CZASOPISMO INŻYNIERII LĄDOWEJ, ŚRODOWISKA I ARCHITEKTURY
JOURNAL OF CIVIL ENGINEERING, ENVIRONMENT AND ARCHITECTURE
JCEEA, t. XXXV, z. 65 (2/18), kwiecień-czerwiec 2018, s. 197-218, DOI:10.7862/rb.2018.36

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APPROXIMATED METHOD FOR DETERMINING MOMENT RESISTANCE AND STIFFNESS OF BOLTED BEAM TO COLUMN JOINTS MADE WITH ANGLE WEB AND FLANGE CLEATS

Bolted beam to column joints with angle cleats are often used in braced and unbraced steel frame structures. It is related to their simple technology which does not require expensive welding process. Works on the estimation of moment resistance and stiffness of such connections were already carried out in the 1930s. However, the lack of appropriate computational tools forced researchers to introduce simplifications and some assumptions in determining the strength of a joint in a complex load condition. Currently, computing techniques allow for taking into account the actual resistance and stiffness of connections at the stage of global static analysis of the structure. Advanced computer methods as well as applied analytical models allow for a fairly precise determination of the parameters of this type of joints. However, these methods, due to their complexity, are quite time-consuming and labor-intensive, and they are suitable for verification of resistance of the connection or analysis of the structure in the final design phase. The paper presents simplified formulas for calculating the moment resistance and rotational stiffness of the beams to column joints with the use of angle sections connecting both the flanges and the web of the beam. An outline of the component method for determining the moment resistance and stiffness of such connections is also presented. The analysis of the influence of individual components of the joint on its global resistance and stiffness was conducted. The presented formulas, developed on the basis of the component method, preferred by Eurocode 3, can be used in the preliminary determination of the characteristics of the joints, used in the global structure analysis.

Keywords: bolted joint with angle web and flange cleats, moment resistance, stiffness, joint characteristics, simplified formula, component method

1. Introduction

Bolted beam to column joints with angle cleats (Fig. 1) are often used in steel frame structures. It is related to their simple technology (fast assembly, lack of the expensive welding processes), and the possibility of easy rectification of executive deviations.

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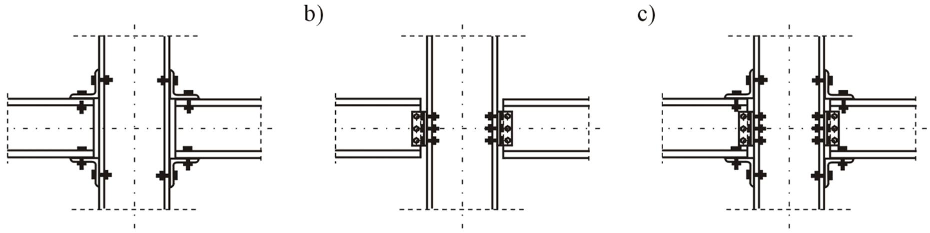

Fig. 1. Angle cleats joint; a) flange angle cleats connection, b) web angle cleats connection, c) flange and web angle cleats connection

Connections of this type are widely used in the USA and Great Britain [1–4] as a natural transition from riveted joints. Angle cleats are also often used to reinforce joints in existing structures, especially in places where welded processes are avoided due to fire reasons. Beam to column joints with using angle cleats connecting the beam web are treated as nominally pined connections, transferring only shear force from the beam to the column [5]. The bending moment created as a result of the eccentric application of this force, causing twisting angles, due to the small value was neglected. Similarly, in bolted beam to column joints using the web and flange angle cleats, it is assumed that the bending moment carries the angle cleats connecting flanges of the beam, whereas the angle cleat connecting the beam web only transmit shear force.

Experimental tests show that the moment resistance of the joint using angle cleats connecting both the flanges and beam web is greater by several dozen percent in relation to the same joint using only angle cleats connecting only the beam flanges [6] (Fig. 2).

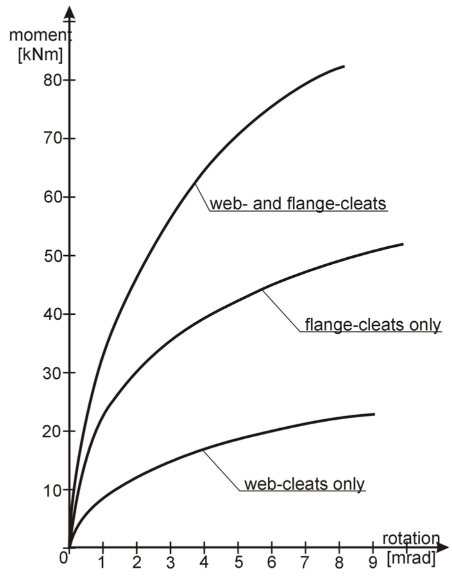

Fig. 2. Moment rotation characteristics for angle cleats joint, based on [6]

2. Moment resistance bolted joints with angle web and flange cleats

The analytical model for calculation characteristics of the joints, introduced in [7], is based on the component method. In this method, the whole connection is composed of basic components. Moment resistance of the joint is based on the lowest resistance of the joint components. In the case of a joint using angle flange cleats only, only the compression zone relative to the bottom angle cleat, the shear zone relating to the column web panel and the tension zone relating to the upper beam angle cleat are distinguished [8]. In the case of connection using angle cleats connecting the flanges and web of the beam, the bolt rows joining the beam web are considered as successive bolt rows in the tensile zone [9]. In addition, some basic components are considered for the individual bolt row and as part of the group of the bolt rows. The list of basic components for individual bolt row is shown in figures 3 and 4.

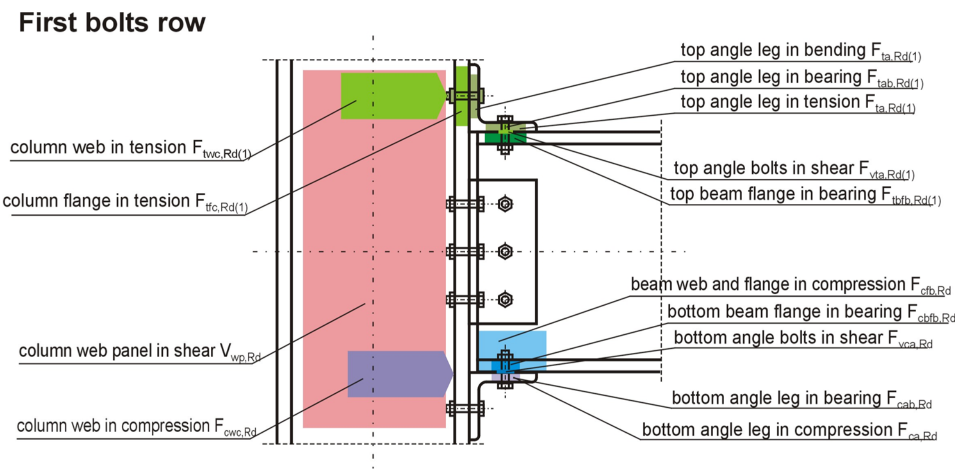

Fig. 3. The basic components of the joint refer to the first bolt row (bolts connecting the column flange with the top angle)

The design resistance of most components can be calculated according to the principles presented in Eurocode [7]. Only, due to the dependence of the collapse mechanism of the web angle cleats from its height, the strength of the angle web cleats in bending are not directly presented in the Standard.

Angle flange cleats in bending can be treated as single T-stubs (Fig. 5). The T-stub should be considered for each individual bolt row as well as for the group of bolt rows. In each case, three possible types of failure are considered.

