Using Functions from Fuzzy Classes of k-valued Logic for Decision Making Based on the Results of Rating Evaluation

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Abstract:
IIn this paper we present an approach based on logic functions with fuzzy conditions for constructing a decision support system for rating evaluation of objects. This approach provides an effective and efficient way for separating rating marks into clusters, associated with a control effect directed at a successful functionality of objects in future.

Keywords: *rating marks, linguistic statements, fuzzy logic functions, decision support.*

1. Introduction

Ratings are widely used in diverse areas of human activities (education, engineering, economics, ecology, etc.) and allow acquiring available and up-to-date information in some sort of a neutral integral index that can be used in decision making. A number of difficulties in acquiring a rating estimate is discussed in detail in [1]. These difficulties are connected with the heterogeneity of characteristics, unstable final results resulting from different scales, and results recognition required for decision making. Thresholds, separating range of values into intervals, are used for rating the recognition of estimates. Control effects are applied while a rating estimate gets into certain intervals. The task of acquiring threshold values is resolved experimentally or based on experts' opinion which is not always possible. A posteriori statistical information may be missing which may lead to significant difficulties and mistakes. Besides that, rating estimates always hold uncertainty zones, leading to a complication of control effect selection. Usually these zones are located near threshold values or "mean-values-zones" that are hard for the recognition because of an ambiguity of the situation cinsidered.

The use of linguistic statements makes it possible to define rating estimates with heterogeneous characteristics and to prevents incorrect arithmetic operations, common for the traditional rating estimation models [2], [3]. But the problem of rating estimates' recognition by purposely acquiring control effects has its flaws, as discussed before. The reason of the flaws is a lack of formalized approach that would be able to reduce experts' mistakes coming from an incomplete or illegible information. This article proposes an approach for the recognition of rating estimates, and the a decision making support based on fuzzy logic functions developed and adapted for fuzzy conditions and goals [4]–[5].

2. Construction of Functions from Fuzzy Classes of k -valued Logic

Consider characteristics $X_j, j=\overline{1,m}$, with corresponding values X_{lj}, $l=\overline{1,m_j}, j=\overline{1,m}$, characterizing their state. Assume these characteristics depend on characteristic Y with values range $Y_l, l=\overline{1,k}$, if Y is associated with some information aggregating operator, allowing to compute value of Y with $X_j, j=\overline{1,m}$. An information aggregation operator O_Y is a function defined on a set of all possible values $X_j, j=\overline{1,m}$ and accepting values on set of values $Y_l, l=\overline{1,k}$:

$$O_Y : X_{1l_1} \times X_{2l_2} \times \dots \times X_{ml_m} \to Y_l .$$

Historically, the first approach for selecting an information aggregation operator is geometric based on the representation of an operator as a surface in the $(m+1)$-dimensional space [5]. A flaw of this approach is the necessity of knowing the value of an aggregation operator on at least $(m+1)$ values of characteristics $X_j, j=\overline{1,m}$ and an inability of applying additional experts' information.

A logical approach to selecting an information aggregation operator is applicable when some conditions on operator O_Y can be applied. If we have k values of characteristic Y, we can introduce an information aggregation operator as some k-valued logic function. If the amount of an dependent characteristic X_j equals m, the information aggregation operator can be represented as a k-valued logic function of m variables.

Suppose that an expert can formulate fuzzy conditions of an unknown function's behavior like: «Function slightly decreases when the first variable strongly increases», «When arguments 3 and 5 simultaneously increase, the function value strongly increases», etc. In this case we can speak of fuzzy classes of k-valued logic [5] or k -valued logic functions of m variables with fuzzy conditions.

A fuzzy condition formulated by an expert describes the membership of function to certain fuzzy class (for example, slightly increasing or slightly decreasing) based on values of a function at points i and $i+1, 0 \le i \le k-1$. Value $\mu_{\tilde{S}}(p,q)$ is a degree of membership to a certain fuzzy class given that $f(i)=p,\ f(i+1)=q,\ \ 0\le p\le k-1,\ 0\le q\le k-1$. All detailed explanations will be given below.

