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Integrated maintenance, inventory and quality engineering decisions for multi-product systems

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ABSTRACT
It is essential for manufacturers to consider the interrelation among quality, inventory, and maintenance decisions to detect imperfect quality products, keep the production system in good operating condition, and manage quality and inventory costs. Hence, this paper aims to develop an integrated model of inventory planning, quality engineering, and maintenance scheduling in which the expected total cost per time unit is minimised by determining the sample size, sampling interval, control limit coefficient, along with production cycle time. In this regard, an imperfect multi-product manufacturing system is considered, in which the inventory shortage in satisfying the demand for each product type and the idle time during the production cycle are not allowed. It is assumed that the process starts in an in-control condition where most produced units are conforming. However, due to the occurrence of an assignable cause (AC), the process mean moves to an out-of-control condition in which a significant fraction of non-conforming units is produced. The efficiency of the proposed mathematical model is evaluated by a numerical example, and then the sensitivity of the proposed model to important inputs is analysed. Finally, a comparative study based on the Taguchi design approach is given to confirm the capability of the proposed model to achieve remarkable cost savings.

KEY WORDS
multi-product system, production planning, maintenance scheduling, control chart, inventory planning, imperfect production system

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Introduction

Increasingly more companies are moving towards producing several items by a single machine to increase production efficiency, stock different produced items and reduce the total cost over the planning horizon. Using a multi-product manufacturing system increases manufacturing productivity by satisfying customer orders faster and more economically. A multi-product production planning problem aims to reach the optimum order and configuration

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of products by minimising the expected total cost and requires an all-around look at different aspects of the problem. Since multi-product systems usually use expensive and complex equipment, it is imperative to keep them in a suitable operational condition by implementing efficient maintenance scheduling. Maintenance activities usually require the suspension of the manufacturing cycle which increases the total production cost. This issue is clearly in conflict with manufacturer cost objectives. However, overhauling the system components reduces the rate of non-conforming products, and consequently, the quality-related costs are decreased remarkably. As another important issue, it is essential to detect process anomalies quickly to satisfy customer expectations. By optimising the sample size, sampling interval, and control limit coefficient, quality control charts lead to decreasing production costs due to a reduction in non-conforming items. Classical economic production quantity (EPQ) models have some technical drawbacks as follows: (1) they fail to consider imperfect production, which may be caused by the fault of machines/equipment, labour mistakes, and deficiency of raw materials; (2) they fail to consider quality control decisions; and (3) they only consider single output whereas, in practice, several products are often produced by a unique set of machines to reduce the system's idle time. Because of mentioned drawbacks and the interrelation among production planning, maintenance scheduling, and quality engineering concepts, the related decisions must be made simultaneously.

Most studies in the context of statistical and economical process quality control, inventory planning, and maintenance scheduling have neglected their interrelation. However, only a few papers in the literature have integrated these concepts, particularly in multi-product systems. In the late 1950s, different researchers attempted to formulate production scheduling to minimise production costs. Rogers (1958) extended a computational basis for economic lot scheduling and established a set of general equations for different scheduling situations. Taking setup time and setup cost into account, Bomberger (1966) provided a dynamic programming model for manufacturing different products in a multi-product production system. By modifying the drawbacks of previous research efforts, Madigan (1968) proposed a simple method for solving scheduling issues of multi-product, single-machine companies. Stankard et al. (1969) suggested a heuristic algorithm to improve the dynamic programming provided by Bomberger (1966). After that, Hodgson (1970) extended a grouping procedure to obtain better results than Stankard et al. (1969), particularly in moderate and high-loading cases. Then, Backer (1970) presented some corrections to improve the method proposed by Madigan (1968) for solving deterministic multi-product inventory problems. Rosenblatt et al. (1986) explored the effect of an imperfect manufacturing system on the optimal production cycle length. They considered the condition in which the process deteriorates over time and produces a certain percentage of defective products.

The 1980s was the decade of quality discussions in industries, and researchers attempted to deal with the economic-statistical design (ESD) of control charts. In this context, Lorenzen and Vance (1986) introduced a general formulation for the economic design of control charts to obtain optimum values of sample size, sampling interval, and control limit coefficient. Amiri et al. (2014) developed a multi-objective ESD of the exponentially weighted moving average (EWMA) chart. They used two evolutionary algorithms, the non-dominated sorting genetic algorithm (NSGA-II) and the multi-objective genetic algorithm (MOGA), to obtain the optimum parameters of the EWMA monitoring scheme. Nenes et al. (2015) considered multiple assignable causes to probe the ESD of a variable-parameter (VP) Shewhart mean chart. Salmasnia et al. (2019a) established a multi-objective ESD model for the Hotelling T^2 monitoring strategy with double warning lines. They employed the NSGA-II algorithm to achieve proper nondominated solutions. Ershadi et al. (2021) suggested a tri-objective mathematical programming model for the ESD of simple linear profile charts. They combined the multiple objective particle swarm optimisation (MOPSO) algorithm with the response surface methodology (RSM) to optimise three objective functions of the total cost, in-control average run length (ARL_0), as well as out-of-control average run length (ARL_1). A multi-objective economic-statistical model for simple linear profile charts based on a hybrid NSGA-II/ RSM/TOPSIS framework was introduced by Roshanbin et al. (2022).

Regarding maintenance scheduling, Cho and Parlar (1991) reviewed the literature associated with the optimal maintenance and replacement models for multi-unit processes. Wang (2002) categorised different maintenance strategies for both single-unit and multi-unit processes and addressed the relationships among the existing models. To design optimal preventive maintenance (PM) and replacement strategies in repairable and maintainable systems, Moghadam et al. (2011) established a programming model to ascertain optimisation of the overall cost and system reliability during the planning horizon. Aiming at minimising the cost rate subject to a reliability constraint, Liu et al. (2019) focused on a maintenance planning problem for single-component systems. They provided a comparative study to evaluate and compare two age-based and reliability-based maintenance strategies using the degradation data of a real system. In contrast to Liu et al. (2019), Kamel et al. (2020) developed a maintenance scheduling approach for complex repairable systems. They optimised the maintenance cost, including random failure cost, repair cost, replacement cost, and total planned downtime cost, in a way that the system availability satisfies a pre-specified level. Under the uncertainty of product demands, a maintenance planning model for the flexible multistage processes in multi-specification and small-batch production was developed by Zhou et al. (2021). Hu et al. (2022) suggested a linear Programming-enhanced RollouT (LPRT) for online maintenance scheduling, which optimises total maintenance cost by satisfying the system's reliability.

The integrated quality-maintenance models have received increasing interest in the literature. In this context, Tagras (1988) employed a Markov model and established general economic programming by incorporating quality monitoring and maintenance scheduling. Under the assumption that the mean parameter deviates from its nominal value due to equipment failure, Cassady et al. (2000) proposed a hybrid quality-maintenance policy by combining the chart and the PM strategy. To achieve lower costs related to quality, maintenance, and inspection activities, Linderman et al. (2005) studied joint optimisation of quality control and adaptive maintenance scheduling. Zhou and Zhu (2008) presented an integrated quality-maintenance model and employed a grid-search methodology to obtain the optimum model values that minimise hourly costs. Gouiaa-Mtibaa et al. (2018) integrated maintenance and quality decision by considering the impact of the system degradation on product quality. Salmasnia et al. (2018a) incorporated the ESD of an adaptive non-central chi-square monitoring scheme for simultaneous monitoring of mean and variability parameters with maintenance scheduling. Salmasnia et al. (2020a) proposed a unified model research by combining the ESD of a VP monitoring scheme with condition-based maintenance for two-unit series processes. They utilised the particle swarm optimisation (PSO) method to optimise the expected total cost per time unit under some statistical constraints. Interested readers may refer to Chen et al. (2011), Mehrafrooz and Noorossana (2011), Liu et al. (2013), and Xiang (2013) for detailed information on joint investigation of quality and maintenance management.

