Application the GPS Observations in SPP Method for Aircraft Positioning in Flight Experiment in Dęblin, Poland (01.06.2010)

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Abstract:
In this article, the results of GPS positioning in civil aviation are presented. The flight test was conducted using Cessna 172 aircraft in Dęblin on 1st of June 2010. The aircraft position was determinated using Single Point Positioning method for GPS code observations. The numerical computations were executed in Aircraft Positioning Software (APS) in Scilab 5.4.1 language. The average accuracy of aircraft position is higher than 11 m in horizontal coordinates and about 13 m in vertical plane, respectively.

Keywords: *GPS, SPP method, HPL, VPL, least square estimation*

1. Introduction

Since few years the development of GNSS technique in precise air navigation is visible in Poland. Especially, the GNSS technique is implemented in air navigation to improve the aircraft position in real time or post-processing. The implementation of GNSS sensor is focused on applied of SBAS data (e. g. EGNOS correction) in civil aviation [2, 3, 5, 6]. The EGNOS system is utilized in civil aviation for non-precision approach (NPA) or approach (APV-I) to landing procedure [7]. The typical accuracy of NPA procedure is equal to 220 m in horizontal plane whereas NPA procedure for vertical plane is not available. In case of the APV-I procedure, the accuracy of aircraft position amounts to 16 m for horizontal plane and 20 m for vertical plane [8]. The requirement of EGNOS system is also concerned to integrity, time of alarm, continuity and availability parameters [1]. Only two parameters (accuracy and availability) are suitable in aviation procedures for the GPS system. Average accuracy of aircraft position in GPS system is recommended by ICAO annex and it is equals to 9 m for horizontal plane and 15 m for vertical plane respectively. The critical accuracy with probability 95% can reach up to 17 m for horizontal plane and 37 m for vertical plane respectively. The availability parameter should be more than 99% for all air operations for whole area of the Earth [10].

In this paper, the accuracy results of aircraft positioning in GPS system are presented. The flight experiment was conducted on 1st of June 2010 in Dęblin, Poland using Cessna 172 aircraft. The aircraft position was recovery based on Single Point Positioning (SPP) method for GPS code observations. The computations of aircraft position was executed in Aircraft Positioning Software (APS), which source code was written in Scilab 5.4.1 language. The final results of aircraft position were compared with accuracy of NPA and APV-I procedures. The structure of article was divided into 5 sections: introduction, methodology of research, experiment and results, discussion and conclusions.

2. Mathematical Model for Designation the Aircraft's Position Using GPS Observations

The SPP method is applied in standalone positioning in geodesy and navigation also. The basic equation of SPP method is described as below [17]:

$$l = d + C \cdot (dtr - dts) + Ion + Trop + Rel + TGD + \\ + RDCB_{L1} + M_{C/A} \quad (1)$$

where:
l – the pseudorange value (C/A code) at 1st frequency in GPS system,
d – the geometric distance between satellite and receiver; include information about: the Earth rotation, the Sagnac effect, the Satellite and Receiver Phase Center Offset, time of pseudorange travelling through atmosphere,

$$d = \sqrt{(x - X_{GPS})^2 + (y - Y_{GPS})^2 + (z - Z_{GPS})^2},$$

(x, y, z) – aircraft's coordinates in ECEF frame,
$(X_{GPS}, Y_{GPS}, Z_{GPS})$ – GPS satellite coordinates,
C – speed of light,
dtr – receiver clock bias,
dts – satellite clock bias,
Ion – ionosphere delay,
$Trop$ – troposphere delay,
Rel – relativistic effect,
TGD – Time Group Delay,
$RDCB_{L1}$ – Receiver Differential Code Bias, referenced to L1 frequency,
$M_{C/A}$ – multipath effect.