According to [5], the fuzzy condition S is represented as a fuzzy binary relation $\tilde{S}$ between sets X,Y which is a fuzzy set $\tilde{S}:\ \forall(x,y)\in X\times Y\ \ \mu_{\tilde{S}}(x,y)\in[0,1]$, and $X=\{x\}, Y=\{y\}$ are non-fuzzy sets.

If sets X,Y are finite $X=\{x_1,x_2,\dots,x_n\}$, $Y=\{y_1,y_2,\dots,y_m\}$ then a fuzzy binary relation $\tilde{S}$ may be represented in a matrix form, the rows and columns of which are associated with elements of sets, and on the intersection of *i*-th row and *j*-th column is an element $\mu_{\tilde{S}}(x_i,y_j)$, i.e.

$$R=\begin{pmatrix}\mu_{\tilde{S}}(x_1,y_1) & \mu_{\tilde{S}}(x_1,y_2) & \dots & \mu_{\tilde{S}}(x_1,y_m)\\ \mu_{\tilde{S}}(x_2,y_1) & \mu_{\tilde{S}}(x_2,y_2) & \dots & \mu_{\tilde{S}}(x_2,y_m)\\ \dots & \dots & \dots & \dots\\ \mu_{\tilde{S}}(x_n,y_1) & \mu_{\tilde{S}}(x_n,y_2) & & \mu_{\tilde{S}}(x_n,y_m)\end{pmatrix}.$$

A fuzzy binary relation $\tilde{S}$ on set X is a fuzzy set $\tilde{S}$: $\forall(x,y)\in X\times X\ \mu_{\tilde{S}}(x,y)\in[0,1]$.

Consider a fuzzy condition S on the behavior of a function f of one variable. Fuzzy relation $\tilde{S}$, corresponding to S, describes the membership of a function to a certain fuzzy class (for example, "slightly increasing" or "slightly decreasing") based on function values at points i and $i+1, 0\le i\le k-1$. Value $\mu_{\tilde{S}}(p,q)$ is a degree of membership to a certain fuzzy class given that $f(i)=p,\ f(i+1)=q,\ \ 0\le p\le k-1,\ 0\le q\le k-1$. Fuzzy relation matrix $\tilde{S}$, corresponding to fuzzy condition S, looks like:

$$R=\left(\mu_{\tilde{S}}(p,q)\right).$$

Let the function behavior depends on several fuzzy conditions – $S^r, r=\overline{1,s}$. Each of these conditions has a corresponding fuzzy relation matrix $\tilde{S}^r, r=\overline{1,s}$. The matrix, generalizing all these conditions, is obtained via a T–norm:

$$\tilde{\tilde{S}}^s=\mathop{\mathrm{T}}_{r=1}^{s}\tilde{S}^r.$$

The triangular norm (T– norm) is real-valued function $T:[0,1]\times[0,1]\to[0,1]$, satisficing these conditions:

1) $T(0,0)=0, T(\mu_{\tilde{A}},1)=T(1,\mu_{\tilde{A}})=\mu_{\tilde{A}}$ (boundedness);

2) $T(\mu_{\tilde{A}},\mu_{\tilde{B}})\le T(\mu_{\tilde{C}},\mu_{\tilde{D}})$ if $\mu_{\tilde{A}}\le\mu_{\tilde{C}},\mu_{\tilde{B}}\le\mu_{\tilde{D}}$ (monotonicity);

3) $T(\mu_{\tilde{A}},\mu_{\tilde{B}})=T(\mu_{\tilde{B}},\mu_{\tilde{A}})$ (commutativeness);

4) $T(\mu_{\tilde{A}},T(\mu_{\tilde{B}},\mu_{\tilde{C}}))=T(T(\mu_{\tilde{A}},\mu_{\tilde{B}}),\mu_{\tilde{C}})$ (associativity).

