17th SYMPOSIUM
ON
HYDROACOUSTICS
Jurata May 23-26, 2000

FINITE-AMPLITUDE ACOUSTIC WAVES IN LIQUID-FILLED RESONATORS

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The paper is focussed on the influence of the shape of the resonator upon the phase and amplitudes of harmonic components of the nonlinear standing wave in order to reduce the nonlinear effects. An one-dimensional model equation was developed to analyze the nonlinear standing waves. If the harmonics are not frequency coincident with the modes of a resonator then we can observe the suppression of the nonlinear effects. The degree of dissonance is influenced by the dispersive effects. Therefore this contribution is dedicated to the study of the dispersive effects on the behaviour the nonlinear standing waves

INTRODUCTION

This paper deals with the problem of finite-amplitude acoustic waves in confined geometries if one take into account the phase velocity dispersion caused by a boundary layer. Renewed interest in nonlinear standing waves has been stimulated by developments in acoustic compressors, thermoacoustic engines and refrigerators, etc. which store energy in the form of an acoustic standing wave in a resonant cavity.

When a standing wave is driven to high amplitude in an acoustic resonator, nonlinear effects couple energy from low- to high-frequency modes, ultimately resulting in shock wave formation and heightened dissipation. These nonlinear effects can be suppressed with the use a of dissonant resonance, in which modal frequencies are not integer multiples of fundamental mode frequency. This method of suppression enable to store energy in the form of an acoustic standing wave in a resonant cavity more effectively, see [2]. For theoretical prediction of the waveform within an arbitrary shaped resonator, which is filled a dispersive medium, model question were derived.

1. DISPERSION EFFECTS

In a nonlinear medium the waves interact with each other, giving rise to harmonics and waves at combination frequencies. In the case of weak nonlinearity the most efficient energy exchange between various spectral components of the field is observed when the synchronism conditions are satisfied.
When describing such processes in acoustics certain difficulties arise that are associated with lack of dispersion. In this case one can seldom deal with two- or three-wave interaction, as the synchronism conditions are simultaneously satisfied at many frequencies.
The process of nonlinear distortion of the profile on an initially harmonic wave can be teated as the interaction of a large number of synchronously propagating harmonics.
In more general cases, provided that quadratic nonlinearity is the most significant, a wave triplet may be considered as "basic" for interactions, i.e. a triad of waves with frequencies $\omega_{1,2,3}$ and wave vectors $\vec{k}_{1,2,3}$ for which the following resonance conditions are satisfied:

$$\omega_1 \pm \omega_2 = \omega_3 \, , \quad \vec{k}_1 \pm \vec{k}_2 = \vec{k}_3 \, . \tag{1}$$

In the absence of dispersion, when $k_i = \omega_i / c$, the resonance relations (1) will be fulfilled only for collinearly propagating waves, when all $\vec{k}_i$ are parallel. Under this restriction, however, these equalities hold true for triads of any harmonics with frequencies $n\omega$ provided n is the same for all waves. Futhermore, cascade processes can easily arise in which, for example, a wave with frequency ω_3 gives birth to a new one with frequency $\omega_4 = \omega_3 + \omega_1$, and so on. All this not only renders the problem more complicated but, which is more important, changes the physical results: energy transfer toward the higher wave numbers, into the small-scale part of spectrum, leads to a nonlinear damping and at the same time to saturation effects; this is exactly the case of discontinuity formation.
Along with this, the losses due to higher harmonics may frequently be avoided by introducing phase velocity dispersion, or selective losses. Many applications are connected with interactions within a bounded space region.

2. MODEL EQUATION

When waves are excited in a bounded system, or resonator, then provided the latter is of high quality, the oscillation amplitude in it appears many times higher than that of the exciting source, which favours the development of the mentioned nonlinear effects.
If we consider the one-dimensional acoustic wave field in a resonator of arbitrary axisymmetric shape, we can derive the following model equation (see [1], [2]):

$$\frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial x}\right)^2 + \frac{\gamma - 1}{2c_0^2}\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial t} + ax\right)^2 + \frac{2\sqrt{2}Bc_0^2}{r}\frac{\partial^{\frac{3}{2}}\varphi}{\partial t^{\frac{1}{2}}\partial x} + x\frac{\mathrm{d}a}{\mathrm{d}t} + a\frac{\partial \varphi}{\partial x} =$$
$$\frac{b}{\rho_0 c_0^2}\left(\frac{\partial^3 \varphi}{\partial t^3} + x\frac{\mathrm{d}^2 a}{\mathrm{d}t^2}\right) + \frac{c_0^2}{r^2}\frac{\partial}{\partial x}\left(r^2\frac{\partial \varphi}{\partial x}\right) \, . \tag{2}$$

