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Mass transfer analysis of two-phase flow in a suspension of microorganisms

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Abstract The aim of present work is to investigate the mass transfer of steady incompressible hydromagnetic fluid near the stagnation point with deferment of dust particles over a stretching surface. Most researchers tried to improve the mass transfer by inclusion of cross-diffusion or dust particles due to their vast applications in industrial processes, extrusion process, chemical processing, manufacturing of various types of liquid drinks and in various engineering treatments. To encourage the mass transport phenomena in this study we incorporated dust with microorganisms. Conservation of mass, momentum, concentration and density of microorganisms are used in relevant flow equations. The arising system of nonlinear partial differential equations is transformed into nonlinear ordinary differential equations. The numerical solutions are obtained by the Runge-Kutta based shooting technique and the local Sherwood number is computed for various values

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of the physical governing parameters (Lewis number, Peclet number, Eckert number). An important finding of present work is that larger values of these parameters encourage the mass transfer rate, and the motile organisms density profiles are augmented with the larger values of fluid particle interaction parameter with reference to bioconvection, bioconvection Lewis number, and dust particle concentration parameter..

Keywords: Gyrotactic microorganisms; Dusty fluid; Mass transfer; Stretching sheet; Bio convection; Boundary layer flow

Nomenclature

b – chemotaxis constant
C – concentration of fluid
C_p – concentration of dust particles
C_w – concentration of the fluid
C_∞ – ambient concentration
c_m – specific heat of dust particles
c_p – specific heat of fluid
D_m – mass diffusivity coefficient
D_n – diffusivity of micro-organisms
Ec – Eckert number
f_0 – suction parameter
K – Stokes drag coefficient
k – thermal conductivity of the fluid
l – dust particles mass concentration parameter
Le – traditional Lewis number for concentration
Lb – bio-convection Lewis number
M – magnetic parameter
m – mass of dust particles per unit volume
N – density of motile micro=organisms
N_p – density of dust particles
N_w – density of motile microorganisms
N^+ – number density of dust particles
N_∞ – ambient density
Pe – bioconvection Peclet number
Re_x – local Reynolds number
r – radius of dust particles
Wc – maximum cell swimming speed
u, v – velocity components of fluid along x- and y-axes
u_p, v_p – velocity component of dust phase along x- and y-axes
x, y, z – Cartesian coordinates

Greek symbols

β_m – fluid particle interaction parameter with reference to bio-convection
β_v – fluid particle interaction parameter
β_c – fluid particle interaction parameter with reference to concentration
λ – ratio of free stream to sheet stretching velocity
ρ – density of fluid
ρ_p – density of dust particles
μ – dynamic viscosity of fluid
τ_v – relaxation time of dust particles
τ_c – time taken by the dust particles to alter its concentration relative to the fluid
τ_m – time taken by the motile organisms to alter its concentration relative to the fluid
Ω – microorganisms concentration difference parameter

1 Introduction

Very recently, bioconvection flow pattern has received great attention by the researchers. Bioconvection is a phenomenon which occurs when microorganisms which are 5%–10% denser than water swim upward on average. The self-impelled microorganisms thus improve the base fluid density in a particular direction causing the bioconvection in the flow. According to the grounds of impellent, the motile microorganisms can be classified into different types of microorganisms such as chemotaxis or oxytocic and gyrotactic microorganisms and negative gravitaxis. The stimulators of these microorganisms are negative gravity, oxygen concentration gradient and the displacement between the centre of buoyancy and mass. The addition of motile microorganisms to the suspension enhance the mass transport rate and concentration of species, which can find industrial applications in enzyme biosensor, extrusion process, polymer sheets, chemical processing and biotechnological applications. For further details see [1–10].

The fluid flow, heat and mass transfer over a stretching sheet has gained researchers attention from the last decade due to the fact that most of the processes such as casting, drawing wires of copper, hot rolling, glass blowing, polymer industries, extrusion of plastic sheets, etc. have benefited from it. Sakiadis discussed the boundary layer flow of a viscous, incompressible fluid over stretched moving solid surfaces [11]. For two dimensional symmetric flows he formulated the boundary layer equations. Later, Crane investigated two-dimensional flow over a linearly stretching sheet [12]. Further researchers focused on flow over a stretching sheet by considering parameters such as thermal radiation, MHD, suction/injection, permeable surface and slip effects [13–17].