If relation matrix $\tilde{\tilde{S}}^s$ has at least one zero row, then the set of conditions $S^r, r=\overline{1,s}$ is contradictory because f satisfies it with the zero value. Next, an algorithm of subset selection is suggested for such contradictory sets. All pairs of conditions are tested for inconsistency. Inconsistent pairs found are removed from the consideration. In the next step all triplets of conditions are tested for inconsistency. Inconsistent triplets found are removed from the consideration. This operation is repeated until step $l, 1\le l\le s$, in which all subsystems consisting of $l+1$ fuzzy conditions, are inconsistent. Then, the consistent subsystems are all subsystems found in step $l-1$ and consisting of l fuzzy conditions. Hereby, any number of fuzzy conditions for one variable may be easily reduced to one fuzzy condition for this variable.

Consider now *k*-valued logic functions with *m* variables and imposed on fuzzy conditions *S*. Assume $m=2$ and $|S|=2$, for simplicity. Let S_1 to be first fuzzy condition being determined by the first variable, and the second condition S_2 – by the second variable. Compute relation matrices $\tilde{S}_1$ and S_2. The satisfaction of conditions S means that conditions S_1 and S_2 are satisfied simultaneously. This means that, for (i_1,i_2), as the x_1 and x_2 values $(i_1,i_2\in\{0,1,\dots,k-1\})$ we should use the values resulting from the use of a T – norm of the (i_1+1)-th row of relation matrix $\tilde{S}^1$ and the (i_2+1)-th row of matrix $\tilde{S}^2$, they are the row values of matix $\tilde{\tilde{S}}$. This matrix has all properties of a relation matrix required.

Assume that the function behavior satisfies some fuzzy condition S and furthermore some initial condition (for example $f(0)=0$). Fuzzy relation $\tilde{S}$, corresponding to a fuzzy condition S, becomes the basis for the fuzzy relation $\tilde{\tilde{S}}$, formalizing both function behavior conditions. According to [4, 5], there holds

$$\mu_{\tilde{\tilde{S}}}(l,j)=\mathop{K}_{i=1}^{k}\mu_{\tilde{\tilde{S}}}(l-1,i)\times\mu_{\tilde{S}}(i,j),\ (1\le j\le k, 2\le l\le k)$$

with the initial value $f(0)=0$. The first row of the relation matrix $\tilde{\tilde{S}}$, corresponding to value 0, equals (1, 0, ..., 0) – as the initial condition. The elements of $\tilde{\tilde{S}}$ are acquired by the multiplication of the previous row of the relation matrix $\tilde{\tilde{S}}$ on columns of the matrix $\tilde{S}$ but instead of the multiplication a triangle norm is used.

A triangle norm is a real function of two variables, $K:[0,1]\times[0,1]\to[0,1]$, satisfying the following conditions:

1) $K(1,1)=1, K(\mu_{\tilde{A}},0)=K(0,\mu_{\tilde{A}})=\mu_{\tilde{A}}$ (limited);

2) $K(\mu_{\tilde{A}},\mu_{\tilde{B}})\ge K(\mu_{\tilde{C}},\mu_{\tilde{D}})$ if $\mu_{\tilde{A}}\ge\mu_{\tilde{C}},\mu_{\tilde{B}}\ge\mu_{\tilde{D}}$ (monotonicity);

3) $K(\mu_{\tilde{A}},\mu_{\tilde{B}})=K(\mu_{\tilde{B}},\mu_{\tilde{A}})$ (commutativity);

4) $K(\mu_{\tilde{A}},K(\mu_{\tilde{B}},\mu_{\tilde{C}}))=K(K(\mu_{\tilde{A}},\mu_{\tilde{B}}),\mu_{\tilde{C}})$ (associativity).