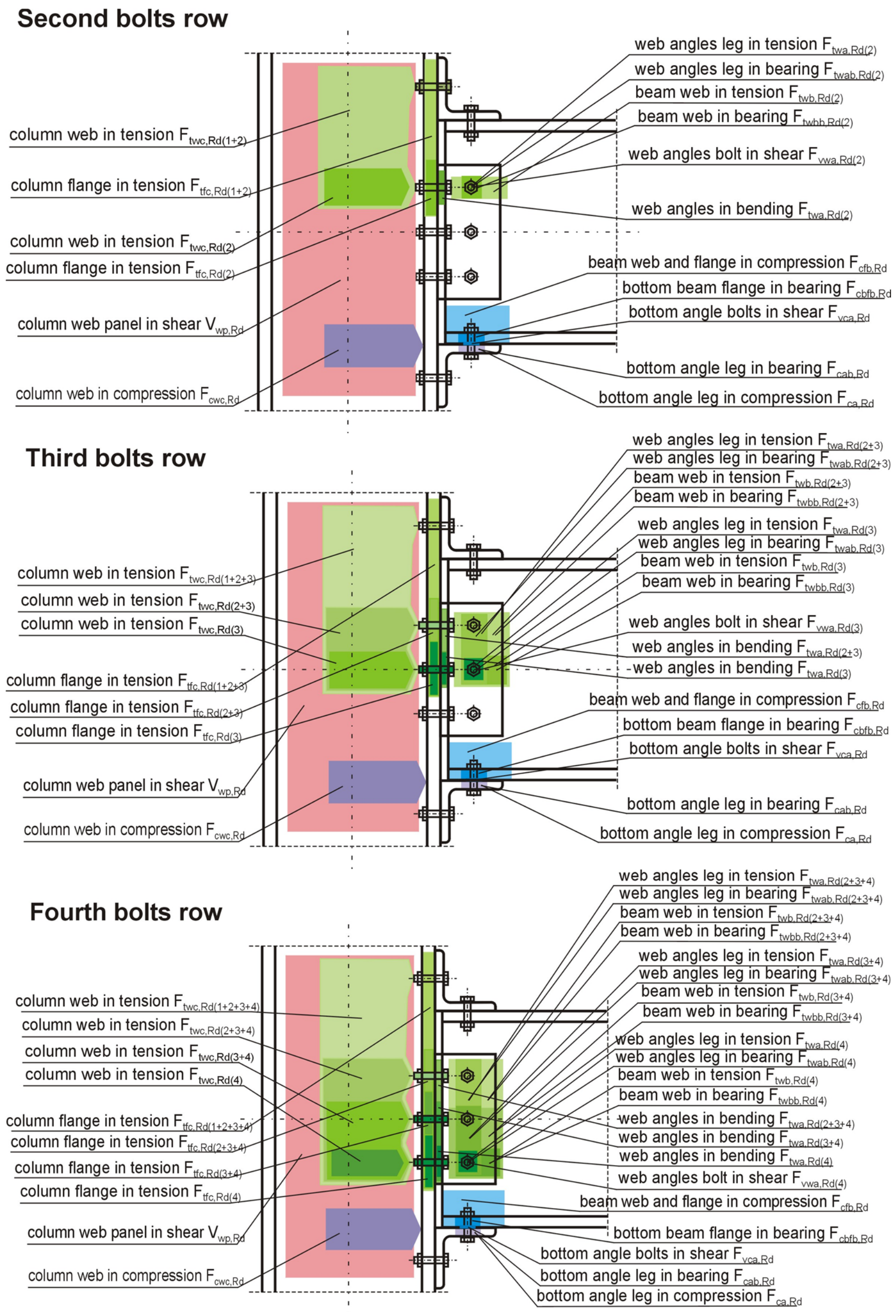

Fig. 4. The basic components of the joint refer to the second, third and fourth bolt row (bolts connecting the column flange with the angle web cleats)

With reference to the equivalent T-stub failing with complete yielding mode, the resistance of the single bolt row correspondingly assumes that the resistance of a couple web angles is equivalent, for each bolt row with the parameter $m`_i$ (Fig. 5) [9].

$$m`_i = \frac{3}{2}\frac{t_{wa}}{\left[\left(\frac{m_{max}\cdot h_i}{t_{wa}\cdot l_{wa}}\right)^2+3\right]^{\frac{1}{2}}-\frac{m_{max}\cdot h_i}{t_{wa}\cdot l_{wa}}} \tag{1}$$

where: t_{wa} – thickness of legs of the angle web cleats,
m_{max} – maximum distance between angle leg and bolt axis,
l_{wa} – height of angle web cleats,
h_i – lever arm i-bolt row.

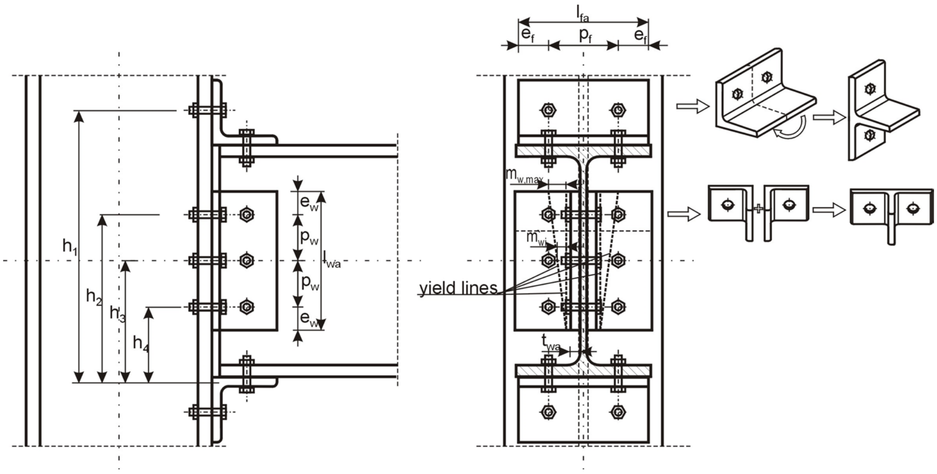

Fig. 5. Method of determining the parameters of the T-stub of the angle web cleats

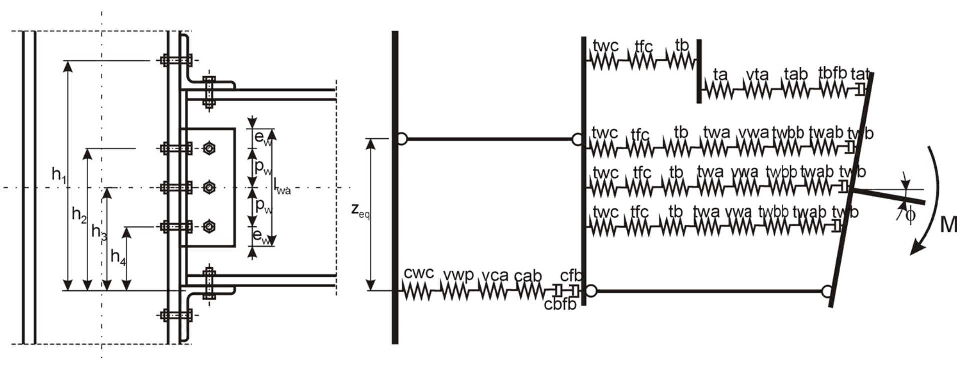

Fig. 6. Mechanical model for evaluating initial stiffness of joint with flange and web angle cleats

The initial stiffness of the joint is calculated on the basis of the mechanical model. This procedure is able to account for all sources of deformations, including the angle cleats, bolts, beam and column components. Mechanical model to determine the initial stiffness of the joint with angle web and flanges cleats is presented in figure 6.

3. Worked example

For the performance of the procedure, was presented a worked example. This example illustrated beam to column joint with use angle web and flange cleats (Fig. 7).

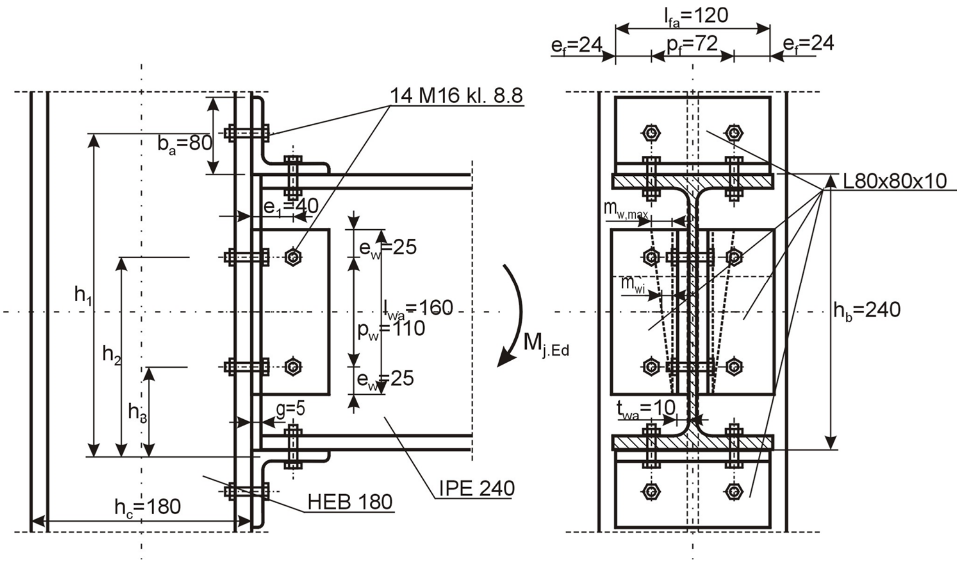

Fig. 7. Geometrical details of joint for calculation example

Basic data:

Steel grade S275 $f_y = 275\ N/mm^2$, $f_u = 430\ N/mm^2$, $\varepsilon = 0{,}92$

Beam IPE 240: $W_{y,pl} = 367 \cdot 10^3 mm^3$, $A_b = 39{,}1 \cdot 10^2 mm^2$, $h_b = 240\ mm$, $b_b = 120\ mm$, $t_{fb} = 9{,}8\ mm$, $t_{wb} = 6{,}2\ mm$, $r_b = 15\ mm$.