The phenomenon of fluid flow with embedded micro- or millimeter sized dust particles is encountered in various advanced engineering problems such as combustion, lunar ash flows, nuclear reactors, corrosive particles in engine oil flow, rain erosion, waste water treatment, paint spraying, polymer technology, etc. Saffman initiated the study on fluid particle suspension and stability of laminar flow of dusty gas [18]. Agranat studied the influence of pressure gradient on friction and heat transport in a dusty fluid [19]. Recently, Lakshmi *et al.* [20] considered the heat and mass transfer of a dusty fluid over a stretching sheet by incorporating thermal radiation. Mamatha *et al.* [21] studied magnetohydrodynamic (MHD) Carreau dusty nanofluid by considering the Cattaneo-Christov heat flux. Recently, Saleem *et al.* [22–29] discussed the nanofluid models in different configurations. Durga Prasad *et al.* [30] examined the cross diffusion and multiple slips in MHD Carreau fluid in a suspension of microorganisms over a variable thickness sheet.

In the present study, we have made an attempt to analyze the mass transfer characteristics of dusty fluid with the incorporation of gyrotactic microorganisms over a stretching sheet. The governing equations of the flow are solved numerically with the Runge-Kutta based shooting process in the commercial Matlab package for different values of parameters of interest.

2 Description of the problem

We have considered a steady 2D viscous, laminar, incompressible convection boundary layer flow of dusty fluid near the stagnation point containing microorganisms over a permeable stretching sheet. Physical flow configuration of the problem along with Cartesian coordinates and considered flow assumptions are presented in Fig. 1 and listed as below:

- The sheet coincides with the plane $y = 0$ and the flow is restricted to $y > 0$.
- The flow is generated owing to the linear stretching of the sheet due to the action of two equal and opposite forces along the x-axis.
- The sheet is stretched with velocity $U_w(x) = dx$, $d > 0$ is the stretching rate.
- The free stream velocity $U(x) = ex$.

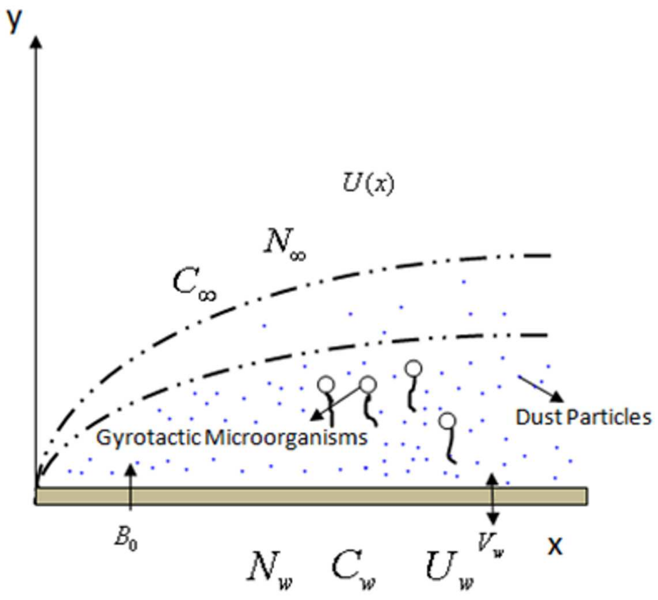

Figure 1: Flow configuration and coordinate system.

- The dust particles (spherical in shape) are considered to be dispersed uniformly throughout the fluid.
- Along the y-axis a constant magnetic field B_0 is applied.
- The number density of dust particles is considered to be constant and the volume fraction is neglected.
- The Stokes linear drag theory is used to model the drag force.
- The motile microorganisms and the dusty fluid have a similar velocity.

In view of the above mentioned assumptions, the governing equations for conservation of mass, momentum, concentration and density of gyrotactic microorganisms for both the fluid and particle phase can be expressed as (see [20,4,31]):

Continuity equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0\,. \tag{2}$$

Momentum equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{KN^+}{\rho}(u_p-u)+U\frac{dU}{dx}+\frac{\sigma B_0^2}{\rho}(U-u)\,, \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\frac{K}{m}(u-u_p)\,. \tag{4}$$

Concentration equations:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{\rho_p}{\rho\tau_c}(C_p-C)\,, \tag{5}$$

$$u_p\frac{\partial C_p}{\partial x}+v_p\frac{\partial C_p}{\partial y}=\frac{1}{\tau_c}(C-C_p)\,. \tag{6}$$

Density of gyrotactic microorganism equations:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=D_n\frac{\partial^2 N}{\partial y^2}-\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial N}{\partial y}\frac{\partial C}{\partial y}+N\frac{\partial^2 C}{\partial y^2}\right]+\frac{\rho_p}{\rho\tau_m}(N_p-N)\,, \tag{7}$$

$$u_p\frac{\partial N_p}{\partial x}+v_p\frac{\partial N_p}{\partial y}=\frac{1}{\tau_m}(N-N_p)\,. \tag{8}$$

