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Models of logistic support systems

Keywords

logistic support system, maintenance modelling, inventory models, procurement process

Abstract

We present an overview of some recent developments in the area of inventory planning and maintenance scheduling issues. The emphasis is on spare part inventory models, which authors divided into four main groups of models: models of optimal spare part inventory policy for system under PM, number of spare parts optimization models, storage reliability models, multi-echelon systems models. Later, in the paper there is considered the time dependent system of systems where the system total task must be executed during the constrained time resource. There is used the simulation approach to investigate the influence of parameters of the procurement process (order quantity, critical inventory level, lead-time length) on the system of systems behavior.

1. Introduction

The principal objectives of a technical system are to ensure the realization of continuous operational processes of its components. However, a population of units (aircraft engine components, computer modules, means of transport, etc.) that randomly fail but are completely repairable requires an effective maintenance infrastructure and a logistic system, that will be available when required. As a result, reliability and effectiveness of a technical system, being worked in changeable environment, cannot be analyzed in isolation, without taking into account the numerous links with its logistic support system.

According to the definition [2], [7], a logistic support system is the process-oriented subsystem of the technical system which supports its operational processes through the integration of all activities, being necessary to assure the effective and economical flow of needed materials and related information, and which supports the maintenance processes of this system in aspect of providing the necessary maintenance and support infrastructure. The emphasis here is on logistics as it pertains to systems throughout their planned life cycle. It provides the means to realize many supportability functions for a set of operational requirements within the intended operations and support environment. The most important functions, which must be assessed in order to fulfil the intended mission, are connected with [73]:

- providing the necessary supplies and services, including:
  - spare parts and components of technical equipment providing,
  - ensuring on-time realization of repair processes according to the requirements and maintenance policy
- sustainment of the operational objects in the functional up-state, what includes:
  - providing necessary equipment to assess the efficient functions of maintenance and support,
  - realization of maintenance processes of support equipment (see *Figure* 1).

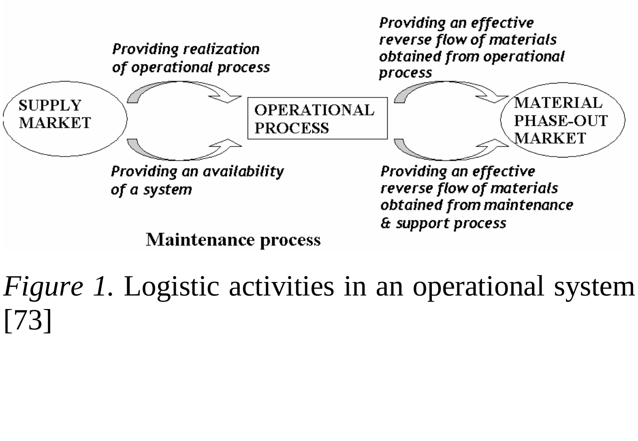

Figure 1. Logistic activities in an operational system [73]

The interest in development and investigation of maintenance problems has been extensively discussed in the literature since the early 1960s. The basic review in the area of maintenance modelling is prepared by Pierskalla & Voelker [53], where authors investigated discrete time vs. continuous time maintenance models, later updated by Valdez-Flores & Feldman [67]. For other surveys see e.g. [10], [48], [49], [50], [52], [57], [65], [71], [73].

However, most of the maintenance models investigated in the literature on reliability theory assume, that all the necessary logistic support resources, which include maintenance resources, support personnel, logistic information and data, spares and repair parts, and facilities, are immediately provided when it is desired. In practice, the repair capacity is not infinite, and logistic information may be unreliable. Moreover, the influence of a spare provisioning policy on the maintenance policy also cannot be ignored, since spares are ordered and carried in the limited quantity, and the procurement lead time is not negligible [73].

Throughout years, the importance of the logistic support functions, and therefore also of logistic support management has grown. The plethora of studies, which have addressed the problem of logistic support systems modelling, can be divided into four main groups being presented in *Figure 2*. A bibliography of the work done can be found in [73].

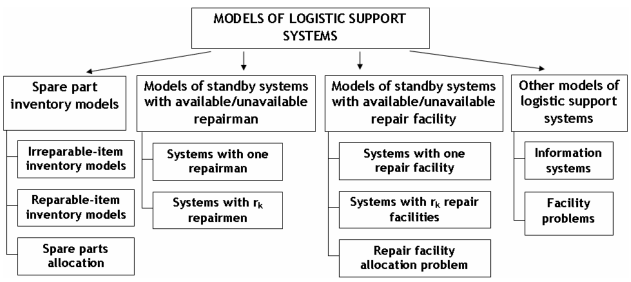

Figure 2. Classification scheme of logistic support system models [73]

The problem of providing an adequate and efficient supply of spare parts, in support of maintenance and repair of operational systems, has been researched for many decades.

Sufficient inventory level of spare parts has to be maintained to keep the system in operating condition. When the spare elements are under stocked, it may lead to extended system downtime and, as a consequence, have a negative impact on the system availability level. On the other hand, maintenance of excessive spares can lead to large inventory holding costs. Moreover, the requirements for planning the logistics of spare elements necessary in maintenance actions of operational systems differ from those of other materials in several ways: the service requirements are higher, the effects of stock-outs may be financially remarkable, the demand for parts may be sporadic and difficult to forecast, and the prices of individual parts may be very high.

Consequently, one of the most important decisions faced by maintenance managers is the determination of optimal stocking levels which involves finding the answer to the following questions, such that the total expected inventory costs are minimized:

- When to (re)order?
- How many items to (re)order?

There is a plethora of models in the literature on reliability theory regarding to the spares supply process optimization. A significant portion of them base on a classic inventory theory, where the procurement process parameters are usually optimized taking into account the cost constraints (see e.g. [55]).

Recently, many inventory papers which treat stock replenishment problems for stochastically failing equipment/systems are surveyed in [53] and updated by [10].

A survey of inventory system models, made in 1991 by Cho & Parlar [10], divides the existing models into three topical categories:

- irreparable-item inventory models,
- repairable-item inventory models: single-echelon case,
- repairable-item inventory models: multi-echelon case.

Irreparable-item inventory modelling have been narrowed down mostly to the cases of Markov processes implementation (see e.g. [63], [78]. Early studies in the area of repairable-item inventory modelling had primarily been focused on the military problems [15]. More recently, other applications have appeared.

Recent overview of models integrating spare part management and repair capacity is made by Guide Jr & Srivastava [25], which examines the various models and classifies them according to their solution methodology, single versus multi-echelon, and exact versus approximate solutions. Worth taking a note is also a survey done by Kennedy et al. [36] in which literature is reviewed according to management issues, multi-echelon problems, problems involving obsolescence, or repairable spare parts.

Consequently, the main problems being solved in the area of inventory planning and maintenance scheduling issues, taking into account the main factors influenced the optimal ordering policy definition (*Figure 3*), are:

- supply process parameters optimization taking into account the chosen maintenance policy constraints (see e.g. [44],
- storage reliability (see e.g. [31], [43],
- service level optimization in order to minimize the inventory costs (see e.g. [41],
- multi-echelon problems (see e.g. [5], [13].

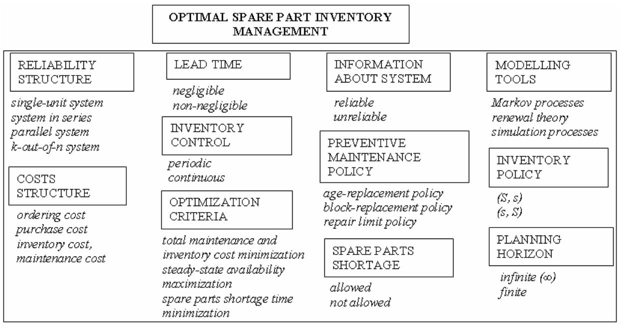

Figure 3. Main factors influenced definition of optimal ordering policy for a technical system [73]

Following the introduction, this paper is focused on spare part inventories optimization problem. Consequently, the paper is organized as follows: in the Section 2 we present an overview of the most often applied models. We do not aim to give a list of all papers that have appeared. Instead, we want to investigate the main ways of the inventory models development, presented in the recent literature. Later, there is provided an example of time dependent system of systems where the system total task must be executed during the constrained time resource, and a briefly summary.
There is used a "system of systems" conception to model the interactions between operational system and its logistic support system. According to the definition [14], the system of systems context arises when a need or a set of needs are met with a mix of multiple systems, each of which are capable of independent operation but must interact with each other in order toprovide a given capability. More information can be found in [74], [75, [76].