Here φ is the velocity potential, t is time, ρ_0 correspond to the equilibrium state of the medium, $a(t)$ is the acceleration of the resonator and r is the radius, which is a function of x

$$r = r(x) \tag{3}$$

and x is a coordinate along the resonator.
The coefficient of sound diffusity b is given by this formula

$$b = \left(\zeta + \frac{4}{3}\eta\right) + \kappa\left(\frac{1}{c_V} - \frac{1}{c_p}\right) , \tag{4}$$

where η and ζ are coefficients of shear and bulk viscosity, κ is the heat conductivity coefficient, The coefficient B is

$$B = \sqrt{\frac{\eta}{2\rho_0 c_0^2}}\left(1 + \frac{\gamma - 1}{\sqrt{Pr}}\right) , \tag{5}$$

where Pr is Prandtl number, $\gamma = c_p/c_V$, c_p and c_V are the specific heats at constant pressure and volume. The fractional derivative of order $3/2$ represents this integrodifferential operator (see [?])

$$\frac{\partial^{\frac{3}{2}}\varphi}{\partial t^{\frac{1}{2}}\partial x} = \sqrt{\frac{1}{\pi}}\frac{\partial}{\partial x}\int_{-\infty}^{t}\frac{\partial\varphi(t', x, r = R_0)}{\partial t'}\frac{\mathrm{d}t'}{\sqrt{t - t'}} , \tag{6}$$

We can assume that the entire resonator is oscillated along its axis by an external force with the acceleration $a(t) = A_0\sin(\omega_0 t)$ and for the solution of model equation (2) it is necessary take into account these boundary conditions

$$\left.\frac{\partial\varphi}{\partial x}(x, t)\right|_{x=0} = 0 , \tag{7}$$

$$\left.\frac{\partial\varphi}{\partial x}(x, t)\right|_{x=L} = 0 , \tag{8}$$

where L is the length of the resonator and ω_0 is the angular frequency of the lowest mode of a resonator. In case that standing waves are excited by a piston, located at $x = L$ then we can boundary condition write in the form ([3], [5]):

$$\left.\frac{\partial\varphi}{\partial x}(x, t)\right|_{x=0} = 0 , \tag{9}$$

$$\left.\frac{\partial\varphi}{\partial x}(x, t)\right|_{x=L} = v_0\sin(\omega_0 t) , \tag{10}$$

All of these equations are written in the coordinate system that is moving with the resonator body, with exception of the case when standing waves are excited by the piston.
There are different variants of phase velocity dispersion in acoustics, for instance the dispersion connected with relaxing fluids, gas bubbles in liquids, boundary layers or the presence of lateral boundaries (in nodispersive media), see [4]. Further we will focus on the only of the mentioned variants, namely the boundary layer. The model equation (2) includes this manner of dispersion.
The phase velocity c_F of plane sound waves in axisymetrical waveguide if one take into account the boundary layer, we can write as

$$c_F = \frac{c_0}{1 + \frac{c_0 B}{r\sqrt{\omega}}} . \tag{11}$$

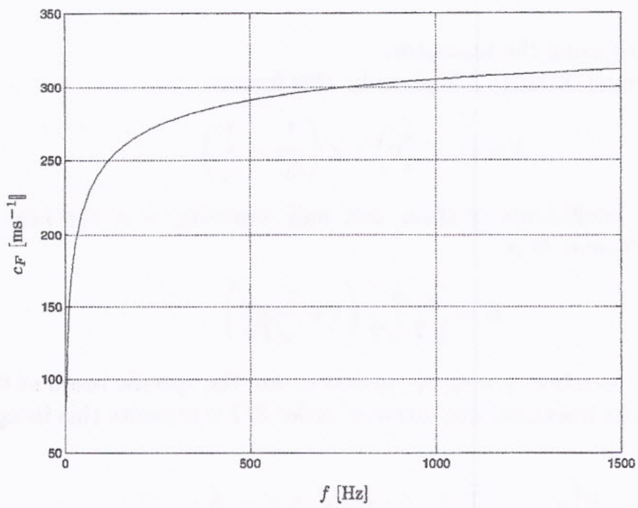

Figure 1: The dependence of the phase velocity on frequency.