The boundary conditions for the velocity, concentration and density of gyrotactic microorganisms are defined as:

$$\begin{aligned}
&\text{at} \quad y=0, \quad u=U_w(x),\ \ C=C_w,\ \ v=-V_w(x),\ \ N=N_w\\
&\text{as} \quad y\to\infty,\ \ u\to U(x),\ \ u_p\to U(x),\ \ v_p\to v,\ \ C\to C_\infty,\\
&\qquad\ \ C_p\to C_\infty,\ \ N\to N_\infty,\ \ N_p\to N_\infty\,.
\end{aligned} \tag{9}$$

Introducing the following similarity transformations:

$$\begin{aligned}
&\zeta=y\left(\tfrac{d}{v}\right)^{0.5},\ \ u=dxf'(\zeta)\,,\ \ v=-(vd)^{0.5}f(\zeta)\,,\ \ u_p=dxF'(\zeta)\,,\\
&v_p=-(vd)^{0.5}F(\zeta)\,,\ \ \rho_p=mN^+,\ \ C=(C_w-C_\infty)\,\phi(\zeta)\,,\\
&C_p=(C_w-C_\infty)\,\phi_p(\zeta)\,,]\ \ N=(N_w-N_\infty)\,\chi(\zeta)\,,\\
&N_p=(N_w-N_\infty)\,\chi_p(\zeta)\,,\ \ K=6\pi\mu r\,,
\end{aligned} \tag{10}$$

the partial differential equations (PDEs) are converted into the nonlinear ordinary differential equations (NODEs) as:

$$\begin{aligned}
&f'''(\zeta)+f''(\zeta)f(\zeta)-[f'(\zeta)]^2\\
&\qquad+\beta_v lF'(\zeta)-\beta_v lf'(\zeta)+\lambda^2+M(\lambda-f'(\zeta))=0\,,
\end{aligned} \tag{11}$$

$$F(\zeta)\,F''(\zeta) - F'(\zeta)^2 + \beta_v\left[f'(\zeta) - F'(\zeta)\right] = 0\,, \tag{12}$$

$$\frac{1}{\mathrm{Le}}\phi''(\zeta) + f(\zeta)\,\phi'(\zeta) + l\beta_c\left[\phi_p(\zeta) - \phi(\zeta)\right] = 0\,, \tag{13}$$

$$F(\zeta)\,\phi_p'(\zeta) + \beta_c\left[\phi(\zeta) - \phi_p(\zeta)\right] = 0\,, \tag{14}$$

$$\begin{aligned}\chi''(\zeta) - \mathrm{Pe}\left[\chi'(\zeta)\,\phi'(\zeta) + (\Omega + \chi)\,\phi''(\zeta)\right]&\\ + Lbf(\zeta)\,\chi'(\zeta) + l\beta_m Lb\left[\chi_p(\zeta) - \chi(\zeta)\right] = 0\,,&\end{aligned} \tag{15}$$

$$F(\zeta)\,\chi_p'(\zeta) + \beta_m\left[\chi(\zeta) - \chi_p(\zeta)\right] = 0\,, \tag{16}$$

where the prime symbol, double prime, and triple prime denote the first, second, and triple derivative.

The dimensionless form of boundary conditions (10) is:

$$\begin{aligned}&f'(0) = 1\,, \quad f(0) = f_0 = \frac{V_w}{(vd)^{1/2}}\,, \quad f'(\infty) = 0\,, \quad F'(\infty) = \lambda\,,\\ &F(\infty) = -f(\infty)\,, \ \phi(0) = 1\,, \ \phi(\infty) = 0, \ \chi(0) = 1\,, \ \chi_p(\infty) = 0\,.\end{aligned} \tag{17}$$

In Eqs. (11)–(16)

$$\begin{aligned}&l = \frac{mN^+}{\rho}, \ \beta_v = \frac{1}{a\tau_v}, \ \mathrm{Le} = \frac{\nu}{D_m}, \beta_c = \frac{1}{a\tau_c}, \ \mathrm{Pe} = \frac{bWc}{D_n}, Lb = \frac{\nu}{D_n},\\ &\Omega = \frac{N}{(N_w - N_\infty)}, \beta_m = \frac{1}{a\tau_m}, \tau_v = \frac{m}{K}, M = \frac{\sigma B_0^2}{d\rho}, f_0 = \frac{V_w}{(vd)^{1/2}}, \lambda = \frac{e}{d}\,.\end{aligned} \tag{18}$$

The shear stress τ_w, local mass flux j_w, motile microorganisms flux q_n on the surface can be expressed as:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad j_w = -D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}, \quad q_n = -D_n\left(\frac{\partial N}{\partial y}\right)_{y=0}. \tag{19}$$

From the engineering point of view, the important parameters which illustrate flow characteristics are the friction coefficient, Nusselt number, Sherwood number and local density number of the motile microorganisms. The Sherwood and Nusselt numbers are defined as:

$$\mathrm{Sh} = \frac{xj_w}{D_m\left(C_w - C_\infty\right)}, \quad \mathrm{Nu} = \frac{xq_n}{D_n\left(N_w - N_\infty\right)}\,. \tag{20}$$

With the use of similarity transformations we get:

$$\mathrm{Sh}\left(\mathrm{Re}_x\right)^{-0.5} = -\phi'(0)\,, \quad \mathrm{Nu}\left(\mathrm{Re}_x\right)^{-0.5} = -\chi'(0)\,. \tag{21}$$

In the above equation $\mathrm{Re}_x = \frac{u_w x}{\nu}$ refers to the local Reynolds number.

