SAFETY FORMAT FOR NON-LINEAR PSEUDO-STATIC RESPONSE OF THE RC-STRUCTURAL SYSTEMS IN ACCIDENTAL DESIGN SITUATIONS

Viktar Tur
professor, D. Sc., PhD, Department of Concrete Technology, Brest State Technical University, Bialystok University of Technology, profturvic@gmail.com

Andrei Tur
Ass. professor, PhD, Department of Building Structures, Brest State Technical University, aturphd@gmail.com

Abstract

The non-linear structural analysis is considered as a basic design procedure, which is used for checking of the structural robustness in accidental design situation. It is explained by following reason: a non-linear structural analysis based on realistic constitutive relations for basic variables (average values) makes possible a simulation of a real structural behavior. It should be pointed that, implementation of the non-linear structural analysis in design of concrete structures requires an alternate approach to safety verification. The paper presents a new approach to safety format for non-linear analysis of RC structures subjected to accidental loads

Keywords: non-linear analysis, progressive collapse, safety format, robustness.

Introduction

In accordance with EN 1992 [4] "*structural analysis shall be carried out using idealization of both the geometry and the behavior of the structure. The idealization selected shall be appropriate to the problem being considered*".

Common idealizations of the structural behavior under applied actions used for structural analysis are: (1) *linear elastic behavior;* (2) *linear elastic behavior with limited redistributions;* (3) *plastic behavior, including strut and ties models*; (4) ***non-linear*** *behavior*.

In resent years structural engineers try to use non-linear analysis while designing a new complex structural system as well as for checking of the existing structures.

Non-linear analysis (static and dynamic) is most widely used as a main computational tool for checking of robustness of the structural systems in accidental design situations (Accidental Limit States Checking).

As it was stated in [1], "*evaluations of the non-linear analysis is supported by rapid increase of computational power as well as new capabilities of the available tools for numerical simulations of structural performance*".

Non-linear analysis take into account the non-linear deformation properties of RC-sections, based on realistic constitutive relations ("$\sigma - \varepsilon$" for material properties) and makes possible a simulation of a real structural behavior. If reflects an integral response, where all local sections interact and therefore it requires an adequate approach for safety verification (note, that in partial safety factor (PSF) method [22] we assume a failure probabilities of separate materials, but do not evaluate the failure probability on the structural level). It should be underlined, that non-linear analysis offers a verification of ***global resistance*** and requires a safety format for global resistance [2]. In accordance with [2], the term ***global resistance*** (global safety) is used for "*assessment of structural response on higher structural level than a cross-section*". The term ***global resistance*** is introduced in [2] in order to distinguish the newly introduced check of safety on global level, as compared to local safety check in the partial safety factor method (PSF-method) in accordance with EN 1990 [22].

1. Pseudo-static response of the structural system with a removed vertical load bearing elements

As was stated in [24] prevent and mitigation of progressive collapse can be achieved using two different methods: (1) TF-method (indirect tie-force method); (2) AP – method (direct alternate path method). The indirect (TF - method) consists of improving the structural integrity of building by providing redundancy of load path and ductile detailing. Currently, the EN 1991-1-7, allows the use of indirect method

and some guidance is contained in the EN 1992-1-1. In this case criteria are devised to check the local resistance to withstand a specific postulated accidental load.

The direct method, referred to as “Alternate Load Path” (AP - method), is most widely used in the practical design and based on criteria for evaluating the capability of a damaged structure to bridge over or around the damaged volume of area without progressive collapse developing from the local damage. The AP-method consists in considering internal force (effect of the actions) redistributions throughout the structure following the loss of a vertical support element [26].

