ANALYSIS OF A ROAD ACCIDENT IN THE ASPECT OF MECHANICS OF A FRONTAL CRASH BETWEEN TWO VEHICLES

Jarosław ZALEWSKI

Faculty of Administration and Social Sciences,
Warsaw University of Technology, Nowakowskiego 18/20,
00-668 Warsaw, Poland, j.zalewski@ans.pw.edu.pl

Summary

In this paper a computer simulation of a frontal oblique collision between two motor vehicles was carried out, using the PC-Crash 8.0 software. The simulation repeated several times for specific values of the coefficient of restitution. Then the results were compared with analytical calculations, which were based on the planar collision mechanics, including the theory of collision between the bodies with smooth surfaces.

Analysis of the influence of the selected coefficient was made primarily in terms of change in the after crash vehicle speed. The aim of the study was to determine whether the value of the coefficient of restitution in a frontal impact play an important role in the analysis of the process of such phenomenon.

Keywords: frontal crash, coefficient of restitution.

ANALIZA WYPADKU DROGOWEGO W ASPEKCIE MECHANIKI ZDERZENIA CZOŁOWEGO SAMOCHODÓW

Streszczenie

W artykule przeprowadzono symulację komputerową zderzenia czołowego skośnego wybranych pojazdów w programie PC-Crash 8.0. Symulację przeprowadzono kilkakrotnie dla określonych wartości współczynnika restytucji. Następnie jej wyniki porównano z obliczeniami analitycznymi, których dokonano w oparciu o mechanikę zderzenia z uwzględnieniem teorii zderzenia ciał gładkich w ruchu płaskim.

Analizy dotyczącej wpływu wybranego współczynnika dokonano przede wszystkim pod kątem zmian prędkości pozderzeniowych samochodów biorących w nim udział. Celem pracy było wykazanie czy wartości współczynnika restytucji w zderzeniu czołowym odgrywają ważną rolę w analizie procesu tego zjawiska.

Słowa kluczowe: zderzenie czołowe, współczynnik restytucji.

1. INTRODUCTION

Modeling of road vehicle collisions, as a tool to assist the understanding and analysis of this process, can be carried out in several different directions [6, 7, 8, 10, 14], respectively simplifying or complicating the existing collision models or creating new ones. In this paper, a simulation of a frontal collision, as one of the most common occurring in Poland [9], was carried out. The aim of this study was to demonstrate the influence of the adopted values of the coefficient of restitution, as a decisive parameter on the plasticity of a collision, on the post crash vehicle speeds and the resulting consequences. Restitution coefficient analysis was previously dealt with, among others, in [1, 2, 4, 5], but mainly in terms of reconstruction or analysis of the process of collision and not its effect on the received post crash values.

The simulation of a frontal oblique impact was prepared, using the PC-Crash 8.0. The stuck vehicle is a 2005 Audi A8, while the striking one – a Mazda 6 from the production period between 2002 and 2005. The simulation of a frontal impact collision reflects the accident on a single-lane road section.

Such approach enabled verification whether the adoption of the specific values of the coefficient of restitution faciliates obtaining of the real parameters of the modelled collision.

2. ASSUMPTIONS

Before carrying out the simulation of a frontal impact, certain simplifying assumptions were made, while enabling the use of a mathematical model of a collision, based on considerations from the work [3], for the analytical calculations:

- bodies of both vehicles are treated as rectangles (Fig. 1) with constant mass and stiffness;

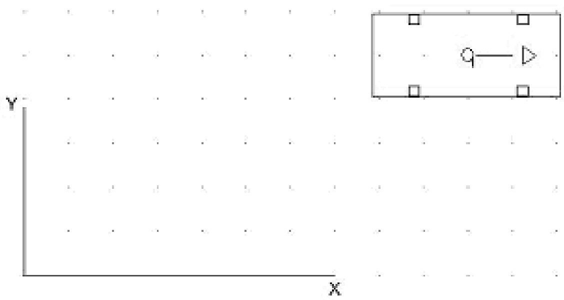

Fig. 1. Presentation of a vehicle body as a cuboid in the PC-Crash software. Source: PC-Crash

