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BUILDING OPTIMAL BOUNDARY CONTROL BY THE SUCCESSIVE APPROXIMATIONS METHOD

Marat Orynbet, Gulmira Bayandina, Gula Tolebayeva
K.I. Satpayev Kazakh National Technical University. Institute of Information and Communication Technologies

Abstract. *The article suggests a method for calculating the boundary control in the tape bearings optimal program using the method of successive approximations.*

Keywords: optimal control, boundary conditions

OPRACOWANIE OPTYMALNEGO STEROWANIA Z OGRANICZENIAMI Z WYKORZYSTANIEM METODY KOLEJNYCH APROKSYMACJI

Streszczenie: *Przedstawiono metodę opracowania optymalnego sterowania z ograniczeniami w taśmociągach z wykorzystaniem metody kolejnych aproksymacji.*

Słowa kluczowe: sterowanie optymalne, warunki graniczne

Introduction

Technical systems with gas dynamic lubrication processes have been widely applied at modern enterprises of mechanical engineering, textile industry and instrumentation. Gas lubrication has interesting and important applications in different types of bearings, in particular in the so-called tape bearings (TB). Lubrication of tape bearings by air or some other gas provides advanced technical solutions giving significant advantages over other engineering systems. In technical systems lubricated by air friction is much lower. The wear and tear of tape bearings lubricated by air or gas considerably decreases. The use of an external gas or air supply under pressure significantly expands the scope of application of such tape bearings, as they successfully work at both low and high speeds and efforts.

Tape bearings are often used in processes of continuous production and processing of plastic film, metal ribbon, paper, textile materials and fibers. Tape bearings are used in self adjusted supports of increased stability for high-speed rotors.

The processes taking place in tape bearings are generally described by systems of partial combined equations relative to the main inter-related parameters characterizing the state of tape bearings. These parameters are primarily: thickness of the air film between the flexible and rigid surfaces: overpressure generated by the relative motion of two surfaces or external power; and tension of the flexible tape in their area of interaction through a thin layer of air. The processes taking place in tape bearings are complex physical and mechanical processes; so the desired state of tape bearings with the required values of the basic parameters in the area of interaction of flexible and rigid surfaces is not always achieved in practice. Specific numerical values of these parameters depend on the most number of factors and may go beyond the permitted values in the process of work. The task of more accurate calculation of tape bearing parameters and keeping them within the necessary range is of current interest.

The performance of specified technical systems can be achieved through continuous search for the best conditions of the processes on the basis of fast processing of information about their condition, as well as an optimal process control strategy in real time.

Solving these problems is possible on the basis of the apparatus of mathematical physics and the theory of optimal control of processes with distributed parameters. The mathematical aspects of this problem were dealt with in the papers by A.G. Butkovsky [3, 4], and T.K. Sirazetdinov [9], K.A. Lurye [7], J.L. Lions [6], and other researchers.

Many problems of control of objects with distributed parameters are characterized by the fact that the spatial variations of the object parameters in the evaluation of dynamic properties of processes in the object are fully determined by the boundary conditions of the boundary value problem. The control problem is reduced to the problem of control of the border state or boundary control problem. The mechanism of any boundary control is reduced to the formation of such boundary conditions under which the processes occurring in the object give the desired result.

The mathematical theory of optimal boundary control in systems with distributed parameters was developed in works [5, 6, 8, 9, 10, 11,12, 13].

The problems of programming open-loop control have been considered in hydrodynamics in connection with the fluid flow stabilization problem in the boundary layer. It was proposed to control the flow in the boundary layer by drawing heat off from the surface of the stream-lined body [1], the suction of fluid through porous streamlined body surface [2], as well as traveling elastic wave generated on the surface of the body. The issues of boundary control in closed-loop control systems with distributed parameters have been considered in the problems of stabilization of unstable states in plasma, hydrodynamics, charged particle beams with feedback operators implemented in the boundary conditions of distributed control systems and provide suppression of these unstable states [7].

However, the range of solved problems of boundary control of complex physical and mechanical processes does not include complex technical systems consisting essentially of the physical processes of different nature. Tape bearings described by the Reynolds equation and the equation of the moment-free shell state may be an example.