As was shown in [26], an AP-method analysis may be performed using of the following basic nonlinear procedures: Nonlinear Dynamic (**NLD**) and Nonlinear Static (**NLS**) procedures. In case of the Nonlinear Static procedure after materially-and-geometrically nonlinear model is built, the accidental load combination are magnified by a dynamic increase factor (**DIF**) that accounts for inertia effects and the resulting load is applied to model with removed vertical load bearing elements. If a dynamic increase factor (**DIF**) is known, for deformation-controlled actions, the resulting deformations are compared to the expected deformation capacities; for the force controlled action, the member strength is not modified and shell not be less than the maximum internal member forces (demands). Otherwise, calculation procedure based on the energetic approach should be used. The basic provisions of this procedure are described in detail in [27]. The purpose here is to analyze the structural response of RC-structural systems subjected to a sudden column loss.

The procedure, which is used for obtaining of the pseudo-static nonlinear response of the structural system, consists of the following main steps:

(1) Calculate the static non-linear response “$F-\delta$” for the modified structural system with a removed vertical load bearing element according to certain rules [27, 28] (see Figure 1, line 1);

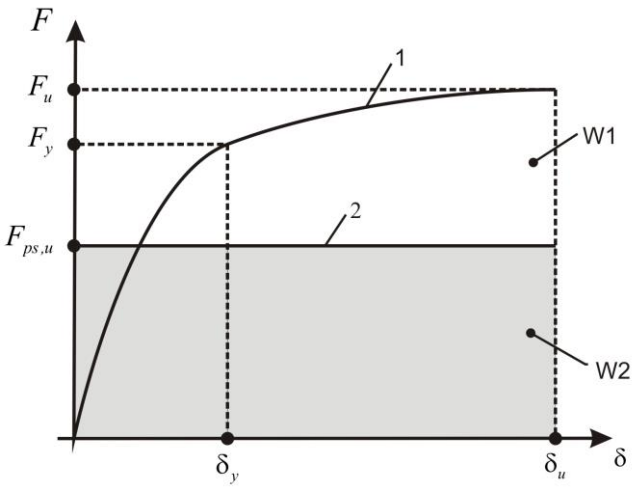
(2) Calculate the pseudo-static response, that taking into account inertia effects, caused by suddenly applied gravity load.

In general case, based on energetic consideration (see Figure 1):

$$\int_0^{\delta_u} P(\delta)d\delta = F_{ps,u}\cdot\delta_u \ , \tag{1}$$

Pseudo-static response is equal:

$$F_{ps,u} = \frac{1}{\delta_u}\int_0^{\delta_u} P(\delta)d\delta \, , \tag{2}$$



1- nonlinear response
2- pseudo-static response

$$W_1 \equiv W_2$$

$$W_1 = \int_0^{\delta_u} P(\delta)d\delta$$

$$W_2 = F_{ps,u}\cdot\delta_u$$

Figure 1 – To assessment of the pseudo-static response of the structural system in accordance with [27, 28]

2. Required level of reliability for accidental design situation

In general case, the probability of structure collapse due to postulated abnormal event can be written as:

$$P(F) = P(F|DH_i)P(D|H_i)P(H_i), \tag{3}$$

As was shown in [24], in a "*specific local resistance*" design strategy, the focus is on minimizing probability $P(F|DH_i)$, that is, to minimize the likelihood of initiation of damage that may lead to progressive collapse.

This strategy may be difficult or uneconomical, and may leave some significant hazards unaddressed.

Accordingly, it is likely that $P(D|H_i)$ will very close to 1,0 in many practical cases, meaning that the collapse probability becomes, approximately:

$$P(F) = P(F|DH_i)P(H_i), \tag{4}$$

It is in minimizing the conditional probability $P(F|H_i)$, that the science and art of the structural engineer becomes paramount [24].

It may be assumed that the occurrence of the abnormal event, H_i, can be modeled as a Poisson process, with yearly mean rate of occurrence, λ_i. The probability of occurrence of this abnormal event during some reference period *T*, is thus approximately $P(H_i) = \lambda_i T$ (for small λ_i) [24]. In the case of fire, gas explosion and some other accidental loads, parameter λ_i may be related to building floor area ($\lambda_i = p \cdot A_f$, in which A_f - floor area and $p = p_1 \cdot p_2$, where term p_1 - probability of occurrence of hazard per unit area and $p_2 < 1.0$ represents effect of warning and control systems).