Suppose, the initial condition is formulated for $f(k-1)$ instead of $f(0)$. Let it be $f(k-1)=q$, $(1\le q\le k-1)$, defining the *k*-th row of the relation matrix $\tilde{\tilde{S}}$. The row consists of zeros except for 1 in column *q*. The other rows of matrix $\tilde{\tilde{S}}$ are

$$\mu_{\tilde{\tilde{S}}}(l,j)=\mathop{K}_{i=1}^{k}\mu_{\tilde{\tilde{S}}}(l+1,i)\times\mu_{\tilde{S}}(j,i),\ (1\le j\le k, 1\le l\le k-1).$$

Assume that the initial condition is set for some intermediate value $f(l^*-1), (1<l^*-1<k)$. Let it be $f(l^*-1)=q, (1\le q\le k-1)$, defining the l^*-th row of

matrix $\tilde{\tilde{S}}$. This row consists of zeros except for 1 in column q. A common way of obtaining the matrix $\tilde{\tilde{S}}$ is defined by the formula:

$$\mu_{\tilde{\tilde{S}}}(l,j)=\begin{cases} \underset{i=1}{\overset{k}{K}} \mu_{\tilde{\tilde{S}}}(l+1,i)\times\mu_{\tilde{S}}(j,i) & \text{for } 1\le l<l^* \\ \underset{i=1}{\overset{k}{K}} \mu_{\tilde{\tilde{S}}}(l-1,i)\times\mu_{\tilde{S}}(i,j) & \text{for } l^*<l\le k \end{cases}$$

$$(1\le l\le k, 1\le j\le k).$$

Suppose some function of one variable has one fuzzy condition and t initial conditions. Therefore that if there are several initial conditions, then $\tilde{\tilde{S}}_i$ is computed for each i-th condition separately ($1 < i \le t$), and the and resulting matrix $\tilde{\tilde{S}}^1$ is given as follows

$$\tilde{\tilde{S}}^1=\underset{i=1}{\overset{t}{\mathrm{T}}}\tilde{\tilde{S}}_i$$

As a result of the formalization of all conditions of k-valued logic functions a fuzzy relation matrix is produced. Functions, defined by forming fuzzy relation, applied to a real task. The posterior information obtained is compared with prior information from experts. Functions with disagreements are discarded. The remaining functions are left. If no functions is left, then we require an additional specification about function conditions from experts and we should retry the computations.

3. Rating Evaluation and Functions from Fuzzy Classes of k-valued Logic

Consider N objects with some quality characteristics (characteristic features) evaluated, $X_j, j=\overline{1,m}$, with corresponding values X_{lj}, $l=\overline{1,m_j}, j=\overline{1,m}$. Assume that the characteristics $X_j, j=\overline{1,m}$ have a significant impact on the characteristic Y (with values $Y_l, l=\overline{1,k}$) that implies a successful functioning of objects in the future.

As a result of a rating evaluation of objects with respect to $X_j, j=\overline{1,m}$, control effects on Y occur. To define rating estimates of objects, results of scoring are formalized using complete orthogonal semantic spaces [1].

A linguistic variable is a quintuple

$$\{X, T(X), U, V, S\},$$

where X – is a name of a variable; $T(X)=\{X_i, i=\overline{1,m}\}$ – a term-set of variable X, i.e. a set of terms or names of linguistic values of variable X (each of these values is a fuzzy variable with a value from a universal set U); V – is a syntactical rule that gives names of the values of a linguistic variable X; S – is a semantic rule that gives to each fuzzy variable with a name from $T(X)$ a corresponding fuzzy subset of a universal set U [7].

A semantic scope is a linguistic variable with a fixed term-set [7]. A theoretical analysis of the properties of semantic scopes aimed at adequacy improvement of the expert assessment models and their usefulness for practical tasks solution has made it possible to formulate the valid requirements for the membership functions $\mu_l(x), l=\overline{1,m}$, of their term-sets $T(X)=\{X_l, l=\overline{1,m}\}$ [8]:

1. For each $X_l, l=\overline{1,m}$, there is $\hat{U}_l \ne \emptyset$, where $\hat{U}_l=\{x\in U: \mu_l(x)=1\}$ is a point or an interval.

2. If $\hat{U}_l=\{x\in U: \mu_l(x)=1\}$, then $\mu_l(x), l=\overline{1,m}$, does not decrease to the left of $\hat{U}_l$ and does not increase to the right of $\hat{U}_l$.

3. $\mu_l(x), l=\overline{1,m}$ have maximally two points of discontinuity of the first type.