3 Results and discussion

For analyzing the approximate solutions of concentration and density of the motile organisms profile for the fluid phase and dust phase, the nonlinear ordinary differential Eqs. (11)–(16) with reference to the boundary conditions (17) are solved numerically by employing the Runge-Kutta based shooting technique. For numerical solutions we have considered the values of non-dimensional parameters as: $Lb = 0.3$, $\beta_m = 0.3$, $\mathrm{Pe} = 0.2$, $\Omega = 0.1$, $\mathrm{Le} = 0.1$, $\beta_c = 0.3$, $l = 0.4$, $\beta_v = 0.3$, $\beta_t = 0.3$, $\mathrm{Pr} = 2$, $\mathrm{Ec} = 0.2$. These values are considered as constant in this study besides the varied parameters as mentioned in the respective figures and Tab. 2.

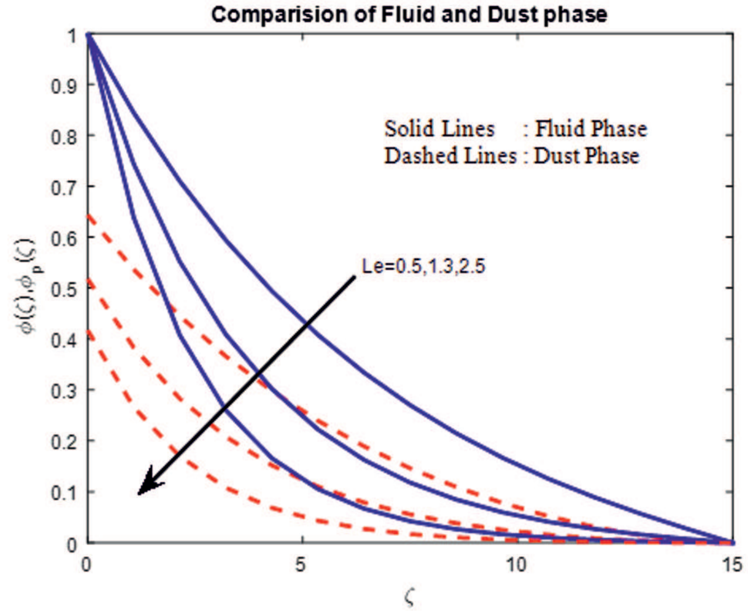

Figure 2: Effects of Le on concentration profiles.

In Fig. 2 we can notice that an increase in Lewis number, Le, decreases the concentration profiles of both the fluid phase and dust phase. As expected the mass diffusivity increases with the increase of Le, consequently the concentration boundary layer thickness decreases. Likewise, in Fig. 3 we observe an increase in Lewis number with the decreasing density of motile microorganisms for both the fluid phase and dust phase. Figure 4 depicts the influence of Peclet number, Pe, on both the fluid phase and dust phase profiles of motile organisms density profile. It is apparent that the density

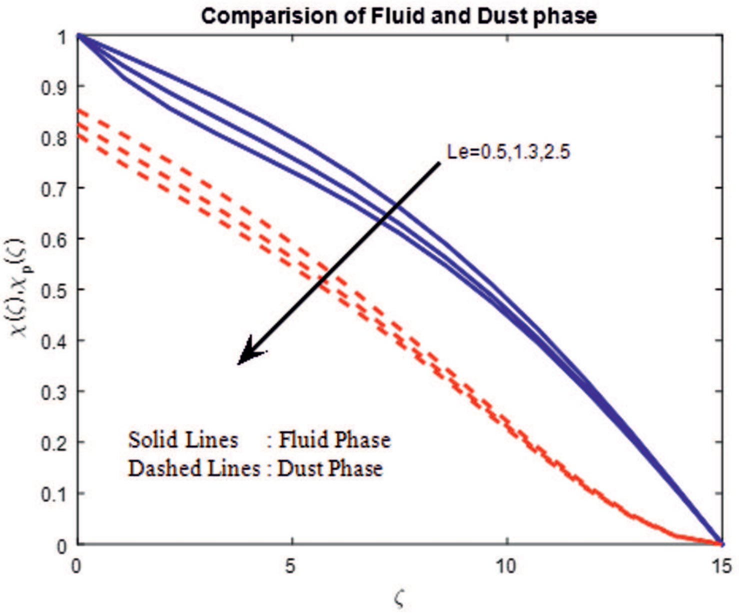

Figure 3: Effects of Le on density profiles of motile organisms.

