

# Voltage induced by currents in power-line sagged conductors in nearby circuits of arbitrary configuration

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(Received: 18.11.2014, revised: 12.12.2014 )

**Abstract.** The study presents a calculation method of the voltage induced by power-line sagged conductor in an inductively coupled overhead circuit of arbitrary configuration isolated from ground. The method bases on the solution utilizing the magnetic vector potential for modeling 3D magnetic fields produced by sagging conductors of catenary electric power lines. It is assumed that the equation of the catenary exactly describes the line sag and the influence of currents induced in the earth on the distribution of power line magnetic field is neglected. The method derived is illustrated by exemplary calculations and the results obtained are partially compared with results computed by optional approach.

**Key words:** power line, catenary, vector potential, overhead circuit, induced voltage, mutual inductance

## 1. Introduction

Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Nowadays of special concern is the possibility of detrimental environmental effects arising from the electrical and magnetic fields formed adjacent to the overhead transmission lines. These fields may affect both operation of near electric and electronic devices and appliances and also various living organisms [1-3].

The problem of the magnetic fields produced by overhead power lines is gaining more significance in recent years. In this context, efforts are continuously being done in order to maximize the utilization of the available line corridors without exceeding the tolerable limits of the lines' magnetic fields, e.g. [2-10].

Power-frequency inductive coupling between overhead power lines and nearby circuits is a matter of concern in electrical engineering/electromagnetic compatibility, as in case of the coupling with earth return circuits and overhead circuits isolated from ground as well.

The power line conductors are periodic catenaries, the sag of which depends on individual characteristics of the line and on environmental conditions. The effect of the catenary on the amplitude of the magnetic field can be significant in some cases [11-17].

The work [15] addressed the question if the ordinary approach of substituting a horizontal conductor at average height above the ground for a real sagged conductor would be a fairly accurate procedure as far as the evaluation of voltages induced in neighboring circuits is concerned. To assess the accuracy of such procedure two approaches are developed: a segmentation method (stair-case approximation) and a corrected segmentation method (improved by assessing a height oriented current to each segment). Using the methods approximate formulas for the mutual impedance between a sagged overhead conductor and an inductively coupled two parallel conductors closed at one end (forming the letter U), located at ground level parallel to the power line and isolated from ground have been obtained.

The objective of the paper is to present an alternative, more general method of calculation of the voltage induced by power-line sagged conductor in an inductively coupled overhead circuit of arbitrary configuration isolated from ground (rails, gas and oil pipelines, measuring circuit, etc.). The method presented bases on the solution utilizing the magnetic vector potential for modeling 3D magnetic fields produced by sagging conductors [11]. It is assumed that the equation of the catenary exactly describes the line sag and the influence of currents induced in the earth on the distribution of power line magnetic field in the air can be neglected. It should be pointed out, that the neglecting of earth currents is acceptable only in case of power frequencies, whereas this simplification is not correct in the case of higher frequencies [18] or in the analysis of transients. The method derived is illustrated by exemplary calculations and the results obtained are partially compared with results computed by optional approach [15].

## 2. Magnetic vector potential produced by sagging conductor

Consider the electromagnetic field in the air produced by a current carrying conductor hanging over the earth surface ( $x, y$  plane), as shown in Figure 1. It is assumed that the current  $I_k$  flows along the  $0x$  axis, that the length of the span is  $L$ , that the maximum and minimum heights of catenary are  $H_k$  and  $h_k$  respectively and the displacement currents both in the air and in the earth are neglected. Furthermore, it is supposed that the influence of the induced earth return currents on the magnetic field has negligible practical importance, as shown in [11].

The analytical calculation of the magnetic field in the free space (air) generated by a time-harmonic current  $I_k$  can be obtained by the application of the Poisson equation:

$$\Delta \vec{A} = -\mu_0 \vec{J}, \quad (1)$$

where  $\vec{A}$  – vector potential,  $\mu_0$  – magnetic permeability of the vacuum and  $J$  denotes the current density.

