FOUNDATION OF THE NEW METHOD OF NUMERICAL CALCULATIONS OF THE NONLINEAR ACOUSTICS FIELDS

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We explain, motivation behind this work and briefly describe foundation of new method which we have developed for efficient solution in PC environment of the nonlinear propagation equation with the boundary conditions applied for both circular and not circular transducers (like array). Comparison between new and old method will be presented for strongly nonlinear disturbance. At the end we will demonstrate the results of the numerical calculations of the nonlinear field propagating from the array.

INTRODUCTION

Theoretical analysis of the nonlinear scalar wave equation, describing the propagation of sound , made it possible to develop a very efficient numerical code solving this equation in the PC domain for one-side boundary problems. The new method applied for axially symmetrical (2D+t) problems makes the calculation times at least several times shorter for weak nonlinearities. In the boundary cases without the axial symmetry, so in fact 3D+t, numerical costs – demanded memory size and calculation time become two orders of magnitude smaller in relation to methods used previously. This enables in general to solve this kind of problems by means of computers of the PC class in the case of a strong nonlinearity[1].

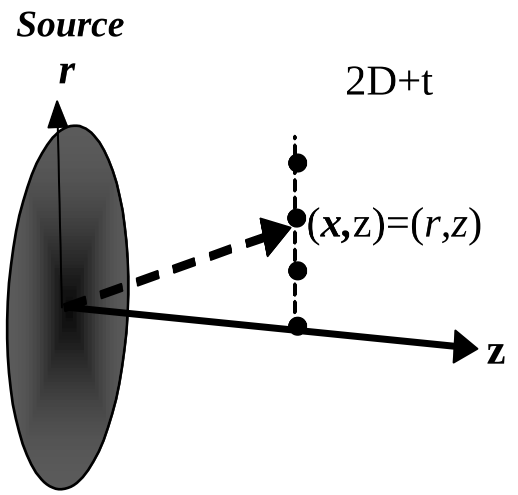
In the ultrasonography nowadays more often used are multielement transmitting probes. Fig.1 shows for example the diagram of arrangement of active piezoelectric elements (antennas) applied in such a heads (convex). This is also a good illustration showing the geometry of boundary conditions which are characteristic for 3D+t problems. The electronic control of the phase and amplitude, stimulating the piezoelectric active elements, makes it possible to deflect and to form time-space characteristics of the ultrasonic beam.

The developed numerical code provides information, which make it possible to obtain all stationary and dynamic characteristics of the field generated by such a probe and especially its time-spatial (4D) visualization.

In this way such a solver, besides of pure scientific applications of solving equations of nonlinear acoustics, can be used as a basic tool to support and rationalize the process of design of the probe as a source of the acoustic field with finite amplitudes. It allows us 1) To test the scientific ideas for possible practical applications. 2) To evaluate the materials. 3) To choice working conditions to optimize the beam shape. 4) Determination of the influence of technological defects on the distribution of the acoustic field. 5) Identification of properties of transmitting probes by comparison of measured and computed fields. 6) Calibration of hydrophones. 7) Determination of secondary effects – positive like hiperthermia and negative thermal effects, mechanical effects - determination of safe radiation doses.

The results obtained in solving of some problems mentioned above by means of the solver, designed by the present authors, were already published [2],[3],[4] and presented during many conferences [5],[6]. Therefore we will reduce our presentation, firstly to show the fundamental theoretical idea which is the basis of a new method of solving nonlinear problems of acoustics and the design of the mentioned solver. Secondly to compare the computation results which were obtained by means of the old method (OLD METHOD) and the new one (NEW METHOD).