of the motile microorganisms is decreased for increasing values of Pe owing to the unbalancing ratio of time scales. In Fig. 5 it is observed that an increase in fluid particle interaction parameter with reference to bioconvection β_m improves the fluid and dust phase density of motile organisms. It is apparent from Fig. 6 that increasing the fluid-particle interaction parameter for concentration, β_c, decreases the fluid phase concentration profiles and improves the dust phase profiles. This may be due to the fact that when we incorporate the dust particles in the flow, the interaction between the fluid and dust phase is high, then there is an improvement in dust phase concentration profile.

In Fig. 7 we can notice that increasing the fluid-particle interaction parameter for concentration decreases the fluid phase as well as dust phase profiles of density of motile organisms. From Fig. 8 it is observed that the density of motile microorganisms increases as bioconvection Lewis number Lb increases for both the fluid and dust phase. In fact, an increase in Lb means an increase of microorganisms diffusion, hence, both the density and boundary layer thickness for motile microorganisms increases. The influence of dust particles mass concentration parameter, l, on fluid phase and dust phase concentration profiles are exhibited in Fig. 9. An introduction

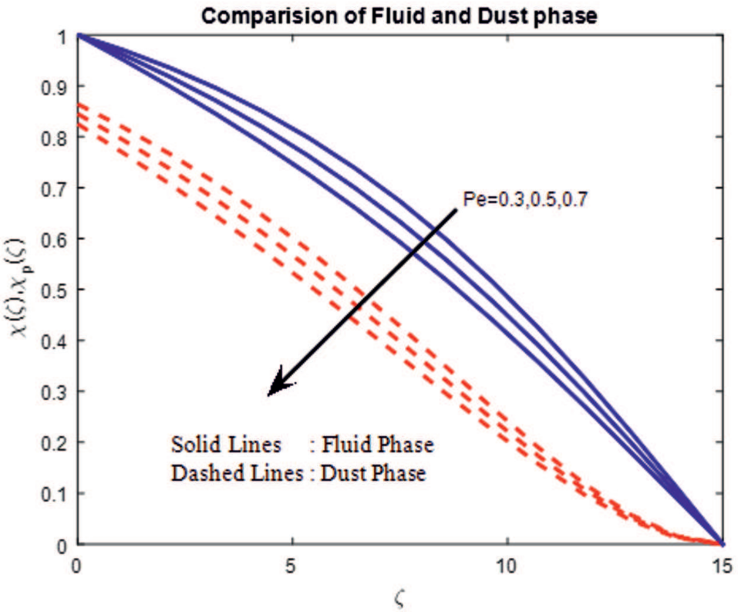

Figure 4: Effects of Pe on density of motile organisms.

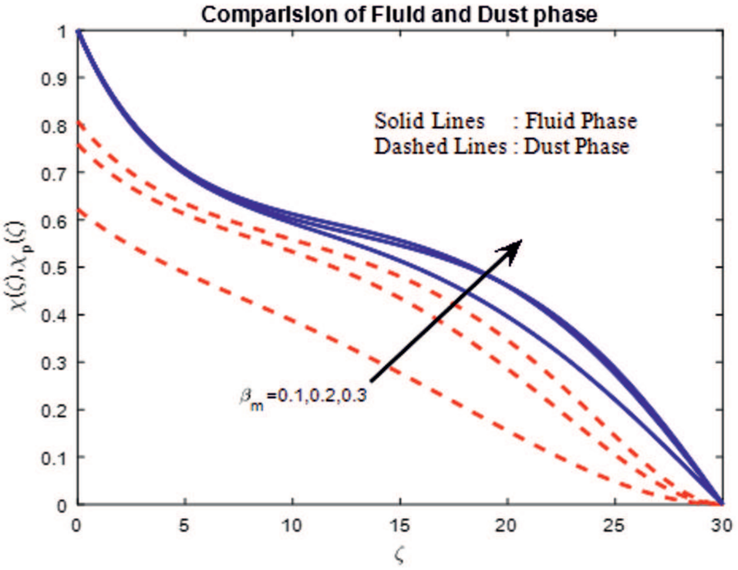

Figure 5: Effects of β_m on density of motile organisms.