In the recent decade, joint consideration of production planning and maintenance strategy has gained growing attention within both academia and industry. In this regard, taking safety stock and maintenance into account, Pal et al. (2014) proposed a hybrid multi-echelon production-inventory model consisting of a manufacturer, supplier, and retailer. To achieve the values of production planning and maintenance planning variables in multi-state systems, Saeidi-Mehrabad et al. (2017) combined production and PM schedule. Considering a multi-product system, Liu et al. (2015) introduced an integrated model by combining the EPQ model and PM strategy.

Concerning hybrid maintenance-inventory-quality models, Ben-Daya and Makhdoum (1998) narrowed their focus to probe the impact of different PM strategies on the combination of the EPQ model and the economic design of the control chart. Taking an imperfect production system into account, Pan et al. (2012) combined the concept of statistical process monitoring (SPM) and maintenance planning with the classical EPQ model. Nurelfath et al. (2016) designed an optimisation model to obtain the optimum values of production planning, maintenance strategy, and SPM-related variables. They found that increasing the PM level leads to a reduction in quality monitoring costs. Salmasnia et al. (2017) extended a hybrid model based on production run length, maintenance schedule, and SPM based on multiple assignable causes. A hybrid maintenance-production model under the VP-T2 monitoring scheme was proposed by Salmasnia et al. (2018b). Fakher et al. (2018) focused on a multi-period multi-product capacitated lot-sizing problem and analysed the trade-off among maintenance, quality, and production. By employing a non-uniform sampling strategy, Salmasnia et al. (2020b) recommended another unified model based on production planning, maintenance management, and ESD of the control chart by taking the time value of money and the stochastic shift magnitude into account. Salmasnia et al. (2019b) studied the ESD of an adaptive non-central chi-square control chart considering production planning and maintenance scheduling. Salmasnia et al. (2022) proposed a unified production-maintenance-quality

Tab. 1. Summarised literature review

Author(s)	Concept(s)			Number of products		Solution approach	Time to shift distribution
	SPM	Maintenance	EPQ	Single	Multiple		
Rogers (1958)			✓		✓	Manual incremental	-
Salmasnia et al. (2018b)	✓	✓	✓	✓		PSO	Exponential
Salmasnia et al. (2019a)	✓			✓		NSGA-II/DEA	Exponential
Salmasnia et al. (2019b)	✓	✓	✓	✓		PSO	Weibull
Bomberger (1966)			✓		✓	Dynamic programming	-
Salmasnia et al. (2020a)	✓	✓		✓		PSO	Exponential
Madgan (1968)			✓		✓	Innovative algorithm	-
Rosenblatt et al. (1986)			✓	✓		Analytical	Linear / exponential
Lorenzen and Vance (1986)			✓	✓		Numerical technique	-
Liu et al. (2015)		✓	✓		✓	Integer programming	Exponential
Nourelfath et al. (2016)	✓	✓	✓		✓	mixed integer programming	Weibull
Chen et al. (2011)	✓	✓	✓	✓		Loss function	Exponential
Salmasnia et al. (2017)	✓	✓	✓	✓		PSO	Weibull
Lee et al. (2012)	✓			✓		Genetic Algorithms	Exponential
Nenes et al. (2015)	✓			✓		Markov chain	Erlang
Yu et al. (2010)	✓			✓		-	Exponential
Chen and Yang (2002)	✓			✓		Optimization model	Weibull
Makis and Fung (1998)	✓		✓	✓		Optimization model	Weibull
Linderman et al. (2005)	✓	✓		✓		Design of Experiments (DOE)	Weibull
Zhou and Zhu (2007)	✓	✓		✓		Grid search approach	Weibull
Yin et al. (2015)	✓	✓		✓		-	Weibull
Le Tang (2012)	✓	✓		✓		-	Exponential
Xiang (2013)	✓	✓		✓		Markov processes	Exponential
Mehrafrooz and Noorossana (2011)	✓	✓		✓		-	Exponential
Kuo (2006)	✓	✓		✓		Markov decision process	Binomial
Pan et al. (2012)	✓	✓	✓	✓		Hessian matrix.	Weibull
Rahim and Ben-Daya (1998)	✓		✓	✓		Non-Markovian Model	Weibull
Ben-Daya and Makhdoum (1998)	✓	✓	✓	✓		x-bar	Weibull
Salmasnia et al. (2020b)	✓	✓	✓	✓		PSO	Weibull
Beheshti Fakher et al. (2018)	✓	✓	✓		✓	Linear mixed-integer programs	Weibull
Gouiaa-Mtibaa et al. (2017)		✓	✓	✓		Lemma	-
Saidi-Mehrabad et al. (2017)		✓	✓		✓	CPLEX	Exponential
Salmasnia et al. (2022)	✓	✓	✓	✓		PSO	Weibull
This paper	✓	✓	✓		✓	PSO	Exponential

model when multiple assignable causes may affect mean and/or variance parameters. Table 1 represents an overview of the literature review considering important features of related previous studies.

Considering a multi-product system, the present article study presents a hybrid mathematical formulation to optimise the expected total production cost unit (ETCU) during each cycle. The proposed model involves the inventory cost, including holding and ordering costs, corrective and preventive maintenance costs, as well as sampling and quality control costs. Moreover, the optimal values of chart parameters, including the sample size, sampling interval between two consecutive subgroups, control limit coefficient, and cycle time, are obtained through the minimisation of ETCU. To do so, it is assumed that the system starts with an in-control situation and shifts to an out-of-control state when an assignable cause occurs.

The structure of this paper is organised as follows: the next section is devoted to theories, notations, and methodical aspects used for problem definition. The suggested mathematical formulation that combines production planning, maintenance scheduling, and quality control decisions is presented in Section 2. A solution approach for solving the proposed mathematical programming model is presented in Section 3. A numerical example, along with comparative study, is given in Section 4. In addition, the model behaviour is investigated through a sensitivity analysis across the important parameters of the proposed mathematical formulation in this section. Finally, the last Section is dedicated to the concluding remarks and recommendations for future studies.

1. Problem definition

To improve the accuracy of existing models, this study attempts to integrate inventory management, control chart, and repair-maintenance scheduling in a multi-product production process. The demand pattern for each produced item is assumed to be constant, and the developed programming model attempts to minimise the total manufacturing cost through optimisation of production cycle time and other decision variables. It is supposed that the production process starts with an in-control situation and may move to an out-of-control one when an unobservable assignable cause occurs. The occurrence of the assignable cause can be the consequence of the process nature, machine deterioration, change in machine setting, a bad batch of raw material, and operator errors which lead to a higher rate of non-conforming products.

The $\bar{X}$ Shewhart chart is used to trace the disturbances of the study quality characteristic. This chart issues an out-of-control signal when an assignable cause affects the process mean level. The study quality characteristic is a normally distributed variable with the in-control mean and standard deviation of μ_0 and σ_0, respectively. Due to the occurrence of assignable causes, the process mean deviates from its target value to $\mu_0 + \delta_u\sigma_0$ where δ_u denotes the shift magnitude. Note that the occurrence of the assignable cause significantly increases the rate of produced non-conforming items. In this case, if the $\bar{X}$chart detects the process deviation, then corrective maintenance (CM) activities are undertaken to restore the process to as-good-as-new condition. Otherwise, preventive maintenance (PM) operations are performed during set-up time to return the process mean to normal condition. Fig. 1 illustrates a different situation during each production cycle and maintenance policy for the system's renovation.

To formulate the objective function, the costs of each condition shall be calculated properly. The cost types during the production process are categorised into one of the following classes: 1) sampling costs, including variable and fixed costs; 2) quality costs for both in-control and out-of-control situations; 3) the inventory cost, including holding costs and other costs associated with storing produced items in a warehouse; 4) maintenance costs imposed by CM and PM activities; 5) false alarm costs; and 6) the set-up cost.

A manufacturing process that produces multiple items is investigated, where a production cycle is a complete run of all produced units. Since the production of a given item more than once during a production cycle may lead to shortage or system idleness, the focus is restricted to cases where each item is produced exactly once within a production cycle. In addition, the production and consumption time intervals for each product can be different. However, the sum of these two time intervals during a production cycle are equal for all products. Consider a company that produces three different products, each of which is produced once within each cycle. Let t_{p_i} denotes the production time of the i^{th} produced item. Fig. 2. depicts the ideal condition in which all items are produced exactly once, and the production cycle ends without shortage or idle time. In contrast to the ideal scenario, the shortage (Fig. 3) or system idleness (Fig. 4) occurs.

