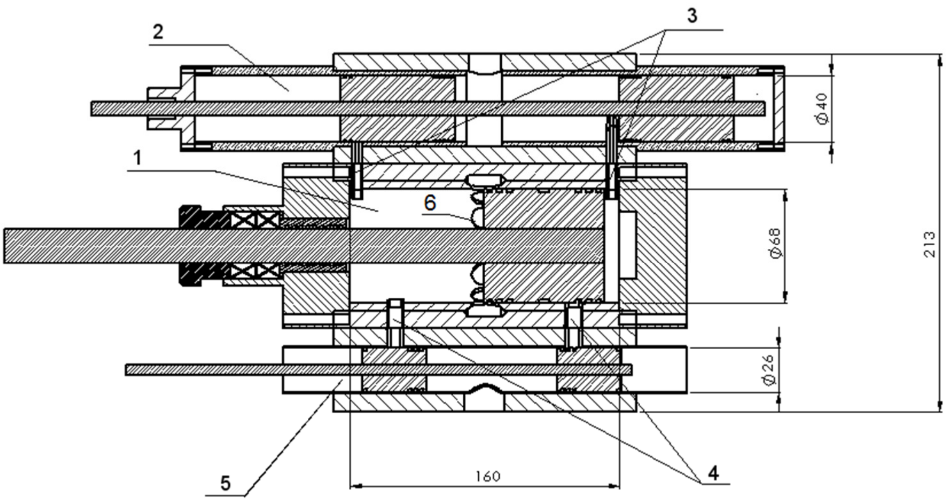

Fig.1. The engine cylinder. Threads omitted for visibility. Dimensions are a product of the analysis being the subject of the article. Noted are: 1 – cylinder, 2 – inlet valves, 3 – inlet ports, 4 – auxiliary outlet ports, 5 – aux. outlet valves, 6 – main outlet ports

Uni directional expansion is based on separating the exhaust and inlet ports, and placing them on the opposite ends of the piston stroke, as shown in the drawing (Fig.1) Where the outlet is through the ports in the middle, and inlet throught the piston valves at the top. Engines utilizing the principle are further referred to as „Uniflow" engines. Uni-directional expansion is advantageous as it prevents, according to Stumpf [1] the cooling of walls by exhaust flow. It does not completely prevent the heat transfer, as the temperature of steam in the cylinder will still decrease with expansion. However due the nature of the arrangement the steam velocity will be the highest close to the exhaust port, therefore greatly diminishing the convection heat transfer to the walls. Literature often quotes the disadvantage of uniflow working in form of high return stroke compression. This is a problem that requires careful engineering calculations, as to not allow the pressure after the return stroke to exceed inlet pressure. This is further discussed in the calculations for the proposed design. An interesting way of accounting for a possible exceeding of the inlet pressure in engines was utilized in Fredrikstad marine engines, where a safety valves were fitted in cylinder heads, releasing the excess pressure into regenerative pre-heaters for feed water heating [9]. The issue is especially profound when the vacuum is diminished due to condenser malfunction. Stumpf proposed additional clearance spaces activated by spring loaded valves [1]. However this increases clearance losses. It is therefore proposed to include a separate auxiliary exhaust valve, activated only if the vacuum is diminished. As mentioned before, the wall losses are lower at higher exhaust pressure, therefore there is less advantage to full uni-directionality in that case. However. it is still advantageous to put those valves closer to the center, with the compression accounted for with calculations explored later. Similar arrangements were used by the Skinner Engine Company [2] with considerable success.

Uni-directional expansion can, if needed be utilized for working on saturated steam, as the temperature gradient in the cylinder follows the expansion [1], keeping the heads hot. Also, reciprocating engines do not suffer from droplet erosion issues. Furthermore, the compression temperature rises above the admission temperature. This can be considered an exergy loss, however it prevents a much greater one in form of the aforementioned condensation. This makes uniflow engines also possible to utilize with organic Rankine cycle power plants at small scale. Interest for their use in CO_2 based refrigeration has also been expressed [8]. Because of their periodical operation they can be utilized for the Misselhorn (isochoric evaporation [18]).

For illustration, a table of isentropic engine efficiencies for examples of different types of engines is provided in table 1. The isentropic efficiency of a reciprocating steam expander is equivalent to the internal efficiency of a steam turbine.

Table 1. Examples of isentropic efficiency values (compiled from "Teoria Maszyn Cieplnych II" and Skinner company materials)

Type	**P_{in} [MPa]**	**P_{out} [kPa]**	**T_{in} [°C]**	**η_i**
compound, two stage	1,00	6,86	179,88	0,656
compound, two stage	1,00	6,86	352,88	0,734
uniflow	1,18	19,61	191,20	0,572
uniflow	1,18	19,61	280,20	0,629
compound uniflow	3,13	8,00	393,00	0,710
simple	1,18	19,61	191,20	0,491

2. Design methodology

2.1. Design objectives for the prototype

The design being the subject is made for the smallest practical size, the prototype being intended for a domestic CHP setup. The assumptions are as follows:

Table 2. Design parameters of the proposed engine

Nominal cut-off ε	30	%
Rotational speed n	600	r.p.m.
Required power N	~1,4	kW
Inlet pressure P_1	0,8	MPa abs
Outlet pressure	0,03	MPa abs
Allowable counter-pressure	0,13	MPa abs

2.2.Thermodynamic principles

In order to ascertain the engine dimensions, calculations pertaining to its working cycle need to be performed for the given demands. Certain authors [10, 4] propose to draw an indicator diagram in such a way, as if the expansion was carried out on an ideal gas with or without making certain corrections. It has been decided to employ corrections, proposed by Neuman [10] and Mozer [11] for the pressure drop at the inlet valve, in form of empirical equations.

An indicator diagram, or a p-V diagram shows the relations of pressure and volume inside a volumetric machine [12]. The field inside the diagram is the indicated work of one engine cycle.

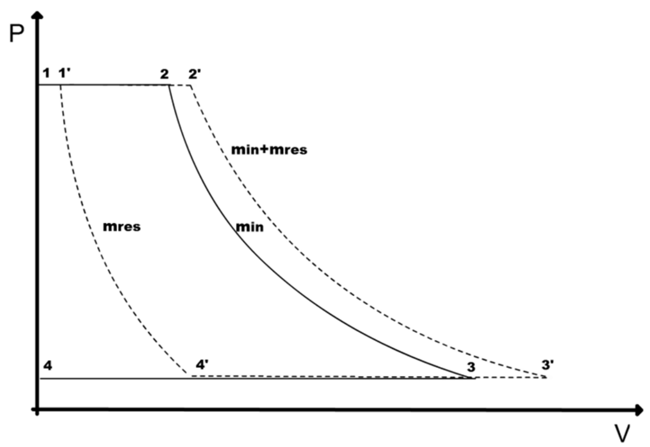

Fig.2. Ideal engine and pseudo-ideal pV diagram

The pseudo-ideal cycle is one of the same isentropic efficiency as in an ideal one, but including the clearance volume that real engines must practically have [4].