- vehicle models are linear, the mass-inertia parameters were adopted according to the program database;
- the vehicles realise a plane motion during the impact, which takes place on a dry surface with a coefficient of friction $\mu = 0.8$;
- before the impact the vehicles moved at speeds: Audi – 60km/h, Mazda – 50km/h;
- the total mass of the unladen Audi was 2020kg, and has been increased by adding the masses of a vehicle driver and three passengers to 2300kg, while the total mass of Mazda (originally 1500kg), was also increased, as in Audi, to 1750kg. In both cases the vehicle does not include any baggage;
- as in the work [13] the height of the center of mass in a laden vehicle was adopted, for Audi equal to 0,558m and for Mazda – 0,573m. It was done so the crash would be simulated as a model of the 3D objects realising a plane motion;
- the values of the coefficients of restitution were assumed at R = 0.1, R = 0.05 and R = 0.01;
- the simulated collision of the vehicles is oblique and frontal, so the occurrence of the crashing force impulses in a tangential direction, relatively the impact plane, was ommitted. The impact plane is understood to be perpendicular to the road surface, being at the same time tangent to both surfaces of the colliding vehicles. It was assumed that the model of a vehicle collision can be simplified to a form of a smooth bodies collision in a plane motion [3].

3. DESCRIPTION OF THE SIMULATION

In PC-Crash 8.0 a vehicle body is treated as a single rigid body (Fig. 1) having a predetermined mass, moments of inertia and stiffness [12]. It is also treated as a quasi-stiff deformable body, however lack of mass loss as a result of the collision was assumed.

Stiffness of the vehicle models in PC-Crash is different for different parts of a vehicle, because the wheels were adopted to be half less stiff than the body, whereas the roof and the side pillars are 75% less stiff than the lower part of the body.

In the analytical calculations the aspect of stiffness was not taken into account, since the main problem was to determine the parameters of the collision.

The main objective was to conduct a computer simulation of motor vehicles collision with certain parameters, and to verify the simulation results through analytical calculations, taking advantage of already established values of the coefficient of restitution.

The simulation of a frontal collision for all three adopted values of the coefficient of restitution was prepared. These values were adopted in accordance with the considerations e.g. in [15]. For each simulation the run time was 2 seconds, while the time of the collision was about 0.25s. In Fig. 2 the location of both vehicles before the collision is presented.

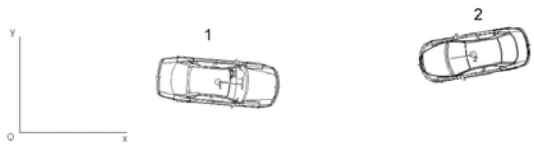

Fig. 2. Location of the vehicle before the collision. 1 – Audi, 2 – Mazda. Source: PC-Crash

In Tab. 1 the results of simulations for each of the accepted values of the coefficient of restitution is presented, containing the translational and angular velocities before and after the collision, whereas the sought values were the post-crash ones. Also the mass and moments of inertia of both vehicles is presented, as the parameters necessary in the analysis of traffic collisions in a plane motion. On the basis of these results a validation based on analytical calculations was prepared in p. 4.

It should be noticed that the speed values from before the collision appearing in the protocol of the collision are less than the value adopted in the assumptions (p. 2), due to the characteristics of the PC-Crash-called "collision detection". By this it is meant that the collision models of vehicles perform braking or partially precipitate speed, which increases the realism of the simulation, because it reflects a slight deceleration caused by removing a leg from the accelerator pedal, which is typical.