Remember that the decision variables in this article are the production cycle time (T) which is equal to the sum of t_{p_i}, the sample size (n), the sampling interval between two consecutive subgroups (h), and the control limit coefficient (l).

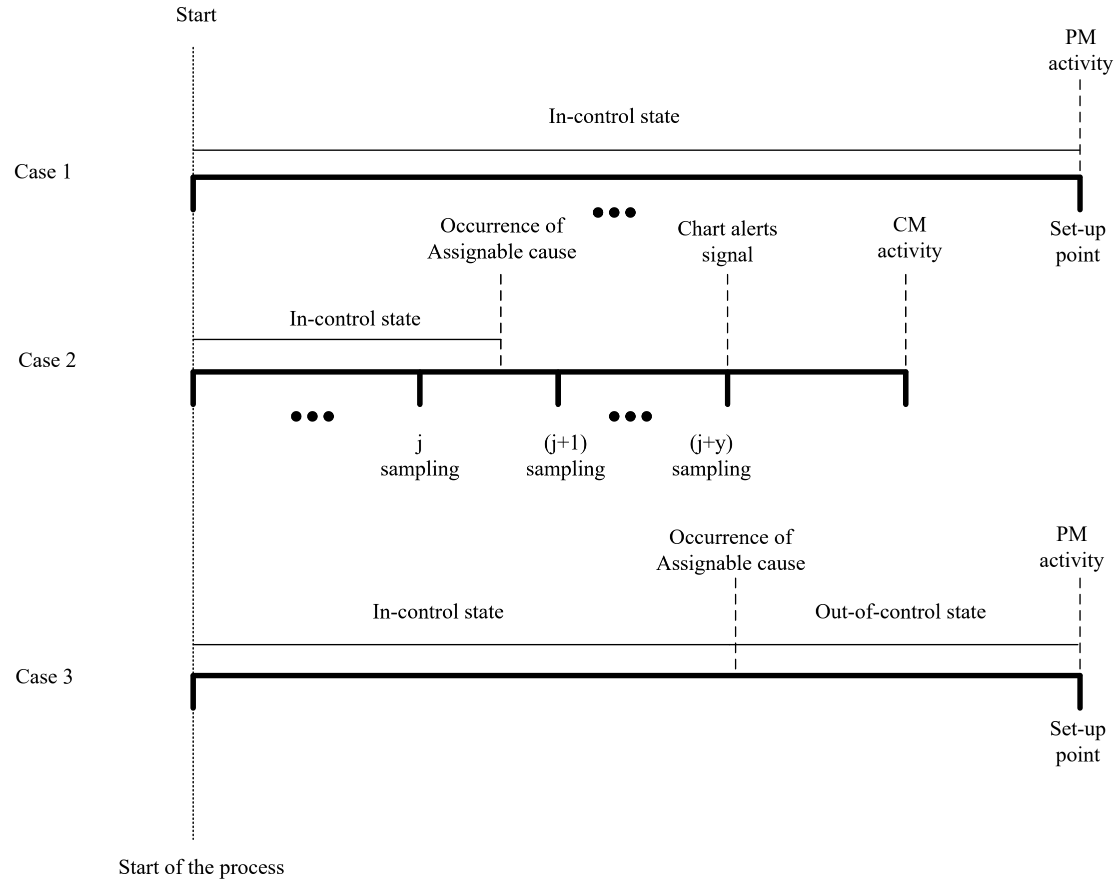

Fig. 1. Different scenarios in an operational cycle

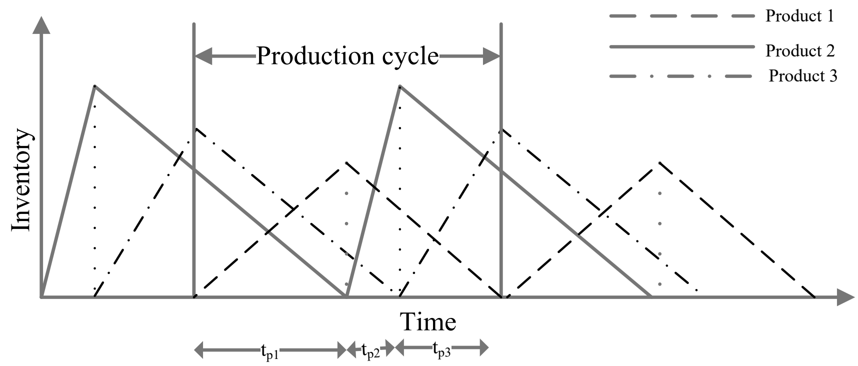

Fig. 2. Ideal condition, which is limited by $t_{p_i} = t_{p_j}$

1.1. Notations

This subsection defines all parameters used to formulate the proposed mathematical model. As illustrated in Table 2, the notations are classified into three general categories of indices, parameters, and decision variables.

1.2. Model assumptions

The assumptions in formulating the proposed mathematical programming model are outlined as follows:

1) The PM activity is performed before set-up time.

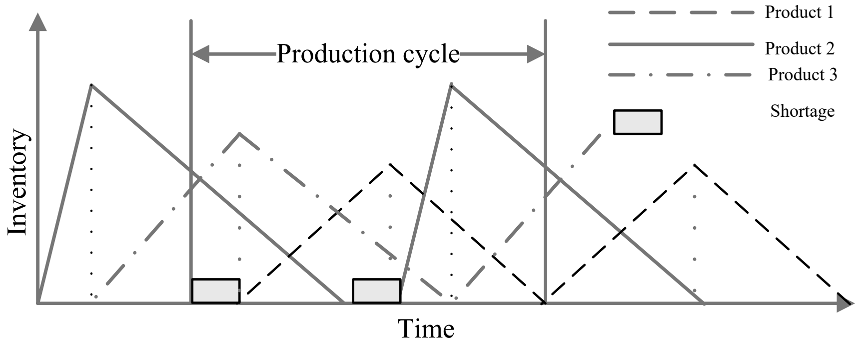

Fig. 3. Shortage case within the production cycle when $\boldsymbol{t_{p_i}} < \boldsymbol{t_{p_j}}$

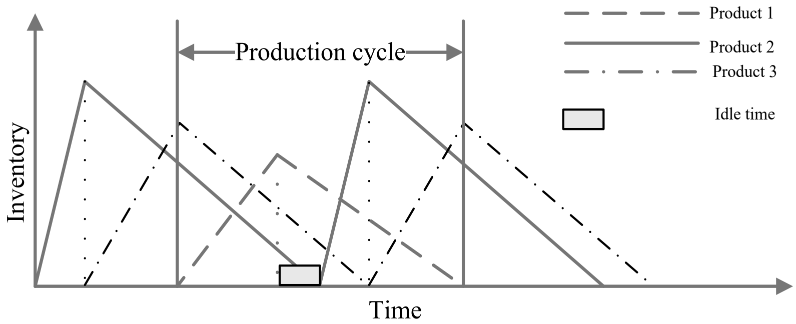

Fig. 4. System idleness within the production cycle when $\boldsymbol{t_{p_i}} > \boldsymbol{t_{p_j}}$

2) Time interval between the occurrence of two consecutive assignable causes follows an exponential distribution.

3) A production cycle is a complete run of all produced items, and each item is produced exactly once within a production cycle.

4) To avoid inventory shortage and system idleness within a production cycle, the sum of production and consumption time periods for all products are equal to each other. That is to say:

$$t_{p_i} + t_{d_i} = T_i = T \tag{1}$$

$$\sum_{i=1}^{k} t_{p_i} = T \tag{2}$$

5) To avoid inventory shortage and system idleness within a production cycle, we have $T_i = T$.

6) To avoid inventory shortage within each cycle, the following constraint is established:

$$p_i t_{p_i} = d_i \sum_{i=1}^{k} t_{p_j}, \forall i \tag{3}$$

7) The required times for implementing PM and CM activities, detecting ACs, and conducting system set-up are negligible.