For an ideal steam engine the diagram will look as shown in figure 2. The cycle is as follows:
- 4 to 1 - isochoric filling of the cylinder
- 1 to 2 - isobaric filling of the cylinder
- 2 to 3 - isentropic expansion
- 3 to 4 - isobaric exhaust

De-facto, this ideal cycle is impossible to reach in a real machine. This is due to losses discussed in paragraph 1.2. Also, a real engine needs re-compression after the exhaust is carried out. For that reason the diagram will be shaped differently. A realization of this is shown in figure 3. In that diagram it can be seen that expansions stops at certain pressure at point 3, before the piston reverses direction. This is mechanically beneficial, as it provides piston cushioning, and impossible to avoid. Release happens from point 4 to 4', after which compression begins. Compression historically was mainly a means of providing mechanical cushioning at reversal of the piston direction, it was also understood it filled, in part, the clearance spaces [7]. The additional benefit to compression is raising the cylinder temperature. It is understood, that compression filling all of clearance space would be utmost beneficial [4]. The points 5 to 1 represent the filling of the remaining clearance space (clearance loss).

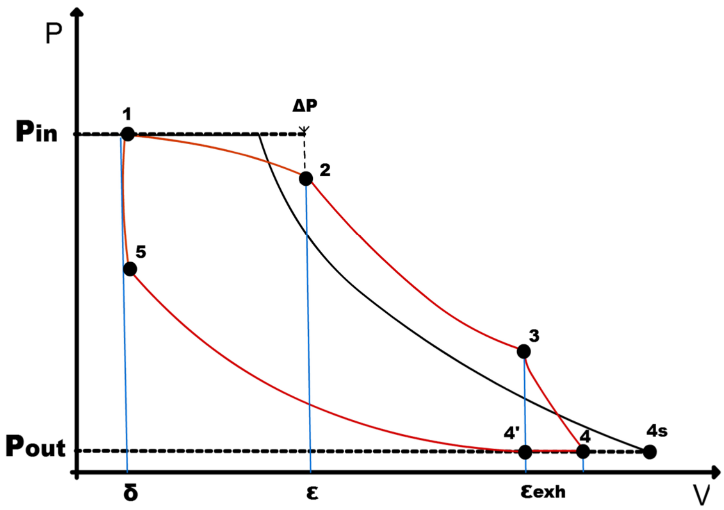

Fig.3. Uniflow and ideal engine pV diagram. Noted are: ΔP – throttling loss during inlet, ε_v (ε_{exh}) – fraction of cylinder volume besides the outlet ports, ε – cut-off, δ – clearance volume

In a uniflow engine the release only happens by the end of the stroke (around 90%) [1]. For that reason, there is also almost 90% of compression. This allows to fill the clearance volume completely. A uniflow engine indicator diagram is shown in figure 3 in red.

Even the given diagrams are however idealized. A question exists of wiredrawing during the valve opening at points 1 to 2. An analytic solution of the problem presents complications.

Empirical relations are therefore used.

The next step, after ascertaining the engine dimensions is to verify them using a more comprehensive model (real gas model). A comprehensive, dynamic model was created in EES software, making use of its libraries (steam parameters). This model is described in chapter 3.

2.3. Ascertaining engine dimensions by simplified process analysis

2.3.1. Compression 4-5, finding the clearance space

For a uniflow engine, due to its construction, the compression stage needs to be evaluated first. This is because the pressure at the end of the stage, cannot exceed inlet pressure p_1. A value of $p_5<p_1$
is to be assumed to account for throttling. This condition enables to calculate the non-dimensional clearance volume δ, defined as:

$$\delta = \frac{V_{clear}}{V_{total}} \tag{2.1}$$

The simplest way to perform this calculation, is to assume a ε_v (90% is recommended [1]), this being the fraction of cylinder volume after the exhaust port area is subtracted, and assume the compression to be a polytropic process, which is how Ochęduszko [4]Neuman [10]describe it based on empirical indicator (p-v) diagrams. Then, as: $p \cdot v^m = idem$, it can be shown that:

$$\delta = \frac{(1-\varepsilon_v)}{\left(\frac{p_5}{p_4}\right)^{\frac{1}{m}}} \tag{2.2}$$

where m = 1,125 for steam, for other working fluids the isentropic exponent at the outlet parameters can be used in the first approximation, and the average exponent over the course of compression in the second one.

Compression can be re-evaluated with the real-gas model after completing the evaluation of the exhaust phase. The principle of doing that is the same as for expansion (chapter 2.1.2), but with decreasing volume.

2.3.2. Admission 1-2, finding the inlet port area

The admission line is not horizontal due to throttling, therefore the throttling must be evaluated.
It was decided, based on empirical data that maximum steam velocity at admission should be set at around 35 m/s [11]. Assuming a constant stream, this allows to write down:

$$\frac{w\pi D^2}{4} = ca, c_{max} = 35\ m/s \tag{2.3}$$

where D is the diameter of the piston, w is the piston linear velocity, c is the steam velocity in the port, and a is the port area. Neglecting the length of the connecting rod, that is assuming it has an infinite length, it can be said, that $w = \omega \cdot R \cdot \cos\alpha$ where R is the radius of the crank, ω the angular velocity of the piston, α is the crank angle. The maximum value is therefore, for α = 90°, w = ωR where ω=πn/30, n being the rotational speed of the engine. From those relations the area of the port a can be deduced. Thus, in approximation, $a = 1{,}98\,cm^2$ for the diameter D = 68 mm, at n = 500 rpm, R = 36,5 mm.

Based on those calculations, it was decided to make the ports minimally larger than the calculated value of a, which will decrease losses. this is with port dimensions 6x40 mm, giving the area of $a = 2{,}4\,cm^2$. It has also to be noted that the engine operating at its nominal cut-off of 30% will not reach maximal velocity at port opening (however it can during overload).