The unknown parameters (e.g. aircraft's coordinates and receiver clock bias) from equation (1) are estimated using least square solution in stochastic processing, as follows [14]:

$$\begin{cases} \mathbf{Qx} = \mathbf{N}^{-1} \cdot \mathbf{L} \\ \mathbf{v} = \mathbf{A} \cdot \mathbf{Qx} - \mathbf{dl} \\ m0_{post} = \sqrt{\dfrac{[\mathbf{pvv}]}{n-k}} \\ \mathbf{C_{Qx}} = m0_{post}^2 \cdot \mathbf{N}^{-1} \\ \mathbf{m_{Qx}} = diag\left(\sqrt{\mathbf{C_{Qx}}}\right) \end{cases} \quad (2)$$

Qx – vector with unknown parameters,
$\mathbf{N} = \mathbf{A}^T \cdot \mathbf{p} \cdot \mathbf{A}$ – matrix of normal equation frame,
A – full rank matrix,
p – matrix of weights,

$$\mathbf{p} = \frac{1}{m0^2_{priori} \cdot \mathbf{ml}^2} \quad [19],$$

$m0_{priori}$ – standard error of unit weight a priori,
$m0_{priori} = 1$,
ml – accuracy of pseudorange,

$$\mathbf{ml} = \frac{ml_0}{\sin(El)} \; [18],$$

ml_0 – standard deviation of code C/A in GPS system,
$ml_0 = 3\ m$ [20],
EL – elevation angle,
$\mathbf{L} = \mathbf{A}^T \cdot \mathbf{p} \cdot \mathbf{dl}$ – misclosure vector,
dl – vector include difference between observations and modeled parameters,
$m0_{post}$ – standard error of unit weight a posteriori,
n – number of observations,
n>4, for each measurement epoch,
k – number of unknown parameters,
k = 4, for each measurement epoch,
V – vector of residuals,
$\mathbf{C_{Qx}}$ – covariance matrix,
$\mathbf{m_{Qx}}$ – standard deviations for unknown parameters, parameter $\mathbf{m_{Qx}}$ is referenced to ECEF frame.

The parameters $\mathbf{m_{Qx}}$ can be expressed in geodetic frame BLh, as below [15]:

$$\begin{cases} mB = \sqrt{\mathbf{m_{BLh}}(1,1)} \\ mL = \sqrt{\mathbf{m_{BLh}}(2,2)} \\ mh = \sqrt{\mathbf{m_{BLh}}(3,3)} \end{cases} \tag{3}$$

where:
$\mathbf{m_{BLh}}$ – covariance matrix in geodetic frame (BLh),
$\mathbf{m_{BLh}} = \mathbf{R} \cdot \mathbf{m_{Qx}} \cdot \mathbf{R}^T$,
R – transition matrix from geocentric (XYZ) to geodetic frame (BLh),
mB – standard deviation of Latitude,
mL – standard deviation of Longitude,
mh – standard deviation of ellipsoidal height.

The mathematical scheme in equation (2) is solved in iterative procedure in adjustment processing for each measurement epoch. In addition, the results from equation (2) are checked and controlled using global test $\chi^2_{f,1-\alpha}$, as follows [19]:

$$\sum pvv = m0^2_{post} \cdot (n-k) < \chi^2_{f,1-\alpha} \tag{4}$$

where:
f – number of freedom degrees, $f = n - k$,
$(1 - \alpha)$ – significance level, $\alpha = 0.05$.

If $m0_{post}$ is much higher than 1, then blunder errors from pseudoranges are detected and removed. In connection with this criterion, the adjustment processing of GPS observations is executed again, as in equation (2).

3. The Experiment and Results

The mathematical formulations from section (2) were utilized for determination of aircraft position in flight experiment on 1st June 2010 in Dęblin (see Figs. 1 and 2). The aircraft trajectory was recovery using GPS code observations from dual-frequency Topcon HiperPro receiver [4]. Raw GPS observations in RINEX file were collected in memory disc in the receiver which was installed in pilot's cabin in Cessna 172 aircraft. The time of flight test was equal to 3361 measurements epochs with interval of 1 second.