8) CM activities start immediately after the detection of the assignable cause to restore the process mean to its in-control condition. This condition continues up to the end of the production cycle.

9) The process cannot return to its in-control condition when the assignable cause occurs.

10) Only one assignable cause exists during the production cycle.

11) The production process starts with an in-control state and moves to an out-of-control state when the occurrence ensues.

12) The process shifts are occurred independently according to homogeneous Poisson distribution.

13) The quality characteristic of interest is supposed to be a normally distributed variable. The occurrence of the assignable cause changes the process mean from its target value μ_0 to $\mu_0 \mp \delta_u \sigma_0$. Only increasing shifts are considered in this study because the probability of detecting increasing and decreasing mean shifts is the same.

14) The variance of the quality characteristic is not changed when the assignable cause occurs and remains unchanged during the production cycle.

2. Model description

In this section, the proposed mathematical model based on the integration of production planning, quality control, and maintenance schedule for a multi-product system is described. As previously mentioned, we assume that the process is in-control at the beginning of the cycle and may deviate from its in-control mean value during the production process when the assignable cause occurs. It is further supposed that the study quality characteristic, X, follows the normal distribution as $N(\mu_0, \sigma_0)$, where μ_0 and σ_0 are the target mean and standard deviation parameters, respectively. When a shift takes place, the mean level of the process changes from μ_0 to $\mu_0 + \delta_\mu \sigma_0$, and the model tries to use this variation for detecting the shift via a control chart and the process move to the out-of-control state. In such situations, it is crucial that the $\bar{X}$ chart detects the process disturbances as soon as possible. The total expected cost involves five cost elements, including the quality cost, sampling cost, inventory holding cost (IHC), maintenance cost, as well as set-up one.

2.1. Quality costs

The quality costs are incurred to ensure the conformance of produced items with the quality specification and/or to compensate for non-conforming outputs to achieve desired technical requirements. When the process mean level moves to an out-of-control condition, the percentage of nonconforming items extremely increases, and consequently, more quality costs are imposed on the manufacturer. For i^{th} product, when the mean parameter is out-of-control (in-control), the expected number of produced items is multiplied by the quality cost per unit under the out-of-control (in-control) condition. Thus, the expected quality cost per production cycle is given as:

$$E(QC) = \sum_{i=1}^{k} Q_{in_i} \times p_i \times ET_{in_i} + \sum_{i=1}^{k} Q_{out_i} \times p_i \times ET_{out_i} \quad (4)$$

where ET_{out_i} and ET_{in} denote the expected time which process stays in-control and out-of-control per production cycle. The value of ET_{out_i} is achieved by the following formula:

$$ET_{out_i} = TTD_i \times E(AC_i) \quad (5)$$

where $E(AC_i)$ is the expected occurrence number of assignable causes during producing i^{th} product per cycle while TTD_i is the time to assignable cause detection by the control chart. These values are given by Equations (6) and (7), respectively.

$$E(AC_i) = \int_0^{t_{p_i}} r(t)\,dt = t_{p_i}\lambda \quad (6)$$

$$TTD_i = h \times \min(ARL_{out}, N_i) - \tau \quad (7)$$

where $r(t) = \frac{f(t)}{1 - F(t)}$ represents the defect rate whereas, N_i is the number of taken samples for i^{th} product during a production cycle, and τ denotes the expected in-control time within the sampling interval in which the AC takes place.

$$N_i = \left\lceil \frac{t_{p_i}}{h} \right\rceil \quad (8)$$

$$\tau = \frac{\int_{ih}^{(i+1)h} (t - ih) f(t)\,dt}{\int_{ih}^{(i+1)h} f(t)\,dt} = \frac{\left[1 - (1 + \lambda h) e^{-\lambda h}\right]}{\left[\lambda \left(1 - e^{-\lambda h}\right)\right]} \quad (9)$$

Considering all products in a production cycle:

$$ET_{out} = \sum_{i=1}^{k} ET_{out_i} \quad (10)$$

$$ET_{in} = \sum_{i=1}^{k} ET_{in_i} \quad (11)$$

where:

$$ET_{in_i} = t_{p_i} - ET_{out_i} \quad (12)$$

2.2. Sampling cost

The expected sampling cost per production cycle, $E(S)$, consist of two variable and fixed elements. To obtain this cost element, the cost of taking each sample $\left(C_v \times n + C_f\right)$ must be multiplied by the total number of subgroups taken within a production cycle $\left(\sum_{i=1}^{k} N_i\right)$ as:

$$E(S) = \left(\sum_{i=1}^{k} N_i\right) \times \left(C_v \times n + C_f\right) \quad (13)$$

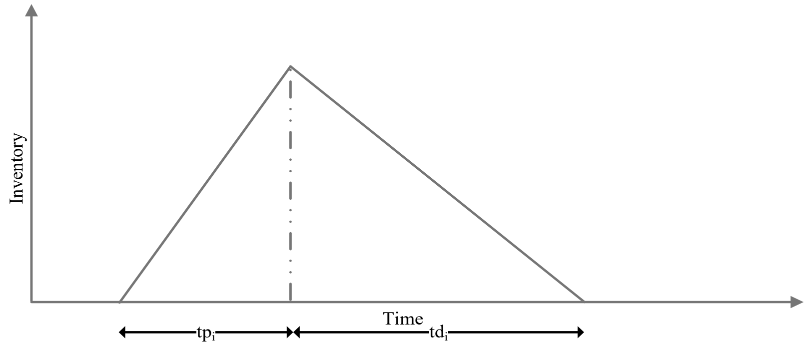

Fig. 5. Inventory approximation for product i

2.3. Inventory holding cost

As seen in Fig. 5, the inventory level for the i^{th} product increases during the production period and decreases since the consumption period begins. The inventory holding cost per production cycle (IHC) for i^{th} product can be derived by multiplication of inventory holding cost per time unit (H_i), production duration (t_{p_i}), the difference between production and demand rates ($p_i - d_i$), and the total duration of the production cycle (T). For all products, the IHC is calculated as follows:

$$E(H) = \sum_{i=1}^{k} \frac{(p_i - d_i) t_{p_i} T}{2} \times H_i \qquad (14)$$

2.4. Maintenance cost

The total maintenance cost involves the costs incurred by the implementation of PM and CM activities, along with the cost of the false alarm. As mentioned, the CM tasks are implemented whenever the $\overline{X}$ chart detects the assignable cause (Fig. 6). Otherwise, according to Fig. 7, the PM tasks are undertaken at the beginning of the production cycle if the shift is not recognized by the $\overline{X}$ chart. The expected CM and PM costs per production cycle are obtained according to Equations (15) and (16), respectively.

$$E(CM) = \left(\sum_{i=1}^{k} E(AC_i) \right) \times C_{cm} \times (1-\beta) \qquad (15)$$

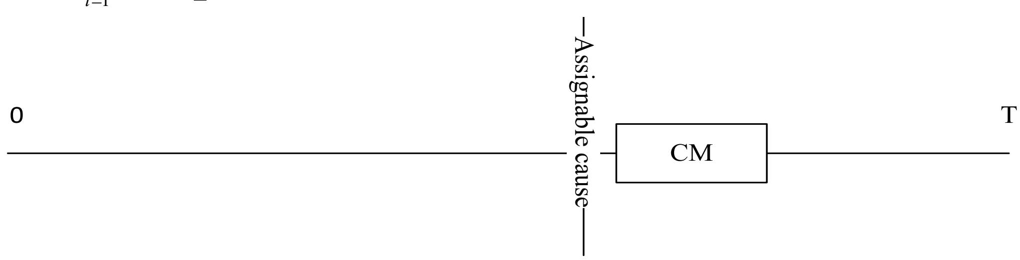

Fig. 6. Time of implementing CM tasks within the production cycle

Fig. 7. Time of implementing PM tasks

$$E\left(PM\right)=k\times C_{pm} \tag{16}$$

The expected false alarm cost per production cycle ($E(f_a)$) can be obtained by multiplication of three elements of the expected number of subgroups under the in-control situation $\left(ET_{in}/h\right)$, probability of Type I error (α), the cost of each false alarm (C_{fd}):

$$E\,(fa)=\frac{ET_{in}}{h}\times\alpha\times C_{fa} \tag{17}$$

The expected maintenance cost will be computed as:

$$E(M)=E(CM)+E(PM)+E(fa) \tag{18}$$

2.5. Set-up cost

The set-up cost within each production cycle is the total cost required to prepare all products and is calculated as:

$$E\left(SC\right)=\sum_{i=1}^{k}S_i \tag{19}$$

2.6. Cost function and constraints

Using the results of the previous subsections, the objective function and the constraints is written as:

$$\text{Min } ETCU=\left[E\left(S\right)+E\left(Q\right)+E\left(H\right)+E\left(M\right)+E\left(SC\right)\right]/T \tag{20}$$

$$\text{S.t: } p_i t_{p_i}=d_i\sum_{j=1}^{k}t_{p_j},\forall i \tag{20.1}$$

where shows the total expected cost per time unit within a production cycle while constraint (20.1) prevents inventory shortages during cycle time. See Fig. 8 for more clarification regarding the significant features of the proposed multi-product mathematical model in comparison with the classical models.