Equations denoting a possible pressure drop have been given by Strahl [11]. Those have been made for steam locomotive engines based on empirical data. The equation has the form noted below:

$$\Delta p = \frac{4 \cdot \varepsilon \cdot (1-\varepsilon) \cdot p_1}{2 \cdot \varepsilon \cdot (1-\varepsilon) + \sqrt[3]{\frac{4C}{10^6}\left[\frac{(\delta+\varepsilon)\cdot(v_e + 2e \cdot \varepsilon)\cdot b \cdot \mu}{(0{,}71+1{,}5\varepsilon)\cdot J \cdot u \cdot \pi}\right]^2}} \tag{2.4}$$

where:

- Δp [at] is the pressure drop in
- ε is the cut-off
- μ is a flow coefficient of the port
- b [mm] is the port length
- m is the fraction of clearance space
- u [1/s] is crank rotations per second
- v_e [mm] is linear lead (valve opening before the direction change of the piston)
- e [mm] is the steam lap
- J [dm^3] is the cylinder stroke volume
- p_1 [ata] is the absolute inlet pressure
- C is a steam constant, denoted as $C = 2g \vee p_1 \square 10^4$, where g is the gravitational acceleration, ν is the specific volume of steam at the given pressure and superheat

The empirical formula is not given in SI units, so a conversion was carried out. The values of the quantities noted above are given in table 3

Table 3. Flow parameters for nominal working

ε	30,00	%	**u**	8,33	*1/s*	**p_1**	7,14	*ata*
μ	0,60	-	**v_e**	24,00	*mm*	**C**	1400478,25	-
b	6,00	*mm*	**e**	9,00	*mm*	**Δp**	1,92	*at*
m	0,09	-	**J**	0,27	*dm^3*		**1,96**	**bar**

The specific volume was read from steam tables. Lap and lead are a product of a second iteration of a process of calculations included here and in later chapters, namely the calculations of valve gear geometry. This estimates the drop at Δp = 1,92 at = 1,96 bar

As a line on the indicator diagram, admission starts at the volume corresponding to the non-dimensional clearance space δ at boiler pressure P_1 and continues to the non-dimensional cut-off ε, where pressure equals $P_2=P_1-\Delta p$. The actual line of admission can be approximated as a parabola, preferably one that smoothly joins the expansion line. For these calculations, it can simply be assumed as a straight line.

2.3.3. Expansion

The expansion stage in the utmost pre-eliminary calculations may be assumed to be a polytropic process $p \cdot v^m = iden$, with the exponent m = 1,1 to 1,35 for superheated steam, and m = 1 for saturated, the values being suggested by Ochęduszko, and Neuman based on empirical data [4, 10]. It is recommended that this be done first, to provide estimate values for further calculations. As in compression, the isentropic exponent can be used for different fluids.

2.3.4. Exhaust – finding the outlet valve area

Having assumed a ε_v, knowing the total volume V and exhaust pressure p_4 the exhaust phase can be graphically approximated as a straight line between points (V_3, P_3) and (V_4, P_4). V_4 corresponds to the non-dimensional exhaust valve location ε_v, based on assumption. This step is also when the exhaust port width can be ascertained and therefore an actual value of ε_v found. An empirical function was created for the purpose based on data cited by Neuman [10], which gives the unitary port area f_0, defined as the area necessary to exhaust 1m^3 of steam per minute based on the ratio of pressure at the end of expansion to the condenser pressure. This is shown in figure 4. It can be shown that: $f = f_0\, d^2\, s\, n$, s being the stroke in meters, d being the piston diameter in meters, f being the necessary cross-section area. It is necessary to perform this calculation again, after the correct dimensions are chosen at the next step.

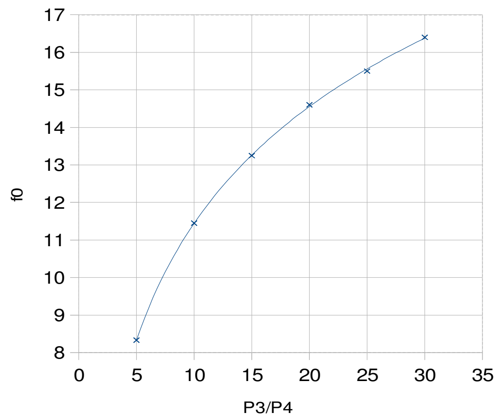

Fig.4. Minimum unitary port area as a function of pressure differential

$$f_0 = 4.49 \ln \frac{p_3}{p_4} + 1{,}11 \left[\frac{m^2}{m^3 min}\right] \tag{2.5}$$

By knowing the valve area, a minimum length of the cylinder to accommodate the ports can be found, this length corresponding to a fraction of the cylinder volume equal to $1-\varepsilon_v$.

2.3.5. Power rating – finding the swept volume

By drawing a predicted indicator diagram, the engines indicated power rating can be predicted. As all the parameters characterizing volume were assumed to be non-dimensional in aforementioned calculations, the result in form of the field under the diagram will be given as a specific value, of work output, per stroke, per volume unit, or :

$$L_{spec} \cong \frac{P_1+P_2}{2} \cdot (\varepsilon - \delta) + \frac{P_2\varepsilon - P_3\varepsilon_v}{m_{exp}-1} + (1-\varepsilon_v) \cdot \frac{P_3-P_4}{2} - \frac{P_5\delta - P_4\varepsilon_v}{m_c-1} \tag{2.6}$$

$$L_{spec} = N_i \cdot \left(\frac{1}{V}\right) \cdot \left(\frac{30}{n}\right) \tag{2.7}$$

Values of V and n can then be adjusted to produce the desired power output N_i. As shown in paragraph 2.1.2., increasing the rotational speed n will increase throttling loss and therefore the specific power output. The equation 2.6. giving the specific work is based on calculating the absolute work of processes mentioned in previous paragraphs with the mentioned approximations.

3. Modeling the engine

3.1. Mass and energy balances for the inlet and outlet

In actuality, the working cycle of the engine includes open processes. Because of that, it must be noted:

$$m_2 = m_{res} + m_{in} \tag{3.1}$$
$$m_{res} = m_5 = m_4, \tag{3.2}$$
$$m_3 - m_4 = m_{out} \tag{3.3}$$

m_{res} being a residual mass left after discharge, m_{in} and m_{out} being the intake and discharge masses respectively. In a steady state, it can be said:

$$m_{in} = m_{out} \tag{3.4}$$

Note, that this is not true for when the engine is starting cold and empty. For calculation convenience, it can be said:

$$m_{in} = m_{inV} + m_{inP} \tag{3.5}$$

Where the indexes V and P denote isochoric (5-1) and quasi-isobaric (1-2) intakes respectively.
Nusselt [5] proposed the following energy balance equation for an intake process in a quasi-ideal engine:

$$m_{in}i_1 + m_r u_5 = m_2 u_2 + P(V_2 - V_5) \tag{3.6}$$

This can be re-assigned and re-written for an engine with port throttling as:

$$m_{in}i_{in} = m_2 u_2 - m_r u_5 + \int_{v2}^{v1} p(V)dV \tag{3.7}$$

As the intake has been divided, new balances must be written for the separate phases. Those can be expressed as:

$$u_1(m_{inV} + m_{res}) = m_{inV} i_{in} + m_{res} u_5 \tag{3.8.a}$$

$$u_2(m_{in} + m_{res}) + F_0 = u_1(m_{inV} + m_{res}) + m_{inP} i_{in} \tag{3.8.b}$$

$$\text{Where: } F_0 = \int_{v2}^{v1} p(V)dV \tag{3.8.c}$$

Which sums to 3.7 when 3.5. is taken into account.

