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APPLYING CHAOS THEORY TO RISK MANAGEMENT IN A GAME OF CHANCE

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Purpose: The purpose of the paper is to illustrate the usage of techniques known from chaos theory to analyze the risk
Design/methodology/approach: In this case the objects of application are winnings graphs of different poker players. Two types of players are presented; winning players (those with positive expected value) and breaking even players (expected value close to zero).
Findings: Charts were analyzed with a fractal dimension calculated with the box method.
Originality/value: Relation between fractal dimension and Hurst exponent is shown. Relation between risk in sense of chaos theory and players' long-term winning is also described. Further applications of chaos theory to analyze the risk in games of chance are also proposed.

Keywords: risk analysis, game theory, hazard game, chaos theory, fractal dimension.

Category of the paper: Research paper.

1. Introduction

Applying chaos theory to risk management is a new trend in game theory. However, it has already developed so far that no analyst can ignore the new paradigm. The strand started by Edgar Peters (Peters, 1994), motivated the application of this new approach to market analysis, and Ehlers (Ehlers, 2005) and Borowski (Borowski, 2019) finally suggested practical application of fractals in FRAMA (fractal adaptive moving average). It is used by stock analysts to find signs of upcoming upswings or downswings.

For many people poker is considered as a pure game of luck, where the only payee is the organizer of the game (e.g. casino) due to rake. However, there are others who indicate that the casino is not the only winner. They claim that choosing a proper strategy concludes long-term winnings being equal to other player's loss. According to this definition poker may be considered as gambling. Risk in poker can be minimized by applying a proper strategy. Usually the strategy is based on knowledge about opponents' style, manners, behaviour etc. So far, risk management based on chaos theory usually referred to capital markets and stock exchange. In this paper another application is shown. Chaos theory can be used to analyse risk of poker play of different players.

2. Chaos theory

Chaos theory describes phenomena, where little changes of parameters cause large effects. The equations are very responsive to these changes. Commonly such chaotic behaviour of systems is called the butterfly effect. Little change of starting conditions may cause huge changes of its influence in long-term scale. Good illustration of this effect can be Edwards Lorenz question: "Predictability: does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" (Alejandrino, 2011).

Weather analysis gave a good example of importance of chaos theory in science. As shown in (Kalnay, 2003) non-linear differential equations are used to weather forecasting. These equations are being solved by supercomputers. As Lorenz proved (Lorenz, 1963) in 1963 exact weather calculation is impossible. It was also shown that equation which describes weather also tends to show chaotic behaviour. This means that in long-term we can expect randomness. Therefore, we can expect a correct forecast for the next day, however, it is impossible to predict exact values of temperature or pressure for the upcoming month.

The chaos theory also describes geometrical objects called fractals. Fractals are self-similar or infinitely subtle objects (Kudrewicz, 2007). In this paper we focus on self-similar objects which means that a subset of such set is similar to the entire set. More specific description can be found in (Falconer, 2003) and (Kudrewicz, 2007). Image 1 shows an example of a self-similar fractal.

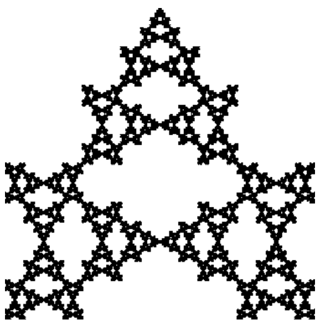

Figure 1. Banach fractal (Fiałkowski, 2006).

As Peters showed in (Peters, 1994) graphs in capital market also tend to become self-similar.

Let us imagine that a three dimensional sheet of paper has no thickness. Assuming that, we can consider it as a two-dimensional object which can be described by means of Euclidean geometry. If we crumple this paper sheet into a paper ball it does not become a three-dimensional object but it is also hard to consider it as two dimensional one.

In such case chaos theory comes up with a fragmentary dimension or fractal dimension, which cannot be described with an integer value. Describing crumpled sheet of paper with Euclidean geometry is a difficult task, even for a computer program. Fractal dimension is much help in this case. It helps us describe how do objects fill the space.

Usually an object exists in a space which has higher dimension than the object itself. Crumpled paper is a good example, which can be considered three dimensional but obviously does not fill the entire three-dimensional space. Likewise, a graph of a function is called two dimensional but also does not fill the entire two-dimensional space. The same can be said about a square or circle.

Mandelbrot (Mandelbrot, 1982) gives an example of limited usage of Euclidean geometry when calculating length of Great Britain coastline. He claims that it is impossible to calculate the exact length of the coastline. The accuracy of the calculation will always depend on the length of the measure we use. We can imagine trying to measure the coastline using two different measures; one kilometre long and one meter long. Obviously, using longer measure will make us ignore many gaps and bays in the coastline. However, they will be included when using shorter measure. This leads to the conclusion that the shorter measure we use the more precise value we gain. Therefore, the length of a coastline depends on the length of used measure. Mandelbrot proposed the fractal dimension to calculate Great Britain coastline.