For axially symmetrical disturbances (2D + t) Fig.1, generated by circular sources, the above mentioned problems can be and are solved by means of codes used in PC environments on the basis of methods known since many years [7]. In such a case the expanding power of processors have a distinct effect on the performance of these solvers. However even then it can occur that the computation takes a long time of many hours. Description of the fields generated by sources of arbitrary shape, particularly like arrays, requires - in respect to the fields generated by circular sources - to introduce an additional spatial dimension; it means that they are really 3D+time see Fig.1. As a result even in the trivial case the size of the set of samples representing the generated field increases no less than two orders of magnitude in respect to axially symmetrical disturbances. It causes qualitatively different requirements for solving problems of this type. The method used up to now for circular sources are for array not sufficient – excluding the stimulation by means of continuous waves.



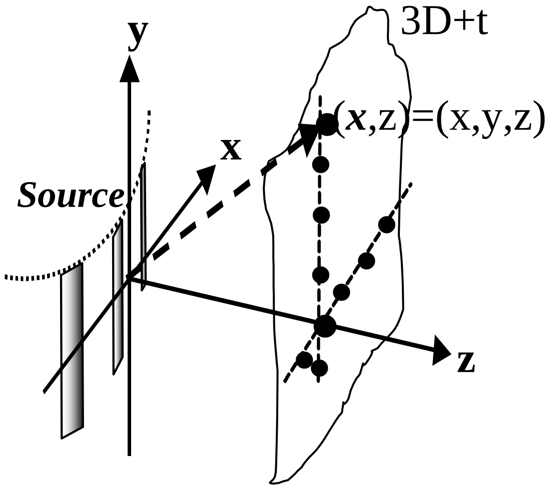


Fig.1. One-dimensional (left) and two dimensional, one-side boundary values problems.

1. NONLINEAR PROPAGATION MODEL

1.1.General description

In the dimensionless system of independent and dependent variables with the retarded time the equation describing propagation of acoustical disturbances in the lossy and nonlinear medium has the following shape

$$\Delta \mathrm{P} - 2\partial_{\tau z}\mathrm{P} - 2\partial_{\tau}\,\mathbf{A}\,\mathrm{P} = -q\partial_{\tau\tau}\mathrm{P}^2 \qquad (1)$$

where: $\Delta \equiv \nabla \cdot \nabla$ Laplace operator; ∇ - operator of gradient; $(\mathbf{x}, z)$ - dimensionless coordinates in space, $\mathbf{x} \equiv (x, y)$ - in the Cartesian coordinate system, $\mathbf{x} \equiv r$ - in cylindrical coordinate system for axially symmetrical problems (in respect to the axis 0–z); $\tau \equiv (t - z)$ – dimensionless retarded time, t– dimensionless time; $\mathrm{P} = \mathrm{P}(\mathbf{x}, z, \tau)$ – dimensionless pressure; A - operator of the convolution type, describing dispersion (absorption), in the time representation

$\mathrm{AP} \equiv A \underset{\tau}{\otimes} P$, $A = A(\tau)$-kernel of the operator A (for more details see [8],[9]). The following normalization in Eq.(1) were used:

$$\tau \equiv \Omega_0 \tau' = \Omega_0 \left(t' - z'/c_0\right), \quad (\mathbf{x}, z) \equiv K_0 \cdot (\mathbf{x}', z'), \quad \partial_\tau \equiv \frac{1}{\Omega_0} \partial_{\tau'}, \quad \nabla \equiv \frac{1}{K_0} \nabla',$$

$$\mathrm{P} \equiv \mathrm{P}'/\mathrm{P}_0, \quad q \equiv P_0 \cdot (\gamma + 1) / \left(2 \rho_0 c_0^2\right), \quad K_0 c_0 = \Omega_0. \tag{2}$$

where: $\mathrm{P}'(\mathbf{x}', z', \tau')$ - pressure; $(\mathbf{x}', z', \tau')$ – dimensional coordinates; P_0-characteristic pressure; ρ_0, c_0 – are equilibrium density and sound speed, respectively, γ - exponent of the adiabate or $\gamma \equiv (B/A) + 1$ - parameter of nonlinearity. Imposing the relation $K_0 c_0 = \Omega_0$ for the values of K_0 and Ω_0, which normalize space and time means the acceptance of a common measure for the distance in space and in time – in radians.