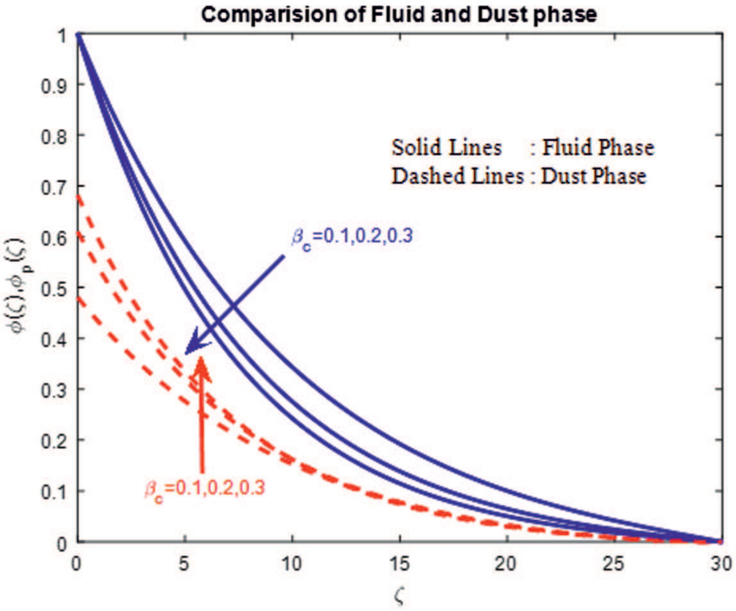

Figure 6: Effects of β_c on concentration profiles.

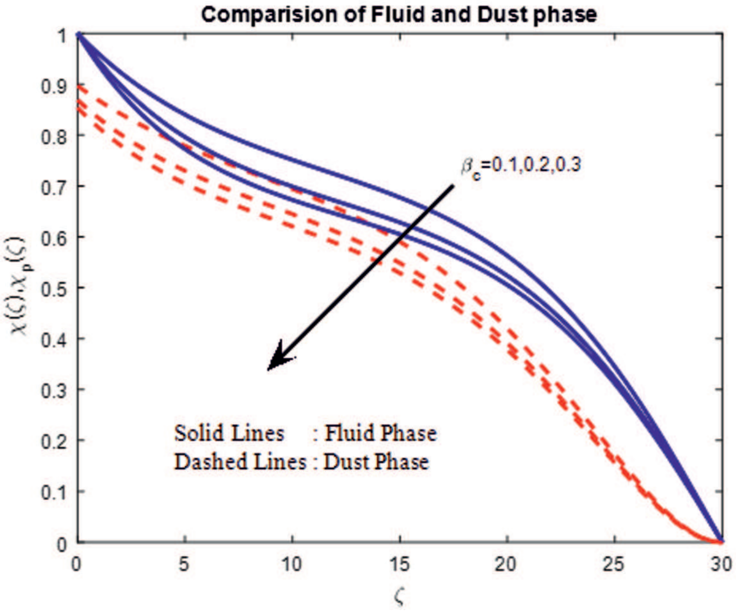

Figure 7: Effects of β_c on density of motile organisms.

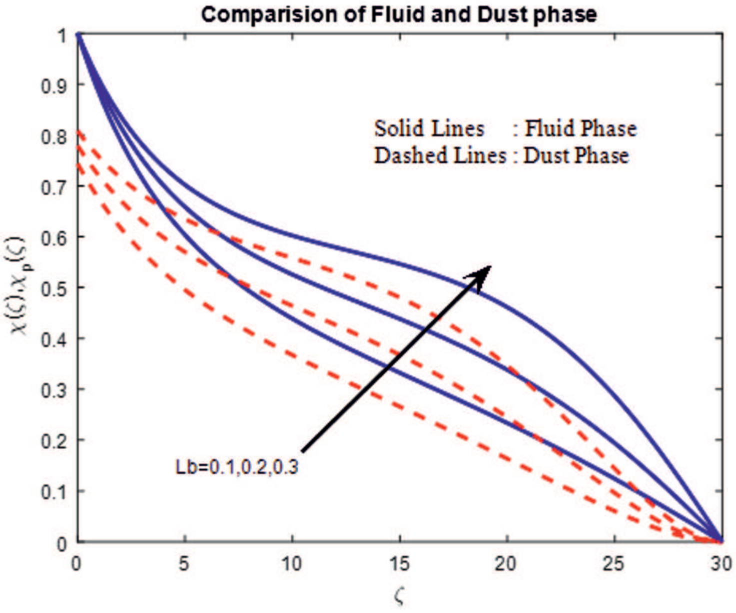

Figure 8: Effects of Lb on density of motile organisms.