Mean rates of occurrence for gas explosions, bomb explosions and vehicular collisions in accordance with [25] are approximately:

- gas explosions (per dwelling): $2x10^{-5}$/yr
- bomb explosions (per dwelling): $2x10^{-6}$/yr
- vehicular collisions (per dwelling): $6x10^{-4}$/yr
- full developed fire (per building): $5x10^{-8}$/yr

As it was shown in [24], to evaluate $P(F|DH_i)$, one must postulate a mathematical model, ***G(X)*** (state model), of the structural system based on principles of mechanics and supplemented, where possible, with experimental data (!). The load and resistance variables are expressed by vector ***X***. We must then determine the probability distribution of each variable and integrate the joint density function of ***X*** over that region of probability space where ***G(X)*** <0 to compute in accordance with EN 1990 conventional limit state probability. But, we must to remember that it is very difficult and complex way (especially for structural systems).

Alternatively, FORM – analysis [24] may be used to compute a conditional reliability index β defined as:

$$\beta = \frac{\mu_G}{\sigma_G}, \tag{5}$$

where μ_G and σ_G is mean and standard deviation of ***G(X)***.

According to *Ellingwood* [24], the reliability index is related to $P(F|DH_i)$ through:

$$\beta = \Phi^{-1}[P(F|DH_i)], \tag{6}$$

in which $\Phi^{-1}[P(F|DH_i)]$ is the percent-point function of the standard Normal probability distribution.

With $P(H_i) = \lambda_i T$, eq. (6) can be rewritten as:

$$\beta = \Phi^{-1}[P(F)/\lambda_i T], \tag{7}$$

As was shown in [24], the first-generation probability-based Limit State Design Criteria (such as, for example, EUROCODES) all are based, to varying degrees, on reliability of individual structural members and components.

However, to implement reliability-based design criteria against progressive collapse in practice sense, the limit state probability (or reliability index) must be evaluated for a **structural system** (!). In contrast to member reliability, this evaluation is difficult (!) even at the present state of art and with computational resources available [24, 27].

Assuming that an analysis of a damaged structure can be performed, an acceptable value of β upon which to base design for conditional limit states is suggested by eqn. (7).

As shown by *Ellingwood* [24], the probability of structural system failure is an order of magnitude less, depending on the redundancy in the system and the degree continuity between members.

For example, if $\lambda_i = 10^{-6}$ to 10^{-5}, than the conditional failure probability for the structural system should be on the order of $10^{-2}\ldots10^{-1}$, and the target value of reliability index β_{tag} should be the order of 1,5. Load and resistance criteria can be developed to be consistent with the reliability.

3. Safety format for nonlinear analysis

The historical review (from CEM MC78 to ***fib*** MC2010) of the non-linear safety format development was in detail described in [23].

With the implementation of the new ***fib*** MC2010 [9], a different perspective was placed on non-linear analysis and safety assessment. The design condition to be used in safety format for non-linear analysis is written in the external actions and resisting internal forces domain:

$$E_d \le R_d\,, \tag{8}$$

where E_d is the design value of the action (effect of the actions) and

R_d is the design value of resistance.

Three different approaches are proposed to evaluate the design resistance R_d (depending on various levels of implementations of probabilistic theory): (1) full probabilistic method, recommended by *JCSS* as a basic method; (2) the global resistance method; and (3) the partial factor method (*PSF*-method).

In the global resistance format, the resistance is considered on a global structural level. Two alternative methods are mentioned in ***fib*** MC2010 [9] for the derivation of the design resistance R_d : (1) global resistance factor method (which was adopted from EN 1992-2, slightly modified); and (2) *ECOV*-method, proposed by Cervenka [2, 6] (estimations of coefficient variation for resistance).

In this case, the safety margin can be expressed by the global safety factor as:

$$R_d = \frac{R_m}{\gamma_R}\,, \tag{9}$$

where R_m is the mean resistance.