Equation (1) should become complete by boundary conditions. In both geometrical cases those conditions can be created as an effect of considerations or a result of measurements carried out near to the source of the disturbance. For circular sources we assume the apodisation in amplitude and in phase along the axis *r*. For non circular sources we suppose the spatial distribution of the orientation of transmitting elements (plane or not plane) and individual amplitude-phase apodisation. In both cases the time excitation can be changed from short pulses up to the continuous waves.

1.2. OLD and NEW methods

It is assumed that the Eq.(1) possesses the only one solution. Of course every function can be represented in many ways. Therefore we suppose that the solution P of Eq.(1) can be represented by the series R^{o} which is defined by the right side of the formula (4) or by the series R given be Eq.(5)

$$\mathrm{P} = \tfrac{1}{2}(R^o + R^{o^*}), \text{ or } \mathrm{P} = \tfrac{1}{2}(R + R^*), \tag{3}$$

$$R^o(\mathbf{x}, z, \tau) \equiv \sum_{n=1}^{N} C_n(\mathbf{x}, z) e^{-i \cdot n \cdot \tau} + o^{N+1}, \tag{4}$$

$$R(\mathbf{x}, z, \tau) \equiv \sum_{m=1}^{M} P_m(\mathbf{x}, z, \tau) e^{-i \cdot m \cdot Nc \cdot \tau} + o^{M+1}. \tag{5}$$

where: $\{C_n\}, n = 1,..., N$ - Fourier spectrum R^o; N – effective dimension of the representation R^o; Theoretically $N = \infty$, then $o^{N+1} \equiv 0$; $\{P_m\}, m = 1,..., M$ - quasi spectrum; M – effective dimension of the representation *R*. Theoretically $M = \infty$, then $o^{M+1} \equiv 0$; Nc - dimensionless filling frequency of the boundary pulse (carrier frequency), number of cycles with the period of $1/fc'$ contained in the window.

Now – we will explain what we mean here by terms “OLD” and “NEW” method or “OLD”, “NEW” representation.

The black bold plots below and above on Fig.2 represent typical pulse time shape and the envelope of their Fourier spectrum obtained from calculations or measurements under conditions of the nonlinear propagation.

The formula (4) represents the decomposition of the disturbance into Fourier series in a given point in space- that means on superposition sin waves unbounded in time. Vertical lines on Fig.2 correspond to coefficients C_n of this decomposition. They are calculated after the substitution of this formula into the nonlinear wave propagation equation (1). And the procedure based on representation (4) we call here as OLD method. This is standard representation used from dozen years especially for description and numerical calculations of the axis symmetrical nonlinear acoustical field propagation ($C_n(\mathbf{x},z)=C_n(r,z)$ see [6]).

By NEW method we mean the treatment which is based on the formula (5). In this case the disturbance is presented as the superposition sin waves of bounded in time (we may say wavelets) $P_m\cdot\exp(-i\cdot Nm\cdot\tau)$ with carrier frequencies being harmonics in respect to the fundamental carrier Nc, $Nm=m\cdot Nc$. Middle part of Fig.2 illustrate symbolically this idea. The series (5) can be interpreted as the quasi Fourier (because $P_m(\mathbf{x},z,\tau)$ depends on time however slowly in respect to $\exp(-im\cdot Nc\cdot\tau)$) superposition of pulses with the envelopes of P_m and carrier frequencies of Nm. The assumed shape of the solution (5) is a mathematical formalization of a observation series of spectra of nonlinear disturbances which are obtained in numerical simulations, observed experimentally, and also after analyzing the influence of the nonlinear term on the disturbance.