3. Solution approach

Because of some complexities, the proposed mathematical model cannot be solved by analytical

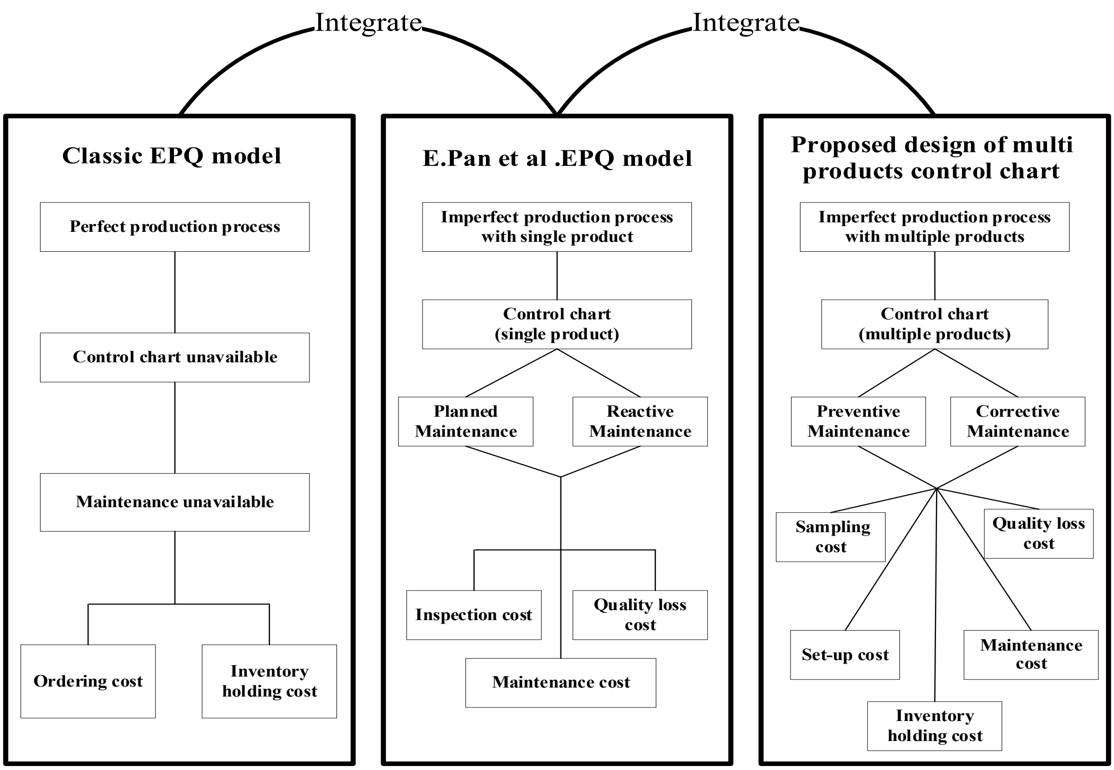

Fig. 8. Main features of the proposed mathematical model

methods. Some major complications are as follows: (1) the feasible area is non-convex and discontinuous; (2) some decision variables in the objective function are in the extreme bound of the integral or in the cumulative density function (CDF) of a (standard) normal distribution; and (3) the model includes both discrete and continuous decision variables.

Meta-heuristic algorithms are one of the up-to-date techniques for solving complex problems which have been extensively used to obtain desired results. Particle Swarm Optimisation (PSO) method is categorised as a meta-heuristic solution technique used to solve the developed numerical model efficiently as it can help in solving non-linear problems. Besides, it is a pioneer searching technique, which is easy to implement and has a simple concept.

3.1. PSO evolutional technique

The PSO, as a meta-heuristic searching algorithm, has been inspired by the social behaviour of a flock of birds looking for food or a bunch of fish.

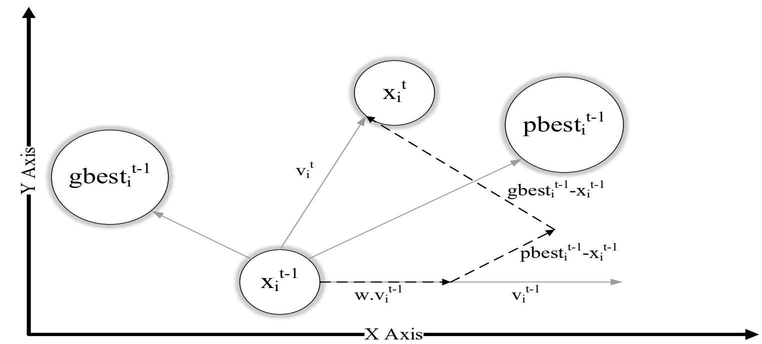

Fig. 9. Searching procedure in the PSO technique

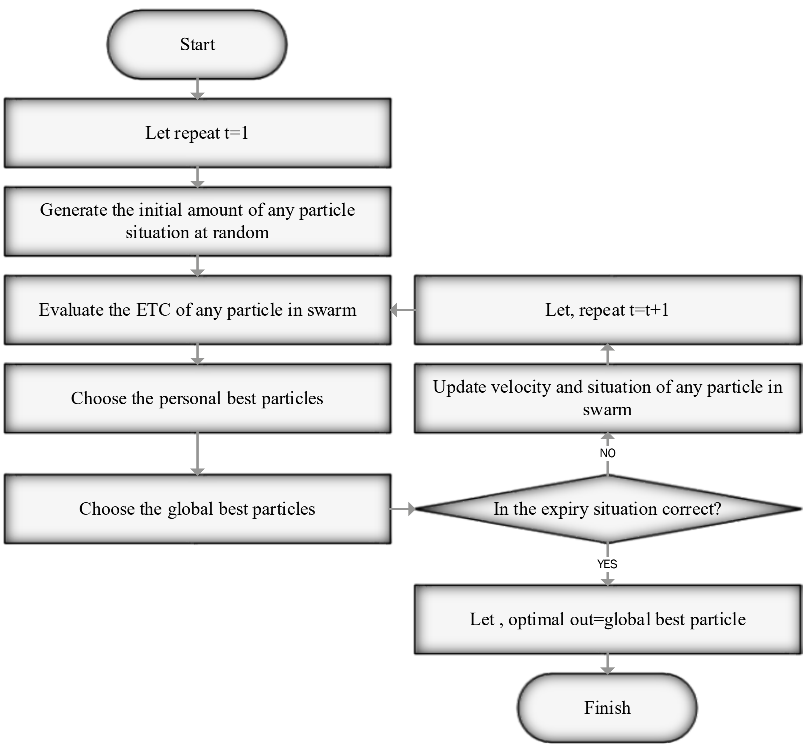

Fig. 10. Computational procedure of the PSO technique

Researchers found that this algorithm has a good performance for finding the optimal value of the decision variable in an optimisation problem. The method produces a random number of particles, and each particle, as one of the members of the population, tries to find the best position based on learning from itself and also from the other members of the group to improve the positions of particles in the solution path and finally finding the target, which is the best position and leads to the optimum value of the objective function. In this algorithm, particles sweep the problem space in accordance with the optimum experienced path of particles. Each particle has a velocity vector that determines the direction of the particle and also the objective function, which calculates the target value based on the particle's position and velocity.