Those equations were implemented in the model for the intake, divided into two phases – isochoric, and quasi-isobaric. F_0 is calculated numerically, based on an approximated admission curve, which in this case is a parabola. The model dynamically calculates the coefficients of the parabola based on the pressure on the inlet and cut-off, based on the Strahl equation, the parameters of the Strahl equation being taken either from the properties of steam or direct user input. The calculation process for the cycle has to be made iteratively, and it is assumed, as in a physical engine that during the first iteration (first engine rotation), $m_{res} = 0$. This value will change with consecutive iterations until a steady state is reached, exactly as in a real engine starting. This erases the term u_5 also, and the same applies, but the process of its calculation is described in the paragraph 3.2.

On the outlet, it is needed to know the parameters of steam at point 4. To calculate them, it was assumed that the exhaust process can be approximated by a open process in form of an iso-energetic process, with u=idem. This is verified by Ochęduszko [4], who arrives at the same conclusions via energy balancing. Therefore, one knows what two independent parameters are – in the form of the specific internal energy u_3 and the pressure P_4 which is the same as the pressure in the condenser. Parameters at 3 are known by the method described in the paragraph 3.2.

Thus, using the two parameters the specific volume v_4 can be obtained. Due to the expansion in the process of emptying the cylinder, $v_4 \gg v_3$, that is to say it can be even 10 times larger (this is a symptom of the incomplete expansion loss). By this it can be calculated:

$$m_4 = m_{res} = \frac{V_4}{v_4} \tag{3.9}$$

This is used as the residual mass in the next iteration.

Analytically, the enthalpy on the outlet can be ascertained from a balance given by Nusselt [5], using the assumption that the outlet is an open, isochoric process (straight vertical line 3-4 on the P-V diagram), which gives about a 2% error as was found by calculation. This is significant when the isentropic efficiency of the machine is being calculated. Therefore, it was decided to arrive at the value by integrating the indicator diagram. The model uses the trapezoid method to calculate the cycle work $L_{cyc.}$

$$m_{in}(i_{in} - i_{out}) = \int_{P_4}^{P_1} V(P)dP = L_{cyc} \tag{3.10}$$

This allows to ascertain the average outlet enthalpy, and the work given by one side of the cylinder in one period. Because the engine is double acting:

$$N_{ind} = \frac{2L_{cyc} \cdot n}{60} \tag{3.11}$$

3.2. Drawing the real gas cycle indicator diagram

The process of drawing the indicator diagram for a real gas engine model employs steam functions built in EES, balances given in 3.1. and the empirical relations discussed earlier.

3.2.1. Admission 1-2

The line p=f(V) is assumed to be a parabola drawn as detailed in 3.1 the parameters of steam at point one are taken as $P_1=P_{in}$, u_1, where u_1 is determined from the balances 3.8, for steady state, and $u_1=f(i_{in}, P_{in})$ for the first, "cold" period. Those parameters are enough to determine others, such as the specific volume v_1, entropy s_1 and the temperature T_1. A similar procedure is utilized for point 2, only that the internal energy u_2 is determined from the equation 3.8.b. Note, that the process 1-2 is an example of non-isenthalpic throttling, that is throttling with extraction of a technical work, where the elementary work is evaluated as L=vdp. This is included in the model via the equation 3.8.b which puts this in term of internal energies and absolute work instead, which is mathematically equivalent. For the next step, the specific entropy is determined, as:

$$s_2 = f(P_2, u_2) \tag{3.12}$$

In this step, also, by the relations 3.12, the mass $m_{inP}=m_2-m_1$ is determined.

$$m_1 = \frac{V_1}{v_1}, m_2 = \frac{V_2}{v_1} \tag{3.13}$$

Note has to be made of the fact that the specific enthalpy at point 5, after compression can be higher than the inlet enthalpy, as can the temperature.

3.2.2. Expansion 2-3

Knowing the parameters P_2, v_2, pressures for increasing volume, for the volume increasing in set increments can be determined, as:

$$\frac{V_i}{V_2} = \frac{v_i}{v_2} \; and \; P_i = f(v_i, s_i) \tag{3.14}$$

It can be assumed that the expansion is isentropic [12], therefore, s_i=idem=s_2. This process is to continue until point 3.

3.2.3. Exhaust 3-4-4'

For the indicator diagram, an approximation is made for the exhaust line between points 3-4-4', drawing a straight line between points 3 (P_3, V_3) and 4 (P_4, V_4), where $P_4=P_{out}$, or the condenser pressure, the line 4-4' being horizontal. At $\varepsilon_v=0,9$, this amounts to 10% of a volume change, therefore a small diagram area, and the errors are tolerable. The short line 4-4' is modeled as an isobaric process at P_{out}, with the parameters of steam being constant, as in the point 4.

For early exhaust, a formula was proposed by Schule [11, 13], determining the pressure P_i at a given point i:

$$log\left[\frac{p_3}{p_i}\cdot\frac{\delta+s_3}{\delta+s_i}\right] = 0.638\cdot\frac{\mu\sqrt{x_3}}{w_{exh}\cdot(\delta+s_3)}\cdot\frac{A_{avg}}{A_{max}}\cdot(\alpha_i - \alpha_3) \tag{3.15}$$

In this formula the symbols s correspond to the non-dimensional swept volume V_i/V, w_{exh} is the average outlet velocity, x in this formula is steam quality (dryness), A_{avg} is the average valve port area, with A_{max} being the maximum and μ is the discharge coefficient (assumed as 0,4). The angles of the crank at the given point in degrees. This formula is valid till $p_i/p_{out}=1,4$ [11], which entails its validity until p=0,42 bar. As in the uniflow engine, there is no real exhaust "stroke", the rest of the curve until the valve closes can be without great error drawn as a straight line or extrapolated further from the formula. The issue of finding the average area A_{avg} can be solved numerically, by drawing a function A=f(h), where h=l-x, and x=f(α) which is a standard piston-crank position equation (3.17) [9]. In these equations x [m] signifies the piston distance dead center. For round valves as in the discussed engine A=f(h) is the formula for circle segment area times the number of port holes, ergo [14]:

$$\frac{A}{n} = r^2\cdot\arccos\left(\frac{1-h}{r}\right) - (r-h)\sqrt{2rh-h^2} \tag{3.16}$$

