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CONIC SECTIONS IN AXONOMETRIC PROJECTION

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Abstract: This paper demonstrates the classification and examples of the conic sections done in axonometry without the help of collineation. Presently there are no collineation transformations in most descriptive geometry programs. Therefore these sections were made without collineation with the use of only common elements.

Keywords: cone, conic sections, axonometric projection, collineation

1 Introduction

Currently, axonometric projection is primarily used for visualization purposes in teaching of descriptive geometry. However, you can also do many construction operations, especially based on the construction of common elements. Eventually, as a result of such operations, we obtain not only the right solution, but also an image which directly refers to imagination.

For this purpose, I would like to propose a classification and determination of conic sections in axonometry.

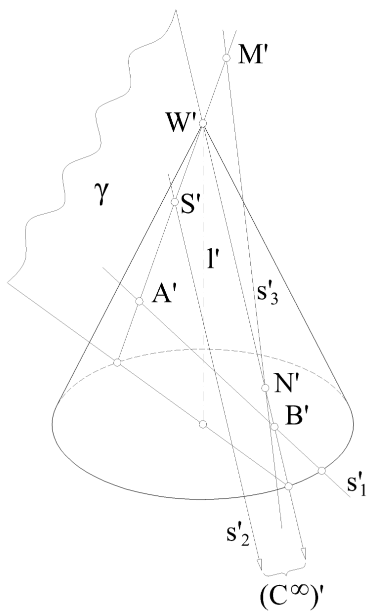

Figure 1: Classification of conic sections

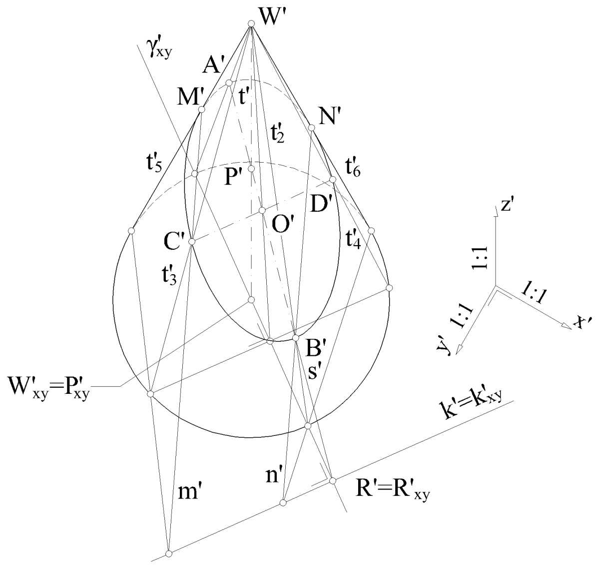

Figure 2: The section of the cone is an ellipse

These sections can be easily done in axonometry with the help of collineation, which is a projection transformation, and the section of the cone is the plane of the collineation image of the circle of its base.

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Now, let's take into consideration the cone and the α plane. The vertex W of the cone does not belong to the α plane. The plane of symmetry of the cone is perpendicular to the α plane. The edge $s=\alpha\cap\gamma$ pierces the cone at two different points.

- If these two points lie on one side of the vertex W then the conic section is an ellipse (line s_1 and points A and B, Fig. 1).
- If one of these points is the infinity point, then the section of the cone is a parabola (line s_2 and points S and C^{∞}, Fig. 1).
- If these two points lie on different sides of the vertex W then the conic section is a hyperbole (line s_3 and points M and N, Fig. 1).

Let me now present examples of sections of the cone, which will be demonstrated in military axonometry.

2 Example 1

The base cone on the xy plane of the coordinate system and the plane $\alpha(k, P)$ are given. The line k is on the xy plane, and the point P is on the axis of the cone. In Figure 2 there is a projection of a cone and a line $k'=k'_{xy}$ and a point $P'\in l'$.

The γ plane of the symmetry of the cone perpendicular to the α plane intersects the cone in lines t_1 and t_2. The orthogonal projection of the γ plane on the xy plane is a line γ'_{xy}. The edge $s=\alpha\cap\gamma$ is determined by the points P and R. The line s pierces the cone at points $A=s\cap t_1$ and $B=s\cap t_2$. Thus the section of the cone with the α plane is an ellipse. Of course, the AB segment is the axis of the ellipse section. The ends C and D of the second axis perpendicular to the AB axis are on the lines t_3 and t_4. Those lines are obtained in the section of the cone with the $\beta\supset WO \wedge \beta\| k$ plane. The projections of the AB and CD axes are conjugate diameters A'B' and C'D' shown in the figure. The points M and N are the points of change of visibility for the ellipse and are on the cones lines t_5 and t_6: $M=m\cap t_5 \wedge m=\alpha\cap\delta(t_3,t_5)$, $N=n\cap t_6 \wedge n=\alpha\cap\varepsilon(t_2,t_6)$.