This algorithm uses global and local searches to achieve high efficiency and is initialised with a random situation of particles that have velocities. Then, the algorithm searches to find the optimal value of the objective function by recalculating particle positions based on the two "best" values and the force of inertia. The first one is the best value, which itself was experienced in the solution path, which is named the personal best ( ), and the second is the best solution observed in the population, which is named the global best ( ). In each iteration, the particle will update its position and velocity after the calculation of the objective function and the related decision variable. The computational procedure of the PSO algorithm is summarised in Fig. 10, and a display of a searching point by the PSO algorithm in a feasible two-dimensional space is depicted in Fig. 9.

3.2. PSO implementation

Four-dimensional particles are considered, each of which is assigned to a certain decision variable, including the production cycle time (T), the sample size (n), the control limit coefficient (l), and the sampling interval between two consecutive samples (h). For producing the initial value of any continuous decision variable, as illustrated in the previous section, a uniform allocation of the random value is generated with regard to the allowable range of the decision variable. The amounts of discrete variables, i.e., the sample size (n), are achieved according to Eq. (21), respectively.

$$n = \min\left[\left(n_{\min} + floor\left(\left(n_{\max} - n_{\min} + 1\right) \times R\right)\right), n_{\max}\right] \quad (21)$$

where, n_{max} and n_{min} are the upper and lower range of n, respectively. Furthermore, R_1 is a uniformly distributed random number within the range (0,1).

4. Experimental results

This section investigates the efficiency of the proposed model to minimise the value of the expected total cost per time unit per production cycle subjected to model constraints. The proposed mathematical model is coded in MATLAB programming software, and its different aspects from both application and theory viewpoints are discussed based on the obtained results. Subsection 5.1 presents an industrial example in which the values of the cost objective function and problem decision variables are optimised through the PSO algorithm. In 5.2, using the Taguchi experimental design, the efficiency of the proposed hybrid model is compared with an alternative mathematical model, in which the quality, maintenance, and inventory decisions are made separately. Finally, in Subsection 5.3, a sensitivity analysis is performed on four parameters of: 1) the production rate of the i^{th} product during a production cycle, 2) the rate of quality cost under the out-of-control situation; 3) PM cost; and 4) cost incurred by each false alarm.

4.1. Numerical example

In this subsection, an industrial example presented by Salmasnia et al. (2017) and Pan et al. (2012) is employed to investigate the ability of the suggested formulation. In this example, a firm sells a certain food product to a wholesaler in packages of specified weights. According to Bisgaard et al. (1984), the weight of packages is selected as the quality characteristic of interest, and assignable causes can change the mean value of package weights. For one of the customers, the production and demand rates for three products are (60,30,20) and (40,20,8), respectively. The quality costs for each conforming and non-conforming product are (160,100,200) and (300,250,400). Each particular product has its own set-up time. To monitor the quality characteristic of interest, planned inspections are carried out periodically. At each inspection point, the fixed sampling cost per subgroup is considered as 25, while the variable sampling cost per item is equal to 4 units. Based on historical data, the occurrence of an assignable cause affects the process outcome and

Tab. 3. Parameter values

Parameter	d_i	p_i	k	Q_{in_i}	Q_{out_i}	S_i	λ
Value	(40‹20‹8)	(60‹30‹20)	3	(160‹100‹200)	(300‹250‹400)	(180‹110‹150)	0.02
Parameter	δ	H_i	C_f	C_v	C_{pm}	C_{cm}	C_{fa}
Value	1	(30‹25‹50)	25	4	2000	1000	100

shifts the mean parameter to $\mu_0 + \sigma_0$. The costs of implementing PM and CM tasks are 2000 and 1000 units, respectively. The cost incurred for each false alarm is 100 units. Finally, the set-up and holding costs are (180,110,150) and (30,25,50) per time unit. The values of the parameters are reported in Table 3.

The optimal values of decision variables obtained by the PSO algorithm are $[h^*, n^*, l^*, T^*] = [0.8831, 9, 1.7956, 3.9742]$, which leads to $ETCU^* = 13454.8552$. These values mean that the first sample with size $n = 9$ is taken after $h = 0.8831$ hour from the beginning of the production cycle. Moreover, the optimum length of the production cycle is calculated as $T = 3.9742$, implying that the PM tasks are conducted at the end of the cycle even though the process stays in-control. In addition, the control limit coefficient shall be considered at 1.7956.

4.2. Comparative study

In this subsection, the effectiveness of the developed hybrid inventory-quality-maintenance model referred to as "Model A", is compared with an alternative, in which the inventory, quality, and maintenance decisions are made separately ("Model B"). To make comparison studies more reliable and the results more defensible, these two models are compared under different scenarios. For this purpose, the Taguchi design is used to generate 27 scenarios under different values of the model parameters. The generated scenarios are presented in Table 4. Note that the values of other parameters are selected from Table 3. The obtained values of the expected total cost per time unit, along with the percentage of cost-saving for models A and B, are reported in Table 5. As expected, for all scenarios, optimising inventory-quality-maintenance decisions significantly increases the total cost imposed on the company. More specifically, an average reduction of about 20 per cent is achieved when the integrated model is replaced by the separated one. Table 6 contains each cost term obtained by Models A and B and their corresponding differences. It can be concluded from Table 6 that the impact of the set-up cost to reduce the cost function is more significant than others. Followed by the set-up cost, maintenance and quality costs have the greatest effect on the reduction of the total cost. On the other hand, since employing Model A leads to larger values of t_{p_i}, the sampling and holding costs obtained by Model A are both greater than those of Model B.

4.3. Sensitivity Analysis

In this subsection, a sensitivity analysis is presented to examine how the variation of four important parameters, including p_i (production rate during the production cycle), Q_{out_i} (quality cost under the out-of-control condition), C_{pm} (PM cost), and C_{fa} (false alarm cost), affects the optimum values of *ETCU* as well as model decision variables. To accomplish this, three different values are considered for each input parameter while other model parameters remain constant. The resulting values are given in Table 7 and in Figs. 11 to 14. The following conclusions are evident from the obtained results reported in Table 7.

1) According to Equations (4) and (14), p_i directly affects both inventory and quality cost formulas and consequently, $ETCU^*$ is considerably affected by the variation of the production rate (Fig. 11). For instance, when $[p_1, p_2, p_3]$ varies from [45 22 10] to [70 60 50], the optimal value of $ETCU^*$increases 1945 units. Moreover, it can be seen from Fig. 11 that by increasing $[p_1, p_2, p_3]$ from [45 22 10] to [70 60 50], the optimal values of h and T reduce. Moreover, as $[p_1, p_2, p_3]$increases from [45 22 10] to [60 30 20], the optimum value of n increases while further increment of $[p_1, p_2, p_3]$to [70 60 50] reduces the optimum value of the sample size from 9 to 6. The results confirm that there is no significant change in the optimum values of the control limit coefficient $l$when $[p_1, p_2, p_3]$ increases from [45 22 10] to [70 60 50].