Where r is the hole diameter. The crank equation is:

$$x = R\left[(1-cos\alpha)+\frac{\lambda}{4}(1-cos2\alpha)\right],\ where\ \lambda = R/L \tag{3.17}$$

Schule [13] recommends to adjust this for steam expansion in the cylinder space, by the formula below:

$$A' = A\cdot\frac{\delta+s_3}{\delta+s_i} \tag{3.18}$$

The average outlet velocity is defined as:

$$w_{exh} = \frac{A_{piston}\cdot c_f}{A} \tag{3.19}$$

Where c_f is the mean piston speed during a single period. This "engine constant" does not correspond to the actual velocity in the outlet, which was estimated as even ten times higher in the model (by evaluating the volume of the residual mass for the average outlet parameters as in paragraph 3.1, and dividing that flow by the valve area in a given valve opening time). In fact, velocities as low as w_{exh}=4 m/s can be found by the equation 3.19 for the given engine. For such low values the pressure p_{out} is actually reached quicker than for higher ones. If the assumption is made that p_{out} is reached at the dead center, then seeing that the equation 3.15 binds p_i in an exponential relation to the other terms, p_i can be approximated by an exponential function going through points 3 and 4. This was the relation implemented in the dynamic model. The assumption is verified by the diagrams provided by Stumpf [1] and by solving 3.15 for a critical value w_c, values of variables for "i" taken as in point 4. It can then be seen that for an engine of the considered dimensions $w_{exh} \approx w_c$. Therefore:

$$p_i = a_{exh} \cdot \exp\left(b\frac{V_i}{V_4}\right) \tag{3.20}$$

Where the constants are to be found by solving two equations for point 3 and 4. However, it is recommended that the formula 3.15 be used for engines with conventional exhaust.

3.2.4. Compression 4'-5

Analogous to expansion, knowing the parameters P_4, v_4, pressures for increasing volume, for the volume increasing in set increments can be determined, as:

$$\frac{V_i}{V_{4'}} = \frac{v_i}{v_{4'}} \; and \; P_i = f(v_i, s_i) \tag{3.21}$$

It can be assumed that the compression is isentropic, therefore, s_i=idem=s_4. This process is to continue until point 5, that is, until $V_i/V_4=\delta$.

3.2.5. Isochoric filling 5-1

This is modeled as a vertical, isochoric line between points 5 and 1 parameters of the steam at point 1 being determined from the balances given in 3.1, namely 3.8.a, the pressure at point 1 being assumed as $P_1=P_{in}$, the volume corresponding to clearance δ.

3.3. Predicted engine characteristics

The cut-off ε and the rotational speed n can be treated as variable parameters. The result of varying cut-off ε at constant n = 600 rpm is shown in figure 5.

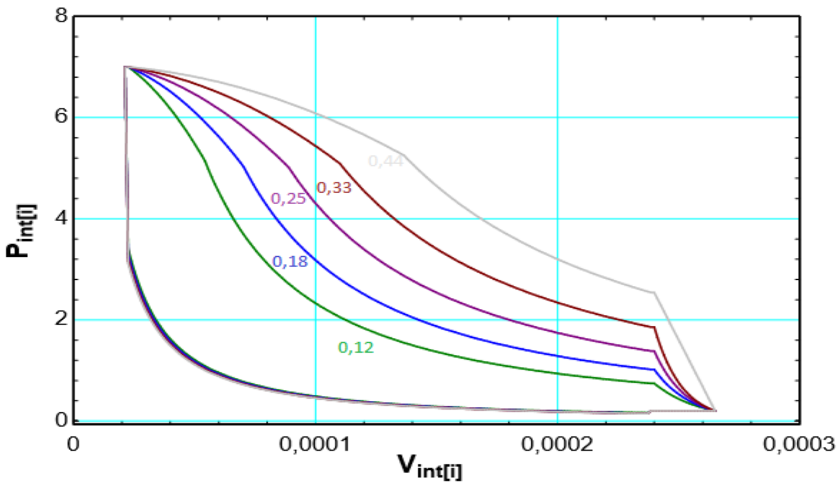

Fig.5. Indicator diagrams for different cut-off (denoted) values at P_{in}=7 bar T=350C, P_{out}=0,2 bar

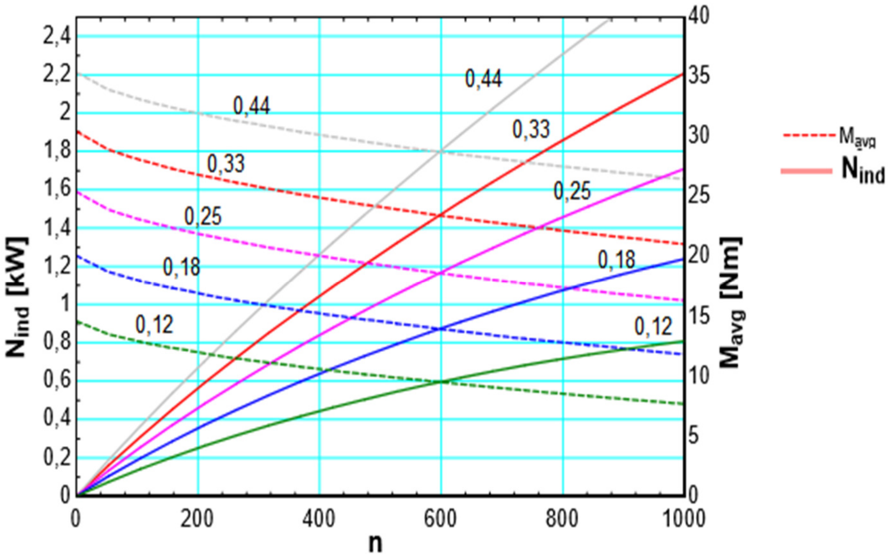

Fig.6. Power (N) and torque (M) by rpm (n) at different cut-offs

Figure 5 shows how the indicated power varies with different cut-off parameters and that with increased cut-off the incomplete expansion loss increases. This has a detrimental effect on the isentropic efficiency of the engine, as shown in figure 7. The torque and power characteristics will show however (fig. 6), that using the engine on low cut-off values is not practical. It can be also seen that the growth of power with shaft speed is not linear, and this is because of the dropping torque. Torque goes down due to the increased loss of throttling in the inlet ports.