2. Spare part inventory models

The general classification scheme for spare part inventory models is presented in *Figure* 4.

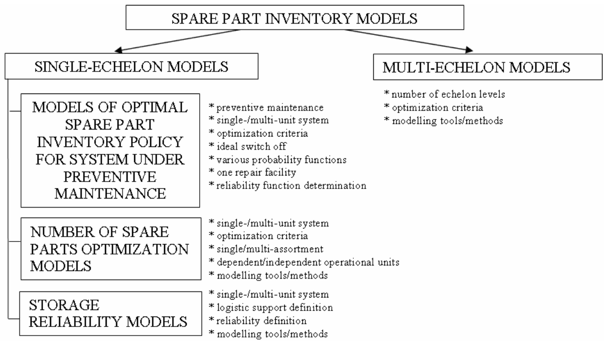

Figure 4. Spare part inventory models classification [73]

2.1. Models of optimal spare part inventory policy for system under preventive maintenance

First group of the models presented in the *Figure 4* regards to the works, which are aimed at spare part provisioning policy parameters optimization when maintenance policy of a technical system is known. The main classification scheme of the investigated models is presented in *Figure 5*.
There are two fundamental types of maintenance – preventive maintenance (PM) and corrective maintenance (CM). For PM demand for spare parts is predictable. For such maintenance it may be possible o order parts to arrive just in time for use, and it may not be necessary to stockpile repair parts at all. In case of unplanned repair, the consequences of stock-outs regards to system unavailability with significant costs that is why some kind of stock policy is necessary. It is natural in technical systems that only spare units which can be delivered by order are available for maintenance/replacement. In this case we cannot neglect a lead time for delivering the spare unit.
The main inventory policy parameters optimization criteria include availability ratio maximization (see e.g. [22]), minimization of maintenance and inventory costs (see e.g. [64]) or minimization of stock-out risk function (see e.g. [26]).
First models which investigate the possibility of spare part shortage due to delivery process performance regards to single-unit systems (see e.g. [64], [66]). In [64] authors consider a one-unit system where each failure is scrapped without repair and each spare is only provided after a lead time by an order. In the presented model the following policy is adopted: order for a spare is made at a pre-specified time instant t_o during an operating period of an original unit which is called a regular order. The lead time entails L_2 time units. After delivery, the original unit is replaced whether is operable or not. However, if the failure of the unit takes place before the time instant t_o emergency order is made at a

failure time instant immediately. After an emergency delivery, which entails L_1 time units, a failed unit is replaced, and the process repeats itself.

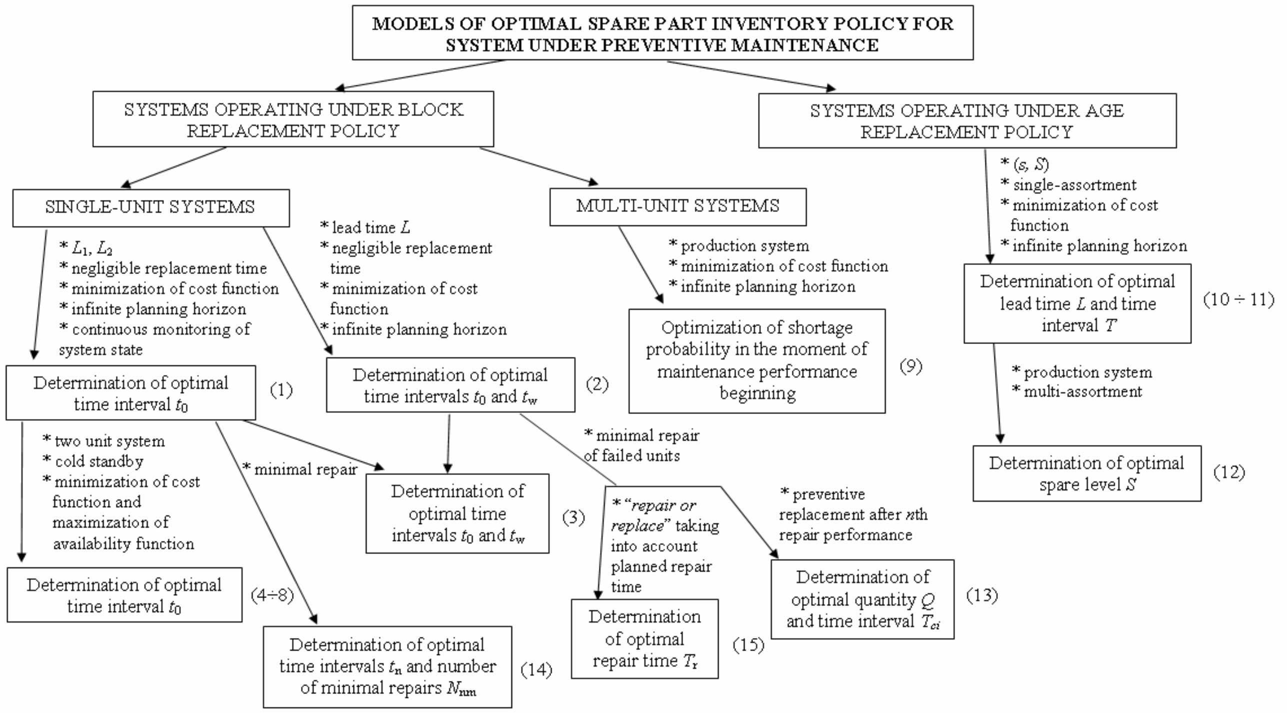

Figure 5. Models of optimal spare part inventory policy for system under PM [73]

Taking into account the following assumptions:

- infinite planning horizon,
- negligible replacement time of operating unit,
- system is continuously kept under constant observation till a pre-specified time instant t_o or till the instant of failure, whichever occurs first,

the expected cost per unit time in the steady state is given by the formula:

$$C_s(t_o) = \frac{c_m \int_0^{t_0} R(t)dt + c_{p1}F(t_0) + c_{p2}R(t_0) + c_{dw}\left[(L_1 - L_2)F(t_0) + \int_{t_0}^{t_0+L_2} F(t)dt\right]}{(L_1 - L_2)F(t_0) + L_2 + \int_0^{t_0} R(t)dt} \quad (1)$$

where:

c_{p1} – cost of spare element expedited order which is made at time instant t

c_{p2} – costs of spare element regular order made at time t_o

c_{dw} – cost of system downtime per unit time

$L_1(L_2)$ – random lead time for emergency (regular) order

c_m – cost of system observation proportional to the expected duration of observation

$R(t)$ – system reliability function

$F(t)$ – cumulative distribution function of unit

Presented model development can be found in [66], where the additional assumption is made: the operational unit replacement is made in one of two situations, whichever occurs first: when unit fails or when time of PM occurs at time instant t_w. The expected cost per unit time in a steady state is given by:

$$C_z(t_0, t_w) = \frac{c_{p1}F(t_0) + c_{p2}R(t_0) + c_{dw}\int_{t_0}^{t_0+L} F(t)dt + c_h \int_{t_0+L}^{t_w} R(t)dt}{\int_0^{t_w} R(t)dt + \int_{t_0}^{t_0+L} F(t)dt} \quad (2)$$

where:

c_h – cost of holding a spare unit in a stock per unit of time

L – random lead time

Another work, made by Dohi et al. [21], presents generalized order-replacement model arising in the spare part inventory management, which bases on the assumptions taken in [64] and [66]. There is considered a replacement problem for one-unit systems where each failed unit is scrapped and each spare is provide, after a lead time, by an order. If the original unit does not fail up to a pre-specified time instant t_o the regular order is made, and after a lead time L_2 the spare unit is delivered. The delivered spare element is put into inventory till the moment of original unit failure or till the moment of PM, whichever occurs first. On the other hand, if the original unit fails before the time instant t_o, the expedited order is made immediately at the failure time, and the spare takes over its operation just after it is delivered after a lead time L_1. In this situation, the regular order is not made. The function of the expected total cost per time unit is given by:

$$
\begin{aligned}
C_z(t_0,t_w) = \Bigg\{ & c_{sp}\left\{\int_0^{\infty}\int_0^{t_0} L_1 dF(t)dE_1(L_1) + \int_0^{\infty}\int_{t_0}^{t_0+L_2}(t_0+L_2-t)dF(t)dE_2(L_2)\right\} + \\
& + c_{p1}F(t_0) + c_{p2}R(t_0) + c_h\int_0^{\infty}\int_{t_0+L_2}^{t_0+L_2+t_w}(t-t_0-L_2)dF(t)dE_2(L_2) + \\
& + c_h\int_0^{\infty}\int_{t_0+L_2+t_w}^{\infty} t_w dF(t)dE_2(L_2)\Bigg\} \times \\
& \times\Bigg(\int_0^{\infty}\int_0^{t_0}(t+L_1)dF(t)dE_1(L_1) + \int_0^{\infty}\int_{t_0}^{t_0+L_2}(t_0+L_2)dF(t)dE_2(L_2) + \\
& + \int_0^{\infty}\int_{t_0+L_2}^{t_0+L_2+t_w} t dF(t)dE_2(L_2) + \int_0^{\infty}\int_{t_0+L_2+t_w}^{\infty}(t_0+L_2+t_w)dF(t)dE_2(L_2)\Bigg)^{-1}
\end{aligned}
\tag{3}
$$

where:

c_{sp} – shortage cost per unit of time

$E_i(t)$ – cumulative distribution function of L_i (i = 1, 2)

The problem of optimal spare ordering policies for two-unit cold standby redundant system with two dissimilar units is considered in [22]. The replacement problem is defined as follows: unit 1 begins working and unit 2 is in standby at time 0, and the planning horizon is infinitive. If unit 1 does not fail up to a pre-specified time t_o, the regular order for spares of both units 1 and 2 is made at time t_o. After a lead time L_2 the spares are delivered, and at the time t_o+L_2 all original units are replaced correctively/preventively by spares, irrespective of the states of original ones. Since two units are not identical. The order for two spares is always needed. On the other hand, if the unit 1 fails before the time t_o, the operation is switched to the unit 2 and the expedited order for spares of both units is immediately made at the failure time. All original units are replaced by spares just after delivery which lasts a lead time L_1. The switchover is assumed to be perfect and instantaneous. The state in which both units fail before delivery of spare units implies the system down.

To obtain the optimal ordering policy parameter, there are developed: the expected cost per unit time in the steady state and the stationary availability.

The expected time for one cycle is defined as:

$$
E[T_{c_j}(t_o)] = \int_0^{t_0}(t+L_1)dF_1(t) + \int_{t_0}^{\infty}(t_0+L_2)dF_1(t)
\tag{4}
$$

The expected inventory cost function for one cycle is given by:

$$
\begin{aligned}
C_z(t_0) = c_{sp}\Bigg\{ & \int_0^{t_0}\int_0^{L_1}(L_1-s)dF_2(s)dF_1(t) + \\
& + \int_{t_0}^{t_0+L_2}\int_0^{t_0+L_2-t}[(t_0+L_2)-(t+s)]dF_2(s)dF_1(t)\Bigg\} + \\
& + c_{rz}\Bigg\{\int_0^{t_0}\int_{L_1}^{\infty}(s-L_1)dF_2(s)dF_1(t) + \\
& + \int_{t_0}^{t_0+L_2}\int_{t_0+L_2-t}^{\infty}[(t+s)-(t_0+L_2)]dF_2(s)dF_1(t) + \\
& + \int_{t_0+L_2}^{\infty}[t-(t_0+L_2)]dF_1(t) + \int_{t_0+L_2}^{\infty}\int_0^{\infty} s dF_2(s)dF_1(t)\Bigg\} + \\
& + c_h\Bigg\{\int_0^{t_0} t dF_1(t) + \int_{t_0}^{t_0+L_2} t dF_1(t) + \int_{t_0+L_2}^{\infty}(t_0+L_2)dF_1(t)\Bigg\} + \\
& + c_{p1}F_1(t_0) + c_{p2}R_1(t_0)
\end{aligned}
\tag{5}
$$

where:

c_{rz} – cost per unit time incurred for the residual lifetime of the original unit, which is still operable

Applying the renewal theorem, the expected cost per unit time in the steady state is given by:

$$
C_s(t_o) = \frac{C_{sj}(t_o)}{E[T_{cj}(t_o)]}
\tag{6}
$$

Moreover, the stationary availability $A(t_o)$, defined as the probability that a system is operative in the steady state, is given by:

$$
A(t_0) = \frac{E[T_{oj}(t_0)]}{E[T_{c_j}(t_0)]}
\tag{7}
$$

where:

$T_{oj}(t_o)$ – effective time of a system for j-th cycle, given by the following formula

$$
\begin{aligned}
E[T_{o_j}(t_0)] = & \int_0^{t_0}\int_0^{L_1}(t+s)dF_2(s)dF_1(t) + \int_0^{t_0}\int_{L_1}^{\infty}(t+L_1)dF_2(s)dF_1(t) + \\
& + \int_{t_0}^{t_0+L_2}\int_0^{t_0+L_2-t}(t+s)dF_2(s)dF_1(t) + t_0 + \\
& + \int_{t_0}^{t_0+L_2}\int_{t_0+L_2-t}^{\infty}(t_0+L_2)dF_2(s)dF_1(t) + \int_{t_0+L_2}^{\infty}(t_0+L_2)dF_1(t)
\end{aligned}
\tag{8}
$$

Another very interesting problem regards to spare parts inventory planning in order to keep a production system in operating condition. Example of such a system consisting of n identical and stochastically independent production machines in k-out-of-n reliability structure is given in [70]. Operational process of the presented system includes planned machines shutdown, during which alee the failed elements in that maintenance cycle are replaced in order to increase system reliability.

The problem considered in the presented paper regards to a priori planning of spares inventories required for the maintenance during a phased mission. In the system, it is possible to replace failed elements only during overhauls performed between two phases. The replacement of the failed elements at the end of phase k may be done by spare parts remaining unused from the proceeding overhauls and by S_k items planned to become available at time point t_k. the failed elements, after replacement, are repaired and put into stock (inventory with returns system). The shortage can occur, when the demand over crosses the number of spare elements being available from stock, and then spares are obtained by an emergency order or by borrowing, penalty cost is paid, and the mission continues. The problem of spare parts planning is to find S_k for which stock out probabilities p_{spk} at time point t_k are smaller than the specified numbers α_{spk}:

$$\begin{cases} \min C_z = \sum_{k=1}^{K} c_z S_k \\ p_{spk}(S_1,...,S_K) \le \alpha_{spk} \end{cases} \tag{9}$$

where:

C_z	– function of expected total purchase and holding cost
c_z	– total purchase and holding cost per unit per unit of time
t_k	– moments of planned overhauls, $k = 1, 2, .., K$
S_k	– planned number of spare elements at t_k
$p_{spk}(S_1,..., S_K)$	– probability of stock out at t_k
α_{spk}	– maximal level of stock out probability in one maintenance cycle

The model is solved with the use of a Markov process whose states are determined by the number of available spares and following assumptions:

- perfect maintenance conditions,
- elements of the system are identical and identically distributed.

Many works which address the problem of determining the optimal ordering policy parameters for technical systems operating under block replacement policy base on using simulation processes (see e.g. [8], [26], [56]). Those models give the solutions to define optimal ordering policy parameters (e.g. order placement moments), define optimal PM parameters (moments of maintenance performance). However, such optimization problems typically entail the use of simplified system models and cannot give optimal solution. More complex real life systems behavior models, where the problem stated regards to multi-criteria optimization issue, are very complicated and are hardly to be put in an explicit analytical form.

Another group of models where the problem of spare inventory optimization is investigated regards to the age replacement policy. The problem of age replacement policy with inventory restrictions can be found in e.g. [46], where authors investigated two inventory policies (s,S).

According to the (1,1) inventory policy, operating element will be replaced in one of two situations: at age T or whenever the minimal repair cost C_{nm} is greater than some predetermined value $C_{nm}^{\max}$, whichever occurs first. When the unit must be replaced it will be ordered and delivered after a lead time L. during the time of waiting for spare element delivery system is in downstate.