4. For each $x\in U$, there holds $\sum_{l=1}^{m}\mu_l(x)=1$.

The semantic scope the membership functions of which meet the requirements mentioned has been termed the Full Orthogonal Semantic Scope (FOSS) [8].

Based on results from [2], let us construct m FOSS's, X_j, $j=\overline{1,m}$, with their corresponding term-sets X_{lj}, $l=\overline{1,m_j}, j=\overline{1,m}$. Let $\mu_{lj}(x)$ be a membership function of fuzzy number $\tilde{X}_{lj}$, corresponding to the l-th term of the j-th FOSS, $l=\overline{1,m_j}$, $j=\overline{1,m}$. A fuzzy number [9] $\tilde{A}$ is a fuzzy set with the membership function $\mu_{\tilde{A}}(x): R\to[0,1]$.

Let $\tilde{X}_j^n$ and $\mu_j^n(x)\equiv(a_{j1}^n, a_{j2}^n, a_{jL}^n, a_{jR}^n), n=\overline{1,N}, j=\overline{1,m}$, be an estimate of a characteristic X_j of the n-th object. Fuzzy value $\tilde{X}_j^n$ with its membership function $\mu_j^n(x)$ is equal to one of the fuzzy values $\tilde{X}_{lj}, l=\overline{1,m_j}$, $j=\overline{1,m}$. The first two parameters in brackets are the abscissas of the apexes of the trapezoid upper bases that are graphs of the corresponding membership functions; while the last two parameters are the lengths of the left and right trapezoid wings, respectively.

We denote the weight coefficients of the evaluated characteristics by $\omega_j, j=\overline{1,k}$, $\sum_{j=1}^{k}\omega_j=1$. A fuzzy rating point of the n-th object [1, 9], $n=\overline{1,N}$, within the characteristics X_j, $j=\overline{1,m}$, is determined as a fuzzy number

$$\tilde{A}_n=\omega_1\otimes\tilde{X}_1^n\oplus\ldots\oplus\omega_k\otimes\tilde{X}_m^n$$

with its membership function

$$\mu_n(x)\equiv\left(\sum_{j=1}^{m}\omega_j a_{j1}^n, \sum_{j=1}^{m}\omega_j a_{j2}^n, \sum_{j=1}^{m}\omega_j a_{jL}^n, \sum_{j=1}^{m}\omega_j a_{jR}^n\right), n=\overline{1,N}.$$

The defuzzification of the fuzzy values $\tilde{A}_n, n=\overline{1,N}$, $\tilde{B}_1=\omega_1\otimes\tilde{X}_{11}\oplus\ldots\oplus\omega_k\otimes\tilde{X}_{1m}$, $\tilde{B}_m=\omega_1\otimes\tilde{X}_{m_1 1}\oplus\ldots\oplus\omega_k\otimes\tilde{X}_{m_m m}$ using the method of gravity center [10] gives us the crisp values $A_n, n=\overline{1,N}, B_1, B_m$. Value $A_n, n=\overline{1,N}$ is called a rating point of appearance of the quality characteristics X_j, $j=\overline{1,m}$, of the n-th object, $n=\overline{1,N}$.

A normed rating point of the n-th object, $n=\overline{1,N}$, is calculated as follows

$$E_n=\frac{A_n-B_1}{B_m-B_1}, n=\overline{1,N}.$$

and the range of values of $E_n, n=\overline{1,N}$ is the unit interval, [0, 1].

To generate control effects as a result of a rating estimation we will use fuzzy logic functions. Suppose that we have *m* variables X_j, $j=\overline{1,m}$, and the desired function takes *k* values (corresponding to the values of s). Our aim is to construct a function with *m* variables from fuzzy classes of a *k*-valued logic. The constructed function make it possible to split estimates on *k* clusters corresponding to the values of *Y*. A control effect aiming at a successful functioning of the object in future is set for each cluster.

The behavior of the desired function is restricted by initial and fuzzy conditions. The construction of such a function is demonstrated in the next section.