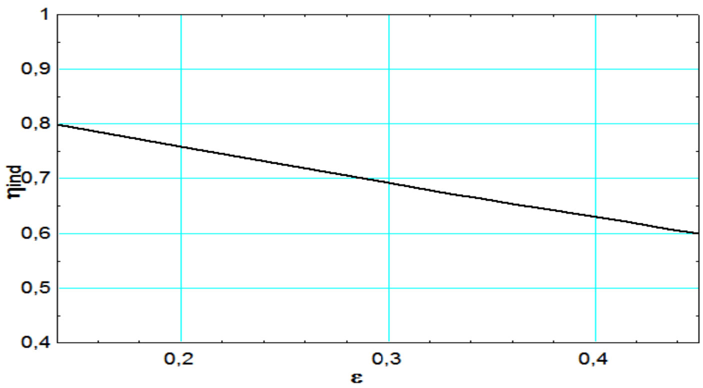

Fig.7. Isentropic efficiency to cut-off at 600 rpm

The efficiency values at different values of n are difficult to compare, because the mass of steam in the cycle also changes with throttling. It is important to note, that the model does not take wall losses into account, therefore in a real engine the values could be lower. They are however within reason, compared to the table 1.

4. Mechanical design of the expander

4.1. Overall design

The engine was designed as a horizontal unit, with a double acting cylinder, piston valves, and an extra outlet valve for counter-pressure work, envisaging for eventual condenser malfunction or the need for process steam. The maximum outlet pressure is therefore a set value. Ease of manufacture was a consideration, therefore most parts are designed to be easily machined, or cut from plate by laser or water jet. A view of a computer model is shown in the illustrations below – fig.8. The engine has been designed as a horizontal, with an open cross-head. The cylinder is to be made of cast iron, the cylinder sleeve of bronze, and the piston of aluminum to minimize inert mass. Using piston rings from PEEK or Teflon is recommended, to decrease the troubles of oil separation from the condenser [16], although their performance will have to be evaluated in an experimental study. Bearings UCP207 [20] are chosen for the prototype design due to their availability. The cylinder dimensions are given in figure 1.

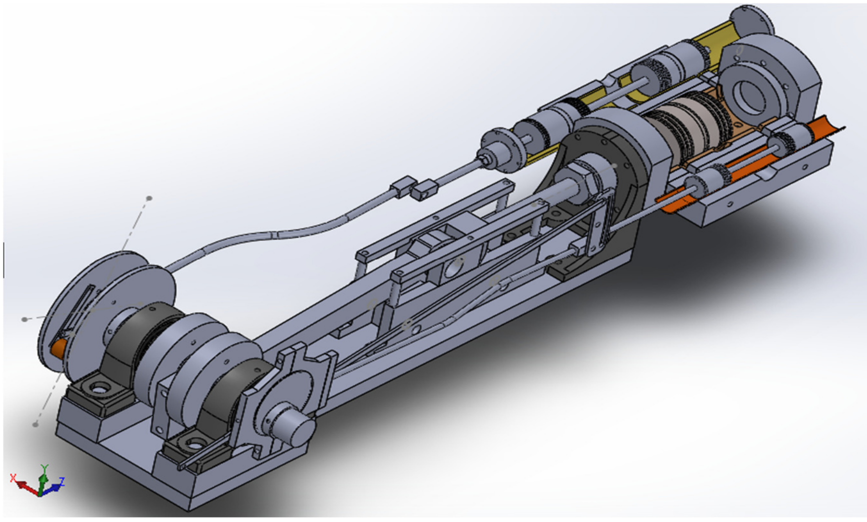

Fig.8. Proposed design cut-away

4.2. Inlet valves

To minimize throttling mentioned in chapters above, the objective should be to provide a full opening of the valve for as much of a part of the duration of admission as possible. One way of facilitating that is to utilize cam-driven poppet valves. In small engines however, they create extra clearance space. Mechanical simplicity was also an objective. Therefore piston valves were chosen. Piston valves are characterized by stroke, exhaust lap, inlet lap, and an angle of advance. Lead (angular and linear) is a by-product of those [11, 10]. In the particular case, only inlet is considered, therefore no exhaust lap is evaluated. The movement of the valve piston is characterized, just like the piston by the equation 3.17. The difference is in the values of R and L. R being the eccentricity of the valve eccentric, and L the valve rod length. Because for the valves L >> R, the rod length term can be neglected. The phase is also different than the phase of the crank by an angle of advance $-\varphi_v+90^o$. Also, it is more convenient to consider the distance from valve chest center, not dead centers. Therefore:

$$x_v = R_v sin(\alpha + \varphi_v) \quad (4.1)$$

Note, that this is a function of the main (piston) crank angle. The valve port opens when the inner edge of the piston reaches the port wall or when x_v=i. A graphical representation of valve kinematics was proposed by Zeuner [15]. This is done by drawing a circle of the radius R_v. A diameter line is then drawn, at angle φ_v to the vertical position. Next two small circles, of the diameter R_v are drawn on the diameter line, each with their center being at ½ R_v from the large circle edge – therefore meeting at the large circle center ("O"). Next from the point O a curve of the radius length is to be drawn within the upper circle, and one of radius e in the lower one. Next, analogously, curves of a i+b and e+b radius are to be drawn. The chords of the small circles between the curves represent valve openings. For the discussed engine, drawing the exhaust circle can be disregarded. The diagram is referred to as a Zeuner diagram, and one for the discussed engine inlet valve is shown in the figure 9.

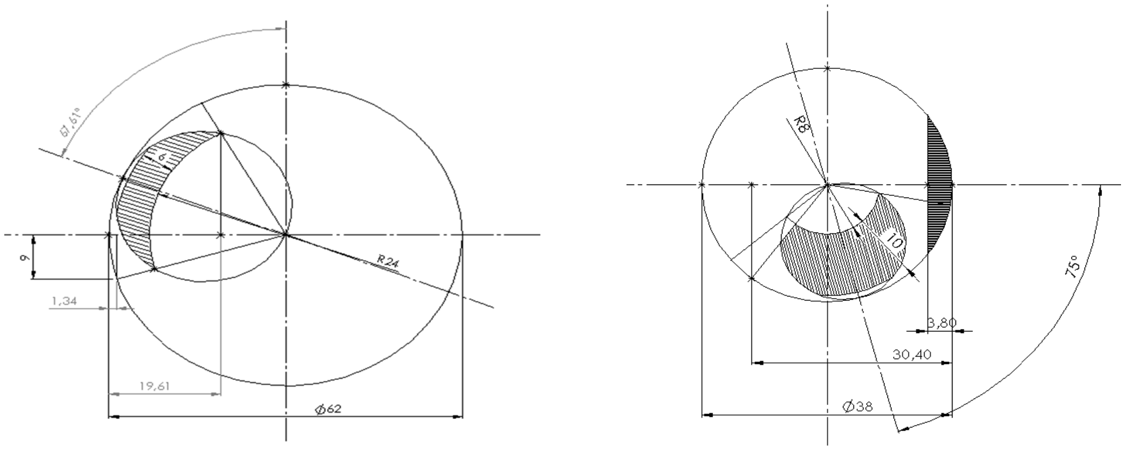

Fig.9. Zeuner valve diagrams for; left – inlet valves, right – exhaust and supplementary exhaust