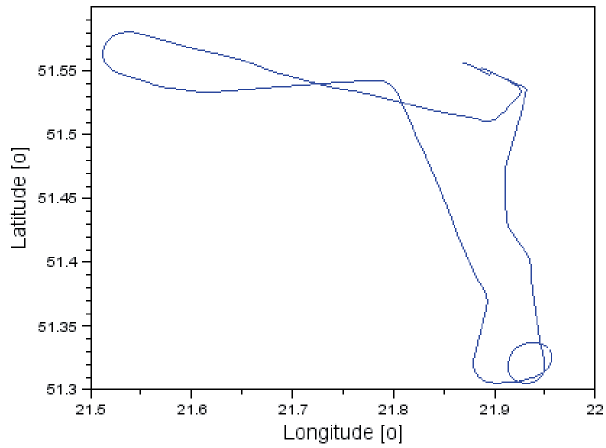

Fig. 1. The horizontal trajectory of Cessna 172 aircraft

The aircraft's coordinates were calculated in Aircraft Positioning Software (APS) in Scilab 5.4.1 language. The APS program can be applied in post-processing mode for determination of aircraft position using GNSS data. Currently, the GPS, GLONASS and GPS/GLONASS observations are implemented for SPP (Single Point Positioning) module, IF LC (Ionosphere-Free linear combination) module, SD-BS (Single Difference Between-Satellites) module and Doppler module in APS program.

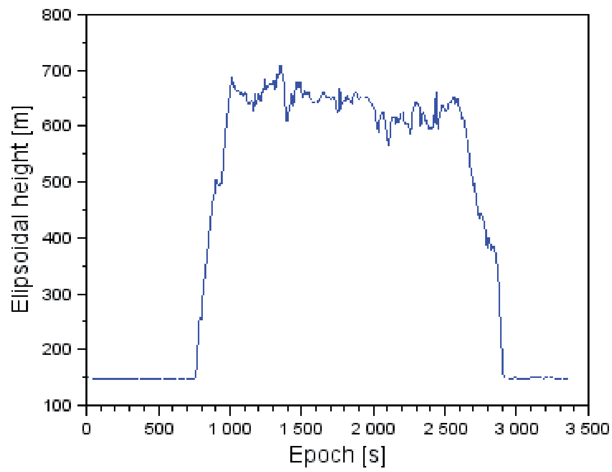

Fig. 2. The vertical trajectory of Cessna 172 aircraft

In this paper, the SPP module in APS program was applied to obtained aircraft position in flight experiment in Dęblin. The basic parameters and input models of SPP module was configurated as below:

- GNSS system: GPS system,
- type of observations: C/A code at 1st frequency,
- type of RINEX file: 2.10,
- source of satellite ephemeris data: precise ephemeris from the CODE Analysis Center [22],
- source of satellite clock data: precise ephemeris from the CODE Analysis Center [22],
- method of satellite position computation: 9-degrees Lagrange polynomial [14],
- method of satellite clock bias computation: 9-degrees Lagrange polynomial [14],
- satellite clock bias correction: satellite clock bias from precise ephemeris is corrected using Differential Code Biases in SPP method [18],
- effect of Earth rotation and time of pseudorange travelling through atmosphere: applied,
- relativistic effect: applied [21],
- ionosphere source: Klobuchar model [12],
- troposphere source: Simple model [13],
- Time Group Delays: applied [16],
- instrumental bias $RDCB_{L1}$: not applied,
- multipath and measurement noise: not applied,
- satellite and receiver phase center offset: based on ANTEX file from IGS service,
- Sagnac effect: applied [14],
- cutoff elevation: 5° [10],
- positioning mode: kinematic,
- mathematical model of solution: least square estimation in iterative scheme,
- adjustment processing: applied,
- maximum number of iteration in single measurement epoch: N=10,
- number of unknown parameters: k=4, for each measurement epoch,
- number of observations: n>4, for each measurement epoch,
- interval of computations: 1 s,
- initial coordinates of aircraft position: based on header of RINEX file,
- receiver clock bias: estimated,
- time of GNSS system: GPS Time,
- reference frame: IGS'08,
- statistical test: test Chi-square,
- value of $m0_{post}$ after adjustment processing: $m0_{post} \in (0.9 : 1.1)$,
- significance level: $(1 - \alpha) = 0.95$,
- maximum value of DOP coefficients: DOP = 6.
- coefficients value for HPL and VPL level: $k_{HPL} = 6$ and $k_{VPL} = 5.33$ [9].