2) As expected, as the value of quality cost under the out-of-control situation Q_{out_i}increases, the

Tab. 4. Generated scenarios through the Taguchi design (27 instances)

Instance	d_i	p_i	Q_{in_i}	Q_{out_i}	S_i	λ	δ	H_i	C_f	C_v	C_{pm}	C_{cm}	C_{fa}
1	[10 32 40]	[45 75 95]	[160 180 130]	[300 250 400]	[180 185 190]	0.1	1	[18 8 28]	15	4	3000	500	90
2	[30 10 40]	[50 80 100]	[180 200 150]	[500 460 600]	[200 205 210]	0.15	0.9	[20 10 30]	20	4	3000	500	90
3	[20 20 50]	[70 100 120]	[200 220 170]	[700 610 560]	[220 225 230]	0.2	1.1	[22 12 32]	25	4	3000	500	90
4	[20 20 50]	[70 100 120]	[200 220 170]	[500 460 600]	[200 205 210]	0.15	1	[18 8 28]	15	5	2400	800	90
5	[10 32 40]	[45 75 95]	[160 180 130]	[700 610 560]	[220 225 230]	0.2	0.9	[20 10 30]	20	5	2400	800	90
6	[30 10 40]	[50 80 100]	[180 200 150]	[300 250 400]	[180 185 190]	0.1	1.1	[22 12 32]	25	5	2400	800	90
7	[30 10 40]	[50 80 100]	[180 200 150]	[700 610 560]	[220 225 230]	0.2	1	[18 8 28]	15	6	4300	1000	90
8	[20 20 50]	[70 100 120]	[200 220 170]	[300 250 400]	[180 185 190]	0.1	0.9	[20 10 30]	20	6	4300	1000	90
9	[10 32 40]	[45 75 95]	[160 180 130]	[500 460 600]	[200 205 210]	0.15	1.1	[22 12 32]	25	6	4300	1000	90
10	[20 20 50]	[50 80 100]	[160 180 130]	[700 610 560]	[200 205 210]	0.1	1.1	[20 10 30]	15	6	2400	500	200
11	[10 32 40]	[70 100 120]	[180 200 150]	[300 250 400]	[220 225 230]	0.15	1	[22 12 32]	20	6	2400	500	200
12	[30 10 40]	[45 75 95]	[200 220 170]	[500 460 600]	[180 185 190]	0.2	0.9	[18 8 28]	25	6	2400	500	200
13	[30 10 40]	[45 75 95]	[200 220 170]	[300 250 400]	[220 225 230]	0.15	1.1	[20 10 30]	15	4	4300	800	200
14	[20 20 50]	[50 80 100]	[160 180 130]	[500 460 600]	[180 185 190]	0.2	1	[22 12 32]	20	4	4300	800	200
15	[10 32 40]	[70 100 120]	[180 200 150]	[700 610 560]	[200 205 210]	0.1	0.9	[18 8 28]	25	4	4300	800	200
16	[10 32 40]	[70 100 120]	[180 200 150]	[500 460 600]	[180 185 190]	0.2	1.1	[20 10 30]	15	5	3000	1000	200
17	[30 10 40]	[45 75 95]	[200 220 170]	[700 610 560]	[200 205 210]	0.1	1	[22 12 32]	20	5	3000	1000	200
18	[20 20 50]	[50 80 100]	[160 180 130]	[300 250 400]	[220 225 230]	0.15	1	[18 8 28]	25	5	3000	1000	200
19	[30 10 40]	[70 100 120]	[160 180 130]	[500 460 600]	[220 225 230]	0.1	1	[22 12 32]	15	5	4300	500	450
20	[20 20 50]	[45 75 95]	[180 200 150]	[700 610 560]	[180 185 190]	0.15	1.1	[18 8 28]	20	5	4300	500	450
21	[10 32 40]	[50 80 100]	[200 220 170]	[300 250 400]	[200 205 210]	0.2	1	[20 10 30]	25	5	4300	500	450
22	[10 32 40]	[50 80 100]	[200 220 170]	[700 610 560]	[180 185 190]	0.15	1	[22 12 32]	15	6	3000	800	450
23	[30 10 40]	[70 100 120]	[160 180 130]	[300 250 400]	[200 205 210]	0.2	1.1	[18 8 28]	20	6	3000	800	450
24	[20 20 50]	[45 75 95]	[180 200 150]	[500 460 600]	[220 225 230]	0.1	1	[20 10 30]	25	6	3000	800	450
25	[20 20 50]	[45 75 95]	[180 200 150]	[300 250 400]	[200 205 210]	0.2	1	[22 12 32]	15	4	2400	1000	450
26	[10 32 40]	[50 80 100]	[200 220 170]	[500 460 600]	[220 225 230]	0.1	1.1	[18 8 28]	20	4	2400	1000	450
27	[30 10 40]	[70 100 120]	[160 180 130]	[700 610 560]	[180 185 190]	0.15	1	[20 10 30]	25	4	2400	1000	450

Volume 14 • Issue 4 • 2022

Tab. 5. Comparison of *ETCU* between Models A and B

Instance	1	2	3	4	5	6	7	8	9
***ETCU* of model A**	17324.82	18866.51	22483.00	22046.67	17926.67	18157.07	19861.57	23113.83	19162.45
***ETCU* of model B**	22023.36	23567.65	27682.53	25494.30	20751.09	22086.29	26353.02	32340.28	26727.00
Cost savings(%)	21.33	19.95	18.78	13.52	13.61	17.79	24.63	28.53	28.30
Instance	10	11	12	13	14	15	16	17	18
***ETCU* of model A**	18458.67	19184.76	20145.33	20845.83	20423.02	20375.45	19960.03	20510.72	18688.63
***ETCU* of model B**	21540.41	22808.86	23537.83	27541.16	28628.56	27429.08	24960.78	25136.27	22723.04
Cost savings(%)	14.31	15.89	14.41	24.31	28.66	25.72	20.03	18.40	17.75
Instance	19	20	21	22	23	24	25	26	27
***ETCU* of model A**	18596.84	21495.39	21995.32	20994.96	17315.42	20790.14	20423.39	20491.49	17217.91
***ETCU* of model B**	26377.63	28383.97	29284.82	26883.18	22076.67	24714.19	23850.23	23470.78	21108.93
Cost savings(%)	29.50	24.27	24.89	21.90	21.57	15.88	14.37	12.69	18.43
Average of cost savings in all of the instances (%): 20.35									

Tab. 6. Cost comparison between Models A and B

	E(S)	E(QC)	E(H)	E(M)	E(SC)
Model A	157.51	16120.07	2761.11	2837.34	116.87
Model B	41.44	17017.88	605.36	14070.68	605.36
Cost savings(%)	-280.02	5.27	-356.10	79.83	80.69

Tab. 7. Range of model parameters in the sensitivity analysis

parameter	value	h	n	l	T	$ETCU^*$
p_i	[45 22 10]	0.95	7	1.66	7.13	12156.64
	[60 30 20]	0.88	9	1.80	3.97	13454.85
	[70 60 50]	0.47	6	1.76	3.32	14101.62
Q_{out_i}	[180 150 250]	0.88	1	5	3.98	13311.15
	[300 250 400]	0.88	9	1.80	3.97	13454.85
	[600 620 510]	0.53	9	1.70	3.99	13633.57
C_{pm}	500	0.72	1	4.87	2.15	11883.23
	2000	0.88	9	1.80	3.97	13454.85
	4300	0.77	8	1.74	5.77	14889.95
C_{fa}	70	0.88	8	1.61	3.97	13449.90
	100	0.88	9	1.80	3.97	13454.85
	450	0.89	13	2.46	3.999	13474.91