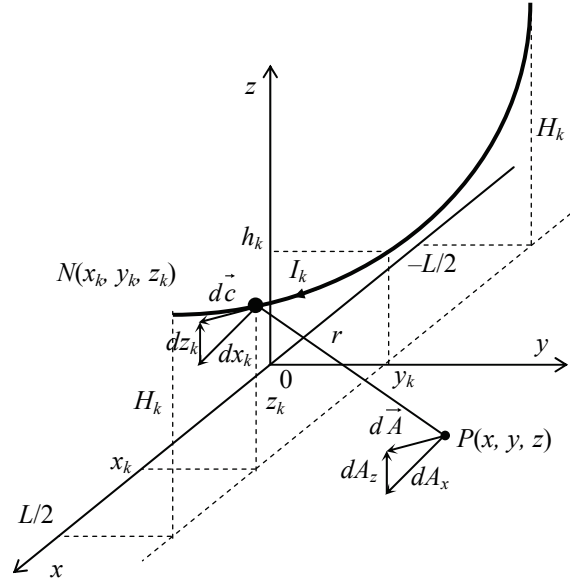


Fig. 1. Current carrying sagging conductor above the earth surface

It can be shown that, assuming that the conductor is very thin, the vector potential produced by a current path  $c$  takes the form:

$$\vec{A} = \frac{I_k \mu_0}{4\pi} \int_c \frac{d\vec{c}}{r}, \quad (2)$$

where the vector element  $d\vec{c}$  coincides with the direction of the current  $I_k$ ,  $r$  is the distance between the source point  $N(x_k, y_k, z_k)$  and the observation point  $P(x, y, z)$  and the distance  $r$  is given by:

$$r = \sqrt{(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2}. \quad (3)$$

Since the modelled conductor is located in the  $x, z$  plane the elementary vector  $d\vec{c}$  in the Cartesian co-ordinates system can be written as:

$$d\vec{c} = \vec{e}_x dx_k + \vec{e}_z dz_k, \quad (4)$$

where  $\vec{e}_x, \vec{e}_z$  are the unit vectors in the direction  $x$  and  $z$  respectively.

A sagging power line conductor has a form of a catenary curve. The approximating equation of the catenary can be written in the form [11, 16, 17]:

$$z_k = h_k + \alpha_k \left[ \cosh\left(\frac{x_k}{\alpha_k}\right) - 1 \right], \quad (5)$$

where  $\alpha_k$  – parameter iterative obtained from the Equation (6):

$$H_k = h_k + \alpha_k \left[ \cosh\left(\frac{L}{2\alpha_k}\right) - 1 \right]. \quad (6)$$

Taking into account the relationship (5) the elementary vector  $d\vec{c}$  in the Cartesian co-ordinates system can be written as:

$$d\vec{c} = \vec{e}_x dx_k + \vec{e}_z \sinh\left(\frac{x_k}{\alpha_k}\right) dx_k. \quad (7)$$

The  $x$ - and  $z$ -components of the vector potential become according to the formula (2):

$$A_x(x, y, z) = \frac{I_k \mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_k}{\sqrt{(x-x_k)^2 + (y-y_k)^2 + \left(z-h_k - \alpha_k \left[ \cosh\left(\frac{x_k}{\alpha_k}\right) - 1 \right]\right)^2}}, \quad (8)$$

$$A_z(x, y, z) = \frac{I_k \mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\sinh\left(\frac{x_k}{\alpha_k}\right) dx_k}{\sqrt{(x-x_k)^2 + (y-y_k)^2 + \left(z-h_k - \alpha_k \left[ \cosh\left(\frac{x_k}{\alpha_k}\right) - 1 \right]\right)^2}} \quad (9)$$

Based on the above formulas for a single span single conductor catenary, the result can be extended to account for the multiple catenary sections of the same conductor along the line as well as for real power lines carrying multiphase arrangements.

### 3. Induced voltage in the circuit of arbitrary configuration

Consider the arbitrary configuration of the overhead circuit, insulated from ground, inductively coupled to a power line conductor. For calculation purposes, the circuit is divided into straight-line segments. Consider next the horizontal circuit segment located underneath a sagging overhead conductor with a current  $I_k$ , as in Figure 2.

The terminating points of the  $i$ -th segment have in the reference system the coordinates  $(x_i, y_i, z_i)$  and  $(x_{i+1}, y_{i+1}, z_{i+1})$  respectively. The segment lies in the  $xy$  plane,  $\varphi_i$  is the angle between the segment and the  $x$  – axis (angle measured anticlockwise and  $\varphi_i (0, \pi)$ ),  $s$  denotes the abscissa along the segment,  $l_i$  is its length and taking into account that the circuit segment is parallel to the  $xy$  plane ( $z_i = z_{i+1}$ )

$$l_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}. \quad (10)$$

The induced electric field intensity in the air can be obtained from the general relation:

$$\vec{E}_i = -j\omega\vec{A}, \tag{11}$$

where  $\omega$  denotes the angular frequency and the imaginary unit  $j = \sqrt{-1}$ .

