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## THE STATISTICS OF FISH ECHO TRACES - MODELS AND VALIDATION


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*Beam pattern probability density function (PDF) plays important role in indirect fish target strength estimation methods as it constitutes the kernel of any form of a "single-beam integral equation" that should be solved for unknown target strength PDF. Typically, the beam pattern PDF is calculated under the assumption of uniform spatial distribution of fish targets in beam pattern cross-section. However, in some cases this assumption can not be justified due to fish or vessel movement, and different number of fish echoes in fish traces.*

*In the paper, two different models of fish traces statistics were investigated: one assuming the vessel movement with stationary fish and the other with stationary vessel and moving fish. Both approaches were modelled numerically and later verified experimentally using data obtained from a dual-beam system.*


### 1. INTRODUCTION

The statistics of so called fish traces - represented by multiple echoes received from the same fish in consecutive echosounder transmissions – and beam pattern probability density function (PDF) seem to be two absolutely separate and unrelated issues. The first one is used in analysis of fish counts estimates [1], whereas the second one plays crucial role in indirect fish target strength estimation [2]. However, it appears that these two seemingly separate problems are closely related when one considers PDF of beam pattern in the context of multiple echoes from individual fish.

Widely used assumption of uniform spatial distribution of fish in water column leads to *sine*-law distribution of the angular position of the fish. This assumption is valid only for the case of the single or non-multiple echoes received from individual fish in consecutive pings. However, when acquiring actual data from acoustic surveys, the multiple or correlated echoes may be collected from the same fish forming the fish traces.

In the paper the analysis of two models of fish traces is presented and the PDF's of the number of multiple echoes occurred in fish traces and angular position of the fish are also derived.

### 2. FORMULATION OF THE PROBLEM

Let us consider two models of fish traces statistics, which geometry is illustrated in Fig.1.

1) model 1 - moving vessel model with stationary fish,

2) model 2 - moving fish model with stationary vessel.

In the first model the uniform vessel movement with stationary fish is assumed as detailed in Fig 1a. The second model assumes fish movement along arbitrary path in the transducer beam pattern cross-section, as shown in Fig 1b.

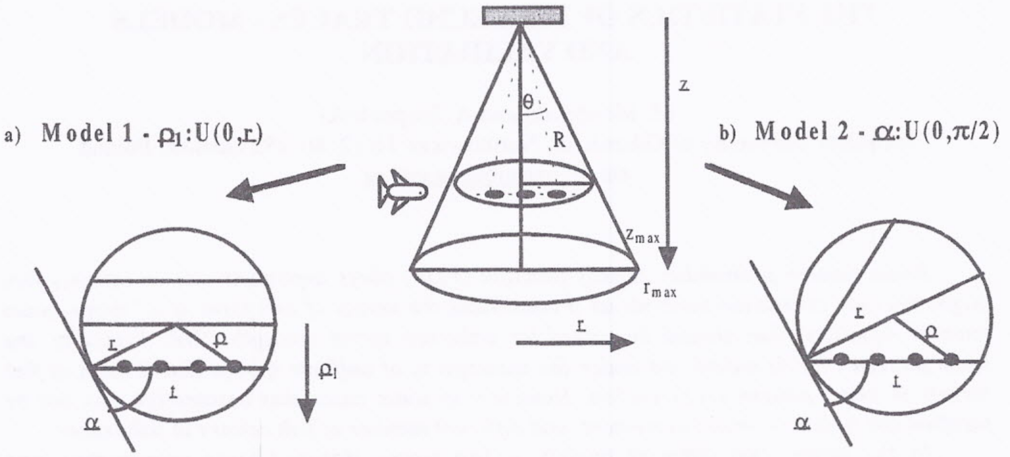


Fig.1. Geometry of two models for number of traces N analysis

Let us further assume that a distribution of variable $z$ representing depth on which fish appears in the conical area defined by observation angle $\theta_{max}$ is uniform, i.e.

$$p_z(z) = \frac{1}{z_{\max}} \qquad (1)$$

where $z_{max}$ represents maximum depth. Due to linear relation between depth and radius of a circular slice $\underline{z}=\underline{r}\ tan\ \theta_{max}$, the distribution of random variable $\underline{r}$ become also uniform, i.e.

$$p_r(r) = \frac{1}{r_{\max}} \qquad (2)$$

where $r_{max} = z_{max}/\sin\theta_{max}$ is maximum possible radius of slice.