Column HEB 180: $W_{y,pl} = 481{,}4 \cdot 10^3 mm^3$, $A_c = 65{,}3 \cdot 10^2 mm^2$, $h_c = 180\ mm$, $b_c = 180\ mm$, $t_{fc} = 14{,}0\ mm$, $t_{wc} = 8{,}5\ mm$, $r_c = 15\ mm$.

Angle cleats L80x80x10: $b_a = 80\ mm$, $t_a = 10\ mm$, $r_a = 10{,}0\ mm$.

Bolts M16 class 8.8: $A_{v,b} = 201\ mm^2$, $A_{s,b} = 157\ mm^2$.

Beam and column sections are in class 1.

Design moment resistance of the joint.

- Column web panel in shear

For single-sided joint, the design plastic shear resistance of unstiffened column web is obtained from:

$$V_{wp,Rd} = \frac{0{,}9 \cdot f_{y,wc} \cdot A_{vc}}{\sqrt{3} \cdot \gamma_{M0}} = \frac{0{,}9 \cdot 275 \cdot 2029}{\sqrt{3} \cdot 1{,}0} = 289932N = 289{,}9kN \qquad (2)$$

where

$$A_{vc} = A_c - 2 \cdot b_c \cdot t_{fc} + (t_{wc} + 2 \cdot r_c) \cdot t_{fc} = 65{,}3 \cdot 10^2 - -2 \cdot 180 \cdot 14{,}0 + (8{,}5 + 2 \cdot 15) \cdot 14{,}0 = 2029\, mm^2 \qquad (3)$$

- Column web panel in compression

$$F_{cwc,Rd} = \frac{\omega \cdot k_{wc} \cdot b_{eff,c,wc} \cdot t_{wc} \cdot f_{ywc}}{\gamma_{M0}} \; and \; F_{cwc,Rd} \leq \frac{\omega \cdot k_{wc} \cdot \rho \cdot b_{eff,c,wc} \cdot t_{wc} \cdot f_{ywc}}{\gamma_{M0}} \qquad (4)$$

effective width of column web in compression:

$$b_{eff,c,wc} = 2 \cdot t_a + 0{,}6 \cdot r_a + 5 \cdot (t_{fc} + s) = 2 \cdot 10{,}0 + 0{,}6 \cdot 10{,}0 + +5 \cdot (14{,}0 + 15{,}0) = 171\, mm \qquad (5)$$

Transformation parameter, for single-sided joint is equal $\beta = 1{,}0$, and reduction factor for $\beta = 1{,}0\; is\; \omega = \omega_1$

$$\omega_1 = \frac{1}{\sqrt{1 + 1{,}3 \cdot \left(\frac{b_{eff,c,wc} \cdot t_{wc}}{A_{vc}}\right)^2}} = \frac{1}{\sqrt{1 + 1{,}3 \cdot \left(\frac{171 \cdot 8{,}5}{2029}\right)^2}} = 0{,}774 \qquad (6)$$

If the maximum longitudinal compressive stress in column $\sigma_{com,Ed}$ not exceed $0{,}7 \cdot f_{y,wd}$, then reduction factor k_{wc} can be adopted as 1, then:

$$F_{cwc,Rd} = \frac{0{,}774 \cdot 1{,}0 \cdot 171 \cdot 8{,}5 \cdot 275}{1{,}0} = 309377\; N = 309{,}4\; kN \qquad (7)$$

The reduction factor for web panel buckling ρ, is depending on the web slenderness:

$$\bar{\lambda}_p = 0{,}932 \cdot \sqrt{\frac{b_{eff,c,wc} \cdot d_{wc} \cdot f_{ywc}}{E \cdot t_{wc}^2}} = 0{,}932 \cdot \sqrt{\frac{171 \cdot 122 \cdot 275}{210 \cdot 10^3 \cdot 8{,}5^2}} = 0{,}573 \qquad (8)$$

where:

$$d_{wc} = h_c - 2 \cdot (t_{fc} + r_c) = 180 - 2 \cdot (14{,}0 + 15{,}0) = 122\; mm \qquad (9)$$

if $\bar{\lambda}_p \leq 0{,}72$ then ρ = 1, and finally: $F_{cwc,Rd} = 309{,}4\; kN$

- Beam flange and web in compression

$$F_{cfb,Rd} = \frac{M_{b,Rd}}{h_b - t_{fb}} = \frac{100{,}9 \cdot 10^6}{240 - 9{,}8} = 438423\ N = 438{,}4\ kN \tag{10}$$

with $M_{b,Rd} = M_{pl,Rd} = \frac{W_{y,pl} \cdot f_y}{\gamma_{M0}} = \frac{367 \cdot 10^3 \cdot 275}{1{,}0} = 100{,}9 \cdot 10^6\ Nmm$

- Bottom angle leg in compression

$$F_{ca,Rd} = \frac{l_{fa} \cdot t_{fa} \cdot f_{ya}}{\gamma_{M0}}\ and\ F_{ca,Rd} \le \frac{\rho \cdot l_{fa} \cdot t_{fa} \cdot f_{ya}}{\gamma_{M0}} \tag{11}$$

The reduction factor for angle leg buckling ρ, is depending on the slenderness:

$$\bar{\lambda}_p = 0{,}932 \cdot \sqrt{\frac{l_{fa} \cdot b_a \cdot f_{ya}}{E \cdot t_{fa}^2}} = 0{,}932 \cdot \sqrt{\frac{120 \cdot 80 \cdot 275}{210 \cdot 10^3 \cdot 10^2}} = 0{,}330 \tag{12}$$

if $\bar{\lambda}_p \le 0{,}72$ then ρ = 1, and:

$$F_{ca,Rd} = \frac{120 \cdot 10{,}0 \cdot 275}{1{,}0} = 330000\ N = 330{,}0\ kN \tag{13}$$

- Bolts in bottom angle leg in bearing

$$F_{cab,Rd} = 2 \cdot \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t_{fa}}{\gamma_{M2}} = 2 \cdot \frac{2{,}03 \cdot 0{,}741 \cdot 430 \cdot 16 \cdot 10{,}0}{1{,}25} = 165586\ N = \\ = 165{,}6\ kN \tag{14}$$

where $k_1 = min\begin{cases} 2{,}8 \cdot \frac{e_f}{d_0} - 1{,}7 = 2{,}8 \cdot \frac{24}{18} - 1{,}7 = 2{,}03 \\ 2{,}5 \end{cases} = 2{,}03$

$$\alpha_b = min\begin{cases} \alpha_d = \frac{e_f}{3 \cdot d_0} = \frac{40}{3 \cdot 18} = 0{,}741 \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0 \end{cases} = 0{,}741$$

- Bolts in bottom beam flange in bearing

$$F_{cbfb,Rd} = 2 \cdot \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t_{fb}}{\gamma_{M2}} = 2 \cdot \frac{2{,}03 \cdot 1{,}0 \cdot 430 \cdot 16 \cdot 9{,}8}{1{,}25} = 218993\ N = \\ = 219{,}0\ kN \tag{15}$$

where $k_1 = min\begin{cases} 2{,}8 \cdot \frac{e_f}{d_0} - 1{,}7 = 2{,}8 \cdot \frac{24}{18} - 1{,}7 = 2{,}03 \\ 2{,}5 \end{cases} = 2{,}03$

$$\alpha_b = min\begin{cases}\alpha_d = \frac{e}{3\cdot d_0} = \frac{\infty}{3\cdot 18} = \infty \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0\end{cases} = 1{,}0$$

- Bolts in shear

Shear plane passes through the unthreaded portion of the bolt

$$F_{vca,Rd} = 2\cdot\frac{\alpha_v\cdot f_{ub}\cdot A_{v,b}}{\gamma_{M2}} = 2\cdot\frac{0{,}6\cdot 800\cdot 201}{1{,}25} = 154368\,N = 154{,}4\,kN \tag{16}$$