4. Decision Making Aimed at Guaranteeing a Commercial Success of Software

Twenty software products designed for retail sales automation, banking, insurance and intercompany accounting were selected for the research. Developed products were used by consumers on a trial mode basis. As input characteristics of software products three qualities were selected: X_1 – modifiability, X_2 – learning curve, and X_3 – functionality.

The modifiability is a characteristic feature of software simplifying a modification of a product, including the modularity, scalability and structuring. A low learning curve makes it possible to reduce efforts on learning and understanding the software and documentation and includes: the informativeness, structuring and readability. The functionality provides a set of functions defined in a product description and satisfying customers' needs. As an output characteristic feature, the success of product was used – *Y*, including its popularity, sales and experts' recognition.

All characteristics mentioned above were compared with respect to three linguistic values: «low», «middle», «high», with their corresponding scores 0, 1, 2, respectively. As a result of a trial usage of software products and their rating points, a recommendation system meant for the improvement of the product's success is presented below.

The experts evaluation results are shown in Table 1.

Table 1. Evaluation results of the software products

n	X_1	X_2	X_3
1	0	1	0
2	0	0	0
3	1	0	2
4	1	1	1
5	2	0	2
6	2	1	1
7	0	1	1
8	1	0	1
9	1	2	0
10	1	2	0
11	0	0	1
12	1	1	0

The data given in Table 1 were formalized using FOSS [2]. The membership functions of linguistic variables «low», «middle», «high» are shown in Table 2. If the membership functions are trapezoid, then the membership function is defined by four parameters. The first two parameters are abscissas of the apexes of the trapezoid upper bases that are graphs of the corresponding membership function while the last two parameters are the lengths of the left and right trapezoid wings, correspondingly. If the membership function is triangular, then it is clearly defined by three parameters. The first parameter is the abscissa of the vertex of the triangle, and the remaining two parameters are the lengths of the left and right wings, respectively.

Table 2. Software products' evaluations given as the trapezoid fuzzy numbers

n	X_1	X_2	X_3
1	(0,0.15,0,0.3)	(0.375,0.425,0.25,0.35)	(0,0.125,0,0.25)
2	(0,0.15,0,0.3)	(0,0.125,0,0.25)	(0,0.125,0,0.25)
3	(0.45,0.55,0.3,0.3)	(0,0.125,0,0.25)	(0.85,1,0.3,0)
4	(0.45,0.55,0.3,0.3)	(0.375,0.425,0.25,0.35)	(0.375,0.55,0.25,0.3)
5	(0.85,1,0.3,0)	(0,0.125,0,0.25)	(0.85,1,0.3,0)
6	(0.85,1,0.3,0)	(0.375,0.425,0.25,0.35)	(0.375,0.55,0.25,0.3)
7	(0,0.15,0,0.3)	(0.375,0.425,0.25,0.35)	(0.375,0.55,0.25,0.3)
8	(0.45,0.55,0.3,0.3)	(0,0.125,0,0.25)	(0.375,0.55,0.25,0.3)
9	(0.45,0.55,0.3,0.3)	(0.775,1,0.35,0)	(0,0.125,0,0.25)
10	(0.45,0.55,0.3,0.3)	(0.775,1,0.35,0)	(0,0.125,0,0.25)
11	(0,0.15,0,0.3)	(0,0.125,0,0.25)	(0.375,0.55,0.25,0.3)
12	(0.45,0.55,0.3,0.3)	(0.375,0.425,0.25,0.35)	(0,0.125,0,0.25)

The rating points have been calculated and are shown in Table 3. The weight coefficients $\omega_j, j=\overline{1,3}$, are equal to $\frac{1}{3}$ according to the experts.

The evaluation results obtained and the ratings are then used to produce control recommendations aiming at achieving a success of the product.

Usually the rating points are split into several intervals obtained in an interaction with the experts. Upon attaining a certain interval, a corresponding control effect is used. We assume that there are 3 intervals corresponding to the quantity of output characteristic feature, and the developed control effects.