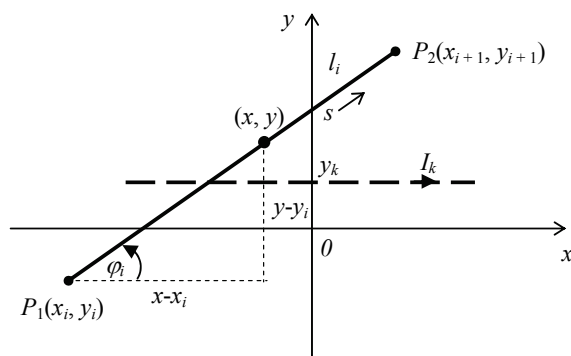


Fig. 2. Current carrying sagging conductor and the  $i$ -th circuit segment – top view

Thus the  $x$ - and  $z$ -components of the electric field intensity can be obtained according to the formulas (8) and (9) taken into account the Equation (11).

The voltage between the termination points  $P_1$  and  $P_2$  of the circuit segment is given by

$$U = -\int_{P_1}^{P_2} \vec{E}_i \cdot \vec{ds}, \tag{12}$$

whereas the integration path lies on the axis of the circuit segment.

In Equation (12) only the  $x$  – component of the induced electric field  $\vec{E}_i$  contributes to the value of the integral since:

$$\vec{E}_i \cdot \vec{ds} = E_{ix}ds_x + E_{iy}ds_y + E_{iz}ds_z \tag{13}$$

and  $E_{iy} = 0$  and  $ds_z = 0$ .

When the relationships (8), (9), (11) and (13) are taken into account, the voltage induced in the  $i$ -th circuit segment will be:

$$U_i = \pm \frac{j\omega I_k \mu_0 \cos \varphi_i}{4\pi} \times \left\{ \int_0^{l_i} \left[ \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx_k}{\sqrt{(x_s - x_k)^2 + (y_s - y_k)^2 + (h_s - h_k - \alpha_k (\cosh(x_k / \alpha_k) - 1))^2}} \right] ds \right\}. \tag{14}$$

It follows from the Figure 2 that for  $0 \leq \varphi_i \leq \frac{\pi}{2}$

$$\begin{aligned}
 x_s &= s \cos \varphi_i + x_i \\
 y_s &= s \sin \varphi_i + y_i \\
 \cos \varphi_i &= \frac{x_{i+1} - x_i}{l_i} \\
 \sin \varphi_i &= \frac{y_{i+1} - y_i}{l_i}.
 \end{aligned} \tag{15}$$

It is easy to show, that for  $\varphi_i > \frac{\pi}{2}$

$$\begin{aligned}
 x_s &= s \cos \varphi_i + x_{i+1} \\
 y_s &= s \sin \varphi_i + y_{i+1} \\
 \cos \varphi_i &= \frac{x_i - x_{i+1}}{l_i} \\
 \sin \varphi_i &= \frac{y_i - y_{i+1}}{l_i}.
 \end{aligned} \tag{16}$$

It should be noted, that the sign plus in Equation (14) concerns the condition

$$0 \leq \varphi_i \leq \frac{\pi}{2}.$$

On the other hand the voltage  $U_i$  can be determined as

$$U_i = j\omega M_{ki} I_k. \tag{17}$$

Therefore

$$M_{ki} = \frac{\mu_0 \cos \varphi_i}{4\pi} \left\{ \int_0^{l_i} \left[ \int_{\frac{L}{2}}^{\frac{L}{2}} \frac{dx_k}{\sqrt{(x_s - x_k)^2 + (y_s - y_k)^2 + (h_s - h_k - \alpha_k (\cosh(x_k / \alpha_k) - 1))^2}} \right] ds \right\} \tag{18}$$

expresses the mutual inductance between the sagging conductor k-th and the i-th circuit segment.

The integrals in formulas (14) and (18) have to be solved numerically.