- Column flange in bending

Tension resistance of the bolt:

$$F_{t,Rd} = min\begin{cases}F_{t,Rd}\frac{0{,}9\cdot f_{ub}\cdot A_{s,b}}{\gamma_{M2}} = \frac{0{,}9\cdot 800\cdot 157}{1{,}25} = 90432 \\ B_{p,Rd} = \frac{0{,}6\cdot\pi\cdot d_m\cdot t_a\cdot f_u}{\gamma_{M2}} = \frac{0{,}6\cdot\pi\cdot 24\cdot 10{,}0\cdot 430}{1{,}25} = 155622\end{cases} = $$

$$= 90432\,N = 90{,}4\,kN \tag{17}$$

Parameter $m = \frac{(p_f - t_{wc} - 2\cdot 0{,}8\cdot r_c)}{2} = \frac{(72-8{,}5-2\cdot 0{,}8\cdot 15)}{2} = 19{,}75mm$, and

$$e = \frac{b_c - p_f}{2} = \frac{180-72}{2} = 54mm, but\ e \leq 1{,}25\cdot m = 1{,}25\cdot 19{,}75 = 24{,}7mm$$

Effective length of equivalent T-stub:

- Circular patterns: $l_{eff,cp} = 2\cdot\pi\cdot m = 2\cdot\pi\cdot 19{,}75 = 124mm$

- Non – circular patterns: $l_{eff,nc} = 4\cdot m + 1{,}25\cdot e = 4\cdot 19{,}75 + 1{,}25\cdot 24{,}7 = 109{,}9mm$,

$$M_{pl,Rd} = \frac{0{,}25\cdot l_{eff,nc}\cdot t_{fc}^2\cdot f_y}{\gamma_{M0}} = \frac{0{,}25\cdot 109{,}9\cdot 14{,}0^2\cdot 275}{1{,}0} = 1480902Nmm$$

The resistance of the column flange in bending is taken as minimum values from three modes of failure.

$$F_{tfc,Rd} = min\begin{cases}\frac{4\cdot M_{pl,Rd}}{m} = \frac{4\cdot 1480902}{19{,}75} = 299930 \\ \frac{2\cdot M_{pl,Rd} + n\cdot\sum F_{t,rd}}{m+n} = \frac{2\cdot 1480902 + 24{,}7\cdot 2\cdot 90432}{19{,}75+24{,}7} = 167135 \\ \sum F_{t,Rd} = 2\cdot 90432 = 180864\end{cases} =$$

$$= 167{,}1\,kN \tag{18}$$

- Column web panel in tension

$$F_{twc,Rd} = \frac{\omega\cdot b_{eff,t,wc}\cdot t_{wc}\cdot f_{ywc}}{\gamma_{M0}} = \frac{0{,}774\cdot 109{,}9\cdot 8{,}5\cdot 275}{1{,}0} = 198834\,N = 198{,}8\,kN \tag{19}$$

- Bolts in top angle leg in bearing

$$F_{tab,Rd} = 2 \cdot \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t_{fa}}{\gamma_{M2}} = 2 \cdot \frac{2{,}03 \cdot 0{,}741 \cdot 430 \cdot 16 \cdot 10{,}0}{1{,}25} = 165586\,N = \\ = 165{,}6\,kN \tag{20}$$

$$\text{where} \quad \alpha_b = min \begin{cases} \alpha_d = \frac{e_2}{3 \cdot d_0} = \frac{40}{3 \cdot 18} = 0{,}741 \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0 \end{cases} = 0{,}741$$

- Bolts in top beam flange in bearing

$$F_{tbfb,Rd} = 2 \cdot \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t_{fb}}{\gamma_{M2}} = 2 \cdot \frac{2{,}03 \cdot 0{,}648 \cdot 430 \cdot 16 \cdot 9{,}8}{1{,}25} = 141908\,N = \\ = 141{,}9\,kN \tag{21}$$

$$\text{where} \quad \alpha_b = min \begin{cases} \alpha_d = \frac{e_f - g}{3 \cdot d_0} = \frac{40-5}{3 \cdot 18} = 0{,}648 \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0 \end{cases} = 0{,}648$$

- Bolts in shear
  Shear resistance of the bolts in top flange is the same as in the bottom flange: $F_{vta,Rd} = F_{vca,Rd} = 154{,}4\,kN$

- Angle cleat in bending

The effective length of the angle flange cleats is taken as half of width of the cleats, then $l_{eff} = 60mm$, value m for large gap $m = e_1 - 0{,}5 \cdot t_a = 40 - 0{,}5 \cdot 10{,}0 == 35\,mm$, and $e = 40mm$

$$M_{pl,Rd} = \frac{0{,}25 \cdot l_{eff} \cdot t_a^2 \cdot f_y}{\gamma_{M0}} = \frac{0{,}25 \cdot 60 \cdot 10{,}0^2 \cdot 275}{1{,}0} = 412500Nmm$$

$$F_{ta,Rd} = min \begin{cases} \frac{4 \cdot M_{pl,Rd}}{m} = \frac{4 \cdot 412500}{35} = 47142 \\ \frac{2 \cdot M_{pl,Rd} + n \cdot \sum F_{t,rd}}{m+n} = \frac{2 \cdot 412500 + 40 \cdot 2 \cdot 90432}{35+40} = 107461 \\ \sum F_{t,Rd} = 2 \cdot 90432 = 180864 \end{cases} = \\ = 47{,}1\,kN \tag{22}$$

- Angle cleat in tension

$$F_{ta,Rd} = min \begin{cases} \frac{l_{fa} \cdot t_a \cdot f_{ya}}{\gamma_{M0}} = \frac{120 \cdot 10{,}0 \cdot 275}{1{,}0} = 330000 \\ \frac{(l_{fa} - 2 \cdot d_0) \cdot t_a \cdot f_{ua}}{\gamma_{M2}} = \frac{(120 - 2 \cdot 18) \cdot 10{,}0 \cdot 430}{1{,}1} = 328364 \end{cases} = 328{,}4\,kN \tag{23}$$

Finally, the design resistance of the first bolt row:

$$F_{1,Rd} = min\,(V_{wp,Rd}, F_{cwc,Rd}, F_{cfb,Rd}, F_{ca,Rd}, F_{cab,Rd}, F_{cbfb,Rd}, F_{vca,Rd}, F_{tfc,Rd}, F_{twc,Rd}, F_{tab,Rd}, F_{tbf,Rd}, F_{vta,Rd}, F_{ta,Rd}, F_{ta,Rd})$$

$F_{1,Rd}$ = *min* (289,9; 309,4; 438,4; 330,0; 165,6; 219,0; 154,4; 167,1; 198,8; 165,6; 141,9; 154,4; 47,1; 328,4) = 47,1 *kN*

Second bolt row:

- Column flange in bending for first and second bolt rows

  Effective length of equivalent T-stub:

  - Circular patterns:

  $l_{eff,cp} = 2 \cdot \pi \cdot m + 2 \cdot p_1 = 2 \cdot \pi \cdot 19{,}75 + 2 \cdot 105 = 334\ mm$

  - Non – circular patterns:

  $l_{eff,nc}=4 \cdot m+1{,}25 \cdot e+p_1=4 \cdot 19{,}75+1{,}25 \cdot 24{,}7+105=214{,}9\ mm$

  where $p_1 = \frac{h_b}{2} + e_1 - \frac{p_w}{2} = \frac{240}{2} + 40 - \frac{110}{2} = 105\ mm$

$$M_{pl,Rd} = \frac{0{,}25 \cdot l_{eff,nc} \cdot t_{fc}^2 \cdot f_y}{\gamma_{M0}} = \frac{0{,}25 \cdot 214{,}9 \cdot 14{,}0^2 \cdot 275}{1{,}0} = 2895778\ Nmm$$

The resistance of the column flange in bending is taken as minimum values from three modes of failure.

$$F_{tfc(1+2),Rd} = min \begin{cases} \frac{4 \cdot M_{pl,Rd}}{m} = \frac{4 \cdot 2895778}{19{,}75} = 586487 \\ \frac{2 \cdot M_{pl,Rd} + n \cdot \sum F_{t,rd}}{m+n} = \frac{2 \cdot 2895778 + 24{,}7 \cdot 4 \cdot 90432}{19{,}75 + 24{,}7} = 331299 \\ \sum F_{t,Rd} = 4 \cdot 90432 = 361728 \end{cases} = 331{,}3\ kN \quad (24)$$

- Column web panel in tension

$$F_{twc(1+2),Rd} = \frac{\omega \cdot b_{eff,t,wc} \cdot t_{wc} \cdot f_{ywc}}{\gamma_{M0}} = \frac{0{,}774 \cdot 214{,}9 \cdot 8{,}5 \cdot 275}{1{,}0} = 388802\ N = 388{,}8\ kN \quad (25)$$