If the rating point attains for the first time $[0, x]$ then the product's success *Y* is low, and a severe refinement is necessary. If the rating point of the software product in question attains the medium interval $[x, y]$ then the software product's success *Y* is medium, and

Table 3. Rating points and rating of software products

n	Rating points	Rating
1	0.248	11
2	0	12
3	0.746	2
4	0.542	6
5	0.816	1
6	0.676	3
7	0.457	8
8	0.433	9
9	0.613	4, 5
10	0.613	4, 5
11	0.329	10
12	0.462	7

the product requires a minor work. If the rating point attains the last interval [y, 1], then the product's success is high and the product is ready for the market. A logic function with fuzzy conditions will be used to generate the control effects.

Function F depending on variables X_1, X_2, X_3, takes on one of three values – «product success is low», «product success is medium», «product success is high». These values correspond to the values 0, 1 and 2 and the corresponding control effects are «product requires severe refinement», «product requires minor refinement», «product is ready for the market». The linguistic values X_1, X_2, X_3 «low», «middle», «high» correspond to the values 0, 1 and 2. The experts form the initial conditions as follows: $F(X_1 = 2) = 2, F(X_2 = 2) = 2,\ F(X_3 = 2) = 2$, and the fuzzy conditions as «slightly-increase» on function behavior for each variable. These conditions are formalized using fuzzy relations. Matrices of these fuzzy relations are shown in Tables 4–6.

Table 4. Fuzzy relation matrix describing «slightly-increase» of logic function F on variable X_1

	0	1	2
0	0.9	1	0.9
1	0	0.9	1
2	0	0	0.9

Table 5. Fuzzy relation matrix describing «slightly-increase» of logic function on variable X_2

	0	1	2
0	0.7	1	0.7
1	0	0.7	1
2	0	0	0.7

Table 6. Fuzzy relation matrix describing «slightly-increase» of logic function on variable X_3

	0	1	2
0	0.8	1	0.8
1	0	0.8	1
2	0	0	0.8

As a result the functions describing the behavior conditions, following fuzzy relation matrices from Tables 4–6, are obtained as shown in Tables 7–9.

Table 7. Fuzzy relation matrix describing logic function F values on variable X_1

	0	1	2
0	1	0.9	0.9
1	0.9	0.9	0.9
2	0	0	1

Table 8. Fuzzy relation matrix describing logic function values on variable X_2

	0	1	2
0	1	0.7	0.7
1	0.7	0.7	0.7
2	0	0	1

Table 9. Fuzzy relation matrix describing logic function values on variable X_3

	0	1	2
0	1	0.8	0.8
1	0.8	0.8	0.8
2	0	0	1

The intersection of the $i+1$-th row and $j+1$-th column of the matrices from Tables 7–9 describes the level of confidence of the function F acceptance value j when variables X_1, X_2, X_3 equal to i, $i = \overline{0,2}$, $j = \overline{0,2}$.

As a result, by taking into account all conditions an equation with a 27 row and 3 column matrix is obtained. The elements of this matrix are levels of confidence of F taking a certain value depending on the values of variables X_1, X_2, X_3. For example, the level of confidence of F being equal to 1 when $X_1 = 0, X_2 = 1, X_3 = 0$, is obtained as follows: take the minimum element on the intersection of the first row and the second column of the matrix shown in Table 7, the element on the intersection of the first row and the second column of the matrix shown in Table 8 and the element on the intersection of the first row and the second column of the matrix shown in Table 9.

After all the computation, a fuzzy relation describing the fuzzy logic function F is obtained, The elements of the entries of the matrix representing this fuzzy relation are shown in Table 10.

Table 10. Fuzzy relation describing function F

	0	1	2
000	1	0.7	0.7
001	0.8	0.7	0.7
002	0	0	0.7
010	0.7	0.7	0.7
011	0.7	0.7	0.7
012	0	0	0.7
020	0	0	0.8
021	0	0	0.8
022	0	0	0.9
100	0.9	0.7	0.7
101	0.8	0.7	0.7
102	0	0	0.7
110	0.7	0.7	0.7
111	0.7	0.7	0.7
112	0	0	0.7
120	0	0	0.8
121	0	0	0.8
122	0	0	0.9
200	0	0	0.7
201	0	0	0.7
202	0	0	0.7
210	0	0	0.7
211	0	0	0.7
121	0	0	0.7
220	0	0	0.8
221	0	0	0.8
222	0	0	1

The interaction with the experts has made it possible to construct the fuzzy function of the 3-valued logic with 3 variables, and this function's values are shown in Table 11.