The global safety factor γ_R cover all uncertainties and can be related to the coefficient of variations of resistance V_R (according a LN- distributions (!) according EN 1992-2[4]) as $\gamma_R = exp(\alpha_R \beta V_R)$.

A simplified formulation was proposed in ***fib*** MC2010, where in denominator of the right hand side in eq. (9) is product of two factors $\gamma_R = \gamma_m \cdot \gamma_{Rd}$ [6]. The first factor γ_m is related to material uncertainty and can be established by probabilistic analysis. The second factor γ_{Rd} is related to model and geometrical uncertainties and recommended value are in range 1.05…1.1 only (!) (as suggested by EN 1992-2 [4]).

As it was stated in [23], after the new ***fib*** MC 2010 [6], although the topic is still controversial, only few contributions were found in literature [3, 8].

The first contribution was presented by Schlune et al [3]. Design resistance R_d is then derived by division of the obtained mean resistance R_m by global resistance factor γ_R :

$$R_d = \frac{R(f_{ym}, f_{cm}, a_{nom})}{\gamma_R}\,, \tag{10}$$

where $\gamma_R = \dfrac{exp(\alpha_R \beta V_R)}{\theta_m}$ again based on the assumption of a lognormal distributed resistance.

Model uncertainties are explicitly taken into accounts thought the use of the bias factor θ_m, which is defined as the mean ratio of experimental to predicted resistance (in accordance with [3] its value varies between 0.7 and 1.2 for failure in compression, bending and shear). The coefficient variation of structural resistance V_R is written as follows:

$$V_R = \sqrt{V_g^2 + V_m^2 + V_f^2}\,, \tag{11}$$

where V_g, V_m, V_f are the coefficients of variations of the geometrical, model and material uncertainties respectively, estimated in accordance with [3].

According with the second contributions, proposed by Allaix et al [7, 8], the design resistance R_d is derived by divisions of obtained resistance factor γ_R and the model uncertainty factor γ_{Rd}:

$$R_d = \frac{R(f_{ym}, f_{cm}, a_{nom})}{\gamma_R \gamma_{Rd}}, \tag{12}$$

In this case, the global resistance factor γ_R is derived from coefficient of variations of the structural resistance V_R (estimated by probabilistic method or based on Cervenka method *ECOV*):

$$\gamma_R = exp(\alpha_R \beta V_R), \tag{13}$$

The model uncertainty factor γ_{Rd} takes into account the difference between the real behavior of the structure and the results obtained based on a numerical model. The model uncertainty factor γ_{Rd} can be derived using the following expression from [3]:

$$\gamma_{Rd} = exp(\tilde{\alpha}_R \beta V_{\vartheta R}), \tag{14}$$

where: $\tilde{\alpha}_R = 0{,}4\alpha_R$ is the sensitivity factor for resistance model uncertainty ($\tilde{\alpha}_R$ <1 in order to account for separate safety assessment of resistance);

$V_{\vartheta R}$ is the coefficient of variations of the resistance model uncertainly ϑ_R. The value of this coefficient of variations can be obtained based on experimental results according to EN 1990 [22].

4. Assessment of the global resistance and global safety factors

At the *first stage* of analysis the value of the global resistance factor γ_R was defined in accordance with [6] from eq. (13). As it was shown above, the *ECOV*-method [1, 2] is based on idea that the random distribution of resistance, which is again described by the coefficient variation V_R, can be estimated from mean R_m and characteristic R_k values of resistance (pseudo-static response of the structural system). In this case, coefficient variations of resistance V_R can be obtained from following equation:

$$V_R \le \frac{1}{1{,}64} ln(\frac{R_m}{R_k}), \tag{8}$$

where R_m, R_k are the mean and design values of resistance (pseudo-static response, as was shown in section 1), obtained by two separate non-linear analysis using mean and characteristic values of input material parameters respectively.