Optimization of the following parameters: lead time L and the age T when a system must be replaced base on minimization of total maintenance and inventory cost, defined by the following formula:

$$C_s(L,T) = \frac{c_d(L) + \left(c_{w1} - c_{w2} + \int_0^{C_{nm}^{kr}} x dF_{cn}(x) \left(1 - F_{cn}(C_{nm}^{\max})\right)^{-1} \right) \left[1 - \left(R(t)\right)^{1-p_{cnm}}\right] + c_{w2}}{L + \int_0^T \left(R(u)\right)^{1-p_{cnm}} du} \tag{10}$$

where:

$F_{cw}(x)$	– cost of repair distribution function
p_{cnm}	– probability that defined minimal repair cost is greater than $C_{nm}^{\max}$
$c_d(L)$	– function of costs associated with delivery performance (e.g. ordering cost, cost of lost production)

This kind of model might correspond to some very critical but expensive piece of equipment where one backup is provided [46].

Second investigated inventory policy called (2,2) is a modification of described (1,1) inventory models. In this model, the system will always contain one unit in operation and one additional unit either in inventory or on order. According to he model assumptions, when an operating unit fails one of two possible situations can happen:

- an additional unit being in inventory is immediately available for replacement,
- an additional unit is on delivery – then the failed unit is repaired at all cost.

No system downtime is ever allowed.

The total cost function is defined as:

$$
\begin{aligned}
C_s(L,T) = \Big\{ & c_d(L) - \ln R(L) C_{nm}^{L} + \left[1 - (R(L))^{1-p_{cnm}} (R(T))^{1-p_{cnm}}\right] \times \\
& \times \left(\int_0^{C_{nm}^{kr}} x dF_{cn}(x) \left(1 - F_{cn}(C_{nm}^{\max})\right)^{-1} + c_{w1} - c_{w2} \right) + c_{w2} \Big\} \times \\
& \times \left(L + (R(L))^{1-p_{cnm}} \int_L^T (R(\tau))^{1-p_{cnm}} d\tau \right)^{-1}
\end{aligned}
\tag{11}
$$

where:

C_{nm}^{L} – expected repair cost function during a lead time

For more complicated problem investigation (see e.g. [69]) simulation processes, dynamic programming, integer programming, and nonlinear programming are the main tools suggested.

Lots of models for the joint optimization of an optimal age-dependent inventory policy and PM policy regard to production systems subjected to random machine breakdowns (see e.g. [23], [35]). An interesting inventory problem is investigated in [62], where authors developed optimal number of inventories $S = S_1, S_2, \ldots, S_r$ when:

- system performs under age replacement policy,
- system failure rate increase with its age.

The optimization problem is stated as:

$$
\begin{cases}
\max \prod_{q=1}^{n_q} p_q(S_q) \\
\sum_{q=1}^{n_q} H_q s_q \le C_z^{\max}
\end{cases}
\tag{12}
$$

where:

n_q – number of types of spare elements in production machine

$p_q(S_q)$ – probability that there will be no stock out of spare elements type q during the overhaul

s_q – initial inventory level

$C_z^{\max}$ – maximal allowed level of inventory costs

Solution of the stated optimization problem is received with the use of dynamic programming.

However, in real life systems, failed element can be replaced or repaired, what needs to give an answer for the following questions:

- when unit should be repaired instead of replacing?
- how many spare parts should be ordered in order to meet demand and the ordering and inventory costs will be minimal?

One of the models, which try to answer for these questions, is presented in [51]. In this paper joint stocking and replacement model with minimal repair at failure is considered. Authors assume, that Q units are purchased per order, operation unit is replaced after using for time interval T_{ci}, if inventory level is (i-1) and minimal repair is performed for any intervening failures. The problem is to select optimal order quantity Q and replacement intervals T_{ci}, so as to minimize the total maintenance and inventory cost per unit time, given by the following formula:

$$
C_s(Q, T_{ci}) = \frac{c_o + c_{wz} Q + c_{nm} \sum_{i=1}^{Q} H(T_{ci}) + c_h \sum_{i=1}^{Q} (i-1) T_{ci}}{\sum_{i=1}^{Q} T_{ci}}
\tag{13}
$$

where:

c_o – cost of an order placing

c_{wz} – replacement cost per unit

i – inventory level

Investigated problem of optimal ordering and maintenance policy parameter definition is continued in many recent papers (see e.g. [58]). In this paper authors defined optimal ordering point t_o and optimal number of minimal repair N_{nm} before PM performance for single-unit system. Assumptions made in this model are the same as presented in [64]. Order for spare element is placed before nth failure of operational unit occurrence (moment t_n), and lasts a lead time L. if operational unit fails before t_n moment occurrence, system is in downstate till the moment t_n+L. However, if unit fails after spare element delivery (kth failure), system is immediately replaced. Other failures are minimally repaired in time (0, t_k).

Optimization of parameters is performed with minimization of total expected cost function given by:

$$
\begin{aligned}
C_s(t_n, N_{nm}) = \Bigg\{ & (N_{nm} - 1) c_{nm} + c_o + \left(\int_0^{\infty} \int_x^{x+L} f_{t_n,t_k}(x,y) dy dx \right) \times \\
& \times \left(c_{sp} \int_0^{\infty} \int_x^{x+L} (x + L - y) f_{t_n,t_k}(x,y) dy dx \right) + \\
& + \left(\int_0^{\infty} \int_{x+L}^{\infty} f_{t_n,t_k}(x,y) dy dx \right) \left(c_h \int_0^{\infty} \int_{x+L}^{\infty} (y - x - L) f_{t_n,t_k}(x,y) dy dx \right) \Bigg\} \times \\
& \times \Bigg\{ \left(\int_0^{\infty} \int_x^{x+L} f_{t_n,t_k}(x,y) dy dx \right) \left(\int_0^{\infty} \sum_{j=1}^{t_n - 1} \frac{\exp(-R(x)) (R(x))^j}{j!} dx + L \right) + \\
& + \left(\int_0^{\infty} \int_{x+L}^{\infty} f_{t_n,t_k}(x,y) dy dx \right) \left(\int_0^{\infty} \sum_{j=1}^{k_{nm} - 1} \frac{\exp(-R(x)) (R(x))^j}{j!} dx \right) \Bigg\}^{-1}
\end{aligned}
\tag{14}
$$

where:

$f_{t_n,t_k}(x,y)$ – probability density function of t_n and t_k

Another interesting solution of the problem ‘replace or repair’ is given in [20], where a simple repair-time limit replacement problem with imperfect repair is considered. Authors investigated a single-unit system in which, when unit fails one estimates the repair time. If the repair can be completed up to a pre-specified time limit $T_{r\max}$, the repair is started immediately, otherwise, the spare unit is ordered with a lead time L. the expected total cost per unit time in the steady state is given by the following formula:

$$C_s(T_r)=\frac{(k_f+c_n)\int_0^{T_r} t\,dG(t)+(k_f L+c_o)\overline{G}(T_r)}{\int_0^{T_r} t\,dG(t)+\frac{1}{\lambda}+\left(L+\frac{1}{\lambda^s}-\frac{1}{\lambda}\right)\overline{G}(T_r)} \tag{15}$$

where:

k_f – penalty cost per unit time when system is in downstate

T_r – random repair time

$G(t)$ – p.d.f. for repair time; $\overline{G}(t)=1-G(t)$

λ^s – failure rate of new unit

λ^n – failure rate of repaired unit

The solution of the presented model is obtained with the use of graphical method based on the Lorenz transform.