Tab. 1. Results of a frontal oblique collision simulations for three different values of the coefficient of restitution
Source: own research

	zderzenie czołowe	
vehicle	Audi	Mazda
mass	m_A=2300kg	m_M=1750kg
moment of inertia relative to the vertical axis passing through the center of mass of a vehicle	I_A=4277kgm^2	I_M=2730kgm^2
coefficient of restitution	R=0,1	
translational speed before the crash	V_A= 54 km/h	V_M= 43 km/h
angular velocity before the crash	ω_A= 0,38 1/s	ω_M= 1,29 1/s
translational speed after the crash	V_A'= 32,8 km/h	V_M'= 10,2 km/h
angular velocity after the crash	ω_A'= -1,76 1/s	ω_M'= 0,77 1/s
impulse of the impact force	18225 Ns	
coefficient of restitution	R=0,05	
translational speed before the crash	V_A= 54 km/h	V_M= 43 km/h
angular velocity before the crash	ω_A= 0,38 1/s	ω_M= 1,29 1/s
translational speed after the crash	V_A'= 34,2 km/h	V_M'= 11,8 km/h
angular velocity after the crash	ω_A'= -1,51 1/s	ω_M'= 0,57 1/s
impulse of the impact force	17607 Ns	
coefficient of restitution	R=0,01	
translational speed before the crash	V_A= 54 km/h	V_M= 43 km/h
angular velocity before the crash	ω_A= 0,38 1/s	ω_M= 1,29 1/s
translational speed after the crash	V_A'= 34,8 km/h	V_M'= 12,6 km/h
angular velocity after the crash	ω_A'= -1,43 1/s	ω_M'= 0,6 1/s
impulse of the impact force	16937 Ns	

4. VERIFICATION OF SIMULATION RESULTS BASED ON ANALYTICAL CALCULATIONS

For comparison of the simulation results from p. 3 calculations were performed using the method for analysis of a collision between the bodies having a smooth surface [3, 8, 9]. Input data were obtained by transforming from the rectangular coordinate system Oxy (Fig. 3), treated as global, to the local system of natural coordinates (normal n and tangent t), as shown in Fig. 4.

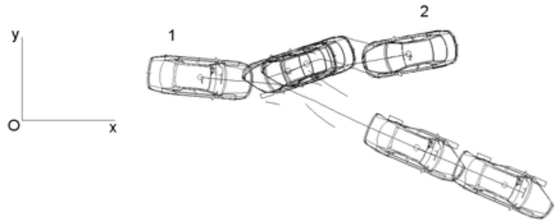

Fig. 3. The coordinate system Oxy for oblique frontal collision. Vehicles during collision along with the deformation of their bodies shown, where 1 - Audi, 2 - Mazda
Source: PC-Crash

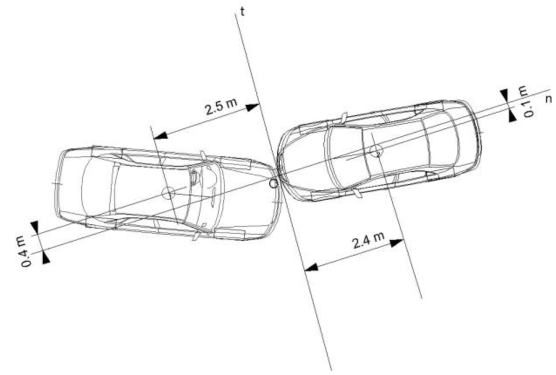

Fig. 4. Location of the vehicles in the local coordinate system for the frontal oblique impact Source: own research

In the protocol of collision generated by the PC-Crash there is a position called the "angle of the vehicle" and the "angle of speed". The angle of the vehicle determines the vehicle gradient towards the x-axis of the global coordinate system Oxy, while the speed angle is the angle between the direction of the velocity vector and the x-axis of the global system. In PC-Crash the occurrence of lateral drift is assumed, and therefore the angles discussed above usually do not have the same values.

To determine the velocity components along the normal and the tangent to the impact plane of the collision, the velocity angle was used, i.e. an angle of the velocity vector with respect to its longitudinal axis. This angle was used then to the distribution of the velocity vector on the normal and tangential direction to the impact plane. In the collision protocol, for each of the simulation the velocity angle was:

- for Audi α1=-5°;
- for Mazda α2=190°.