Based on the above formulas for a single span single conductor catenary, in presence of many current sources (current carrying conductors), the superposition can be applied and the induced voltage calculation in any circuit of complex geometry can be performed.

#### 4. Examples of calculations

*Example 1.* Consider first the case presented in [15] – let us calculate the voltage induced in a two-conductor circuit at ground level and the mutual inductance between a sagging conductor of a power line and the circuit, as shown in Figure 3.

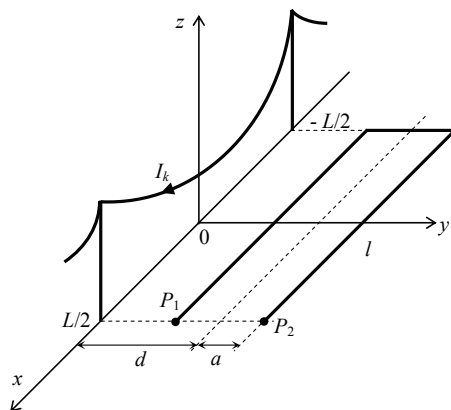


Fig. 3. Current carrying sagging conductor and a two-conductor circuit at ground level (no scale)

The calculation has been carried out numerically for parameters as given in [15]:  $L = 300$  m,  $H_k = 20.45$  m,  $h_k = 12.27$  m,  $y_k = 0$  m (power line sagging conductor);  $l = 300$  m ( $x_1 = -150$  m,  $x_2 = 150$  m),  $h_s = 0$  m,  $d = 150$  m,  $a = 0.75$  m (two-conductor circuit placed at ground level parallel to the power line route). The voltage between the termination points  $P_1$  and  $P_2$  of the circuit is found from the Equation (14) and the mutual inductance according to Equation (18), when the rms current in sagged conductor is assumed  $I_k = 2000$  A and the frequency  $f = 50$  Hz.

A question how do the other line segments (nearby spans) influence the calculation results has been also investigated. The calculation results shown in the Table 1 have been carried out for one, three and five influencing spans taking into account, whereas the influenced circuit was located always under the middle span.

Table 1. Induced voltage (rms value) and mutual inductance module

Number of spans	$U$	$M$	$U$ [15]	$M$ [15]
–	V	$\mu$ H	V	$\mu$ H
1	1.797	2.861	~1.9	~3.0
3	1.896	3.018		
5	1.898	3.021		

It follows from the calculations that results obtained by using the approach developed in this work comply with the results derived in [15]. Moreover it was shown that the value of the induced voltage and the mutual inductance are slightly dependent from the number of spans taken into account, so the influence of nearby spans can be neglected in many practical cases when the influenced circuit is shorter then the span length.

*Example 2.* Next calculations show the influence of the location of the receiving circuit (as in the earlier case) with respect to the power-line route, Figure 4.

Table 2 contains results of calculations of the induced voltage and mutual inductance as function of the angle  $\varphi$  (rotation in the point  $x = 0$ ,  $y = 15$  m). In the calculations one influencing span has been taken into account.

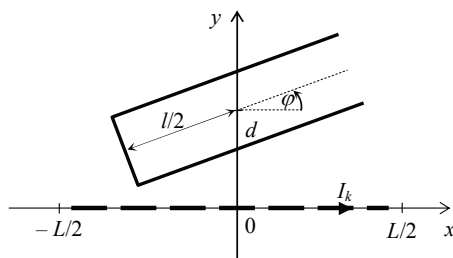


Fig. 4. Current carrying sagging conductor and a two-conductor circuit at ground level – top view (no scale)

Table 2. Induced voltage (rms value) and mutual inductance as function of the angle  $\varphi$

Angle $\varphi$ (degree)	$U$ (V)	$M$ ( $\mu\text{H}$ )
0	1.797	-2.861
15	0.666	-1.060
30	0.256	-0.410
45	0.191	-0.305
60	0.179	-0.286
75	0.178	-0.284
90	0.178	-0.284
105	0.179	-0.286
120	0.181	-0.288
135	0.193	-0.307
150	0.258	-0.412
165	0.667	-1.061
180	1.797	-2.861

As expected the results depend on the angle between the segments of the circuit and the route of the power line conductor. If the circuit parallels the route, the resultant voltage is the superposition of the electromotive forces (*emf*) induced along the parallel circuit segments. For the perpendicular location of the circuit to the power line route the induced *emf* appears in the middle (very short) circuit segment only and in the case considered practically equals zero volts.