When designing an engine, the value of length is an unknown, as is R_v. What is known from previous calculations is b. With the objective being the minimization of port throttling, it can be assumed, that R_v>i+b, so that full opening is reached. Also, it is imperative that full opening is reached for most of the admission phase. What is also known, by assumption is the cut-off point ε. A value of lead can also be assumed (angular lead of around 15° can be recommended). The Zeuner diagram gives the valve opening as a function of crank angle (counted from the point A), therefore A to D represents the piston movement. Lines from point O to the points of intersection of the small upper circle and the "i" curve represent angles of lead and cut-off, therefore the corresponding positions on the line AD corresponds to their linear values – the corresponding distances from a to the cut-off point is therefore known. Ergo, using those assumptions, by trial and error the values of φ_v and length can be reached. In the proposed engine φ_v=67,61° and i=24 mm, which means the valves can be considered "long-lap", R_v=31 mm. It can be noticed by working with Zeuner diagrams, that long lap valves are the only way of preventing throttling, they do however have mechanical disadvantages (large inert masses), therefore are only suitable for smaller units. It should be noted, that lead is also a subject to trial and error calculation, as it is responsible for the admission curve shape, and pressure loss (paragraph 2.3.2).

For the proposed engine, the valve pistons were designed as units turned from aluminum, screwed on a common valve rod to facilitate for adjustment, and with space for several piston rings, to be made of PTFE or PEEK to minimize the necessity of lubrication [16].

4.2.b Supplementary exhaust valves

The method for their design is essentially identical. It is however vital to keep them as close to the center as possible, this is done by solving the equation 2.2 for ε_v with p_4 being the outlet pressure. For the engine this is assumed to be 1.2 bar, therefore the supplementary valve non-dimensional position ε_v'=25%. Those valves are open for a longer duration of time, and their quick opening is not an issue. Therefore the ports were designed to be considerably long. A Zeuner diagram representing their opening and the opening of the proper exhaust ports has been included (Fig.9, right). Using separate valves has the benefit of not tying their phase with the inlet valve phase, and not cooling the inlet valves with exhaust steam. A special kind of Stephensons linkage is proposed, tied to one eccentric and the shaft. Putting the valve link in the "shaft" position effectively entails shutting the supplementary valves down.

4.3. Stress and fit calculations

Several of the engine components are a subject to considerable stress, often due to periodically changing forces. It was decided based on literary data, to perform a stress analysis of the following parts for the noted stresses:

1) Cylinder head bolts – stretching
2) Valve block bolts – stretching
3) Piston rod – buckling
4) Connecting rod – buckling and stretching
5) Crankshaft – torsion and bending
6) Strongback bolts – stretching

The acting forces are analogous for all elements besides 6 and 1, being the maximum force acting on the area by steam pressure, equal to:

$$F_p = P_{max} \cdot A \tag{4.2}$$

Where the area A is the piston cross-section for either the main piston or the valve piston (for "2").

While the force acting on 6 and the extra force on 1 is calculated as the force necessary to keep the element stationary by friction, with the force F_{lin} acting on it, from:

$$F_s = \frac{F_{lin}}{\mu} \tag{4.3}$$

Where μ is the coefficient of friction, assumed as μ=0,4 for cast iron on steel [17]. In element 6, the $F_{lin}=F_p$ whereas on 1, $F_{lin}=G_{cyl}$, or the cylinder weight.

For stretching, the formula used is:

$$\sigma = \frac{F}{nA} < k = k_{mat}/x \tag{4.4}$$

Where the value sought is A or n, n being the number of elements, A their cross-section, k_{mat} the yield point for the given material and x a safety coefficient. To take account for fatigue, the safety coefficients are taken from Neuman [10].

Buckling is calculated in terms of the second moment of inertia being large enough for the critical force not to be reached. The lengths of elements are assumptions based on design geometry.

$$F_{crit} = \frac{\pi^2}{x} \cdot \frac{EI_{el}}{l_{el}^2} \tag{4.5}$$

E is the Young modulus, I_{el} the moment of inertia, l_{el} the elements length x is again given by Neuman. The calculation results from 1, 2, 3, 4 and 6 are presented in tables 4 and 5. Point 5 due to its more complicated nature is outlined in the paragraph 4.3.b. As the engine is a small unit it was seen as fit to design the crank as a press-fit, without keys or any other torque transferring elements. Calculations pertaining to the fit are also presented there.

Table 4. Stretching stresses

Element	Cylinder head bolts	Valve block bolts	Strongback bolts	Connecting rod
x [-]	5	5	5	8
Material/type	5.8 bolt	5.8 bolt	8.8 bolt	S275
Re [MPa]	400	400	640	275
F [N]	2728,57	879,64	6355	2542,17
Dimensions	M6	M6	M3	20x30mm
A_{el} [mm^2]	17,88	4,98	17,88	600
n_{el} [-]	6	6	4	1
A_{sum} [mm^2]	107,304	29,45	71,54	600
σ [MPa]	23,69	29,87	88,84	4,23
k [MPa]	80	80	128	34,38

Table 5. Buckling stresses

Element	Piston rod	Connecting rod
Cross-section shape	Circle	rectangular
Formula for I	$\frac{\pi d^4}{64}$	$\frac{bh^3}{12}$
F [N]	2542,17	2542,17
L [mm]	320	350
E [GPa]	200	200
I_{min} [m^4]	$2{,}5 \cdot 10^{-9}$	$2{,}99 \cdot 10^{-9}$
Dimensions sought	d = 15 mm	b = 12 mm h = 15 mm
Actual dimensions	d = 20 mm	b = 20 mm h = 30 mm

4.3.1. Crankshaft

The crankshaft was evaluated by using the Huber hypothesis [19], to check if the considered dimensions are able to withstand torsion and bending by the maximum piston force (4.2). Huber writes, that:

$$M_{red} = \sqrt{M^2 + \frac{3}{4} M_o^2} \tag{4.6}$$

Where M is the bending moment, M_o the twisting moment, and M_o a "reduced" value, compared to simple bending, therefore:

$$\sigma = \frac{M_{red}}{W} \tag{4.7}$$

Where W is the bending stress factor I/y.