We can see from equation (11) how the phase velocity depends on frequency. This dependence is shown in the figure 1 for $B = 3.10^{-4}$ ms$^{-1/2}$, $R_0 = 0.01$ m and $c_0 = 345$ ms^{-1}. It is clear that the wave number k also depends on frequency and for this reason boundary layer effects influence a degree of dissonance.
If we assume a cylindrical resonator ($r = R_0$), then for given the homogeneous boundary conditions (7) and (8) we can get the following eigenfrequencies

$$f_n = \frac{nc_0}{2L}, \quad n = 1, 2, 3, \ldots \tag{12}$$

The eigenfrequencies of this resonator are shifted if the boundary layer dispersion is taken into account. We can write this relation between eigenfrequencies for the dispersive and nondispersive case

$$f_n' = f_n - \Delta f_n = f_n \left(1 - \frac{Bc_0}{Bc_0 + r\sqrt{\omega_n}}\right) . \tag{13}$$

It is evident from equation (13) that the sound velocity dispersion may prevent the cumulative energy transfer to higher harmonics because there are differences between frequencies of harmonics and eigenfrequencies. The resonance conditions can be satisfied only for selected triads because the difference Δf_n is dependent on frequency. However, the efficiency of interaction between them will be higher.

We can see from the figure 2 and 3 that the boundary layer significantly influences the form of sound waves, especially due to the dispersion. The dispersion make the asymmetry of the primary symmetrical waves; the peak is rounded while the trough remains sharp.

CONCLUSION

The problem of dispersion due to the boundary layer is studied in this paper. The model equation was derived for the analyze of behaviour of nonlinear standing waves in resonators

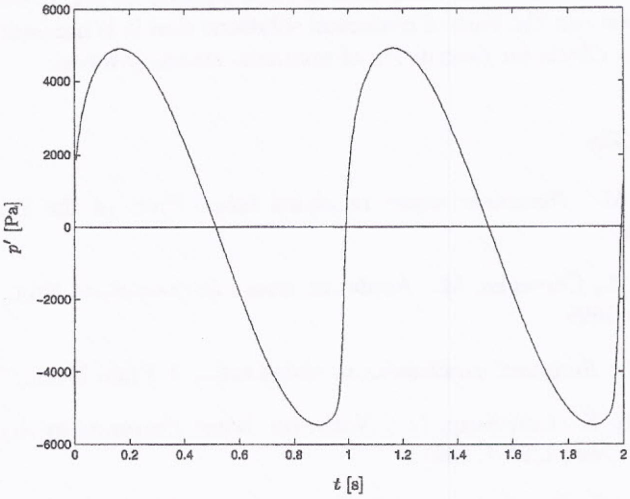

Figure 2: The influence of the boundary layer dispersion on the form of wave when the excitation by vibrating force is weak.

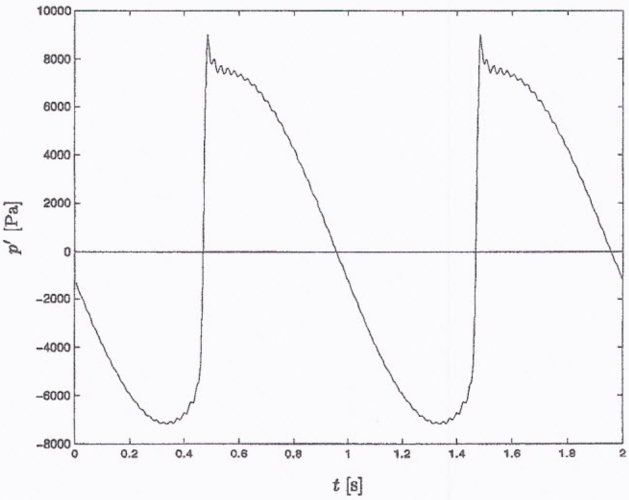

Figure 3: The influence of the boundary layer dispersion on the form of wave when the excitation by vibrating force is strong.

of arbitrary axisymmetric shapes. If we take into account boundary layer effects, we can observe the change of the wave form. Because the phase velocity c_F depends on frequency, we can observe that resonance conditions are not satisfied for higher harmonics of standing waves. It is clear, on the base of numerical solutions that it is necessary take into account boundary layer effects for description of nonlinear standing waves.

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This research has been supported by GACR grant No. 202/98/P240.