1. Statement of the problem

The main content of the optimal control problem is the selection of one of various possible implementations of the considered process which would provide for the best process according to some pre-specified criterion. A possibility of choice of different realizations of a process is conditioned by the presence of controls by modifying which we can interfere in its course and apply the desired trajectory. As a rule, this situation is mathematically expressed by the fact that a set of mathematical relations describing processes includes parameters that can be changed within certain limits. In a particular case these parameters may be included in the boundary conditions of the problem and thus affect the behavior of the solution of the constitutive equation.

Let us consider the quasi-static formulation of the problem of calculating the optimal control program. The calculation of the optimal boundary condition of the program for distributed systems in a quasi-static formulation allows the use of modern numerical methods for solving nonlinear partial differential equations using the necessary extremum conditions. These methods are aimed at finding a function that directly meets the necessary and sufficient conditions of optimality. The problem of finding the minimum of the function by means of the necessary conditions is reduced to the problem of finding the roots of the function. And the problem of calculating the optimal boundary of the program is reduced to the solution of boundary value problems for systems of ordinary differential equations. They are easy for programming and allow the use of simple standard programs. This method is similar to the so-called method of adjoint equations with a free right end, yet taking into account the specifics of the boundary control and the restrictions on the control actions. When constructing optimal boundary control, an efficient method of successive approximations is used. There are many methods of successive approximations, such as Euler method, Ritz method, Kantarovich method, and the so-called direct methods to minimize the functional so that it took the smallest possible value.

$$I = \int_{\Omega 0}^{\Omega k} [Q(\Omega.t) - Q^*(\Omega.t)]^2 d\Omega \quad (1)$$

Here Ω is the current angular coordinate, *t* is the current time, Q (Ω, t) is the actual state of the controlled parameter, Q * (Ω, t) is the desired state of the controlled parameter. In all the above said cases it is necessary to calculate the functional gradient. Since the controlled object is described by partial differential equations, the computation of the gradient is reduced to the calculation of the partial derivatives on all the variables included in the minimized functional. In the numerical implementation these operations are quite time-consuming and take up much CPU time, thus making the use of modern computers inefficient. For efficient calculation of the gradient we use the approach which allows taking into account the specifics of the boundary control and solving the optimization problem while reducing the number of computational procedures. The reduction in computation and consideration of the specificity of a solved problem can be made at the expense of the conjugated system, and more exactly with the use of necessary and sufficient conditions of optimality. For this problem with extra conditions that impose restrictions on control action of the type:

$$V[U_r(t)] < 0 \quad (r = 1, \dots, m) \quad (2)$$

we construct an algorithm of optimal boundary control. Thus we formulate the problem as follows. In the plane Ω, t there is a rectangular area Dab, in which systems of differential equations are given.

$$\frac{\partial Q_i}{\partial \Omega} = f_i(\Omega, Q) \;\; (i = 1, \dots, n) \;\; (Q = Q_1, Q_2, \dots, Q_N) \quad (3)$$

On the boundary Γ function Qi is subject to boundary conditions of the first order

$$Q_i(\Omega, t) = Q_i(t) \;\; t > 0 \quad (4)$$

The control variables in this system are some of the main parameters included in the boundary conditions

$$Q_r(\Omega_0, t) = Q_r(t) \;\; (r = 1, \dots, m < n) \quad (5)$$

Due to the fact that the controlled parameter changes differently with time, the value of Q_i is a function of the two variables $Q_i = Q_i(\Omega, t)$. By varying the boundary values $U_r(\Omega_0, t)$ subject to the condition (2) it is necessary to select their meaning so that the criterion (1) had the smallest value for an arbitrary time t.