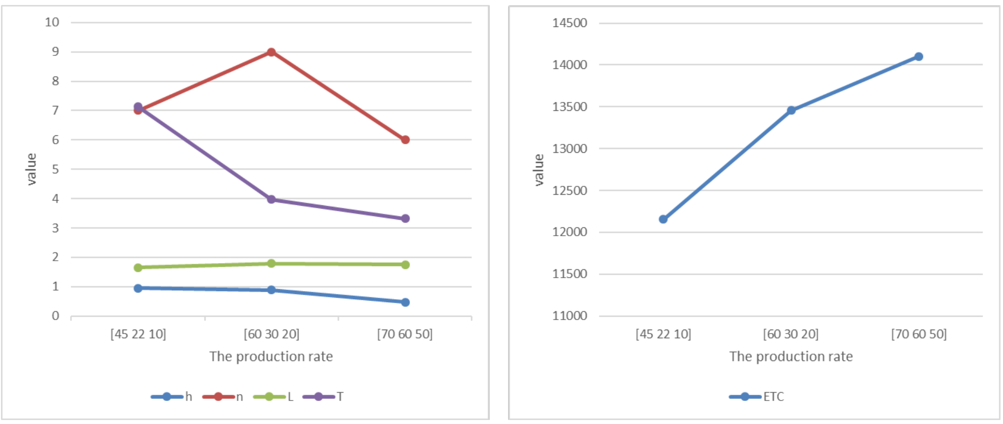

Fig. 11. Effect of p_i on the model output

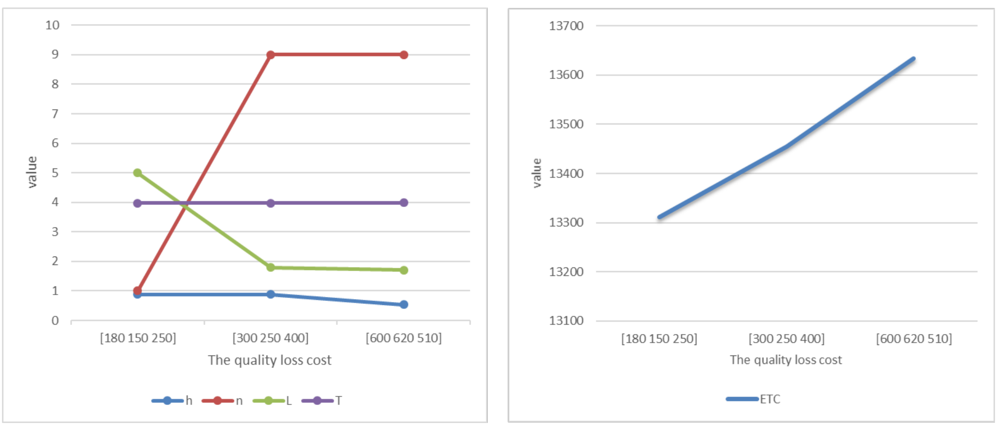

Fig. 12. Effect of Q_{out_i} on the model output

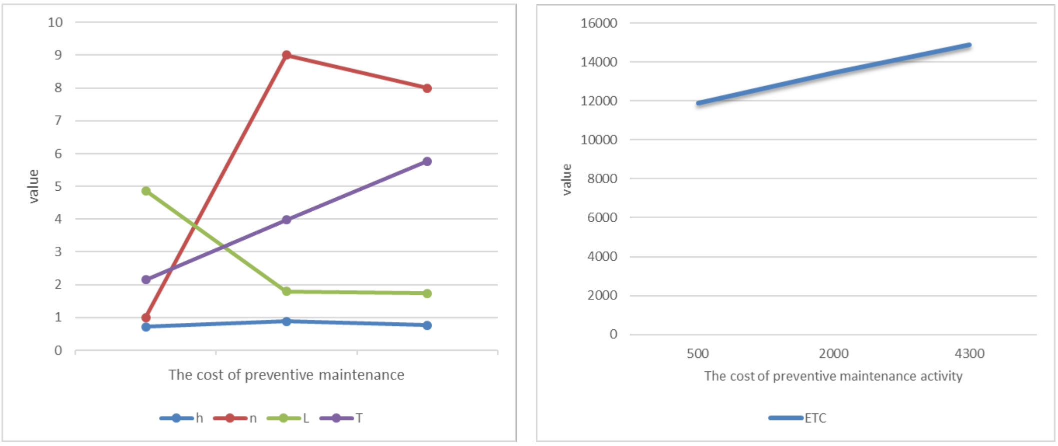

Fig. 13. Effect of C_{pm} on the model output

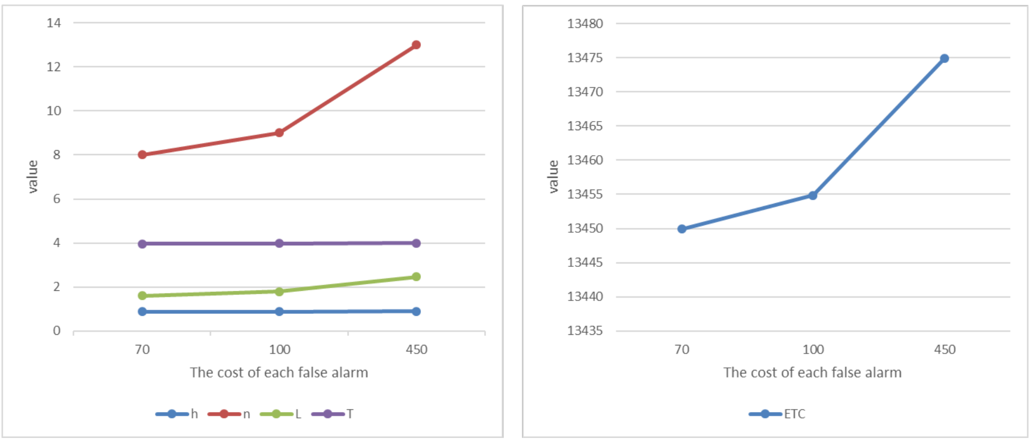

Fig. 14. Effect of C_{fa} on the model output

consecutive subgroups increase when C_{pm} increases from 500 to 2000 and then decreases when C_{pm} increases from 2000 to 4300.

4) As shown in Table 7 and Fig. 14, a slight increase in $ETCU^*$ can be observed since large values of C_{fa} lead to more maintenance-repair costs. However, in contrast to p_i, Q_{out_i} and C_{pm} the dependency of $ETCU^*$ on C_{fa} is not significant. In addition, compared to the other parameters, C_{fa} has a non-linear impact on $ETCU^*$. It can be also concluded that T and h are the least affected variables by variation of C_{fa} .

The obtained results allow for summarising the practical implications of the proposed model, which can be employed by industrial practitioners to optimise the efficiency of their manufacturing systems.

- Integration of maintenance, inventory control, and quality engineering decisions leads to significant cost savings in comparison with making such decisions separately. It is remarkable that the greatest effect of implementing a simultaneous decision-making policy is achieved from the reduction in the set-up cost. Moreover, the maintenance and quality costs are the most affected cost terms by replacing the simultaneous decision-making policy with the separate one.
- In practice, as the value of Q_{out_i} increases, it becomes more necessary to detect the out-of-control condition as soon as possible, which can be achieved by increasing the control chart power as well as reducing the time interval between two consecutive samplings. It should be noted that the control chart power can be improved by taking larger samples and choosing small values of the control limit coefficient. Therefore, in industrial systems, where the production of non-conforming products imposes a significant cost to the manufacturer, it is vital to take larger samples, reduce the time interval between two

consecutive samples, and reduce the distance between the upper and lower control limits.

- In situations when stopping the production process leads to a significant cost for the company, the control chart should be designed so that the probability of issuing an out-of-control alarm under the in-control condition is reduced as much as possible. In this situation, quality practitioners are advised to use larger values for the control limit coefficient to avoid a high false alarm rate.

Conclusions

On the one hand, multi-product systems consist of expensive and complex equipment, and consequently, it is crucial to maintain them in a suitable operational condition by implementing efficient maintenance activities. On the other hand, it is essential to detect process anomalies quickly to satisfy customer expectations and reduce the production rate of non-conforming items by designing statistical quality control techniques. To make more interactive decisions, this paper aimed to integrate three concepts of SPM, maintenance, and inventory planning, for the multi-production processes. The PSO evolutionary technique was implemented to solve the optimisation model and obtain decision variables. The accomplishments of the study displayed that the proposed simulation has better performance for multi-production systems with respect to economic criteria in comparison with the separated model. A comparative study in which 27 different states of inputs were examined based on the Taguchi design of experiment (DOE) method to calculate the cost saving due to different states of input data. The measure was used for comparison between the suggested mathematical model and a separated model. About 20 per cent of cost savings were observed due to the integration of the effective parameter, which almost happened due to the reduction in maintenance and quality cost. Besides, a sensitivity analysis was designed to track the change of results due to four model inputs; the production rate of the product (in one production cycle), the cost of quality per unit when assignable cause occurs, the preventive maintenance cost and the cost of each false alarm. Results demonstrated that the rate of production and the PM cost have significant direct effects on.

The proposed model was established based on some assumptions, such as the normality of the quality characteristic of interest, single-machine production system and the occurrence possibility of only one type of assignable cause, which can be reasonable in small and medium production systems. However, in large and complex industries, such assumptions may not be true. Therefore, to bring the proposed model closer to practice, future studies need to consider multiple assignable causes and multi-machine (parallel or serial) systems. Moreover, developing control charts without the normality assumption of the quality characteristic of interest can be fruitful for future research.

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