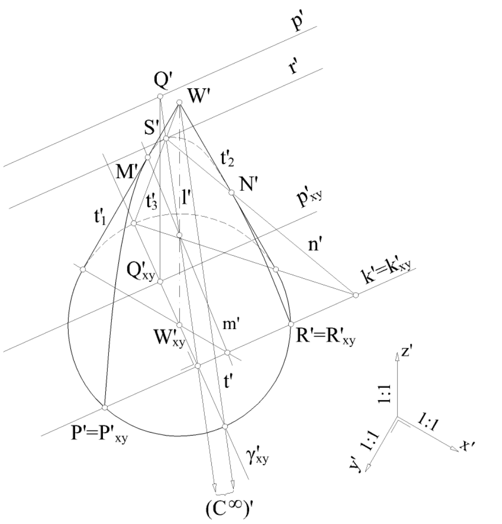

Figure 3: The section of the cone is a parabola

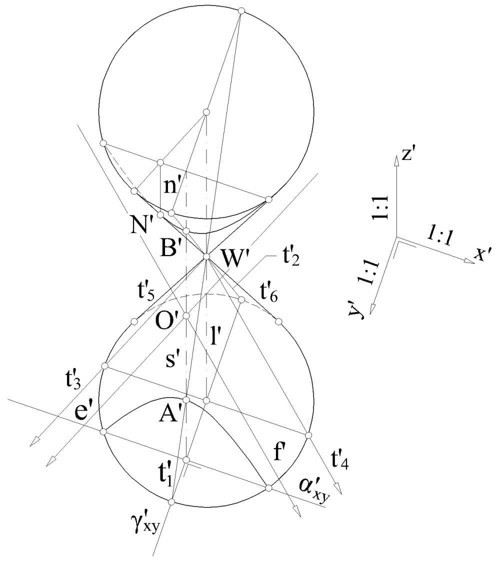

Figure 4: The section of the cone is a hyperbole

3 Example 2

Let be given a cone standing on the xy plane of the coordinate system and an α plane: $\alpha\supset p \wedge \alpha\| t$, $p\|(x,y) \wedge p\perp t$.

The γ plane is the plane of symmetry of the cone perpendicular to the line p. It is the plane of symmetry for the section of the cone with the α plane. The edge $s=\alpha\cap\gamma$ contains the point Q and is parallel to the line t. The line s pierces the cone at points S and C^{∞}. Thus the section of the cone is a parabola. The axis of the parabola is the line s, the point S is the vertex of the parabola, the line r is tangent to the parabola at point S. The edge $k=\alpha\cap(x,y)$ intersects the base of the cone at points P and R. These elements define the parabola of the section.

The points M and N of the visibility changes are on the lines t_1 and t_2 determined by the lines m and n: $M=m\cap t_1 \wedge m=\alpha\cap\delta(l,t_1)$, $N=n\cap t_2 \wedge n=\alpha\cap\varepsilon(t_1,t_2)$.

A parabola in Figure 3 is a projection of the section. Thus, an line r' was obtained tangent at the point S', point $(C^{\infty})'$ and points M', N', P', R' (one of them is enough).

4 Example 3

Given is the cone standing on the xy plane of the coordinate system and the plane $\alpha\perp(x,y)$ be given.

The orthogonal projection of the α plane on the xy plane is the line α'_{xy} (Fig. 4). The γ plane of the symmetry of the cone is perpendicular to the α plane. The edge $s=\alpha\cap\gamma$ is perpendicular to the xy plane and pierces the cone at points $A=s\cap t_1$ and $B=s\cap t_2$. Points A and B are on different sides of the vertex W. Thus the section of the cone is a hyperbole. Points A and B are the vertices of the hyperbole, and the line s is its axis.

Lines t_3 and t_4 are in the β: $\beta\|\alpha \wedge \beta\supset l$ plane. Hyperbolic asymptotes e and f intersect at the center point O of the segment AB and are parallel to the lines t_3 and t_4. The point N of the visibility change on the upper arc of the hyperbola is on the line t_6 and the line n: $N=n\cap t_6 \wedge n=\alpha\cap\varepsilon(t_6,l)$.

5 Conclusions

The classification and sections of the cone in axonometry were presented. It can be used in teaching of descriptive geometry because collineation is not used. These operations are usually done in the orthogonal projection. It is more advantageous to show this in axonometry because it provides also visualization.

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PRZEKROJE STOŻKA W RZUCIE AKSONEMETRYCZNYM

W niniejszym artykule przedstawiono klasyfikację oraz przykłady przekrojów stożka wykonane w aksonometrii bez użycia kolineacji. Obecnie w większości programów geometrii wykreślnej nie ma przekształceń kolineacyjnych. Dlatego przekroje te zostały zrealizowane bez kolineacji z wykorzystaniem tylko elementów wspólnych.

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