2. Solution of the problem of optimal boundary control

We divide the spatial coordinate Ω and the time coordinate t into small segments. The segment $[\Omega_0, \Omega_k]$ into M parts by the points $\Omega_0 = 0, \Omega_1, \Omega_2, \dots, \Omega, m = \Omega_k$; and the segment $[0, t]$ - into the N parts by the points $t_0 = 0, t_1, t_2, t_3, \dots, t_n = t$. Then we draw the right lines parallel to the coordinate axes Ω and t via the ends of each of the segments $[\Omega_0, \Omega_k]$ and $[0, t]$ lines. Then the rectangular area D will be divided into MN small rectangular areas Dab $(a = 1,2,\dots\dots, M)$; $(b = 1,2,\dots\dots\dots, N)$. We shall look for the optimization problem solution in the set of control functions $Q_r(\Omega_0, t) = U_r(t)$ $(r = 1,2,\dots\dots, m < n)$ of the constants in each small area Dab, and uniques $Q(\Omega, t)$ providing condition (4). Thus, the solution of the formulated problem is reduced to minimizing the function $I(U_{rb})$ of Nm variables, where

$$I(U_{rb}) \quad (6)$$

are defined by (1), provided that the system (3) was solved in $U_r(\Omega_0, t) = U_b(\Omega_0, t)$ for D_{ab}. Thus we have the problem of minimizing the function of a finite number of variables. If we use the gradient method, it is necessary to make calculations by the formula (6) Nm times, i.e. to solve a system of differential equations (3) Nm times. However, if we use the necessary optimality conditions for the determination of the first variation of the variables of Functional I on variables U_r^b, it is possible to organize effective computational methods to significantly reduce the number of computations. For their implementation there is a kind of function $I[U_r^b(\Omega_0)]$ used, as well as functions $U_r(\Omega_0, t)$. The solution will be sought in the form of a piecewise constant function

$$U_r(\Omega_0, t) = \sum_{b=0}^{N} {}^* U_r^b \quad (r = 1,2, \dots, m < n) \quad (7)$$

Let us consider the functional

$$I^* = \int_{\Omega 0}^{\Omega k} [Q(\Omega.t) - Q^*(\Omega.t)]^2 + \int_0^t \int_{\Omega 0}^{\Omega k} \sum_{i=1}^{n} £_i [f_i \frac{\partial Qi}{\partial \Omega}] d\Omega dt \quad (8)$$

We shall put the integral (8) as follows:

$$I^* = \int_{\Omega 0}^{\Omega k} [Q(\Omega.t) - Q^*(\Omega.t)]^2 + \\ + \sum_{b=0}^{N-1} {}^* \int_b^{tb+1} \int_{\Omega 0}^{\Omega k} \sum_{i=1}^{n} £\, i[fi \frac{\partial Qi}{\partial \Omega}] d\Omega dt \quad (9)$$

The expression for the variation of Functional I* can be written as follows:

$$\delta I^* = 2 \int_{\Omega 0}^{\Omega k} [Q1(\Omega.t) - Q^*1(\Omega.t)] \quad \delta Q1(\Omega, t) +$$

$$\sum_{b=0}^{N-1} {}^* \int_b^{tb+1} \int_{\Omega 0}^{\Omega k} \sum_{i=1}^{n} £_i [\sum_{s=1}^{n} {}^* f_i \frac{\partial fi}{\partial qs} \delta Qs -_{\partial Qi/\partial \Omega}] d\Omega dt \quad (10)$$

By integrating the expression (10) part by part we obtain:

$$\delta I^* = 2[Q1(\Omega, t) - Q^*1(\Omega.t)] - c + \int_b^{tb+1} £r(\Omega_0, t) dt \quad 11)$$

The following condition should be satisfied:

$$2[Q1(\Omega, t) - Q^*1(\Omega.t)] = c$$

Then the formula (10) to express the variation is presented in the following form:

$$\delta I^* = \int_b^{tb+1} £_r(\Omega_0, t) \delta U_r(t) dt \quad (12)$$

If functions $Q^0(\Omega,t)$, £(Ω,t) at arbitrary control $U^0(t)$ satisfy the system (3), the increment of I under the influence of variation $\delta U(t)$of boundary control U(t) is calculated by the formula (12).