If taken into consideration that the angles are measured, as shown in Fig. 5, then the angles needed for distribution to the tangential and normal component of the velocity for each vehicle can be easily determined. After simple trigonometric transformations the angle required for distribution of the translational velocity into components for Audi is ρA=23°, while for Mazda ρM=8°. In Fig. 5 the normal impulses of the impact force are also applied at the origin of the Ont coordinate system.

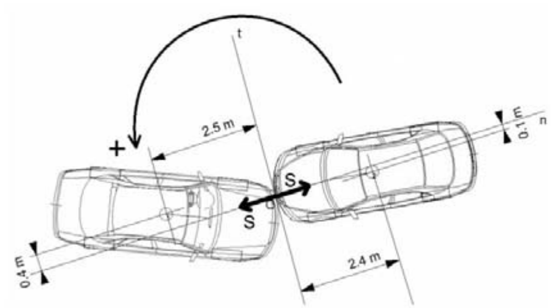

Fig. 5. The positive direction of the angle measurement relative to the x-axis of the global coordinate system and the angle of inclination of the translational velocity relative to the x axis
Source: own research

In Fig. 6 the angles used in the distribution of velocity into components along with the tangential and the normal velocity vectors are shown, as well as the angular velocities respectively for Audi and Mazda. These components were used in analytical calculations, both to determine the relative velocity of the two vehicles at the moment of impact, and the kinematic state after the collision.

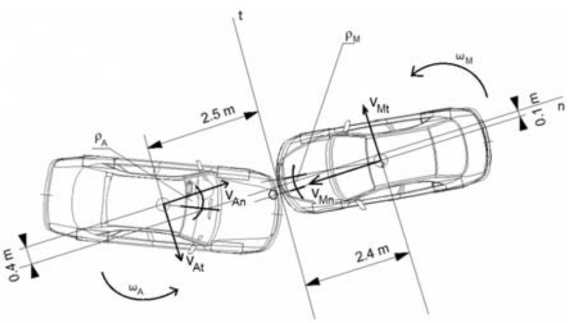

Fig. 6. Velocity components and angular velocities at the moment of collision. Also the distances from the center of the mass of each vehicle to the origin of the local coordinate system Ont are indicated
Source: own research.

Based on the work [3] and Fig. 6 and 8, the problem of collision between the bodies having smooth surface was considered. It was assumed that the tangential velocities during a frontal impact collision change so little that their values before the collision may be equal to the values after it. Thus, in the calculations a model of smooth bodies collision in plane motion was used [3, 8].

In case of an eccentric collision, its normal axis does not pass through the center of mass of any vehicle, so the geometric center of the collision is not on the line joining both centers of mass of the colliding vehicles. For this reason, it is assumed that besides the normal velocity change, eccentric collisions also cause the change in angular velocity. As it is known from the considerations conducted in [3], when considering such type of collision, the impulse of the impact force can be omitted in tangential direction to the collision plane (the plane tangent to the surface of the two vehicles at the time of an instant contact, perpendicular to the road surface at the same time).

The markings, which were adopted in the next part of this chapter, refer to the symbols used in the equations of the analysed collision, where the index "A" indicates the parameters of Audi, and the "M" indicates the parameters of Mazda. Indexes "n" and "t" denote respectively the normal and tangential direction.

Using the markings in Fig. 6 the collision model is represented by the following equations:

a) in the direction normal to the impact plane:

$$\begin{aligned} m_A (v'_{An} - v_{An}) &= -S, \\ m_M (v_{Mn} - v'_{Mn}) &= -S, \end{aligned} \tag{1}$$

b) in the direction tangential to the impact plane:

$$\begin{aligned} m_A (v'_{At} - v_{At}) &= 0, \\ m_M (v_{Mt} - v'_{Mt}) &= 0, \end{aligned} \tag{2}$$

c) in the direction of rotation on a plane of the road, in accordance with the selected angular velocities:

$$\begin{aligned} I_A (\omega'_A - \omega_A) &= -St_A, \\ I_M (\omega'_M - \omega_M) &= St_M, \end{aligned} \tag{3}$$

A set of six equations was obtained with seven unknowns, where the seventh unknown is the impulse of the impact force. Leaving the unknowns in equations (1) – (3) on the left enabled obtaining the kinematic state of the vehicles after collision. It is important to note that the length of a collision is considered to be a very short time, at most a tenth of a second (about 0.1 - 0.2s).