The Fourier spectrum of the disturbance P, $f\mathrm{P}(\mathbf{x},z,n)\equiv F\left[\mathrm{P}(\mathbf{x},z,\tau)\right]$ can be interpreted as the superposition of wavelets Fourier spectra

$$fR(\mathbf{x},z,n)=\sum_{m=1}^{M} fP_m(\mathbf{x},z,n-mNc),\qquad fP_m(\mathbf{x},z,w)\equiv F\left[P_m(\mathbf{x},z,\tau)\right]. \tag{6}$$

The spectral structure of (6) was shown schematically in Fig.2 by separation of components $fP_m(...,n-mNc)$ of the sum (6). In Fig.2 it was shown that the successive components of the sum can of course "overlap – intersect " themselves more or less depending on the relative band width fP_m (calculated in relation to Nc) it means depending on the duration time and on the shape of the envelope P_m of the wavelet. Although the frequencies $n_m=mNc$ are distinguished in the notation (5) and (6) it does not mean that they are the coordinates of local spectral maxima. It results from the experiment, numerical calculation and theoretical considerations that there can exist and exists a small shift of the frequency dn_m of the local spectral maximum to the position $n_{\max}=n_m+dn_m$ caused by the geometry of the source (especially in the case of an array) by dispersion (absorption) in the medium and quasi dispersion. One should stress that the notations (5),(6) do not contain limitations excluding the above described and showed in Fig.2 phenomena, although it clearly accentuated the fundamental sound and its harmonics . In the middle part of the figure there are shown schematically reconstruction of the wavelets $P_m(\mathbf{x},z,\tau)\exp(-imNc\,\tau)$ from their spectra $fP_m(\mathbf{x},z,n-mNc)$ and in bottom the reconstruction of disturbances from the wavelets.

Strictly NEW representation together with nonlinear wave equation produces several methods of different rank of accuracy and complication which permit to determine P_m. It is an interesting however, very wide problem. Therefore in this paper we limit our presentation to the results obtained by means of the simplest method results from (1),(3),(5) .

If R^o and R are solutions of the same boundary problem then of course $R^o-R=0$. For R^o and R obtained numerically it can be only $R^o-R=\vartheta$. However, it can be easily shown that for the acceptable differences of ϑ the relationship between effective dimensions of the disturbance representation (4),(5) is the following $N=M\cdot Nc$.

Cost of calculations of the nonlinear interaction is proportional to P^2 i.e. depends on the actual dimension (N or M) of the representation of P. That means; approximate relation between costs of the NEW and OLD methods is proportional to $1/Nc^2$, memory reservation to $1/Nc$ (for methods presented in this paper)