Figure 3 presents values of $m0_{post}$ and $\chi^2_{f,1-\alpha}$ parameters as a final results of statistical test Chi-square. The mean value of $m0_{post}$ equals to 1.002, with range between 0.915 and 1.099. The term $m0_{post}$ is less than $\chi^2_{f,1-\alpha}$ parameter for all measurement epochs and it can be concluded that Chi-square test was obtained in the experiment.

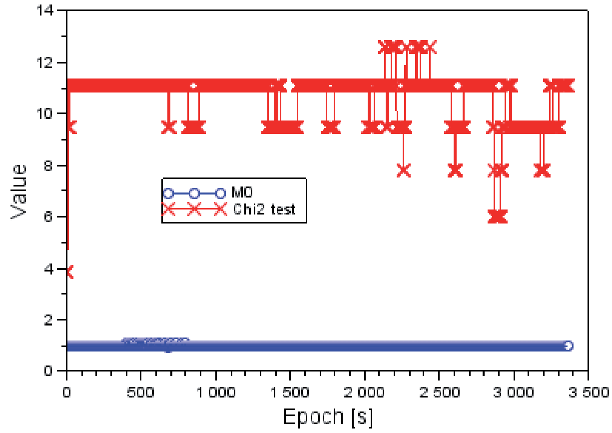

Fig. 3. The values of statistical test Chi-square

Figure 4 presents PDOP and GDOP values for each measurement epoch in flight test in Dęblin. The PDOP parameter is a function of position errors only but GDOP term includes error of receiver clock bias as well. The mean value of PDOP term amount to 1.7 with range between 1.3 and 4.0. In case of GDOP term, the mean value equals 2.0 with range between 1.4 and 4.6. The minimum value of GDOP and PDOP parameters is available if number of GPS satellites reach up to 10. The maximum value of GDOP and PDOP parameters is available if number of GPS satellites decreases to 5. It should be noticed that both values of PDOP and GDOP terms are less than maximum value of DOP coefficient (e.g. DOP = 6).

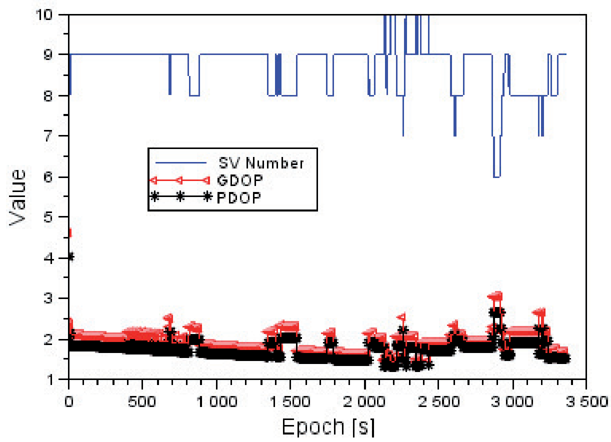

Fig. 4. The values of DOP parameters

Figure 5 presents the accuracy (e.g. standard deviation parameter) values of receiver clock bias for each measurement epoch. The standard deviation of receiver clock bias was calculated as in equation (5) [14]:

$$m_{dtr} = \sqrt{\mathbf{C}_{\mathbf{Qx}}(4,4)} \tag{5}$$

The average accuracy of receiver clock bias is about 38.8 ns (in meter unit: 11.6 m) with range between 3.9 ns and 97.6 ns (in meter unit: 1.1 m and 29.3 m). The accuracy of receiver clock bias is irregularly but still growing up almost to 100 ns.