Main classification of the models regarded to optimization of inventory and maintenance policy parameters is presented in *Table 1.*

2.2. Number of spare parts optimization models

Presented above group of models cannot take into account the problem of obtained system reliability characteristics being influenced by chosen inventory policy. As a result, authors define second group of models which analyze optimization of inventory and maintenance policy parameters in order to provide maximal level of reliability/availability of a maintained system. The main classification scheme is presented in *Figure 6.*

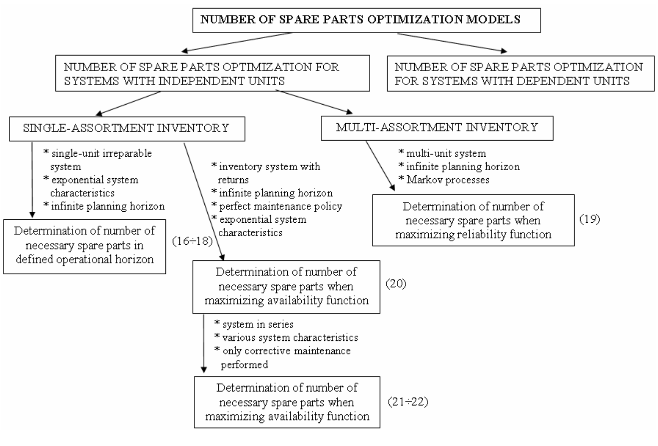

Figure 6. Number of spare parts optimization models classification [73]

The problem of influence of spares provisioning decisions on reliability/availability of an operational system has been investigated for almost 50 years. First papers regard to defence systems modelling - see e.g. [1], where authors presented an analysis of inventory policy parameters and their influence on marine system maintenance process. Another example is presented in [12], where the problem of aircraft supply process is investigated. More recently other applications in civil sector have appeared, with the use of classical inventory methods:

- deterministic models (e.g. Wilson method, (T,s) inventory policy, (s,Q) inventory policy),
- probabilistic models (see e.g. [45]).

Deterministic models are easy to calculate, however, the likelihood of obtained results is usually not satisfied.

Moreover, most of the known models assume that supply process is a Poisson process (see e.g. [2]. According to this author, the problem of determination of optimal number of spare elements for irreparable system which will satisfy the demand during operational process time $(0,t)$ is not possible to solve analytically without making an assumption about exponentially distributed operational time of a system. As a result, for this assumption a solution is given by the known formula:

$$P(N(t)\le n_o)\ge\sum_{i=0}^{n}\frac{(\lambda t)^i}{i!}\exp(-\lambda t) \tag{16}$$

where:

n_o – number of used elements during operational process

Number of necessary spare elements when replacement time is assumed to be negligible, is given by:

$$P(N(t)\le n_o)\ge\alpha_p \tag{17}$$

where:

α_p – rejection level

Presented model can be used only for definition of necessary spare elements for single-unit irreparable system, thus, this is very simplified example, which cannot be applied for real life system optimization problems solving. However, there should be clearly stated that problem of optimal spare parts number definition can be easily obtained only for a small amount of cases, when following conditions are satisfied:

- single-unit system or multi-unit system connected in series,
- independency of operational units,

- time to failure is exponentially distributed.

Taking also into account the following assumptions:

- spare elements cannot degrade in time during being in stock,
- failed elements can only be replaced not repaired,

probability that during operational time (0,*t*) there will be no shortage, can be defined as:

$$P_s(n,t) = \prod_{i=1}^{k} p_{si}(n_i,t) = \exp(-\lambda_i t)\sum_{j=1}^{d_i}\lambda_{ij} \tag{18}$$

where:

$p_{si}(n_i,t)$ – probability that during time (0,*t*) there will be no shortage of elements of type *i*

λ_{ij} – failure rate of *i*th type element, which is in *j*th place in a system

d_i – number of elements of type *i*

Table 1. Models of optimal spare part inventory policy for system under PM – an overview

Inventory policy	Preventive maintenance policy	Operational system type	Optimization criteria	Model parameters	Modelling method	Planning horizon	Papers
(s, S)	Age replacement policy	single-unit	$C_s \to MIN$	L, T	analytical	infinitive (∞)	[46]
				T, S			[23, 35]
		multi-unit		T1, s, S	simulation processes		[80]
				Sk			[62]
			$C_z \to MIN$	sk, Sk	dynamic programming		[69]
(R, S)	Block replacement policy	single-unit	$C_s \to MIN$	t0, t1	analytical		[21, 66]
			$C_s \to MIN$ $A \to MAX$	t0			[64]
		multi-unit	$C_s \to MIN$				[22]
				T, S	analytical/simulation processes		[3]
				Sk	simulation processes		[70]
(S, s)				T, s, S			[56]
(R, S)		single-unit / multi-unit		T, R, s	analytical/simulation processes		[8]
(s, Q)	Block replacement policy with minimal repair	single-unit	$C_s \to MIN$	S, Q	analytical		[51]
ROP in the moment of nth maintenance				t0, Nnm			[58]

s – critical inventory level | *T – time between orders performance* | *R – reordering point*
S – maximal inventory level | | *Q – ordering quantity*

When the presented above assumption cannot be satisfied, other modelling methods (instead of analytical) must be implemented, like e.g. simulation processes (see e.g. [11]), heuristic methods (see e.g. [77]), or databases of exploitation processes (see e.g. [42]).

Problem of optimal number of spare elements definition for multi-unit system is investigated in [72]. Presented model is aimed at maximization of system reliability taking into account the cost constraints:

$$\begin{cases} R(t) = \prod_{j=1}^{m}\left(\sum_{i=0}^{x_j} P_{ij}(t)\right) \\ C_z = \dfrac{\prod_{i=1}^{m} x_i^{q_i}}{\sum_{i=1}^{m} x_i C_{zi}} \end{cases} \tag{19}$$

where:

$P_{ij}(t)$– probability that system is in *i*th state for *j*th type of spare elements

q_i – probability of *i*th type spare element availability

C_{zi} – cost of *i*th type spare element availability

x_i – number of ith type spare elements being in stock
m – number of subsystem components

There can be also found lots of optimization models aimed at defining the number of spare units in order to maximize steady-state system availability. One of interesting models is presented in [6], where is considered inventory model with returns for repairable system with cold standbys and non-zero replacement time. In the investigated multi-unit system, when a failure occurs the defective units are replaced by spare ones, which must be available. The failed units after replacement are sent out for repair. Repaired units are good-as-new and are put in a pool of spares. When the situation occurs, that an i unit fails and there is no good spare, the system is maintained by repairing a failed i unit. The problem is how many spares ought to be allocated and to where. The solution bases on availability function for a mission time (0,T), given by:

$$\begin{cases} \max A(S) = A(S^*) \\ C_{sj}(S^*, T) \le C_{sj}^{\max} \end{cases} \tag{20}$$

where:
S^* – vector of optimum spare units for subsystems
C_{sj} – total jth cost of the system for mission time (0,T)
C^{max}_{sj} – maximal allowed level of jth cost

Taking into account the following conditions:

- operational and repair times are exponentially distributed,
- spare units do not fail during a storage time,
- repaired units are good-as-new,

the computation of steady-state availability, defined by formula (20), is done by using Markovian chains.
Models for spare components with exponential, gamma, normal and Weibull time to failure distribution using a renewal process are developed in [38]. In the presented paper, authors consider a system with n components connected in series. Maintenance process regards to the replacement of failed component by a new identical spar element. The objective is to maximize availability of the system satisfying constraints on costs and weight, what can be written by:

$$\begin{cases} \max A(T_s) = \prod_{i=1}^{n} n\left(\sum_{j=0}^{N_{wi}^{\max}} \sum_{i=0}^{j} \left(F_i^{\,k}(P_{ij}) - F_i^{\,k+1}(P_{ij}) \right) \right) \\ \sum_{i=0}^{n} \sum_{j=0}^{n_{wi}^{\max}} j \times c_{zi} \times z_{ij} \le C_z^{\max} \\ \sum_{i=0}^{n} \sum_{j=0}^{n_{wi}^{\max}} j \times w_i \times z_{ij} \le W_{\max} \end{cases} \tag{21}$$

where:
n^{max}_{wi} – maximum number of spares allowed for component i
$F_i^n(t)$ – n-fold convolution of function $F_i(t)$
$F_i(t)$ – time to failure distribution of component i
c_{zi} – cost of spare for component i
w_i – weight of each spare for component i
W_{max} – maximum allowed weight
z_{ij} – binary variable:

$$z_{ij} = \begin{cases} 1, \textit{ if } j \textit{ spares are selected for component } i \\ 0, \textit{ otherwise} \end{cases} \tag{22}$$

The above optimization problem is solved using an efficient branch and bound procedure.
To sum up, taking into account the presented above considerations, there can be defined the main drawback of existing number of spare part optimization models:

- most of the work is based on an item approach rather than system approach,
- simple solutions can be obtained only for a small amount of cases; models require the use of special purpose algorithms to handle large problems.