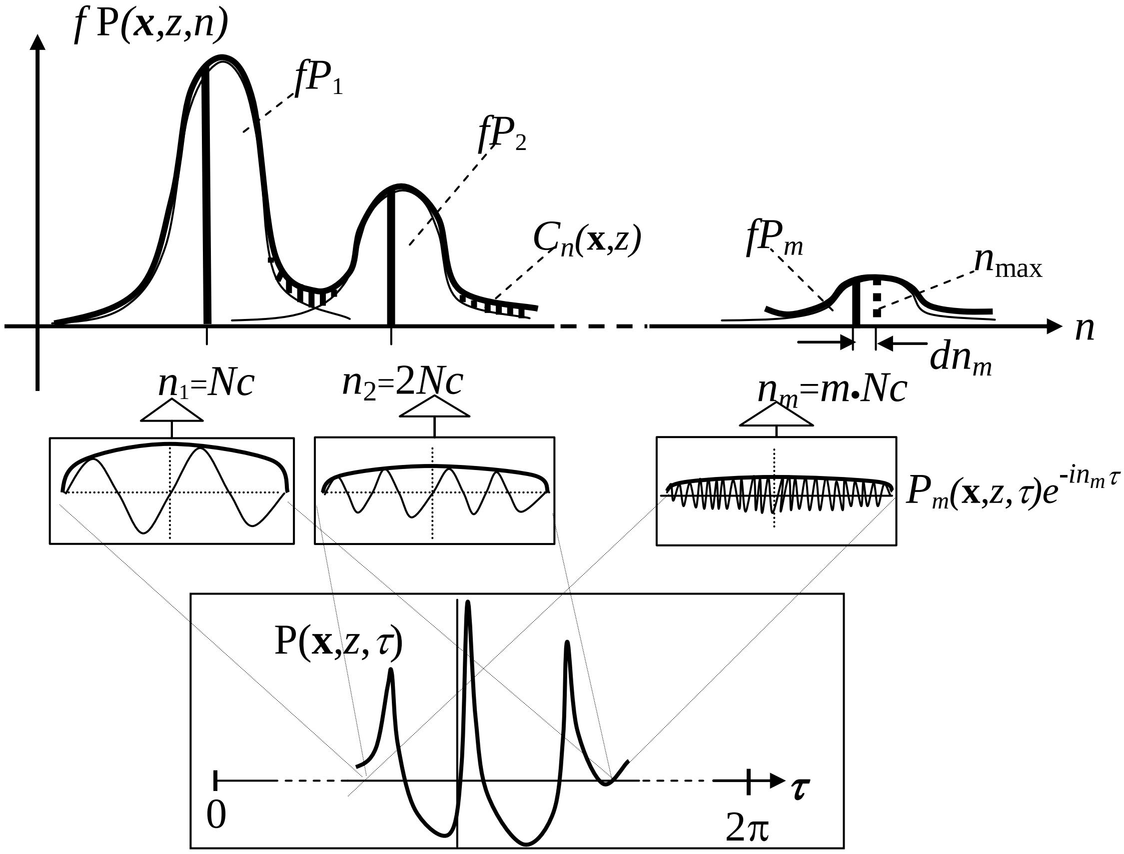


Fig.2. Decomposition of the $\mathrm{P}=\mathrm{Re}(R(\mathbf{x},z,\tau))$, throughout wavelets $P_m(\mathbf{x},z,\tau)\exp(-imNc\,\tau)$, their Fourier spectra $fP_m(\mathbf{x},z,n-mNc)$, in to full spectrum $fR(\mathbf{x},z,\tau)$ and back.

2. COMPARISION NEW/OLD

We show below in common plots the results of numeral solutions of Eq.(1) by means of the method using representations (4) –OLD METHOD and representations (5) – NEW METHOD. Solved was an axially symmetric problem (Fig.1 – left). As the source a circular transducer 30 mm in diameter, with the geometrical focus 85 mm, excited by pulses with rectangular envelope of the frequency 2 MHz with the pressure amplitude Po = 0.25 MPa was used. A uniform apodisation along the axis r was assumed. Two cases were calculated; a short 2 cycles excitation time (Fig.3) and long excitation time of 8 cycles (Fig.4).

The comparison of the two methods, for the case of boundary conditions leading to the problems of 3 D+t (Fig.1 – right), was in our case impossible as explained in the introduction. It should be noted that the requirements of the numerical code, which is based on the NEW method, are much higher in its simplest version for calculations of circular sources than for sources the array type. It results, paradoxically, from the ordering of the field (higher symmetry for the circular source). So, if the numerical code operates successfully for circular sources, it works for other sources too.