- Bolts in web angle leg in bearing

$$F_{twab,Rd} = \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot 2 \cdot t_{fa}}{\gamma_{M2}} = \frac{2{,}19 \cdot 0{,}741 \cdot 430 \cdot 16 \cdot 2 \cdot 10{,}0}{1{,}25} = 178637\ N = 178{,}6\ kN \quad (26)$$

where $k_1 = min \begin{cases} 2{,}8 \cdot \frac{e_w}{d_0} - 1{,}7 = 2{,}8 \cdot \frac{25}{18} - 1{,}7 = 2{,}19 \\ 2{,}5 \end{cases} = 2{,}19$

$$\alpha_b = min\begin{cases} \alpha_d = \frac{e_2}{3\cdot d_0} = \frac{40}{3\cdot 18} = 0{,}741 \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0 \end{cases} = 0{,}741$$

- Bolts in beam web in bearing

$$F_{twbb,Rd} = \frac{k_1\cdot\alpha_b\cdot f_u\cdot d\cdot t_{wb}}{\gamma_{M2}} = \frac{2{,}5\cdot 0{,}648\cdot 430\cdot 16\cdot 6{,}2}{1{,}25} = 55282\ N = 55{,}3\ kN \quad (27)$$

where $k_1 = min\begin{cases} 2{,}8\cdot\frac{h_b-p_w}{2\cdot d_0} - 1{,}7 = 2{,}8\cdot\frac{240-110}{2\cdot 18} - 1{,}7 = 8{,}4 \\ 2{,}5 \end{cases} = 2{,}5$

$$\alpha_b = min\begin{cases} \alpha_d = \frac{e_1-g}{3\cdot d_0} = \frac{40-5}{3\cdot 18} = 0{,}648 \\ \frac{f_{ub}}{f_{ua}} = \frac{800}{430} = 1{,}86 \\ 1{,}0 \end{cases} = 0{,}648$$

- Bolts in shear.
  Shear resistance of the bolt in web is the same as in the flange: $F_{vwa,Rd} = F_{vca,Rd} = 154{,}4\ kN$
- Angle cleat in bending
  Value of m for large gap $m_{w,max} = e_1 - 0{,}5\cdot t_a = 40 - 0{,}5\cdot 10{,}0 = 35\ mm$, and $e = 40mm$.

$$m`_2 = \frac{3}{2}\frac{t_{wa}}{\left[\left(\frac{m_{max}\cdot h_2}{t_{wa}\cdot l_{wa}}\right)^2+3\right]^{\frac{1}{2}} - \frac{m_{max}\cdot h_2}{t_{wa}\cdot l_{wa}}} = \frac{3}{2}\frac{10{,}0}{\left[\left(\frac{35{,}0\cdot 180}{10{,}0\cdot 160}\right)^2+3\right]^{\frac{1}{2}} - \frac{35{,}0\cdot 180}{10{,}0\cdot 160}} = 9{,}57mm \quad (28)$$

where $h_2 = \frac{(h_b+p_w+t_a)}{2} = \frac{(240+110+10{,}0)}{2} = 180\ mm$

Effective length of equivalent T-stub:

- Circular patterns:

$$l_{eff,cp} = min\begin{cases} 2\cdot\pi\cdot m`_2 = 2\cdot\pi\cdot 9{,}57 = 60{,}1 \\ \pi\cdot m`_2 + 2\cdot e_w = \pi\cdot 9{,}57 + 2\cdot 25 = 80{,}0 \end{cases} = 60{,}1\ mm$$

- Non – circular patterns:

$$l_{eff,nc} = min\begin{cases} 4\cdot m`_2 + 1{,}25\cdot e` = 4\cdot 9{,}57 + 1{,}25\cdot 12{,}0 = 53{,}3 \\ 2\cdot m`_2 + 0{,}625\cdot e` + e_w = 2\cdot 9{,}57 + 0{,}625\cdot 12{,}0 + 25 = 51{,}6 \end{cases} = 51{,}6\ mm$$

$$M_{pl,Rd} = \frac{0{,}25\cdot l_{eff}\cdot t_a^2\cdot f_y}{\gamma_{M0}} = \frac{0{,}25\cdot 51{,}6\cdot 10{,}0^2\cdot 275}{1{,}0} = 354750Nmm$$

$$F_{twa,Rd} = min\begin{cases} \frac{4\cdot M_{pl,Rd}}{m`_2} = \frac{4\cdot 354750}{9{,}57} = 148276 \\ \frac{2\cdot M_{pl,Rd}+n\cdot\sum F_{t,rd}}{m+n} = \frac{2\cdot 354750+12{,}0\cdot 2\cdot 90432}{9{,}57+12{,}0} = 133513 = \\ \sum F_{t,Rd} = 2\cdot 90432 = 180864 \end{cases}$$
$$= 133{,}5\ kN \tag{29}$$

- Angle web cleats in tension

$$F_{twa,Rd} = \frac{l_{eff}\cdot 2\cdot t_a\cdot f_{ya}}{\gamma_{M0}} = \frac{51{,}6\cdot 2\cdot 10{,}0\cdot 275}{1{,}0} = 283800N = 283{,}8kN \tag{30}$$

- Beam web in tension

$$F_{twb,Rd} = \frac{l_{eff}\cdot t_{wb}\cdot f_{yb}}{\gamma_{M0}} = \frac{51{,}6\cdot 6{,}2\cdot 275}{1{,}0} = 87978N = 88{,}0kN \tag{31}$$

Finally, the design resistance of the second bolt row:

$F_{2,Rd} = min$ ($V_{wp,Rd}$-$F_{1,Rd}$; $F_{cwc,Rd}$-$F_{1,Rd}$; $F_{cfb,Rd}$-$F_{1,Rd}$; $F_{ca,Rd}$-$F_{1,Rd}$; $F_{cab,Rd}$- $F_{1,Rd}$; $F_{cbfb,Rd}$-$F_{1,Rd}$; $F_{vca,Rd}$-$F_{1,Rd}$; $F_{tfc,Rd}$; $F_{tfc,Rd(1+2)}$-$F_{1,Rd}$; $F_{twc,Rd}$; $F_{twc,Rd(1+2)}$-$F_{1,Rd}$; $F_{twab,Rd}$; $F_{twbb,Rd}$; $F_{twa,Rd}$; $F_{twb,Rd}$; $F_{twa,Rd}$; $F_{vwa,Rd}$)

$F_{1,Rd}$ = *min* (289,9-47,1; 309,4-47,1; 438,4-47,1; 330,0-47,1; 165,6-47,1; 219,0-47,1; 154,4-47,1; 167,1; 331,3-47,1; 198,8; 388,8-47,1; 178,6; 55,3; 133,5; 88,0; 283,8; 154,4) = 55,3 *kN*

Third bolt row:

- Column flange in bending for first, second and third bolt rows

  Effective length of equivalent T-stub:

  - Circular patterns:

  $l_{eff,cp} = 2\cdot\pi\cdot m+2\cdot(p_1+p_w) = 2\cdot\pi\cdot 19{,}75+2\cdot\ (105+110) = 554\ mm$

  - Non – circular patterns:

  $l_{eff,nc} = 4\cdot m+1{,}25\cdot e+p_1+p_w=4\cdot 19{,}75+1{,}25\cdot 24{,}7+105+110=324{,}9\ mm$

$$M_{pl,Rd} = \frac{0{,}25\cdot l_{eff,nc}\cdot t_{fc}^2\cdot f_y}{\gamma_{M0}} = \frac{0{,}25\cdot 324{,}9\cdot 14{,}0^2\cdot 275}{1{,}0} = 4378028\ Nmm$$

The resistance of the column flange in bending is taken as minimum values from three modes of failure.

$$F_{tfc(1+2+3),Rd} = min\begin{cases} \frac{4\cdot M_{pl,Rd}}{m} = \frac{4\cdot 4378028}{19{,}75} = 886689 \\ \frac{2\cdot M_{pl,Rd}+n\cdot\sum F_{t,rd}}{m+n} = \frac{2\cdot 4378028+24{,}7\cdot 6\cdot 90432}{19{,}75+24{,}7} = 498494 = \\ \sum F_{t,Rd} = 6\cdot 90432 = 542592 \end{cases}$$
$$= 498{,}5\ kN \tag{32}$$

- Column flange in bending for second and third bolt rows

  Effective length of equivalent T-stub:

  - Circular patterns:

  $l_{eff,cp} = 2\cdot\pi\cdot m+2\cdot p_w = 2\cdot\pi\cdot 19{,}75+2\cdot 110 = 344\ mm$

  - Non – circular patterns:

  $l_{eff,nc}=4\cdot m+1{,}25\cdot e+p_w=4\cdot 19{,}75+1{,}25\cdot 24{,}7+110=219{,}9\ mm$

$$M_{pl,Rd} = \frac{0{,}25\cdot l_{eff,nc}\cdot t_{fc}^2\cdot f_y}{\gamma_{M0}} = \frac{0{,}25\cdot 219{,}9\cdot 14{,}0^2\cdot 275}{1{,}0} = 2963152\ Nmm$$