2.3. Storage reliability models

Another important problem regards to reliability of the spare elements being in stock and its influence on operational performance of a maintained system. Some elements e.g. electronic equipment can and does fail during the time, when they are awaiting for long periods of time prior to usage. As a result, there can be stated a question: what is the spare element reliability after being in storage for defined period of time (e.g. X years)? The main classification scheme of the models, which investigate problem of storage reliability, is presented in *Figure* 7.
One of the first works developing storage reliability with periodic test model for electronic equipment is presented in [Marti84]. Model gives the possibility to calculate the expected number of failures during one periodic test internal, with the assumption about exponentially distributed processes in a system:

$$N_{in} = \left(\frac{T^4}{T^4 + T^T}\lambda_{S4} + \frac{T^T}{T^4 + T^T}\lambda + \frac{v_{in}}{T^4 + T^T}\lambda_C \right)\left(T^4 + T^T\right) \quad (24)$$

where:

T^4 – time in which the item is in storage between two periodic inspections performance

T^T – test time

λ_{S4} – failure rate of an item being in storage

λ_C – failure rate of a switching process

v_{in} – number of period test cycles during storage process

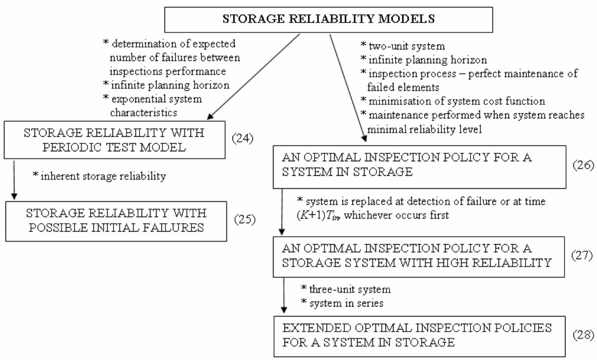

Figure 7. Storage reliability models classification [73]

The storage reliability problem has been of particular interest since 1960s, especially in safety-related systems and in the military industry, where the storage time is often the largest portion of the total life time. These systems have to be very reliable upon demand. In paper [79], authors consider model of storage reliability as a combination of the inherent storage failure probability with the initial failure probability, which is usually assumed to be negligible. The storage reliability at moment t is given by the following formula:

$$R(t) = R_0 R_{op}(t), \quad t \geq 0 \quad (25)$$

where:

R_0 – probability that a unit is good at time zero

$R_{op}(t)$ – inherent storage reliability

The estimation of the initial reliability and the failure rate during the storage is presented in the exponential case.

Recent storage reliability models include also inspection and maintenance operation performance (see e.g. [28], [31]). In the presented article, considered system consists of two units, unit 1 is inspected and maintained to a good-as-new condition at periodic times kT (k=1, 2, ...) to hold a higher reliability than a pre-specified value R_m. Unit 2 is not maintained, i.e. its hazard rate remains unchanged by nay inspections. A system is overhauled if the reliability becomes lover than R_m. For such an inspection model an average cost until overhaul is given by:

$$C_s(T_{in}) = \frac{N_{in1}c_{in} + c_w}{N_{in1}T_{in} + T_{Rm}} \quad (26)$$

where:

N_{in1} – numbers of inspections done during a system storage

T_{Rm} – random time to system reach the level of reliability R_m

Optimization process is given for Weibull distribution of time to failure of unit 1 and exponential distribution of failure process of unit 2.

Calculation of a problem of optimal inspection policy for a storage system with periodic inspection can be also found in [29]. In the considered model, system is replaced at detection of failure or at time $(K+1)T_{in}$, whichever occurs first. The total expected cost until replacement is given by:

$$\begin{aligned} C_s(T_{in}) = {} & \left(c_{in} + c_{dw1}T_{in}\right)\sum_{j=0}^{K}\left(R_1(T_{in})\right)^j R_2(jT_{in}) - \\ & - c_{dw1}\sum_{j=0}^{K}\left(R_1(T_{in})\right)^j \int_{jT}^{(j+1)T_{in}} R_1(t - jT_{in})R_2(t)dt + c_w - c_{in} \end{aligned} \quad (27)$$

where:

c_{dw1} – cost of system downtime elapsed between failure and its detection per unit of time

Extended optimal inspection policy for a system in storage is developed in [30]. In the presented article, system consists of units 1 and 2, where unit 2 consists of units 21 and 22. That is, a system consists of a series system with independent unit 1, unit 21 and unit 22. Authors consider the followed extended inspection policy:

- periodic inspection: when a system is inspected at time interval jT_{in}, unit 1 is maintained and is like new after every time interval T_{in}, unit 2 is not done, i.e. its hazard rate remains unchanged by any inspections,
- periodic replacement: a system is partially replaced at time interval NT_{in} (unit 21 is replaced, unit 22 is not done, i.e. its hazard rate remains unchanged by any replacement),
- overhaul: a system is overhauled if the reliability becomes equal to or lower than R_m.

For the presented extended inspection policy, the average cost until overhaul is given by:

$$C_s(N_{in1}, T) = \frac{(N_w^{21}N_{in1} + N_{in1}^{21})c_{in} + N_w^{21}c_{w1}^{21}c_w}{(N_w^{21}N_{in1} + N_{in1}^{21}) + T_q} \quad (28)$$

where:

N^{21}_{w} – unit 21 replacement number
c^{21}_{w1} – cost of replacement of unit 21
T_q – time to system overhaul since $(N_{21}N_{in1}+n_q)$th inspection
N^{21}_{in1} – number of inspections since finishing N_{21}th replacement till system overhaul

An optimal inspection time and an optimal replacement time are numerically discussed.

2.4. Multi-echelon system models

The reparable-item inventory problem has received much attention in the logistics literature. The models presented above regards to single-echelon systems. However, for advanced technical systems, such as engines, or airplanes, high system availability is enhanced thanks to multi-echelon inventory system, in which usually are two or more echelons equipped with repair and stocking facilities. The lower echelon consists of a series of bases which are first level maintenance stations. The upper level, usually called a depot, provides second level support (*Figure* 8). Usually items are sent to the higher echelon if local repair is technically impossible, i.e. if the local repair shop does not have appropriate equipment or skills.

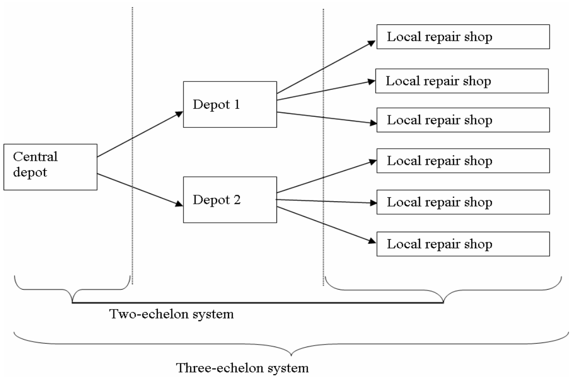

Figure 8. Multi-echelon system

The most important results on divergent multi-echelon inventory systems are reviewed in [19]. Authors concentrate on two types of policies: ordering policies and installation stock policies. In the second literature review, presented in [17], authors focus on describing those models which can be practically applied. Moreover, authors revisit in detail Multi-Echelon Technique For Recoverable Item Control (METRIC) mode and its variations and discuss a variety of more general queuing models.

Repairable items are referred to as components, which are expensive, critically important and subject to infrequent failures. When they fail, they should be repaired and reused after repair since they are too expensive to be discarded. Thus, one way to achieve high operational availability of such system is to acquire enough spare parts to provide immediate replacement of damaged components, what needs to have effective supply system – usually multi-echelon system. This problem is especially important for military systems, where main problems regard to:

- evaluation of time-varying availability in multi-echelon inventory system (see e.g. [39], [46], [40]),
- analysis of supply system of aircraft systems (see e.g. [32], [54]),
- optimization of multi-echelon service part supply system for marine system [60], [61]),
- determination of number of spares in an inventory/repair system which supports equipment with scheduled usage (example of NASA's space shuttle – see [4]).

Described models are aimed at maximization military system availability under the cost constraints. The most commonly used modelling tool is Monte Carlo simulation.

The review of the main allocation models for multi-echelon systems is given in *Table 2*. Moreover, in [5], there is presented a comparison of allocation policies in a two-echelon inventory model.