The results of the non-linear analysis of the statically undetermined an encastre RC-beam and values of the coefficient variations V_R and global resistance coefficient γ_R obtained by calculations are presented in Table 1.

Table 1 – The results of estimation the coefficient γ_R based on ECOV

FE-Program	$\frac{r_l}{r_l^{'}}$ [%]	Resistance, kN/m		$V_R = \frac{1}{1{,}64} ln(\frac{R_m}{R_k})$	$\gamma_R = exp(1{,}2V_R)$
		R_m	R_k		
Program	$\frac{0{,}48}{1{,}05}$	119,5	109,4	0,054	1,07
Notes: Materials properties: concrete class C25/30, f_{cm}=33 MPa, steel B500, f_{ym}=1,1 f_{yk}=550 MPa; section 300x350 mm; $\beta_{tag} = 1{,}5$ for accidental design situation					

The results, presented in Table 1 was obtained with FEM-computer program (Program), which (1) declared about possibilities for non-linear analysis of reinforced concrete structures and (2) most widely used in practical design. As it was declared in software manual, FE-program is capable of a "*realistic simulation of RC-structure"* behavior in the entire loading range with ductile as well as brittle failure modes [3, 6].

As was shown in [7] the result of investigation depends on assumption and criteria underlying the model used in the non-linear analysis. It should be noted that the different FEM-programs (software), which

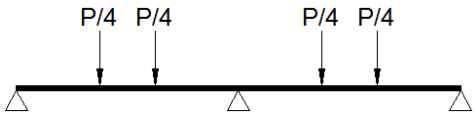
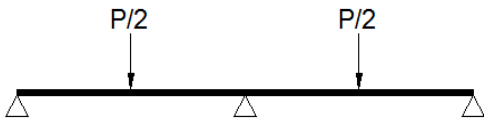
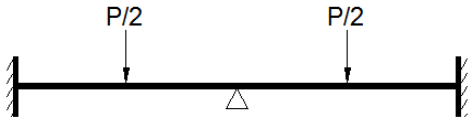
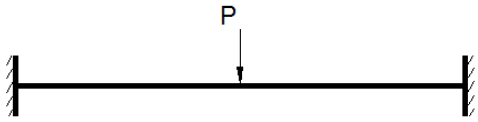
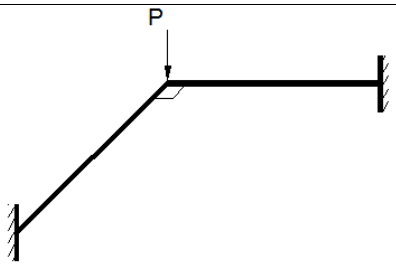
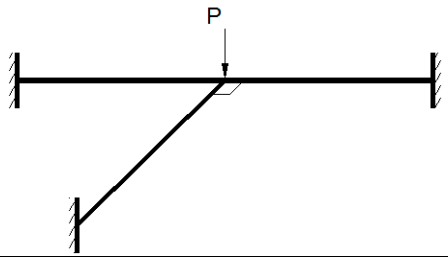
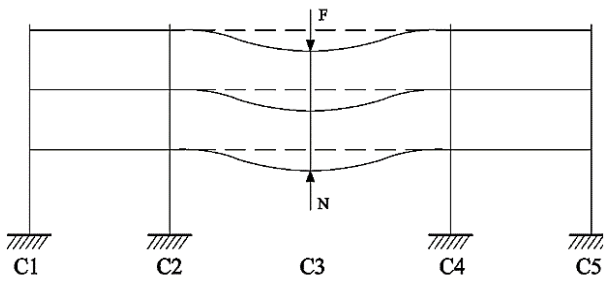
applied for non-linear structural analysis, will have own different level of FEM-model uncertainties in addition to local cross-section resistance model, material and geometry uncertainties. Clearly, the approach is meaningful if structural model covers all relevant failure mechanisms.

So, effects of model uncertainties should be treated separately (!).