## 3. MOVING VESSEL AND STATINOARY FISH MODEL

In the first model all fish traces represents parallel lines, crossing every circular slice of observation cone. Thus, the fish position in consecutive pings can be represented by equidistant points on a parallel chords. The unknown statistics of a number of fish $N_l$ can be evaluated from the geometrical equation:

$$\underline{N}_1 = \frac{2}{\Delta d}\sqrt{\underline{r}^2 - \underline{\rho}_1^{\,2}} \tag{3}$$

where $\Delta d$ represents sampling distance between consecutive points. Random variable $r$ represents radius of circle and random variable $\rho_l$ represents the distance between centre of that circle and the trace of the fish. In this model we assume that a fish may appear in the circle in such a way that the distance form the centre to the trace of the fish is equally probable, so in other words, the distribution of $\rho_l$ is uniform in a range interval $(0,r)$. This allows us to treat the random variable $\rho_l$ as a product of two random variables $\underline{\rho}_l=\underline{r}\ \underline{u}$, where variable $\underline{u}$ is represented by normalised uniform distribution. Substituting this relation to equation (1) we obtain:

$$\underline{N}_1 = \frac{2}{\Delta d}\underline{r}\ \sqrt{1-\underline{u}^2} \tag{4}$$

Now we can treat again the number $\underline{N}$ of fish traces as a product of two random variables $\underline{x}=2\underline{r}/\Delta d$ and $\underline{y}=(1-\underline{u}^2)^{1/2}$ and calculate the probability distribution function as a integral equation $p_z(z)=\int p_x(x)p_y(z/x)/x\, dx$, which gives the PDF of N as:

$$p_{N_1}(N) = \int_{\frac{\Delta d\, N_1}{2r_{\max}}}^{1} \frac{\Delta d}{2r_{\max}} \frac{y}{\sqrt{1-y^2}} \frac{dy}{y} \tag{5}$$

which evaluates into:

$$p_{N_1}(N) = \frac{\Delta d}{2r_{\max}}\left(\frac{\pi}{2} - arc\sin\frac{\Delta d\ N}{2r_{\max}}\right) \tag{6}$$

In Eq. (6) expression $\Delta d/(2r_{max})$ may be treated as the parameter of data measurement system and can be calculated from relation for mean value of the distribution:

$$E\{N_1\} = \frac{\Delta d}{2r_{\max}}\frac{\pi}{8} \tag{7}$$

Probability density functions of the number of echoes in fish traces for this model is presented in Fig. 2.

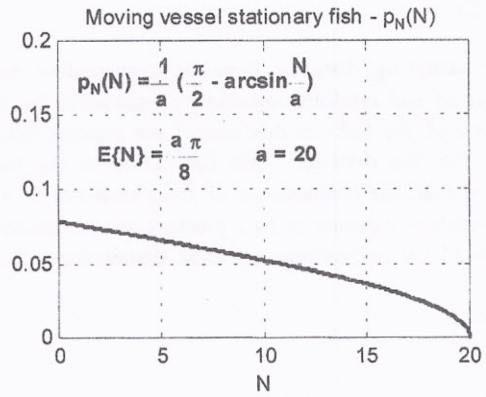


Fig.2. Theoretical distribution of the number N of echoes in fish traces for moving vessel and stationary fish model