The kinematic state of both vehicles after collision (post impact parameters) was described formulas:

$$v'_{An} = v_{An} - \frac{S}{m_A},\ v'_{At} = v_{At},$$
$$v'_{Mn} = v_{Mn} - \frac{S}{m_M},\ v'_{Mt} = v_{Mt},$$
$$\omega'_A = \omega_A - \frac{St_A}{I_A}, \tag{4}$$
$$\omega'_M = \omega_M + \frac{St_M}{I_M}.$$

To determine the kinematic state in this case it is necessary to know the impulse of impact force in the normal direction of the collision. Using considerations from [3] on the collision of smooth bodies, the impulse can be determined using the formula (5).

$$S = (1 + R) \cdot \varepsilon,$$
$$\varepsilon = \frac{[(v_{An} + \omega_A t_A) - (v_{Mn} + \omega_M t_M)]m_A m_M}{(1 + \vartheta_M^2)m_A + (1 + \vartheta_A^2)m_M}, \tag{5}$$

where:

$$\vartheta_A = \frac{t_A}{i_A},\ i_A = \sqrt{\frac{I_A}{m_A}},$$
$$\vartheta_B = \frac{t_B}{i_B},\ i_B = \sqrt{\frac{I_B}{m_B}}.$$

Distances n_A, n_M, t_A, t_M between the center of mass of each vehicle and the geometric center of the collision were measured in PC-Crash when setting the vehicles at the position of a one-point contact, i.e. the initial moment of impact, which is presented in Fig. 6. The PC-Crash 8.0, as a software for accident reconstruction, enables the distance measurement.

In Tab. 2 the results of analytical calculations are presented, for oblique frontal collision and the three selected values of the coefficient of restitution.

Tab. 2. Results of calculations based on the model discussed e.g. in [3]
Source: own research

	frontal oblique crash	
vehicle	Audi	Mazda
coefficient of restitution	R=0,1	
translational speed after the crash	v_A'= 50 km/h	v_M'= 38 km/h
angular velocity after the crash	ω_A'= 0,18 1/s	ω_M'= 1,36 1/s
impulse of the impact force	2013 Ns	
coefficient of restitution	R=0,05	
translational speed after the crash	v_A'= 51 km/h	v_M'= 39 km/h
angular velocity after the crash	ω_A'= 0,19 1/s	ω_M'= 1,36 1/s
impulse of the impact force	2007 Ns	
coefficient of restitution	R=0,01	
translational speed after the crash	v_A'= 51 km/h	v_M'= 39 km/h
angular velocity after the crash	ω_A'= 0,2 1/s	ω_M'= 1,36 1/s
impulse of the impact force	1931 Ns	

5. CONCLUSIONS

From the calculations it can be observed, that after the adoption of the collision model for the bodies with smooth surfaces, the coefficient of restitution influence mainly the impulse of the impact force and indirectly the post-crash parameters. In practice, such a collision model could be considered as a tool for the preliminary impact assessment and testimony, as to the pre-crash speed in the process of accident reconstruction. This is obviously a very simplified model.

Despite the differences in the results obtained, some regularity can be noticed, both in the simulation results and analytical calculations. Firstly, the impact force impulse decreases along with the coefficient of restitution, second, the translational speed of the vehicles after the collision increase with decreasing of this coefficient. As for the angular velocity, a similar regularity can not be ascertained, because as long as it is substantially equal as a result of calculations, so much as a result of the simulation it takes different values.

The differences in the obtained results can be explained by a number of insights. The impact force impulse in the simulation was even nine times greater than in analytical calculations, which in itself is a reason to consider the correctness of the collision model used in PC-Crash. The difference between the speeds before and after the impact in the collision protocol is so extensive, that it indicates rather a crash into a concrete, non-deformable, rigid barrier, rather than contact with another vehicle, where two vehicles can penetrate each other, as well as such significant speed decrease can dispense only on a longer part of the road and during a longer period of time.