At the *second stage* of analysis the coefficient of variations $V_{\vartheta R}$ of the computer model uncertainties was assessed based on theoretical background described in Annex D EN 1990 [22]. From these features, it is suggested to be derived from the comparison of the experimental tests data and numerical calculations results, but though probabilistic consideration.

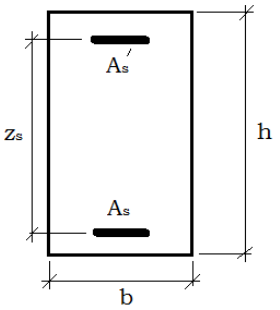
The set of the test results obtained in experimental investigations of the different types of statically indeterminate structures demonstrates different failure mechanism (see Tables 2, 3) was collected from some references [10-21] and used for assessment of the coefficient variations $V_{\vartheta R}$ and model uncertainly factor γ_{Rd}. The model uncertainly factor γ_{Rd} takes into account difference between the real behavior of structure and the results of a numerical modeling suitable for specific structure.

Table 2 – Loading arrangement for experimental specimens

Loading scheme	Beam Series	Reference
	B1, B2, B3	[10]
	B4, B6, B7, B8, B9	[11], [12], [13], [14], [15]
	B5	[11]
	B10, B11, B13	[16], [18]
	B12	[27]
	B14, B15	[19]
	F1, F2	[20], [21]

The real properties of the material and specimens geometry characteristics obtained by testing were used as an input data for non-linear analysis. The main characteristics of the analyzed test specimens are presented in Tables 2, 3.

Table 3 – Basic parameters of the experimental specimens (input data for non-linear analysis)

Beam Series	Cross-section	Dimensions Size, mm					Material properties			
							Concrete		Steel	
		b	h	z_s	r_l	r_l'	f_{cm} / f_{ctm}, MPa	E_{cm}, GPa	f_{ym}, MPa	E_s, GPa
1	2	3	4	5	6	7	8	9	10	11
B1		150	260	236	0,64	0,91	34,4 / 4,51	32	440	200
B2					0,91	0,64	32,4 / 3,95	31,9	435	
B3					0,91	0,64	33,9 / 4,41	33,5	433	
B4			250	210	0,66	0,46	28 / 2,5	28	520 (Ø10)	
B5							29 / 2,64		580 (Ø12)	
B6		200	300	265	0,85	0,85	26,6	28,3	511	
B7		150	250	170	1,28	1,28	74,2	48,3	412	
B8			250	210	0,46	1,17	25	28	445	
B9		375	120	68	0,64	1,85	30,1	31,5	447	
B10		180	180	118	0,59	0,59	30,5	31,6	592	
B11							59	54,3	550	
B12		100	180	150	0,87	0,87	40	41,2	575	
B13		900	150	130	1,06	0,83	33	28,9	450	
B14, B15		200	140	90	0,66	0,44	26 / 1,5	28	530	
F1	columns	200	200	170	0,63	0,63	25	27,3	416	
	beams	100	200		1,26	1,26				
F2	columns, beams	150	400	300	2,39	2,39	30	29,7	418	

As it can be seen from the Table 4, the estimated values of coefficient of variations V_{Rd} for model uncertainties are much higher than recommended in codes (for example, in ***fib*** MC2010, values in range 1,05…1,1).

The same results and conclusions were obtained by Schlune [3]. Schlune concluded that model uncertainties of non-linear analysis are much higher than in standard design based on engineering formulas and are strongly dependent on modes of failure and adopted failure criteria. Reported in [3] coefficient of variation due to model uncertainty for bending failure in range 5…30%, for shear 15…64%. Schlune concluded that due to the lack of data, the choice of model uncertainty often depends on engineering judgment and can be subjective.