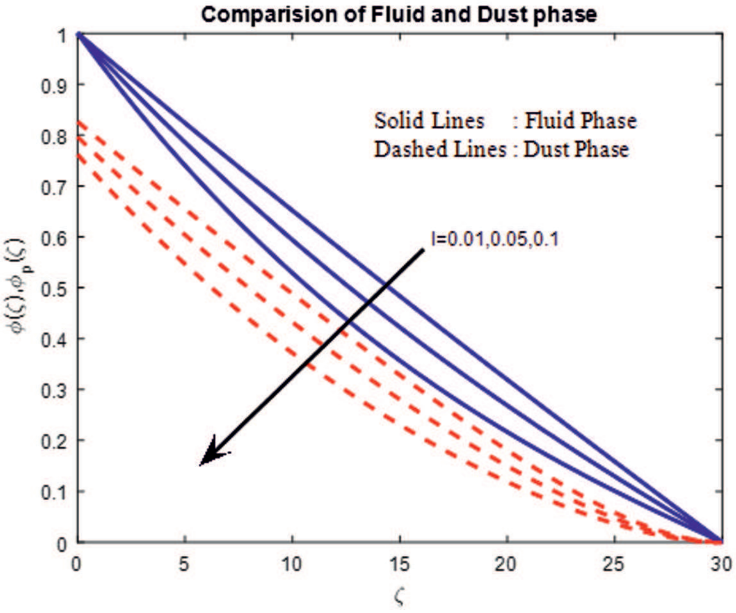

Figure 9: Effects of l on concentration profiles.

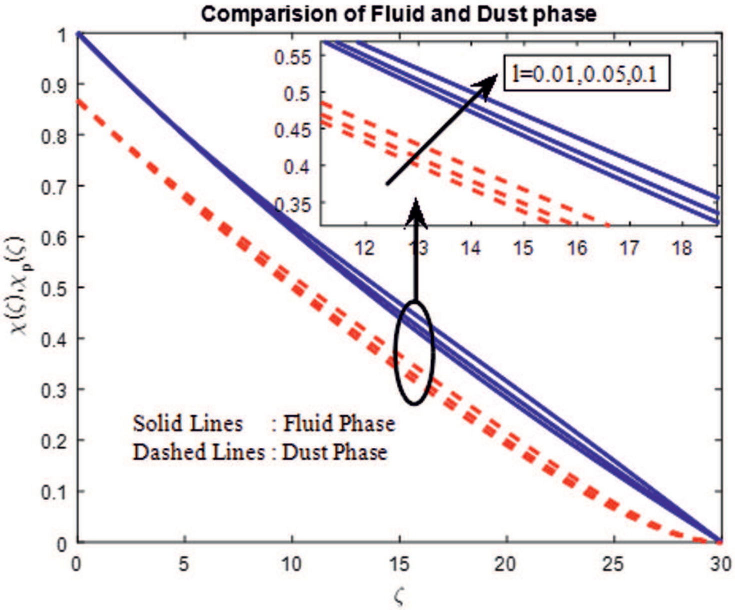

Figure 10: Effects of l on density of motile organisms.

of uniformly sized dust particles in the fluid creates internal friction within the fluid. As a result there is a retardation in the flow and decrease of the concentration boundary layer thickness for both the fluid and dust phase. In Fig. 10 it is observed that larger values of l improve the boundary layer thickness of both the fluid and dust phase motile organisms density profiles. Figures 11 and 12 portray that a growth in magnetic parameter M, acts to decline the concentration and density of motile organisms profiles of both the particle and the fluid phase.

Table 1 displays the $f''(0)$ values obtained in the previously published results and in the current study. It is found that the obtained numerical values are accurate.

Table 2 displays the influence of non-dimensional parameters (Le, Pe, β_m, β_c, Lb, l, M) on the local Sherwood and Nusselt numbers $(\mathrm{Sh}\,(\mathrm{Re}_x)^{-0.5}$, $\mathrm{Nu}\,(\mathrm{Re}_x)^{-0.5})$. It is observed that larger values of $\mathrm{Le}, \mathrm{Pe}, \beta_c, l$ encourage the mass transfer rate, whereas larger values of β_m, Lb, M decrease the mass transfer rate $(\mathrm{Nu}\,(\mathrm{Re}_x)^{-0.5})$. Larger values of Le, β_c, l increase the mass transfer rate $\left(\mathrm{Sh}\,(\mathrm{Re}_x)^{-0.5}\right)$.

Table 1: Comparison of results for $f''(0)$ when $M = \beta = Q = Nr = \mathrm{Pr} = A* = B^* = \Omega = Lb = \mathrm{Pe} = 0$, $\mathrm{Ec} = l = 0$.

λ	Singh *et al.* [32]	Roy and Guota [33]	Ramesh *et al.* [31]	Present study
0.1	-0.9737	-0.9696	-0.9737	-0.9737
0.2	-0.9215	-0.9181	-0.9215	-0.9214
0.5	-0.6676	-0.6673	-0.6676	-0.6676
2.0	2.0174	2.0175	2.0175	2.0175
3.0	4.7290	4.7293	4.7292	4.7293

Table 2: Deviation in mass transfer rates $(-\phi'(0))$ and $(-\chi'(0))$ for different values of non-dimensional governing parameters.