The resistance of the column flange in bending is taken as minimum values from three modes of failure.

$$F_{tfc(1+2),Rd} = min\begin{cases} \frac{4\cdot M_{pl,Rd}}{m} = \frac{4\cdot 2963152}{19{,}75} = 600132 \\ \frac{2\cdot M_{pl,Rd}+n\cdot\sum F_{t,rd}}{m+n} = \frac{2\cdot 2963152+24{,}7\cdot 4\cdot 90432}{19{,}75+24{,}7} = 334330 \\ \sum F_{t,Rd} = 4\cdot 90432 = 361728 \end{cases} = \\ = 334{,}3\ kN \quad (33)$$

- Column web panel in tension for first, second and third bolt rows

$$F_{twc(1+2+3),Rd} = \frac{\omega\cdot b_{eff,t,wc}\cdot t_{wc}\cdot f_{ywc}}{\gamma_{M0}} = \frac{0{,}774\cdot 324{,}9\cdot 8{,}5\cdot 275}{1{,}0} = \\ = 587817\ N = 587{,}8\ kN \quad (34)$$

- Column web panel in tension for second and third bolt rows

$$F_{twc(2+3),Rd} = \frac{\omega\cdot b_{eff,t,wc}\cdot t_{wc}\cdot f_{ywc}}{\gamma_{M0}} = \frac{0{,}774\cdot 219{,}9\cdot 8{,}5\cdot 275}{1{,}0} = \\ = 397849\ N = 397{,}8\ kN \quad (35)$$

- Angle cleat in bending

$$m`_3 = \frac{3}{2}\frac{t_{wa}}{\left[\left(\frac{m_{max}\cdot h_3}{t_{wa}\cdot l_{wa}}\right)^2+3\right]^{\frac{1}{2}}-\frac{m_{max}\cdot h_3}{t_{wa}\cdot l_{wa}}} = \frac{3}{2}\frac{10{,}0}{\left[\left(\frac{35{,}0\cdot 70}{10{,}0\cdot 160}\right)^2+3\right]^{\frac{1}{2}}-\frac{35{,}0\cdot 70}{10{,}0\cdot 160}} = 4{,}87mm \quad (36)$$

where $h_3 = \frac{(h_b-p_w+t_a)}{2} = \frac{(240-110+10{,}0)}{2} = 70\ mm$

  Effective length of equivalent T-stub:

  - Circular patterns:

$$l_{eff,cp} = min\begin{cases} 2\cdot\pi\cdot m`_3 = 2\cdot\pi\cdot 4{,}87 = 30{,}6 \\ \pi\cdot m`_3+2\cdot e_w = \pi\cdot 4{,}87+2\cdot 25 = 65{,}3 \end{cases} = 30{,}6\ mm$$

- Non – circular patterns:

$$l_{eff,nc} =$$

$$= min\begin{cases} 4 \cdot m_3^{`} + 1{,}25 \cdot e^{`} = 4 \cdot 4{,}87 + 1{,}25 \cdot 6{,}09 = 27{,}1 \\ 2 \cdot m_3^{`} + 0{,}625 \cdot e^{`} + e_w = 2 \cdot 4{,}87 + 0{,}625 \cdot 6{,}09 + 25 = 38{,}5 \end{cases} =$$

$$= 27{,}1\, mm$$

$$M_{pl,Rd} = \frac{0{,}25 \cdot l_{eff} \cdot t_a^2 \cdot f_y}{\gamma_{M0}} = \frac{0{,}25 \cdot 27{,}1 \cdot 10{,}0^2 \cdot 275}{1{,}0} = 186312 Nmm$$

$$F_{twa,Rd} = min\begin{cases} \frac{4 \cdot M_{pl,Rd}}{m`_3} = \frac{4 \cdot 186312}{4{,}87} = 153029 \\ \frac{2 \cdot M_{pl,Rd} + n \cdot \sum F_{t,rd}}{m+n} = \frac{2 \cdot 186312 + 6{,}09 \cdot 2 \cdot 90432}{4{,}87 + 6{,}09} = 134497 \\ \sum F_{t,Rd} = 2 \cdot 90432 = 180864 \end{cases} =$$

$$= 134{,}5\, kN \quad (37)$$

- Angle web cleats in tension

$$F_{twa,Rd} = \frac{l_{eff} \cdot 2 \cdot t_a \cdot f_{ya}}{\gamma_{M0}} = \frac{27{,}1 \cdot 2 \cdot 10{,}0 \cdot 275}{1{,}0} = 149050N = 149{,}0kN \quad (38)$$

- Beam web in tension

$$F_{twb,Rd} = \frac{l_{eff} \cdot t_{wb} \cdot f_{yb}}{\gamma_{M0}} = \frac{27{,}1 \cdot 6{,}2 \cdot 275}{1{,}0} = 46205N = 46{,}2kN \quad (39)$$

Due to the fact that $\sum l_{eff} = 51{,}6 + 27{,}1 = 78{,}7\, mm < p_w = 110\, mm$, the resistance of the components for the group of bolts is greater than for individual bolts, so it can be omitted in the calculations. The shear and bearing resistance of the bolts for third bolt row are the same as for the second bolt row.

Finally, the design resistance of the third bolt row:

$F_{3,Rd}$ = *min* ($V_{wp,Rd}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{cwc,Rd}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{cfb,Rd}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{ca,Rd}$-$F_{1,Rd}$--$F_{2,Rd}$; $F_{cab,Rd}$- $F_{1,Rd}$-$F_{2,Rd}$; $F_{cbfb,Rd}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{vca,Rd}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{tfc,Rd(3)}$; $F_{tfc,Rd(1+2+3)}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{tfc,Rd(2+3)}$-$F_{2,Rd}$; $F_{twc,Rd(3)}$; $F_{twc,Rd(1+2+3)}$-$F_{1,Rd}$-$F_{2,Rd}$; $F_{twc,Rd(2+3)}$-$F_{2,Rd}$; $F_{twab,Rd}$; $F_{twbb,Rd}$; $F_{twa,Rd}$; $F_{twb,Rd}$; $F_{twa,Rd}$; $F_{vwa,Rd}$)

$F_{3,Rd}$ = *min* (289,9-47,1-55,3; 309,4-47,1-55,3; 438,4-47,1-55,3; 330,0-47,1- -55,3; 165,6-47,1-55,3; 219,0-47,1-55,3; 154,4-47,1-55,3; 167,1; 498,5-47,1-55,3; 334,3-55,3; 198,8; 587,8-47,1-55,3; 397,8-55,3; 178,6; 55,3; 134,5; 46,2; 149,0; 154,4) = 46,2 *kN*

Moment resistance of the joint:

$$M_{j,Rd} = \sum h_r \cdot F_{r,Rd} = 285 \cdot 47{,}1 + 180 \cdot 55{,}3 + 70 \cdot 46{,}2 = \\ = 26600 kNmm = 26{,}6\ kNm. \quad (40)$$

Rotational stiffness of the joint.

- Column web panel in shear

$$k_{vwp} = \frac{0{,}38 \cdot A_{vc}}{\beta \cdot h_1} = \frac{0{,}38 \cdot 2029}{1{,}0 \cdot 285} = 2{,}71\ mm \quad (41)$$

- Column web panel in compression

$$k_{cwc} = \frac{0{,}7 \cdot b_{eff} \cdot t_{wc}}{d_{wc}} = \frac{0{,}7 \cdot 171 \cdot 8{,}5}{122} = 8{,}34\ mm \quad (42)$$

- Bolts in bottom flange in shear

$$k_{vca} = \frac{16 \cdot n_b \cdot d^2 \cdot f_{ub}}{E \cdot d_{M16}} = \frac{16 \cdot 1 \cdot 16^2 \cdot 800}{210 \cdot 10^3 \cdot 16} = 0{,}98\ mm \quad (43)$$

- Bolts in bottom angle cleat in bearing

$$k_{cab} = \frac{24 \cdot n_b \cdot k_b \cdot k_t \cdot d \cdot f_u}{E} = \frac{24 \cdot 1 \cdot 1{,}25 \cdot 0{,}938 \cdot 16 \cdot 430}{210 \cdot 10^3} = 0{,}92\ mm \quad (44)$$

where $k_b = 1{,}25$, and $k_t = \frac{1{,}5 \cdot t_a}{d_{M16}} = \frac{1{,}5 \cdot 10{,}0}{16} = 0{,}938 < 2{,}5$

- Column web panel in tension

$$k_{twc} = \frac{0{,}7 \cdot l_{eff} \cdot t_{wc}}{d_{wc}} = \frac{0{,}7 \cdot 109{,}9 \cdot 8{,}5}{122} = 5{,}36\ mm \quad (45)$$