Note, that coefficient of variations V_m due to material uncertainty (variability) has not a fixed value. In the case of concrete, the mean value of the concrete compressive strength for different classes according to EN 1992 [4] is calculated as: $f_{cm}=f_{ck}+8\ MPa$ (where $8MPa \approx 1{,}64\sigma_c$, which standard deviation $\sigma_c = 4{,}88MPa$). For fixed value of standard deviation (as a basic characteristic of the production quality control) $\sigma_c = 4{,}5MPa$, coefficient of variation $V_{m,c}$ of concrete compressive strength will be in range from 8,6% (C50/60) to 21% (C16/20) and coefficient of variation for materials V_m will be in range from V_m=10,48% to 21,84% (with fixed value of coefficient of variations V_s=6% for steel).

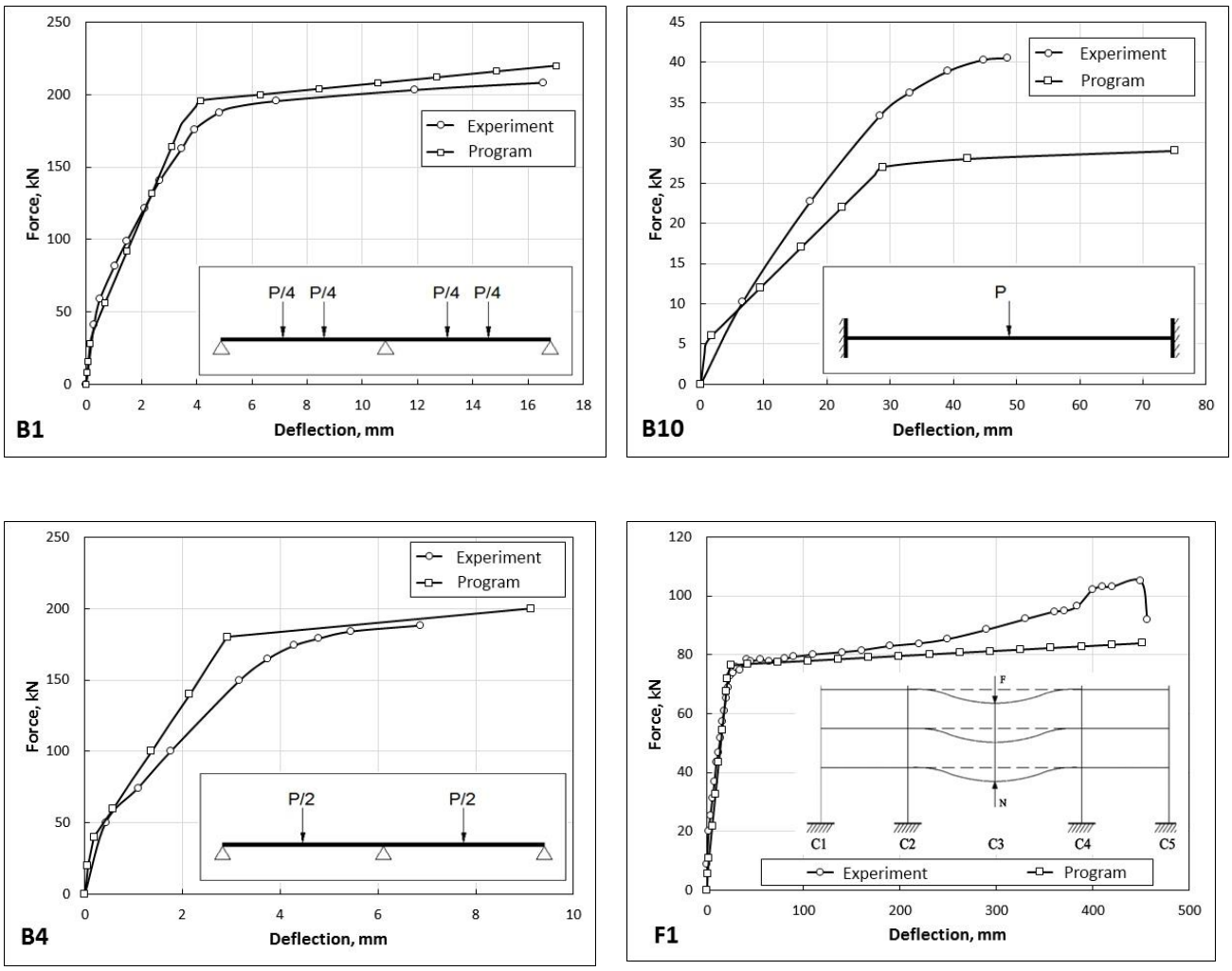


Figure 2 – Some typical examples of the experimental and predicted force-deflection response of the analyzed specimens (see Tables 2, 3 for designation of the specimens)