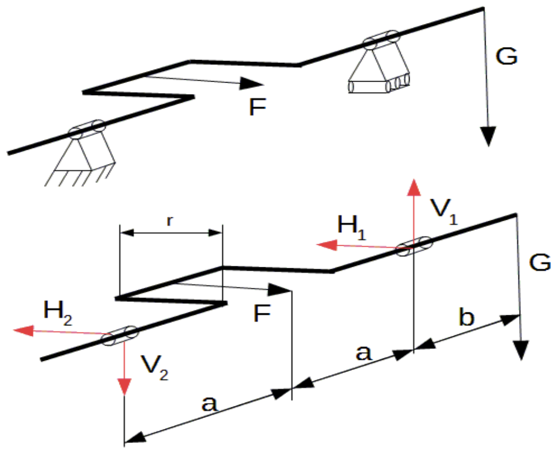

Fig.10. Forces acting on the crankshaft

F is the piston force, G the weight of the flywheel (mass assumed 4,5 kg). Therefore, because the point under most stress is located z=a-0,51 from the mid-crank point [10], it can be said that the bending moment there is:

$$M = \sqrt{[H_2(a+z) - Fz]^2 + [V_2(a+z)]^2} \tag{4.8}$$

And the maximum torsion is the same as maximum torque (not exerted by the crank in this phase)

$$M_o = Fr \tag{4.9}$$

Because the bearings are UCP207, therefore l = 49,2 [20]. Other assumptions include, a = 54,6, r = 36,5, based on the stroke. The cranks are assumed as 20 mm thick. The shaft is assumed to have d=35 mm. Therefore, based on the mentioned equations, **σ = 20,48 MPa**. This is well below the yield point for S275 (275 MPa).

The crankshaft fit is calculated as a press fit necessary to withstand the maximum torsion (this is based on the assumption that the Young's modulus is 210 GPa), by calculating the minimum pressure on it corresponding to the fit [20]. The negative clearance has to facilitate for the pressure on the shaft being at least:

$$p_{min} = \frac{2kM_O}{\mu\pi l d^2} \tag{4.10}$$

Where d is the shaft diameter, k a safety coefficient (here it was assumed k = 2). The minimal negative clearance is given as:

$$W_{min} = p_{min} d \left(\frac{c_1}{E} + \frac{c_2}{E}\right) \tag{4.11}$$

Where the c are factors compensating for the part being drilled through, defined as:

$$c_1 = \frac{1+\Delta_1{}^2}{1-\Delta_1{}^2} - \upsilon_1 \; and \; c_2 = \frac{1+\Delta_2{}^2}{1-\Delta_2{}^2} + \upsilon_2, where \; \Delta = d_{out}/d_{in} \tag{4.12}$$

Assuming the outer diameter of the crank as 90 mm (as the radius has to be larger than the theoretical crank radius plus half the crank thickness), the minimum clearance is W_{min}=23,67μm.

5. Conclusions

A small expander was designed, for the purposes of serving as a part of a possible, future, experimental micro-CHP set-up. The calculations have showed the expander to have useful working characteristics and high efficiency for a small machine. A computational model was created, based on old steam engine related literature and new works for a reciprocating steam expander that can be used to evaluate any future designs or work be a part of a model for a whole CHP system using such an expander. Moreover, basic mechanical design procedures for expanders were outlined to serve as reference in future work.

It was shown, that isentropic efficiencies for small expanders can reach values in the realm of 70%, and even up to over 80% at the cost of power density (as this is facilitated by lower cut-offs that demand the engine to be of a larger size to have a given amount of power). A simple evaluation for a conventional Rankine cycle amounts this to a cycle efficiency of 13,7% for the nominal parameters (table 2), and producing 7,5 kW of usable heat (at 69 °C) and 1,4 kW of (indicated) power.

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Projekt i modelowanie rozprężarki tłokowej typu przelotowego, studium przypadku

Słowa kluczowe: rozprężarka tłokowa, maszyna parowa, obieg Rankine'a, straty ciśnienia, rozprężanie, spadek ciśnienia w zaworze

Streszczenie

Przedmiotem pracy jest projekt tłokowego silnika parowego, jako rozprężarki dla siłowni parowych o małej mocy. Uznano, że dla założonych parametrów pary (7 bar na wlocie, 0,3 bar w skraplaczu) i przyjętej mocy w granicach 1,4 kW, konstrukcją optymalną ze względu na sprawność wewnętrzną i prostotę konstrukcji będzie maszyna o przepływie jednokierunkowym [1, 4]. W celu określenia jej wymiarów, stworzono uproszczony model obliczeniowy bazujący na założeniu, że sprężanie i rozprężanie pary to proces politropowy, w którym nadto wzięto pod uwagę straty ciśnienia pary na wlocie do maszyny, korzystając ze wzorów empirycznych [11]. Następnie, uzyskane w ten sposób wymiary wykorzystano jako dane dla kolejnego modelu, bazującego już na założeniu, że czynnik roboczy stanowi gaz rzeczywisty. Model ten utworzono w programie EES. Opiera się on na rozwiązywaniu równań bilansów w celu określenia parametrów pary w punktach charakterystycznych wykresu p-V. Następnie wykres jest kreślony przez program, zakładając przebieg krzywych wlotu i wylotu jako odpowiednio paraboli i funkcji wykładniczej (co stanowi dopuszczalne uproszczenie funkcji wylotu podanej przez Schulego [13] w tym szczególnym przypadku), oraz krzywych sprężania i rozprężania jako zbioru punktów dla procesu izentropowego (w ten sposób bierze się pod uwagę zmienność wykładnika izentropy). Zmieniając w modelu parametry prędkości obrotowej n i napełnienia ε rysuje się następnie charakterystyki mocy, momentu obrotowego i sprawności izentropowej maszyny. Osiągalne są sprawności izentropowe rzędu 0,8 przy małych mocach, oraz rzędu 0,7 przy mocach około nominalnych (1,4 kW). W dalszej części pracy pokazane są szczegóły obliczeń konstrukcyjnych proponowanej maszyny, w tym obliczenia zaworów wlotowych i obliczenia wytrzymałościowe wału i innych elementów. Na bazie ich wyników stworzono projekt silnika (w oprogramowaniu Solidworks).