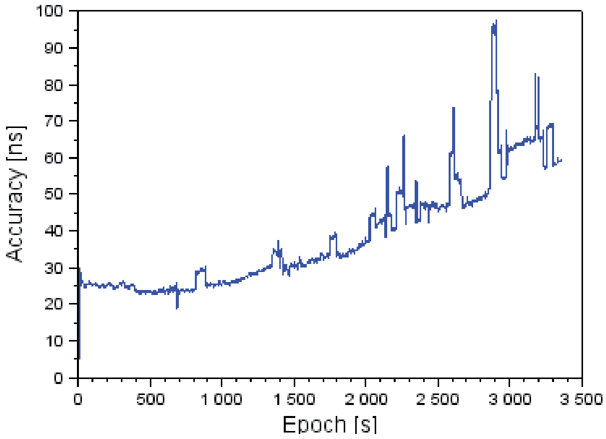

Fig. 5. The standard deviation of receiver clock bias

Figure 6 presents the accuracy (e. g. standard deviation term) values of aircraft position in geodetic frame BLh for each measurement epoch. The average value of Latitude accuracy equals to 10.5 m with range between 1.0 m and 24.6 m. The average value of Longitude accuracy equals to 6.5 m with range between 0.5 m and 14.2 m. The average accuracy of each horizontal coordinates is higher than accuracy of ICAO standard (e. g. 17 m). The average accuracy of ellipsoidal height amounts to 13.1 m with magnitude order between 1.8 m and 31.9 m. The average accuracy of ellipsoidal height for Cessna 172 aircraft is higher than ICAO standard in vertical plane (e.g. 37 m).

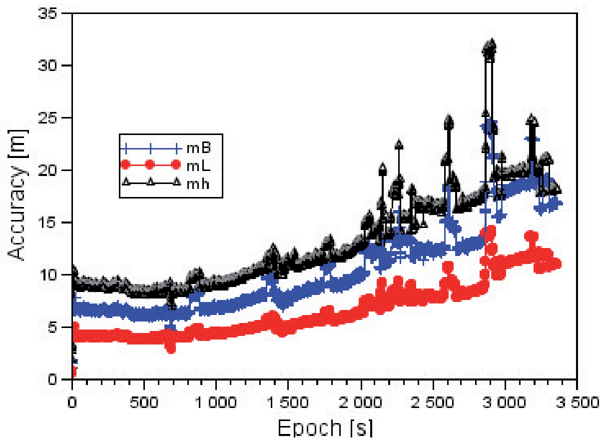

Fig. 6. The standard deviations of aircraft's position in geodetic frame BLh

4. Discussion

In section (4), the accuracy of aircraft' coordinates was verified with accuracy of NPA and APV-I procedures. The NPA and APV-I procedures are implemented in Polish aviation based on SoL (Safety of Life) service in EGNOS system. The majority of accuracy parameters in NPA and APV-I procedures are called HPL and VPL terms. The HPL and VPL parameters are calculated using formula (6) [11]:

$$\begin{cases} HPL = k_{HPL} \cdot \sqrt{mB^2 + mL^2} \\ VPL = k_{VPL} \cdot mh \end{cases} \tag{6}$$

where:
k_{HPL} = 6 – for horizontal plane,
k_{VPL} = 5.33 – for vertical plane.

The values of k_{HP} and k_{VPL} parameters in equation (5) are referenced to landing approach APV-I in EGNOS system.