## 4. MOVING FISH AND STATIONARY VESSEL MODEL

In the second model, we assume that the fish crosses an arbitrary cross-section of conical sampled volume with equally probable angle α. From geometrical relations number of fish traces can be expressed as

$$\underline{N_2} = \frac{2}{\Delta d}\underline{r}\ \sin\underline{\alpha} \tag{8}$$

where random variable $\alpha$ represents crossing angle. The unknown distribution of the number of fish echoes in the fish trace $N_2$ can be derived again from the equation determining the PDF of the product of two random variables $\underline{x}=2\underline{r}/\Delta d$ and $\underline{y}=\sin\underline{\alpha}$. Assuming uniform distribution of angle $\alpha$ we obtain:

$$p_{N_2}(N) = \int\limits_{\frac{\Delta d\, N_2}{2 r_{\max}}}^{1} \frac{\Delta d}{2r_{\max}} \frac{2}{\pi} \frac{1}{\sqrt{1-y^2}} \frac{dy}{y} \tag{9}$$

which eventually leads to:

$$p_{N_2}(N) = \frac{\Delta d}{\pi\, r_{\max}} ar\tanh\sqrt{1-\left(\frac{\Delta d}{2r_{\max}} N_2\right)^2} \tag{9}$$

Distribution from Eq. (9) has mean value described by expression:

$$E\{N_2\} = \frac{\Delta d}{2r_{\max}} \frac{1}{\pi} \tag{10}$$

Probability density functions of the number of echoes in fish traces for this model is presented in Fig. 3.

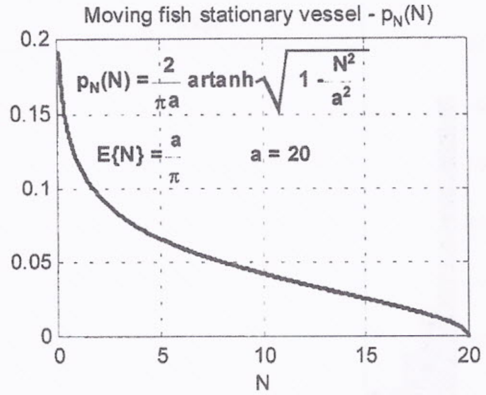


Fig.3. Theoretical distribution of the number N of echoes in fish traces for moving fish and stationary vessel model

## 5. SURVEY RESULTS AND CONCLUSION

In order to justify the correctness of the presented analysis and validate its results the actual fish echo data was used. The data was acquired from acoustic survey on pelagic fish populations (mostly salmon and trout) in Coeur d'Alene Lake, Idaho (provided by J.B. Hedgepeth, Biosonics Inc., Seattle and E. Parkinson, University of Vancouver, Canada) using dual-beam digital echosounder of 420 kHz operating frequency and 0.4 ms pulse length. There were processed records of over 6500 pings from which over 10000 fish echoes were extracted for analysis and using software algorithms the 2009 fish was counted. The distribution of the number $N$ of multiple echoes in fish traces are shown in the Fig. 4, in a form of histogram. The results match model 2 of distribution presented in Fig 2. However, it is also possible that it matches model 1 due to border effect in obtaining PDF estimate by histogram technique (the range between N=0 and N=1 cumulates as only N=1 has physical sense).

Numerical experiments conducted on survey data show good agreement with presented models of fish statistical behaviour during measurements. The range Mean value of distribution is equal 5.3. It is worth to note that in practice it is possible that complicated fish behaviour can be modelled by mixture of model 1 and model 2 due to relative movement of fish and the vessel. Model 2 with uniform distribution of crossing angle represents more "random" case than model 1 with its sine-law distribution of crossing angle.

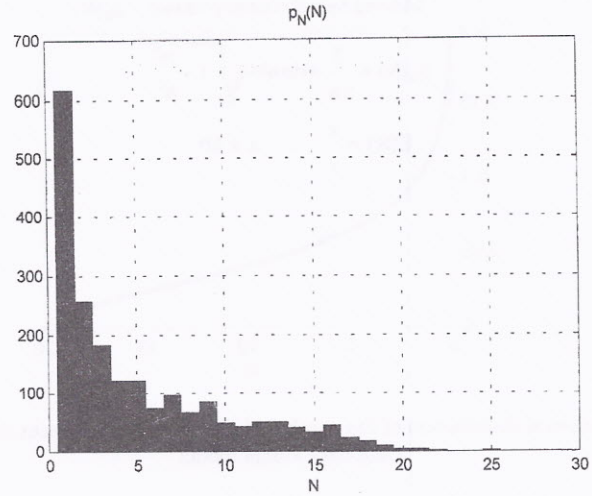


Fig.4. Sample distribution of the number of multiple echoes in fish traces from survey

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