Let the variation of the functional $\delta U(t)$ of boundary control U(t) have the following form:

$$\delta U(t) = \begin{cases} const & tb < t < tb + 1 \\ 0 & tb < 0 \;\; tb > t \end{cases}$$

Taking into account (12) we obtain the following formula to calculate the derivative:

$$\frac{\delta I}{\delta Urb} = \lim_{\Delta Urb \to 0} \frac{\Delta I}{\Delta Urb} = \frac{\partial I}{\partial Urb} = \int_b^{tb+1} £r(\Omega_0, t) dt \quad (13)$$

Or

$$\frac{\partial I(Urb)}{\partial Urb} = \int_b^{tb+1} £r(\Omega_0, t) dt \quad (14)$$

Let us suppose that function $£_r(\Omega,t)$ is continuous along Ω and piecewise continuous along t. The required derivative $\frac{\partial I(U_r^b)}{\partial U_r^b}$ for all

Nm derivatives can be defined by a piecewise constant function U(t) by solving a system of differential equations (3) with the boundary conditions (4) and carrying the Q (t) in sufficient number of points. Then it is necessary to solve the dual system:

$$\frac{\delta £_r}{\delta \Omega} = -\sum_{t=1}^{n} \frac{\delta f_i}{\delta Q_s} £_I \quad (s = 1, \dots, n) \tag{15}$$

To the system (3) with

$$£_s(\Omega, t) = 0 \qquad (s = 1,2 \dots, n) \tag{16}$$

$$£_1(\Omega, t) = c.$$

After that, the desired derivative $\frac{\delta I}{\delta U_r^b}$ can be defined by the formula (14) in cross section $\Omega = \Omega_0$.

In other words, there is no need to solve system (3) Nm times and it is reduced solving this system and its conjugate system (15) just once with the boundary conditions (5) and (16).

Next, using the method of successive approximations it is necessary to solve the system at the next control impact by the formula:

$$U_r^{b,j+1} = U_r^b + \delta U_r^b rb \tag{17}$$

where:

$$\delta U_r^b < \text{e} \quad |e| > 0 \quad (r = 1,2, \dots, m)\ (b = 1,2 \dots, N-1)$$

In formula (17) j is a step of approximations.

If the original system solution (3) under the boundary conditions (4) is stable, it can be expected that when we integrate the adjoint system (15) with the boundary conditions (16) in the negative direction of the axes, its solution will also be stable and it does not cause computational difficulties, which sometimes can take place in nonlinear systems.

If there are restrictions for the control action parameters when the algorithm of calculating the optimum boundary program is made, it is necessary to meet the necessary requirements at each step:

$$U_r^{min} < U_r^{b,j+1}(t) < U_r^{max}.$$

3. Conclusion

The described method is used to derive the necessary and sufficient conditions for optimality of the boundary control of tape bearings. The optimality conditions are derived using indirect methods, by means of the direct satisfaction of optimality necessary conditions based on the classical variations calculus with application of Lagrange multipliers.

The authors suggested a method for calculating the boundary control in the tape bearings optimal program using the method of successive approximations. The calculation method is based on the solution of the so-called systems of coupled equations, taking into account the specifics of the boundary control and using a simplified algorithm for solving a quality boundary function.

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Ph.D. Marat Orynbet
e-mail: orynbet.marat@mail.ru

Ph.D. in Engineering Science, Docent of Automation and Management Department of Kazakh National Technical University after K.I. Satpayev, the famous scientist in the fileld of robotic technology systems management, the author of more than 100 scientific papers. For the long time was engaged in robotic automation problems of technology and non-ferrous metal industry of the Republic of Kazakhstan and Russia.

M.Sc. Gulmira Bayandina

Gulmira Bayandina has the Degree of Master of Engineering with a specialty No. 553650 of Automation and Control. For the current moment Ms. Bayandina occupies the post of senior teacher of the Automation and Control Department. The main scientific field includes the works in such spheres as: optimization of controlled objects and synthesis of automated control systems. She teaches the following courses: "Automation of typical technology processes and manufacturing", "Automation of technology processes", "Design of automated manufacturing systems". She is also the author of 10 teaching and guiding papers.

Gula Tolebayeva
e-mail: gul_bayan@mail.ru

Graduate student of Institute of Information and Telecommunication Technologies of KazNTU named after K.I. Satpayev. Has experience in the field of information technology, particularly in software development. Diploma work related to software development of the simulation model of motion of small space systems around the center of mass.

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