- Column flange in bending

$$k_{tfc} = \frac{0{,}9 \cdot l_{eff} \cdot t_{fc}^3}{m^3} = \frac{0{,}9 \cdot 109{,}9 \cdot 14{,}0^3}{19{,}75^3} = 35{,}23\ mm \quad (46)$$

- Bolts in tension

$$k_{tb} = \frac{1{,}6 \cdot A_{sb}}{L_b} = \frac{1{,}6 \cdot 157}{42{,}4} = 5{,}92\ mm \quad (47)$$

where $L_b = t_{fc} + t_a + 2 \cdot t_w + 0{,}5 \cdot (k + t_n) = 14{,}0 + 10{,}0 + 2 \cdot 3{,}0 + 0{,}5\ (10{,}0 + 14{,}8) = 42{,}4\ mm$

For first bolt row

- Top angle cleat in bending

$$k_{ta} = \frac{0{,}9 \cdot l_{eff} \cdot t_a^3}{m^3} = \frac{0{,}9 \cdot 60 \cdot 10{,}0^3}{35^3} = 1{,}26\ mm \quad (48)$$

- Bolts in top angle cleat in bearing

$$k_{tab} = \frac{24 \cdot n_b \cdot k_b \cdot k_t \cdot d \cdot f_u}{E} = \frac{24 \cdot 1 \cdot 1{,}125 \cdot 0{,}938 \cdot 16 \cdot 430}{210 \cdot 10^3} = 0{,}83\ mm \tag{49}$$

where $k_b = \frac{0{,}25 \cdot e_2}{d} + 0{,}5 = \frac{0{,}25 \cdot 40}{16} + 0{,}5 = 1{,}125 < 1{,}25,$

and $k_t = \frac{1{,}5 \cdot t_a}{d_{M16}} = \frac{1{,}5 \cdot 10{,}0}{16} = 0{,}938 < 2{,}5$

- Bolts in top beam flange in bearing

$$k_{tbfb} = \frac{24 \cdot n_b \cdot k_b \cdot k_t \cdot d \cdot f_u}{E} = \frac{24 \cdot 1 \cdot 1{,}047 \cdot 0{,}919 \cdot 16 \cdot 430}{210 \cdot 10^3} = 0{,}76\ mm \tag{50}$$

where $k_b = \frac{0{,}25 \cdot e`_2}{d} + 0{,}5 = \frac{0{,}25 \cdot 35}{16} + 0{,}5 = 1{,}047 < 1{,}25,$

and $k_t = \frac{1{,}5 \cdot t_{fb}}{d_{M16}} = \frac{1{,}5 \cdot 9{,}8}{16} = 0{,}919 < 2{,}5$

For second bolt row

- Web angle cleats in bending

$$k_{twa} = \frac{0{,}9 \cdot l_{eff} \cdot t_a^3}{m^3} = \frac{0{,}9 \cdot 51{,}6 \cdot 10{,}0^3}{9{,}57^3} = 52{,}99\ mm \tag{51}$$

- Bolts in beam web in bearing

$$k_{twbb} = \frac{24 \cdot n_b \cdot k_b \cdot k_t \cdot d \cdot f_u}{E} = \frac{24 \cdot 0{,}5 \cdot 1{,}047 \cdot 0{,}581 \cdot 16 \cdot 430}{210 \cdot 10^3} = 0{,}24\ mm \tag{52}$$

where $k_b = 1{,}047$, and $k_t = \frac{1{,}5 \cdot t_{wb}}{d_{M16}} = \frac{1{,}5 \cdot 6{,}2}{16} = 0{,}581 < 2{,}5$

For third bolt row

- Web angle cleats in bending

$$k_{twa} = \frac{0{,}9 \cdot l_{eff} \cdot t_a^3}{m^3} = \frac{0{,}9 \cdot 27{,}1 \cdot 10{,}0^3}{4{,}87^3} = 211{,}17\ mm \tag{53}$$

Shear and bearing resistance are the same as for previous bolt rows.

Effective stiffness coefficient for first bolt row

$$k_{eff1} = \frac{1}{\sum \frac{1}{k_{i,1}}} = \frac{1}{\frac{1}{k_{twc}} + \frac{1}{k_{tfc}} + \frac{1}{k_{tb}} + \frac{1}{k_{ta}} + \frac{1}{k_{vta}} + \frac{1}{k_{tab}} + \frac{1}{k_{tbfb}}} =$$

$$= \frac{1}{\frac{1}{5{,}36} + \frac{1}{35{,}23} + \frac{1}{5{,}92} + \frac{1}{1{,}26} + \frac{1}{0{,}98} + \frac{1}{0{,}83} + \frac{1}{0{,}76}} = 0{,}21\ mm \tag{54}$$

Effective lever arm for first bolt row

$$h_{eff1} = \frac{k_{eff,c} \cdot h_1^2 + k_{eff,a} \cdot (h_b + t_a/2)^2}{k_{eff,c} \cdot h_1 + k_{eff,a} \cdot (h_b + t_a/2)} = \frac{2{,}07 \cdot 285^2 + 0{,}23 \cdot (240 + 10{,}0/2)^2}{2{,}07 \cdot 285 + 0{,}23 \cdot (240 + 10{,}0/2)} =$$

$$= 281{,}5\ mm \tag{55}$$

where $k_{eff,c} = \frac{1}{\frac{1}{k_{twc}}+\frac{1}{k_{tfc}}+\frac{1}{k_{tb}}} = \frac{1}{\frac{1}{5,36}+\frac{1}{7,88}+\frac{1}{5,92}} = 2,07\ mm$, and

$$k_{eff,a} = \frac{1}{\frac{1}{k_{ta}}+\frac{1}{k_{vta}}+\frac{1}{k_{tab}}+\frac{1}{k_{tbfb}}} = \frac{1}{\frac{1}{1,26}+\frac{1}{0,98}+\frac{1}{0,83}+\frac{1}{0,76}} = 0,23\ mm$$

Effective stiffness coefficient for second bolt row

$$k_{eff2} = \frac{1}{\sum\frac{1}{k_{i,2}}} = \frac{1}{\frac{1}{k_{twc}}+\frac{1}{k_{tfc}}+\frac{1}{k_{tb}}+\frac{1}{k_{twa}}+\frac{1}{k_{vwa}}+\frac{1}{k_{twbb}}+\frac{1}{k_{twab}}} =$$
$$= \frac{1}{\frac{1}{5,36}+\frac{1}{7,88}+\frac{1}{5,92}+\frac{1}{52,99}+\frac{1}{0,98}+\frac{1}{0,24}+\frac{1}{0,83}} = 0,15\ mm \quad (56)$$

Effective stiffness coefficient for third bolt row

$$k_{eff3} = \frac{1}{\sum\frac{1}{k_{i,3}}} = \frac{1}{\frac{1}{k_{twc}}+\frac{1}{k_{tfc}}+\frac{1}{k_{tb}}+\frac{1}{k_{twa}}+\frac{1}{k_{vwa}}+\frac{1}{k_{twbb}}+\frac{1}{k_{twab}}} =$$
$$= \frac{1}{\frac{1}{5,36}+\frac{1}{7,88}+\frac{1}{5,92}+\frac{1}{211,17}+\frac{1}{0,98}+\frac{1}{0,24}+\frac{1}{0,83}} = 0,15\ mm \quad (57)$$

Equivalent lever arm

$$z_{eq} = \frac{\sum k_{eff,i}\cdot h_i^2}{\sum k_{eff,i}\cdot h_i} = \frac{0,21\cdot 281,5^2+0,15\cdot 180^2+0,15\cdot 70^2}{0,21\cdot 281,5+0,15\cdot 180+0,15\cdot 70} = 230,1\ mm \quad (58)$$

Equivalent stiffness coefficient

$$k_{eq} = \frac{\sum k_{eff,i}\cdot h_i}{z_{eq}} = \frac{0,21\cdot 281,5+0,15\cdot 180+0,15\cdot 70}{230,1} = 0,42\ mm \quad (59)$$

Finally, stiffness of the joint is equal:

$$S_j = \frac{E\cdot z_{eq}^2}{\mu\cdot\sum\frac{1}{k_i}} = \frac{E\cdot z_{eq}^2}{\mu\cdot\left(\frac{1}{k_{cwc}}+\frac{1}{k_{vwp}}+\frac{1}{k_{vca}}+\frac{1}{k_{cab}}+\frac{1}{k_{eq}}\right)} = \frac{210\cdot 10^3\cdot 230,1^2}{1,0\cdot\left(\frac{1}{8,34}+\frac{1}{2,71}+\frac{1}{0,98}+\frac{1}{0,92}+\frac{1}{0,42}\right)} =$$
$$= 34233,9\cdot 10^6\frac{Nmm}{rad} = 34233,9\ kNm/rad. \quad (60)$$

4. Proposed estimation of the resistance and stiffness of the joint

As can see in the example, the procedure of the calculating of moment resistance and initial stiffness of the connection is labor-intensive and complicated. Therefore, this method is used to check the characteristics of existing connections or in computer programs. In the initial phase, designers need fast and approximated methods to estimate the resistance and stiffness of the joints. In such cases polynomial approximation functions are often used.