Table 1. The comparison of HPL/VPL value from GPS solution and NPA procedure

HPL/VPL parameter	Average accuracy of HPL/VPL term based on GPS observations	Theoretical accuracy of HPL/VPL term in NPA procedure
HPL value	HPL = 74.2 m	556 m
VPL value	VPL= 69.9 m	Not available

In Table 1, the HPL/VPL results from APS program were compared with NPA standards. The average value of HPL term equals to 74.2 m based on GPS observations in flight test in Dęblin. The results shows that the average value of HPL term is not exceeded the theoretical accuracy of NPA procedure (e. g. 556 m) in horizontal plane and the average value of VPL term equals to 69.9 m. The theoretical accuracy of NPA is still not active in vertical plane. In connection with it, the comparison of VPL value from GPS solution and NPA procedure is currently impossible. This test shows that the range of HPL parameter is between 6.9 m and 170.5 m, whereas the range of VPL parameter is between 9.7 m and 170.3 m, respectively.

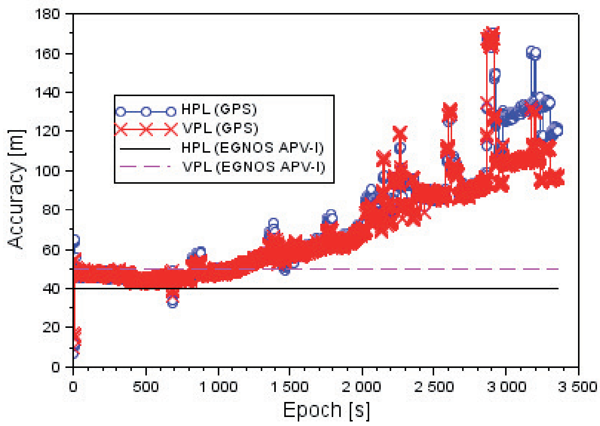

Fig. 7. The accuracy of HPL/VPL parameters in comparison to EGNOS APV-I standards

The values of HPL/VPL terms from APS program was also compared with EGNOS APV-I standards (see Fig. 7). The accuracy of HPL/VPL terms in EGNOS APV-I procedure are equal to 40 m in horizontal plane and 50 m in vertical plane respectively. The values of HPL term in flight test are much more than EGNOS APV-I standard for horizontal plane (e. g. about 99% results of all measurement epochs). In case of the VPL parameter, about 31% results (e. g. 1045 measurement epochs) from APS program is less than EGNOS APV-I standard for vertical plane. The values of HPL/VPL terms are still growing up for that experiment and this is a negative situation in context of safety in air operations. The results of HPL/VPL parameters in flight test in Dęblin are showed that implementation of GPS system in approach APV-I in air navigation is still limited.

5. Conclusions

In this paper, the results of GPS positioning in Polish aviation were presented. The flight test was conducted in military airport in Dęblin on 1st of June 2010 using Cessna 172 aircraft. The aircraft position was recovery based on SPP method for GPS code observations with time interval of 1 s. The raw GPS observations were collected in dual-frequency Topcon HiperPro receiver which was installed in pilot's cabin in Cessna 172 aircraft. The numerical computations of aircraft's coordinates were executed in Aircraft Positioning Software (APS) in Scilab 5.4.1 language. The least square estimation in adjustment processing of GPS observations in applied in SPP module in APS program. Numbers of parameters were used in this article, such as the aircraft's trajectory, standard error of unit weight a posteriori, test Chi-square, GDOP and PDOP, accuracy of aircraft position, accuracy of receiver clock bias, HPL/VPL which were presented in Figure 1 to 7. The results shows that the average accuracy of aircraft position is higher than theoretical accuracy of GPS system for ICAO standard. The values of HPL/VPL terms were compared with NPA and APV-I standards also. The average value of HPL term equals to 74.2 m and it is higher than accuracy of NPA procedure in horizontal plane. In case of the accuracy of EGNOS APV-I standards, the values of HPL parameters are much more than 40 m in horizontal plane. The average value of VPL term equals to 69.9 m and it can be only compared with EGNOS APV-I standards. The values of VPL term from 1045 measurement epochs are less than 50 m in vertical plane. Preliminary results analyzed in this paper, indicate that the GPS system must be still monitored in aspect of landing approach in EGNOS APV-I procedure.

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