Le	Pe	β_m	β_c	Lb	l	M	$(-\phi'(0))$	$(-\chi'(0))$
0.5							0.156766	0.037197
1.3							0.276077	0.063509
2.5							0.418415	0.095562
	0.3						0.084815	0.026348
	0.5						0.084815	0.035530
	0.7						0.084815	0.045499
		0.1					0.183965	0.108973
		0.2					0.183965	0.105902
		0.3					0.183965	0.104438
			0.1				0.103889	0.044245
			0.2				0.126251	0.060505
			0.3				0.139274	0.070232
				0.1			0.183965	0.133555
				0.2			0.183965	0.116955
				0.3			0.183965	0.104438
					0.01		0.035707	0.042425
					0.05		0.045347	0.042925
					0.1		0.057542	0.044018
						0.5	0.125398	0.132372
						1	0.119251	0.125879
						1.5	0.111563	0.116457

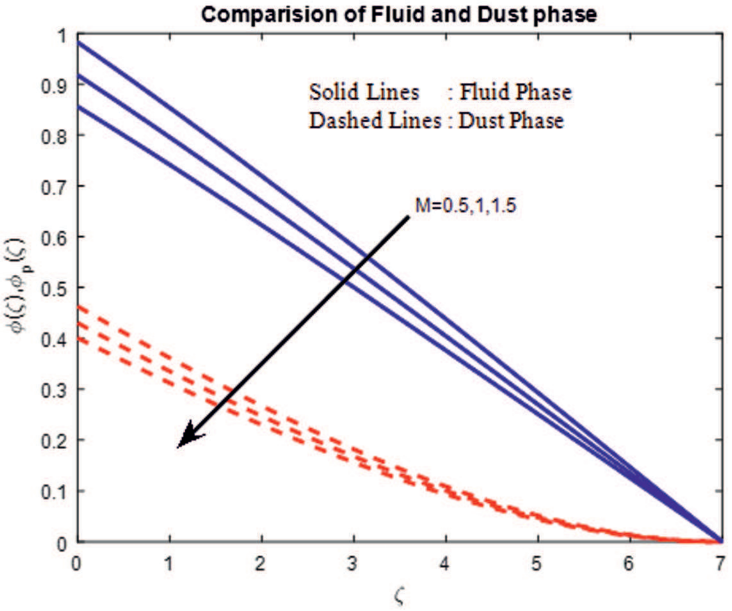

Figure 11: Effects of M on concentration profiles.

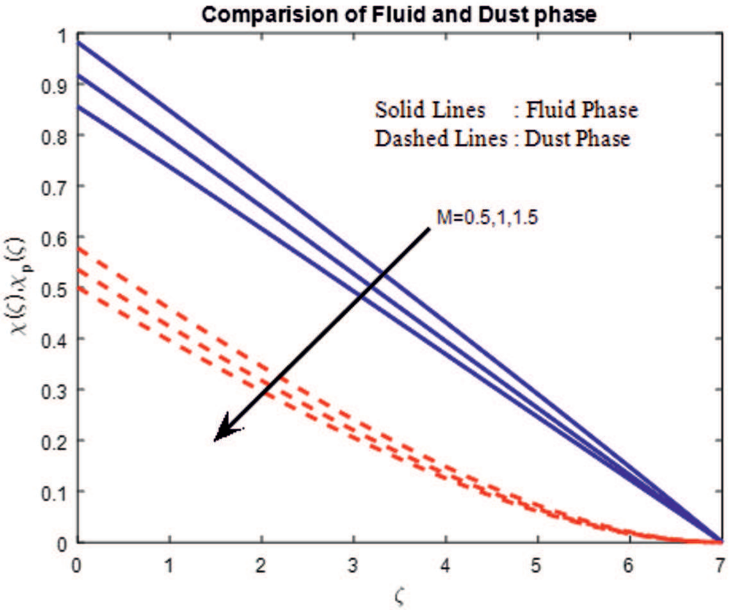

Figure 12: Effects of M on density of motile organisms.

4 Conclusion

In the present investigation, we have numerically studied two dimensional, viscous, laminar, incompressible convection boundary layer flow of dusty fluid containing microorganisms over a stretching sheet. The major findings of the present investigation are as follows:

- The motile microorganisms density profile declines for larger values of Lewis number, Peclet number, fluid-particle interaction parameter for concentration.
- The motile microorganisms density profiles augments with larger values of fluidf particle interaction parameter, bioconvection Lewis number and dust particles mass concentration parameter.
- Larger values of fluid-particle interaction parameter improve dust phase concentration profiles and decline fluid phase concentration profiles.
- Larger values of Lewis number and dust particles mass concentration parameter decline concentration profiles of both the fluid and dust phase.
- Larger values of Lewis number, Peclet number, fluid particle interaction parameter, and dust particles mass concentration parameter encourage the mass transfer rate.

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