Design procedure shown that the moment resistance and initial stiffness are depend on many different parameters both dependent and independent in relation to himself. The influence of some of these parameters on the resistance and the stiffness of the connection is very similar to the joints with only flange angle cleats introduced in [8].

In the case of joints with web and flange angle cleats, one of the additional parameters is the length of the angle web cleat l_{wa}. The influence of length of the angle web cleat on the resistance and the stiffness of the connection are introduced on Fig. 8 and 9.

After analyzing dozens of different configurations of beam to column joints with use of angle web and flange cleats was ascertained that on the resistance and the stiffness the greatest influence had: the height of the beam h_b, the depth of the column h_c, the length of flange of the angle cleats b_a, the diameter of the bolts d, length of the angle web cleat l_{wa}, and geometric dimensions which usually are variable dependent from the elements of the connection.

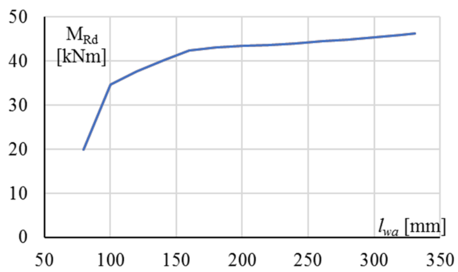

Fig. 8. Influence of length of the angle web cleat l_{wa} on moment resistance of the joint

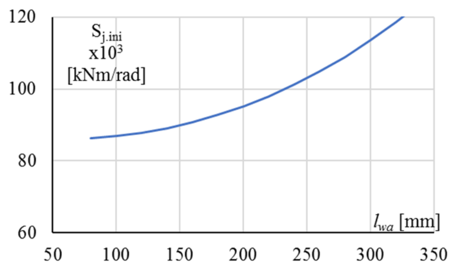

Fig. 9. Influence of length of the angle web cleat l_{wa} on initial stiffness of the joint

For further analysis, typical design assumptions and constant values of the following parameters were established:

- steel grade for all elements S275,
- bolts grade 8.8,
- bolts diameter for web and angle cleats is the same,
- column with HEB - section,
- beam with IPE - section,
- angle cleats and web cleats are made of the same sections,
- thickness of the angle leg is means thickness of accepted angle section,
- angle flange cleats length is equal to width of column flange,
- the spacing of the bolts is depended on connected elements according to constructional recommendations,
- edge distance for angle web cleat is equal $1,2 \cdot d_0$.
- gap g is equal 5 mm.

As a variable parameters, the following joint properties were analyzed:

- beam height $x_1 = h_b$ in range (200 ÷ 450) [mm],
- column depth $x_2 = h_c$ in range (100 ÷ 300) [mm],
- length of flange of the angle cleats $x_3 = b_a$ in range (60 ÷ 200) [mm],

− bolt diameter $x_4 = d$ in range (10 ÷ 24) [mm], and
− length of the angle web cleat $x_5 = l_{wa}$ in range ($4{,}6 \cdot d_0 \div d_b$) [mm].

To finding of the approximation function of was applied the theory of the planning of the experiment [10], and the computer simulation. For determine of characteristics of the joins, was adopted a plan the experiment of Hartley PS/DS-P:Ha5 based at the range of variation on the hypercube, with the arrangement showing in table 1.

Table 1. Planning of experiment for determination of the resistance and stiffness of the joint

	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_5$	x_1: h_b	x_2: h_c	x_3: b_a	x_4: d	x_5: l_{wa}	M_R [kN·m]	$S_{j.ini}$ [kN·m/rad]
1	+1	+1	+1	+1	+1	450	300	200	24	379	103,18	145239
2	-1	-1	+1	+1	+1	200	100	200	24	159	-	-
3	-1	+1	-1	-1	-1	200	300	60	10	51	8,08	58962
4	+1	-1	-1	-1	-1	450	100	60	10	51	22,65	126819
5	-1	+1	-1	+1	+1	200	300	60	24	159	3,43	20213
6	+1	-1	-1	+1	+1	450	100	60	24	379	39,82	109450
7	+1	+1	+1	-1	-1	450	300	200	10	51	33,78	117502
8	-1	-1	+1	-1	-1	200	100	200	10	51	17,84	79079
9	-1	+1	+1	+1	-1	200	300	200	24	120	16,41	16098
10	+1	-1	+1	+1	-1	450	100	200	24	120	63,40	109376
11	+1	+1	-1	-1	+1	450	300	60	10	379	27,13	131015
12	-1	-1	-1	-1	+1	200	100	60	10	159	11,06	42265
13	-1	+1	+1	-1	+1	200	300	200	10	159	18,09	40379
14	+1	-1	+1	-1	+1	450	100	200	10	379	33,78	100128
15	+1	+1	-1	+1	-1	450	300	60	24	120	22,85	116191
16	-1	-1	-1	+1	-1	200	100	60	24	120	3,43	28869
17	+1	0	0	0	0	450	200	130	16	233	73,84	73177
18	-1	0	0	0	0	200	200	130	16	120	15,79	24590
19	0	+1	0	0	0	330	300	130	16	177	54,32	40224
20	0	-1	0	0	0	330	100	130	16	177	41,18	53243
21	0	0	+1	0	0	330	200	200	16	177	63,68	44586
22	0	0	-1	0	0	330	200	60	16	177	25,36	60854
23	0	0	0	+1	0	330	200	130	24	195	83,11	37264
24	0	0	0	-1	0	330	200	130	10	161	23,90	78301
25	0	0	0	0	+1	330	200	130	16	271	55,61	47523
26	0	0	0	0	-1	330	200	130	16	83	50,85	67749
27	0	0	0	0	0	330	200	130	16	177	53,23	47109

After analyzing with the overlay SOLVER of several variants of the approximation function, the function giving the least square error was chose. The resistance of the bolted beam to column joint with the use of angle web flange cleats can be obtain from:

$$M_{Rd} = 4{,}885 \cdot 10^{-3} \cdot h_b^{0{,}636} \cdot h_c^{0{,}048} \cdot b_a^{0{,}508} \cdot d^{0{,}534} \cdot l_{wa}^{0{,}222} \tag{61}$$

The stiffness of the unstiffened joint can be obtained from:

$$S_{j,ini} = 4{,}528 \cdot 10^{-5} \cdot h_b^{3{,}529} \cdot h_c^{0{,}105} \cdot b_a^{-0{,}088} \cdot d^{-0{,}075} \cdot l_{wa}^{-0{,}031} + 30139 \quad (62)$$

where: M_{Rd} – in [kNm], $S_{j,ini}$ - in [kNm/rad], and h_b, h_c, b_a, l_{wa}, d - in [mm].

The equations (55) and (56) refer to one-sided connections of the beam to the unstiffened column. In the initial stage of the designing they can also be used for other similar types of joints. After the achievement of calculations was observed that in the investigated range of joints, the stiffening of the column has not influence on the resistance of the joint, and only enlarges his stiffness. The increase of stiffness of the joint, caused by the ribbing of the column, does not exceed several dozen percent. So, it can be omitted in the preliminary analysis of the joint. In case of two-sided joints, their resistance is the same as the moment resistance of the cantilever joints. The initial stiffnes of two-sided stiffened joints is such alone as stiffness of the stiffened cantilever joints. In case of two-sided not stiffened joints the stiffness is comparable with the arithmetic mean of the stiffness of cantilever stiffened and unstiffened joints.

5. Conclusion

The knowledge of the moment resistance and the initial stiffness of the joint has a basic meaning in the initial stage of the designing, for choice of the suitable method of analysis of the construction. Proposed in this paper, an approximation formulas allowed on the preliminary estimation of the joint characteristics, without the necessity of the usage of arduousness calculations according to the procedure presented in [7].

Comparing results received according to the exact standard method, and calculations by means of simplified formulas, differences did not exceed a dozen percent.

For worked example introduced in the Section 3, from approximate formulas the moment resistance amounts 25,7 [kNm] (26,6 [kNm]), and the initial stiffness 39 394 [kNm/rad] (34 234 [kNm/rad]).

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Przesłano do redakcji: 24.04.2018 r.
Przyjęto do druku: 15.06.2018 r.