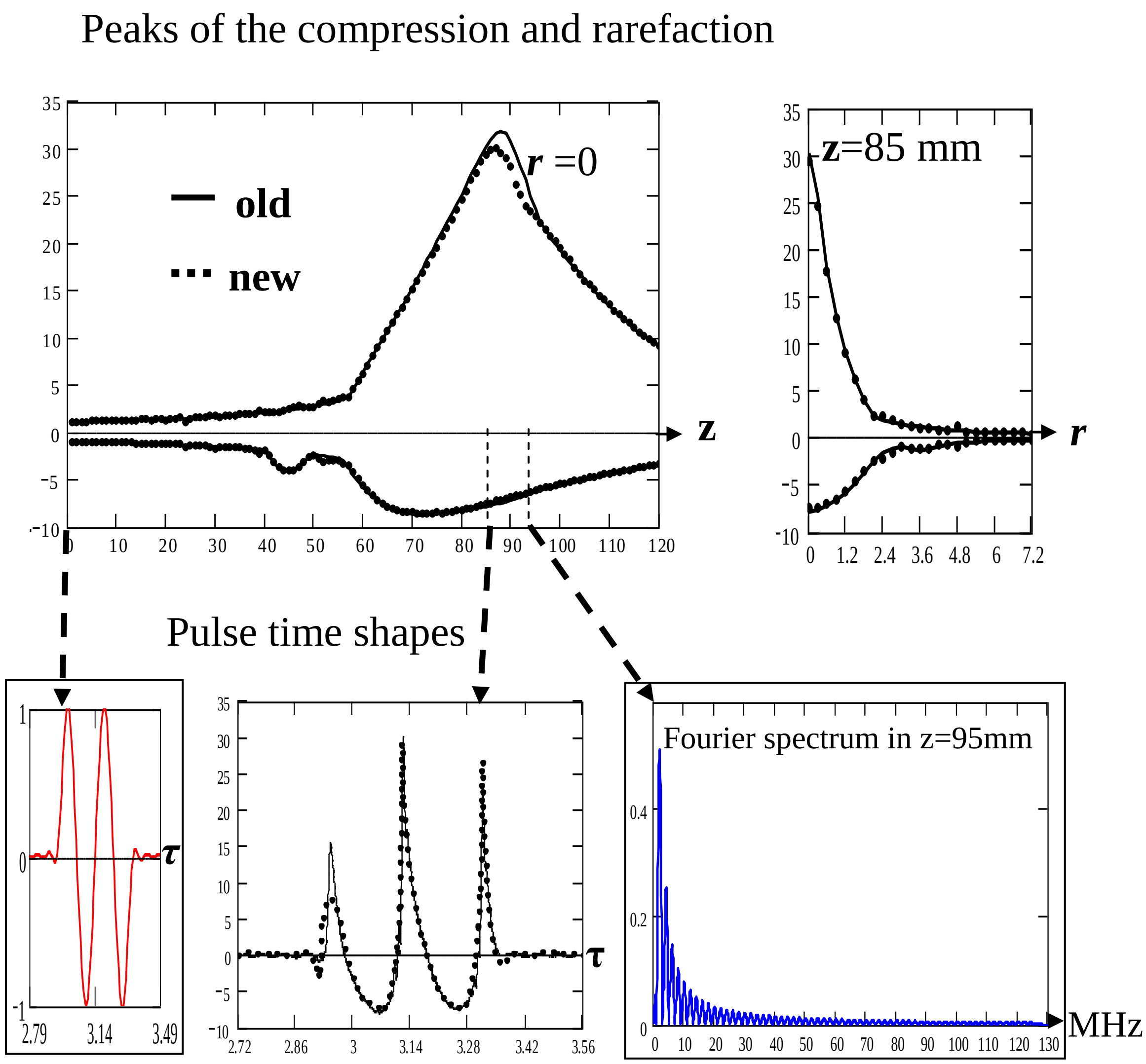


Fig.3. Comparison NEW-OLD methods. Short time excitation- two cycles.