*Example 3.* Consider next the inductive interference of a power line on the circuit of complex configuration located on the earth surface, as shown in Figure 5. The circuit is divided into 4 straight-line segments. The terminating points of the segments are:  $P_1(-80, 20, 0)$ ,  $P_2(-9, 3, 90, 7, 0)$ ,  $P_3(48, 6, 75, 2, 0)$ ,  $P_4(61, 6, 26, 8, 0)$  and  $P_5(8, -18, 1, 0)$ . The overhead power line is suspended on towers H52. With a span length of 450 m and the earth resistivity of 100  $\Omega\text{m}$  the power line represents a typical Polish 220 kV, 3-phase transmission line in flat



configuration with two ground wires. The route of the power line axis lies on the  $x$ -axis, Figure 5. The magnitude of the phase current is chosen to 500 A and symmetrical operating conditions are assumed:  $I_1 = 500$  A,  $I_2 = 500\exp(-j120^\circ)$  A,  $I_3 = 500\exp(j120^\circ)$  A. The geometrical data of the phase conductor  $L_1(y = 7.8$  m),  $L_2(y = 0$  m),  $L_3(y = -7.8$  m) are:  $H_k = 20.5$  m,  $h_k = 5.5$  m (catastrophic sag). The height of the ground wires at the tower is 26.3 m and their  $y$ -coordinates are  $\pm 5.8$  m. The calculated currents induced in the ground wires are  $I_{g1} = 35.3\exp(-j171.85^\circ)$  A and  $I_{g2} = 37.78\exp(j20.77^\circ)$  A, respectively.

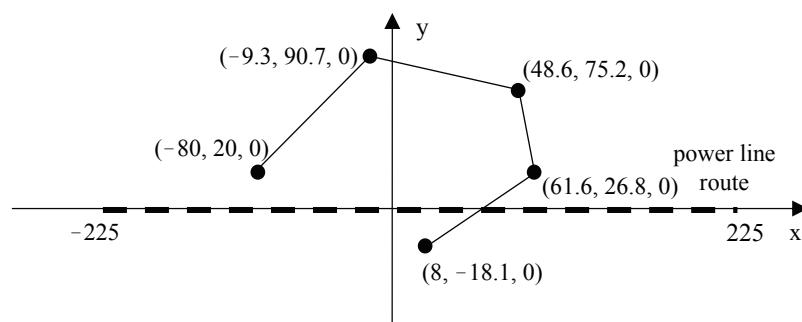


Fig. 5. Sagging power line and a circuit of complex geometry at ground level – top view (no scale)

The voltage between the termination points  $P_1$  and  $P_5$  of the influenced circuit found by the superposition using the eqn. (14) is  $U = 0.762$  V (rms value).

## 5. Conclusions

The paper presents a general method of calculation of the voltage induced by current in a power-line sagged conductor in an inductively coupled overhead circuit of arbitrary configuration isolated from ground. Procedure of determining the mutual inductance between the sagged conductor and an inductively coupled overhead circuit is also presented. The clear and precise method bases on the solution utilizing the magnetic vector potential for modeling the 3D magnetic fields produced by sagging conductors of catenary electric power lines. The method assumes that the line sag is described by the equation of the catenary, effects of earth currents onto magnetic field are negligible and that power-line currents have prescribed values.

The mutual inductance and the voltage induced in a nearby overhead circuit are investigated, whereas the circuit of any complex geometry is divided into straight-line segments horizontal located under the power line. The formulas obtained in the paper require numerical integration which can be performed by the use of freely available tools.

It should be noted, that the method presented can be extended by superposition in order to calculate the induced voltage by a catenary power line in overhead circuits with complex geometry for conditions that are almost always satisfied for power engineering applications.

The necessary data for the calculations are: the number of power line conductors, the current in each conductor, the geometrical data of the sagging power line, the number of segments the influenced circuit is divided into, and the coordinates  $(x_i, y_i, z_i)$  and  $(x_{i+1}, y_{i+1}, z_{i+1})$  of terminating points of each segment.

It should be pointed out, that the capability of the method developed is greater than other analytical methods presented in the literature and the direct analytical approach used, as opposed to the commercialized 3D simulators, enables any physical interpretation of the phenomena being simulated.

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