Further research provide analysis of the influence of the coefficient of restitution in the aspect of tangential velocity (assuming the roughness of the surfaces of the colliding vehicles), enabling more realism of the collision model. Another step in the research is to analyse the influence of the restitution coefficients in a complex motion, as has already been indicated in [7] and [8].

BIBLIOGRAPHY

[1] Brach R.M.: *An impact moment coefficient for vehicle collision analysis*. SAE paper 770014.
[2] Brach R.M., *Impact analysis of two vehicle collisions*. SAE paper 830468.
[3] Gryboś R.: *Teoria uderzenia w dyskretnych układach mechanicznych*. PWN, Warszawa 1969. (in Polish)
[4] Ishikawa H.: *Impact center and restitution coefficients for accident reconstruction*. SAE paper 940564.
[5] Ishikawa H.: *Impact model for accident reconstruction - normal and tangential restitution coefficients*. SAE paper 930654
[6] Kisilowski J., Zalewski J.: *Analysis of the Selected Aspects of a Crash Between Two Vehicles*. Advances in Intelligent Systems Research, Atlantis Press, 2013.
[7] Kisilowski J., Zalewski J.: *Selected Aspects of Analysis and Modelling of a Collision Between Two Vehicles*. International Journal of Information Science and Intelligent System, Vol. 2, No. 4, Martin Science Publishing, 2013.
[8] Kisilowski J., Zalewski J.: *Wybrane aspekty analizy i modelowania zderzenia samochodów*. Technika Transportu Szynowego, 10/2013. (in Polish)
[9] Kisilowski J., Zalewski J.: *Wybrane problemy bezpieczeństwa w ruchu drogowym*. Logistyka, 3/2014. (in Polish)
[10] Mnyazikwiye B. B., Karimi H. R., Robbersmyr K. G.: *Mathematical Modeling of Vehicle Frontal Crash by a Double Spring-Mass-Damper Model*. IEEE, 2013.
[11] Niziński S., Wierzbicki S.: *Zintegrowany system informatyczny sterowania pojazdów*. Diagnostyka, nr 30 t.2/2004. s. 47-52.
[12] *PC-Crash a simulation program for vehicle accidents, Operating Manual*, Version 8.0. Dr Steffan Datentechnik, Linz, Austria 2006.
[13] Reimpell J.: *Podwozia samochodów.* WKŁ, Warszawa 2001. (in Polish)
[14] Rose N. A., Fenton S., J., Beauchamp G.: *Restitution Modeling for Crush Analysis: Theory and Validation.* SAE paper no. 2006-01-0908, 2006.
[15] Wicher J.: *O współczynniku restytucji w zderzeniach samochodów, On the coefficient of restitution in the vehicle collisions.* Zeszyty Naukowe Politechniki Świętokrzyskiej. Mechanika, 2006. (in Polish)

Dr inż. **Jarosław ZALEWSKI.** Obszary zainteresowań naukowych: bezpieczeństwo ruchu drogowego, szczególnie wybrane aspekty rekonstrukcji wypadków drogowych, analiza danych statystycznych dotyczących wypadków drogowych, nowatorskie rozwiązania poprawy bezpieczeństwa ruchu drogowego; dynamika pojazdów samochodowych, a w szczególe modelowanie zderzeń samochodów, badanie dynamiki środków transportu w ujęciu modelowym; mechanika z wyróżnieniem mechaniki zderzeń, elementów związanych z wytrzymałością, współpracy koła z nawierzchnią drogi, a także geometrii mas w nadwoziu samochodowym; elementy informatyki, szczególnie symulacje komputerowe oraz programy wspomagające analizę mechaniczną zjawisk występujących w konstrukcjach. Interesuję się również amatorsko grafiką komputerową (szczególnie komputerowe wspomaganie projektowania, tzw. CAD).