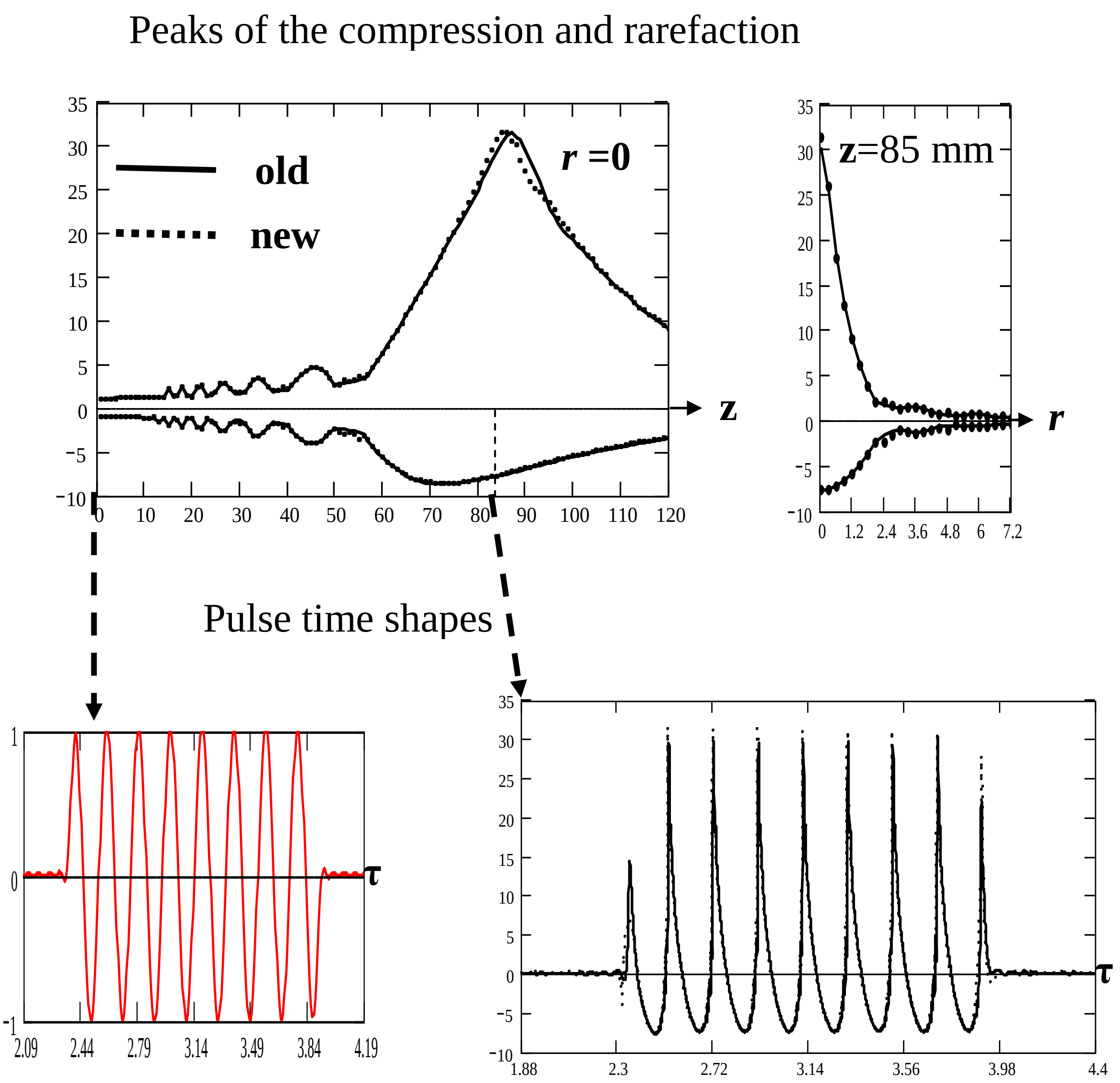


Fig.4. Comparison NEW-OLD methods. Long time excitation- eight cycles.

Time of the calculations for OLD method-6 hours; for NEW method-18 minutes.

3. SOME RESULTS FOR ARRAY

In Figs.5,6 we present the spatial distributions corresponding to the carrier and next two harmonics (in respect to the carrier) of the 20^{o} deflected acoustic beam generated by 128 elements linear array with pitch 0.2mm and the elevation of 6 mm. The transmitted pulse is shown in Fig.3. Carrier frequency was equal to 7.5 MHz.