Using function *F* we can conclude that: the software products Nr 1, 2, 8, 11, with the rating points in the first interval [0, 0.45] require severe improvements, the software products Nr 4, 7, 12, with the rating points in the second interval [0.45, 0.55] require slight improvements, the software products Nr 3, 5, 6, 9, 10, in the third interval [0.55, 1] are ready for the marked. Thus, function *F* has made it possible to split the rating points into three intervals and set a control effect for each interval that is focused on the product's success in the future. These results match the experts' opinion based on experience and knowledge.

Table 11. The 3-valued logic function F of 3 variables

Variables	Function value
000	0
001	0
002	2
010	0
011	1
012	2
020	2
021	2
022	2
100	0
101	0
102	2
110	1
111	1
112	2
120	2
121	2
122	2
200	2
201	2
202	2
210	2
211	2
121	2
220	2
221	2
222	2

5. Concluding remarks

The rating evaluation is applied in a vast number of human activities and used to form control effects aiming at an effective and efficient functioning of objects under consideration. Difficulties in the selection of control effect come from incomplete information or possible errors in the experts' decisions. Furthermore indefinite zones of the rating score values exist and make decision making ambiguous.

This paper proposed an approach to decision making on rating points using functions from fuzzy classes of the k -valued logic. The derivation of such functions is performed using initial conditions and fuzzy conditions on their behavior. The functions obtained make it possible to cluster the rating points with their corresponding control effect, for a better functionality success. An example of a practical application confirms the effectiveness and efficiency of the developed approach.

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REFERENCES

[1] Poleshchuk O., "The determination of students' fuzzy rating points and qualification levels", *International Journal of Industrial and Systems Engineering*, 2011, vol. 9, no. 1, 13–20.

[2] Poleshchuk O., Komarov E. "The determination of rating points of objects with qualitative characteristics and their usage in decision making problems", *International Journal of Computational and Mathematical Sciences*, 2009, vol. 3, no. 7, 360 – 364.

[3] Poleshchuk O., Komarov E., "The determination of rating points of objects and groups of objects with qualitative characteristics." In: *Annual Conference of the North American Fuzzy Information Processing Society*, 2009, p. 5156416.

[4] Ryjov A., "Fuzzy data bases: description of objects and retrieval of information." In: *Proceeding of the First European Congress in Intelligent Technologies*, 1993, vol. 3, 1557– 1562.

[5] Rogozhin S., Ryjov A., "Fuzzy classes in k-valued logic." In: *V National conference "Neurocomputers and applications"*, 1999, 460–463.

[6] Darwish A., Poleshchuk O., "New models for monitoring and clustering of the state of plant species based on sematic spaces", *Journal of Intelligent and Fuzzy Systems*, 2014, vol. 26, no. 3, 1089–1094.

[7] Zadeh L.A., "The Concept of a linguistic variable and its application to approximate reasoning", Part 1, 2 and 3, *Information Sciences*, 1975, vol. 8, 199–249, 301-357, 1976,vol. 9, 43–80.

[8] Ryjov A., "The Concept of a Full Orthogonal Semantic Scope and the Measuring of Semantic Uncertainty." In: *Fifth International Conference Information Processing and Management of Uncertainty in Knowledge-Based Systems*, 1994, 33–34.

[9] Poleshchuk O., Komarov E., "Expert Fuzzy Information Processing", *Studies in Fuzziness and Soft Computing*, 2011, 1–239.

[10] D. Dubois, H. Prade, "Fuzzy real algebra: some results," *Fuzzy Sets and Systems*, 1979, vol. 2, no.4, 327–348.

[11] Yager R., Filev D.P., "On the issue of defuzzification and selection based on a fuzzy set", *Fuzzy Sets and Systems*, 1993, vol. 55, no. 3, 255– 272.