Research on multi-echelon inventory models has gained importance over the last 30 years. For large multi-component system the problem of optimal allocation policy definition cannot be solved analytically. Monte Carlo simulation (see e.g. [59]), branch and bound algorithm (see e.g. [9]), or queuing theory (see e.g. [13], [16], [34], [37]) are the main tools suggested.

3. An example

Consider a repairable system of systems under continuous monitoring, in which there are integrated two independent systems: operational and its supporting system. Both systems have only two states: upstate, when they are operable and can perform its specified functions, and downstate, otherwise.

The system of systems total task is defined as the continuous performing of exploitation process. Moreover, in the presented model the logistic support functions are narrowed down only to providing the necessary spare parts to the operational system. As a result, the logistic support system is inoperable when there is no capability of supplying the operational processes with necessary spares.

The operational system is composed of M identical elements working in a reliability structure, which determines the moments when the system goes to a down-state.

Table 2. Multi-echelon system models – an overview

Number of echelons	Optimization criteria	Modelling method	Papers
two-echelon system	$C_s \to MIN$	push i pull strategies	[27]
		analytical with queuing theory	[37]
	$W_0 \to MAX$		[16]
		simulation processes	[59]
	$C_s \to MIN$ $\alpha_o \to MIN$		[34]
		analytical with queuing theory	[33]
	$\bar{n}_{ocz} \to MIN$	branch & bound algorithm	[9]
three-echelon system	$C_s \to MIN$ $A \to MAX$	Markov processes	[13]
four-echelon system	$C_h + C_f \to MIN$	analytical	[18]
n-echelon system	$\alpha_o \to MIN$		[68]

$\bar{n}_{ocz}$ – mean number of awaiting orders in a system
W_0 – fill ratio
α_o – service level

Let's also assume that elements' failures are random in time, and each failure entails a random duration of repair before the element/system is put back into service. Let's also assume that any information about failures in this system is reliable and comes immediately to the logistic system.

In the investigated model, when logistic support system is in up-state, the ability of the system of systems depends only on:

- the time, when the operational system is operable,
- the time of technical system repair.

In the situation, when the supporting system is inoperable due to the lack of spare parts, the system of systems availability also depends on the logistic delayed time, which is necessary to solve logistic problems.

Moreover, if there is restricted the system of systems total task completion time, defined as the time of operational system recovery process, the system of systems remains in upstate if this defined time will be shorter than time resource. Otherwise, the system of systems will fail and remain in downstate till the end of delivery process.

Consequently, the following additional model assumptions are taken into account to define the system of systems performance process:

- randomness and independency of all the performed processes,
- critical inventory level (CIL) used as a stock policy,
- the individual time redundancy used to model the system of systems performance [doktorat].

To the best authors' knowledge, an effective way for achieving the reliable operational systems logistic support especially bases on meeting two targets: reliability/availability and cost constraints:

$$\begin{cases} C_s = \dfrac{C_{sj}}{E[T_j]} \\ A \le A_{\min} \end{cases} \tag{29}$$

where:
C_{sj} – the expected total system of systems costs in a jth procurement cycle
T_j – the random time of the jth cycle
A – the system of systems' availability ratio
A_{min} – the limiting availability ratio of system of systems performance

For the investigated model, the expected total system of systems cost in a jth procurement cycle may be calculated as [73]:

$$C_s = c_f H(t) + c_o + c_p Q + c_h E\left[\int_0^{T_j} I(t)dt\right] + \\ + P_{nnj}\left(k_f H(t) + c_{dw} E[\xi_1]\right) \tag{30}$$

where:
c_f – operational element replacement cost
c_p – cost of one spare purchasing
$I(t)$ – the quantity on-hand at time t
P_{nnj} – probability that system of systems fails

The expected procurement cycle time is defined as follows [73]:

$$E[T_j] = Q\left(E[T_o] + E[T_r]\right) + E[\tau] \tag{31}$$

where:
E[T_o] – expected time to failure of operational system, E[T_r] – expected replacement time of operational system
E[τ] – expected supply task performance time

The basic formula for steady-state availability ratio assessment is expressed as follows [Gniedenko]:

$$A = \frac{E[T_o]}{E[T_o] + E[T_r]} = 1 - \frac{E[T_r]}{E[T_o] + E[T_r]} \tag{32}$$

For the presented system of systems with time dependency, the availability ratio in one procurement cycle is expressed as [73]:

- for the system of systems with negligible replacement time of operational element:

$$A = 1 - \frac{E[\xi]}{QE[T_o] + E[\tau]} \tag{33}$$

where:
E[ξ] – expected system of systems downtime caused by the time of operational system recovery
Q – order quantity

- for the system of systems with non-negligible replacement time of operational element:

$$A = 1 - \frac{E[\xi_1]}{Q(E[T_o] + E[T_r]) + E[\tau]} \tag{34}$$

More information can be found in [73].

3.1. Simulation model and obtained results

The analytical model of performance of the presented system of systems with time dependency is investigated in e.g. [74], [75, [76].
The analytical results of the modeled problem can be obtained only for a small amount of cases, when the operational system is a single-unit system, the performed processes are modeled according to the exponential distributions, *etc*. (see. [73]). Thus, there can be written the following conclusion, that this analytical model is an oversimplified version of the real system behavior, so the obtained results are not traceable to practical situations.
To overcome this problem, there is proposed a simulation model of time dependent system of systems performance, which has been developed with the use of *GNU Octave* program. The simulation algorithm of the modeled system of systems is given in *Figure* 9.
The system of systems level of availability ratio, the probability of system of systems downtime occurrence, or economic results strongly depend on the operational system reliability structure. That is why, the model of time dependent system of systems performance was created for the three various system reliability structures – series, parallel and "*k* out of *n*".

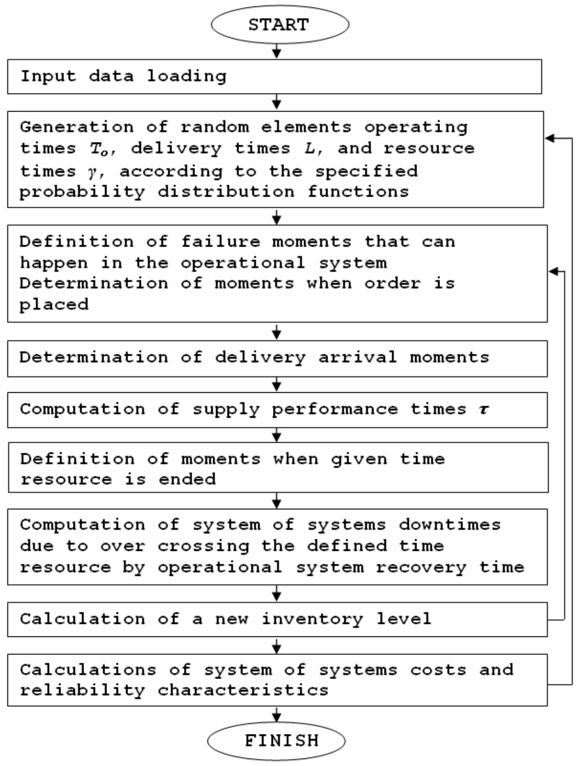

Figure 9. Simulation algorithm of time dependent system of systems performance [73]

The simulation results of the modeled system of systems have been carried out for the input parameters, presented in *Table 3*.
The main reliability and economic results are presented in *Figures 10 - 15.*

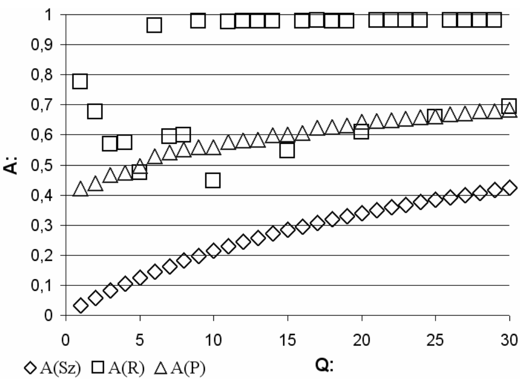

Figure 10. System of systems availability ratio for various levels of order quantity of spare elements