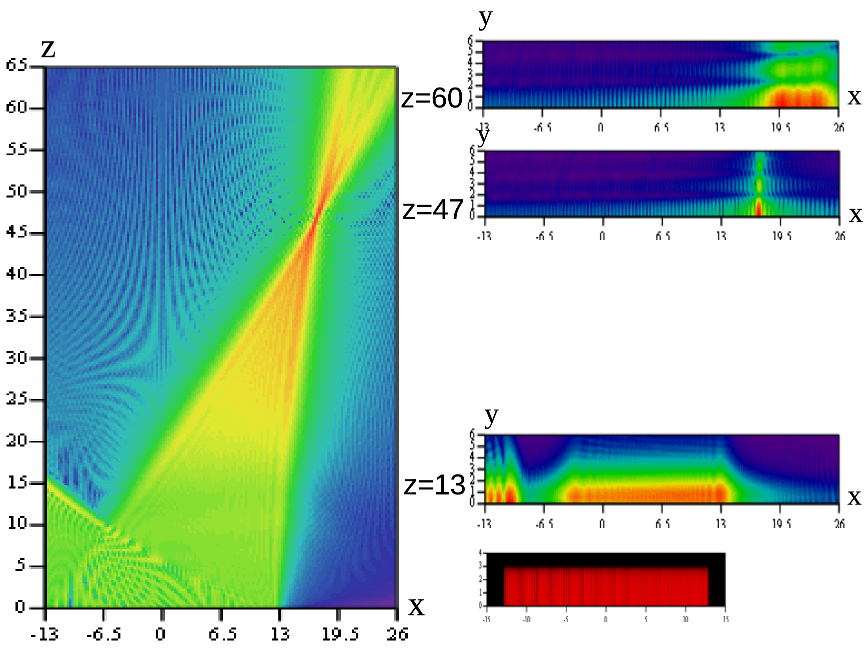


Fig.5. Spatial distributions of the acoustic beam corresponding to the carrier frequency 7.5 MHz. Spatial scales in mm. Logarithmic value scale.

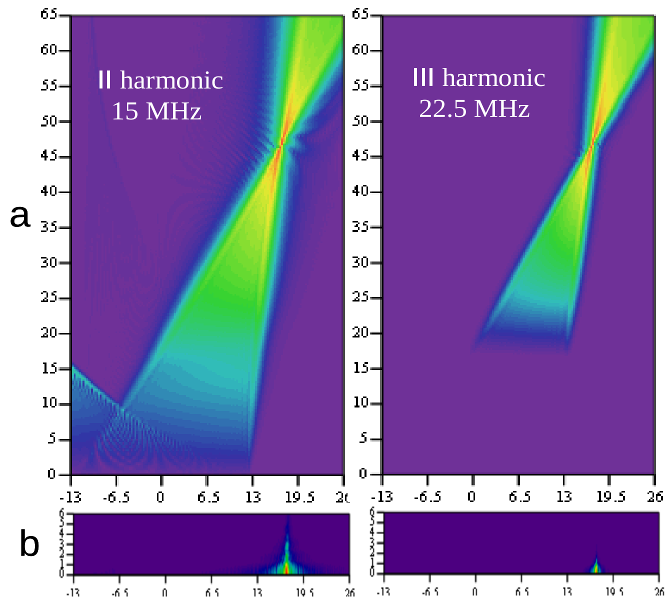


Fig.6. Spatial distributions of the second and third harmonics: a) in z-x plane b) in focal x-y plane z=47mm. Spatial scales in mm. Logarithmic value scale.

4. CONCLUSION

At the begin we noted that the OLD method is used in our considerations as the reference method. The obtained results show a satisfactory agreement in amplitude and an excellent in phases, independently of the pulse duration. Differences between them are visible in the small time scale, it means that they are formed at very high frequencies. One can observe that the agreement rapidly increases with the distance from the beam axis.

ACKNOWLEDGMENTS

This work has been supported by the Ministry of Science and Higher Education, Poland (Grant No. N N518 503339).

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