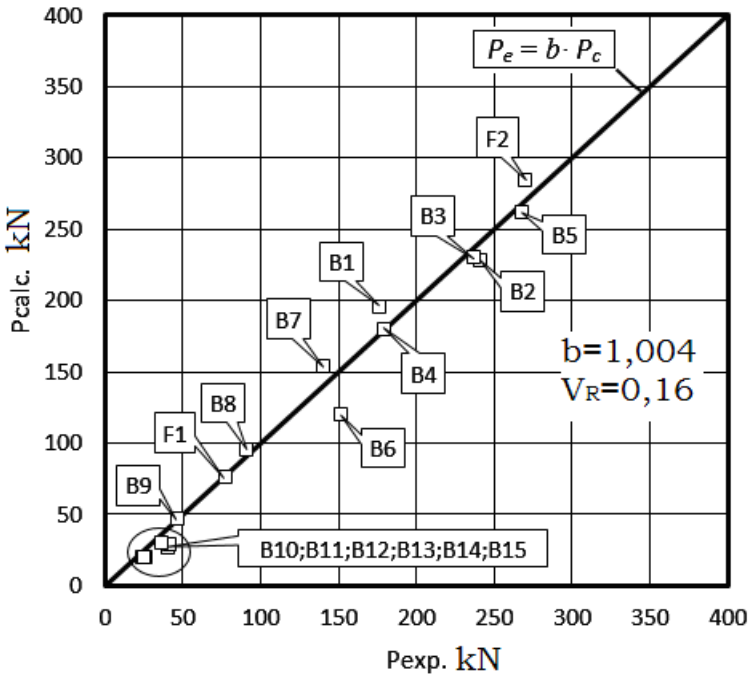


Figure 3 – For estimatiation of the coefficient V_R for FEM-model (see with tables 2, 3)

Further research is need to recommended appropriate values of the model uncertainty for numerical simulation. It should be noted, that for different FEM-programs values of γ_{Rd} will be different. These values for FEM-program should be estimated based on full probabilistic approach, taking into account statistical parameters of the FEM-model uncertainties and consists of in Program Manual.

Table 4 – Estimated values of the global coefficient γ_R

Program	Shlune model [3]						Allaix model [8]				
	Coefficient of variation, %				θ_m	γ_R	coeff. var., %		factors		
	V_m^*	V_g	V_{Rd}	V_R			V_{R0}	V_{Rd}	γ_{R_0}	γ_{Rd}	γ_R
Program	var	det	15,7	17,8… 30,6	1,004	1,55… 1,97	5,8	15,7	1,19	1,21	1,44
Note: Value of V_m due to material variability in range from 8,6% (C50/60) to V_m=21% (C16/20).											

Conclusions

Safety format suitable for non-linear analysis (pseudo-static response) that based on global resistance in accordance with ***fib*** MC2010 concept are presented.

The following conclusions can be adopt: (1) the differences between proposed methods are not significant; (2) fixed value of global safety factor $\gamma_R = 1{,}27$ in accordance with ***fib*** MC2010 and EN 1992-2 is not good approach for safety assessment and sometimes can be unconservative results; (3) the values of the global resistance factor γ_R should be estimated separately for different computer programs, which are used for non-linear analysis (pseudo-static response of the structural system), based on experimental results. These values for separate computer programs should be estimated based on full probabilistic approach, taking into account statistical parameters of the *FEM*-model uncertainties.

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