Table 3. Input parameters of modeled system of systems

	Initial value	Explanation of denotation
A_o	1	Weibull's shape parameter of single operational element time to failure
$1/B_o$	100	Weibull's scale parameter of single operational element time to failure
A_r	1	Weibull's shape parameter of single operational element replacement time
$1/B_r$	10	Weibull's scale parameter of single operational element replacement time
A_L	1	Weibull's shape parameter of lead-time time
$1/B_L$	1000	Weibull's scale parameter of lead-time time
A_γ	1	Weibull's shape parameter of time resource time
$1/B_\gamma$	10	Weibull's scale parameter of time resource time
k	1	"k" out of "M"
M	5	number of elements working in an operational system
s	5	critical inventory level
Q	30	order quantity
c_w	50	replacement cost of a unit
c_o	50	ordering cost
c_p	50	purchase cost of one unit
c_h	100	inventory unit holding cost per unit time
k_f	1000	penalty cost of system of systems failure occurrence
c_{dw}	1000	cost of system of systems downtime unit

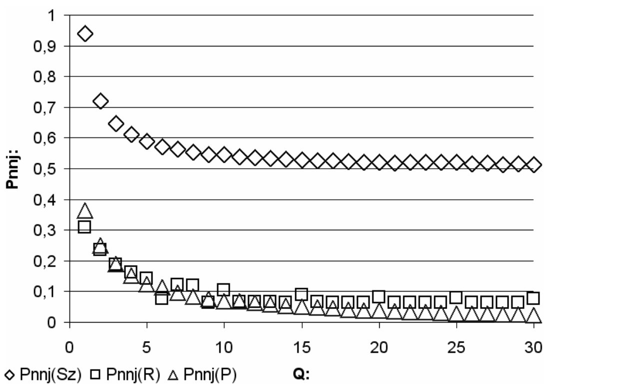

Figure 11. The system of systems downtime probability for various levels of order quantity of spare elements

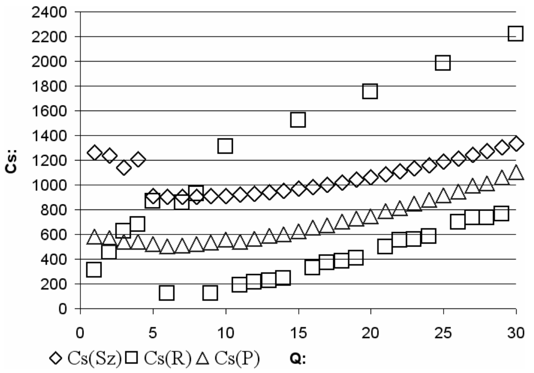

Figure 12. Expected cost per unit time function for various levels of order quantity of spare elements

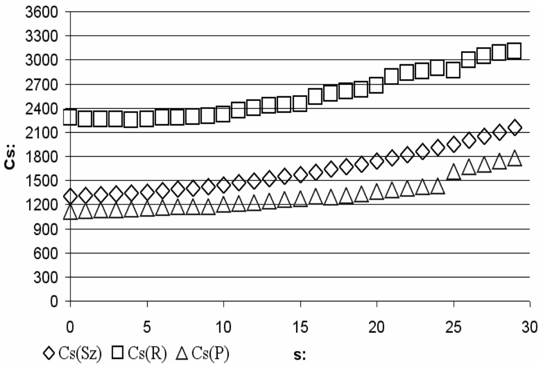

Figure 13. Expected cost per unit time function for various critical inventory levels

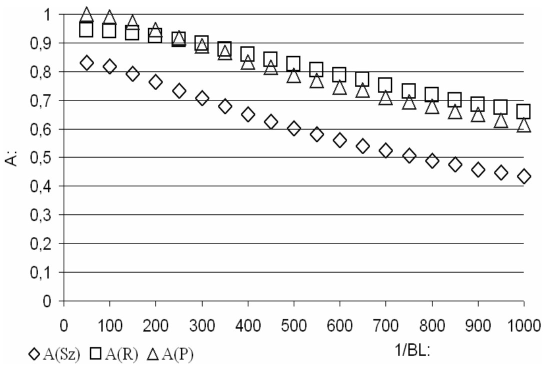

Figure 14. System of systems availability ratio for various lead-time lengths

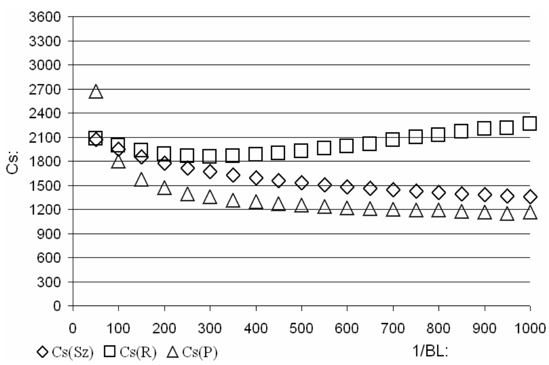

Figure 15. Expected cost per unit time function for various lead-time lengths

The ordered and delivered spare parts quantity determines the length of a single procurement cycle (time that elapses between the two consecutive moments when the inventories on-hand drop to a critical level). As a result, the bigger the ordered quantity, the higher mean stock level in the system and rarer deliveries performed.

The expected costs incurred by the system of systems with a different-structured operational system are mainly determined by the inventory holding costs and system of systems' downtime costs (*Figure* 12). When the level of ordered quantity rises, the expected costs function has the local minimum in case of series and k out of M systems. It is a result of rarer deliveries and downtimes occurrence that arise from inventory lack. On the other hand, the more spare elements are purchased, the higher inventory holding costs are incurred.

The worst solutions for the system of systems with operational system in parallel occur when the ordered quantity Q is a multiple of M. If all M elements are replaced and there is no spares remaining in a logistic system, there is a higher downtime probability and its economical consequences, than if there are some elements in a stock. This downtime costs together with the inventory holding costs have the greatest influence on the system of systems economic results.

The same effect can be seen when availability of system of systems with operational system in parallel is analyzed. The system of systems reaches the lowest availability ratio level when Q is a multiple of M. For system of systems with other reliability structures of operational systems, the rarer and bigger deliveries, the higher availability ratio is achieved.

The level of ordered quantity has also the influence on the probability of system of systems downtime occurrence, what is especially evident for a series structure case (*Figure 11*). A lower level of ordered quantity forces frequent deliveries, and as a result, there is higher probability that the possible delays of the delivery cause the system of systems downtime.

The next parameter of the procurement process, which affects the system of systems performance, is the critical inventory level (*Figure 13*). The higher critical inventory level, the higher the mean inventory level in a system what incurs higher inventory holding costs. However, the higher s level gives also a possibility to reduce the delivery delay consequences what has a positive impact on system of systems reliability results.

Moreover, there also can be seen the influence of lead-time length on the system of systems behavior. The longer lead-time affects especially the reliability characteristic of the system of systems (*Figure* 14). On the other hand, the longer the lead-time, the lower inventory holding costs and the higher downtime costs incurred, what is connected with the bigger probability of system of systems downtime occurrence. This relation can be seen in the *Figur*e 15 as a local minimum of the C_s value.

In order to model the time dependencies between the operational system and its logistic support system, there have to be identified basic relations, which result from the system of systems structure, components' parameters, or processes' execution times.

In other words, the presented model especially can support decision processes in the area of supply task performance requirements. Especially gives a convenient tool to decide which supplier can provide the desirable time of supply delivery in order to achieve a defined system of systems' operational capability.

On the other side, the developed model can be helpful to assess the reliability requirements of operational system elements in order to provide the continuous system of systems' total task performance

4. Conclusion

To sum up, all the presented models from the area of supply process parameters optimization, when system is maintained according to defined PM, can be divided into two groups:

- searching effective optimization methods for already known models (see e.g. [6], [47]),
- searching for system models in which new assumptions are made (e.g. new reliability structure, dependent elements in a system) (see e.g. [21], [22]).

Moreover, literature on modelling relations between logistic and operational systems is scarce. Up to now, the interactions between operational system and its supporting system have not been clearly investigated. The research has focused on the evaluation of reliability and economical characteristics for both systems in the separate way.

Moreover, the logistic systems have been evaluating and designing mostly in terms of: inventory modelling, supply processes organization, and transportation processes modelling.

However, the simultaneous setting of all structural parameters (e.g. redundancy, repair shop capacity) and control variables (e.g. spare part inventory levels, maintenance policy parameters, repair job priorities, time resource) is mathematically a hard problem, and cannot be done without many simplified assumptions taken.

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