There are many ways to calculate the fractal dimension. One of them is the box method. Algorithm of calculating the fractal dimension with box method is the following. Firstly, we take the starting size of the box and denote it as „r". Secondly, we draw in the space a net of boxes sized „r". Then we count the boxes which contain any part of examined object and obtain the number N(r). Then we decrease the size of the box linearly or exceptionally depending on the chosen method (Weisstein, 2018), (Weisstein, 2019) and obtain consecutive numbers „r" and N(r). We draw the function y = log (N(r)) as function from log(1/r). We designate the line of regression. We measure the gradient of the regression line obtaining the fractal dimension. The formula for the box dimension is the following:

$$D = \frac{log\left(N(r)\right)}{log\left(\frac{1}{r}\right)} \tag{1}$$

This algorithm was elaborated basing on (Peters, 1994), (Weisstein, 2019) and (Sutherland, 2002).

Similarly we can calculate the fractal dimension with the circle method obtaining dimension of Bouligand-Minkowski described in (Weisstein, 2019) and (Sutherland, 2002). According to measures (Peters, 1994) fractal dimension of Great Britain coastline is 1.3. In the case of Norway it is 1.52.

Fractal dimension describes how torn the object is against a space one dimension higher. Having that said, we can claim that Norway in comparison to Great Britain has got much more cragged coastline. From (Peters, 1994) we know that the more cragged the coastline is, the closer to 2 its fractal dimension is.

In probability theory it is common to colligate risk with variance. The chaos theory does not negate this idea but introduces a new approach to problems connected with risk. Following Markowitz (Markowitz, 1952) we can talk about higher risk on a stock market when we observe higher variability on stock prices. This variability can be described with standard aberration. However, we can use standard aberration only if we have got to do with a random system. Studies described in (Peters, 1994) showed that risk calculated with standard aberration often does not give the proper sense of risk in terms of economy.

Figure 2 illustrates the example found in (Peters, 1994) showing the difference between understanding the risk in terms of chaos theory and probability theory.

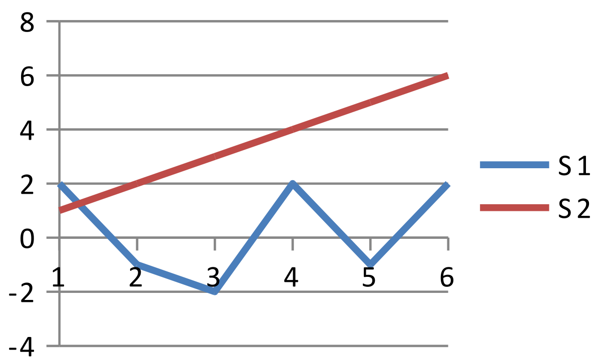

Figure 2. Rates of return S1 and S2 (Peters, 1994).

For stocks S1 and S2 the standard aberration is respectively 1.86 and 1.87; almost equal in both cases. S1 fractal dimension is 1.42 and in S2 it is 1.13 (Peters, 1994). In terms of economy Lower risk is observed when investing in S2. Statistical analysis proves that the risk is equal in both cases, however the fractal dimension points unambiguously to S1 as the stock with higher risk.

In fractal analysis Hurst exponent is used very commonly. The formula which enables us to calculate the Hurst exponent is given in (Peters, 1994). It has a key meaning in designating the risk from a given graph. Strict relation between fractal dimension and Hurst exponent has been shown in (Peters, 1994) and is given by the formula:

$$H = 2 - D \tag{2}$$

Therefore, in this paper we focus on evaluating the risk only with the fractal dimension.

3. Games of chance

3.1. Definition of a game of chance

In this paper we define game of chance following (Act of 2009) as a random slot machine game or mutual bets, where more or less the result of the game is random. Many people do claim that such games with random factor do not always have to be gambling games. In such games, having proper skills, it is possible to obtain expected value above zero. These people use 'gambling' expression to describe games where independently to players' skills their expected value is always below zero.

Good example of such a game can be roulette. Let us assume that player bets S on a field which has probability of hitting k/n. For standard 37 roulette fields the expected value is – S/37 for S money put at risk. It happens because when the ball hits our filed we gain S*n/k and when it misses, we get zero. This makes the casinos expected value positive.

Poker can be an example of a game where expected value depends on players' skills. The game itself is too complex to prove it formally. Therefore a hypothetical game of luck which is generally based on the same idea as poker is introduced.

Let us imagine a game for two players who draw randomly 0 or 1 and play using the following rules:

1. Each player draws a number independently. Both can have the same number. Probability of each number is 0.5.
2. Each player can add 1 PLN to the pot. The players do not know of the other player's actions.
3. If one player added 1 PLN to the pot and the other did not, the one who added wins the pot (1 PLN).
4. If both players added 1 PLN the winner is the one with higher number and they take the pot (2 PLN). If the numbers are equal they split the pot (1 PLN for each of them).
5. If none of them addend 1 PLN next turn is played (go back to 1).

Let us assume that player A has got the following strategy; if I draw 1, I add to the pot, if I draw 0, I don't. Player B has any other strategy which can be expressed this way; if I draw 1, I add in x% of cases; if I draw 0, I add in y% of cases. Situation where player drew 0 and adds to the pot will be called a bluff.

Let us show that the expected value of player A is:

$$\begin{aligned} &EV(playerA) = ½ * ½ * x * 0 \\ &+½ * ½ * (1 - x) * 0 \\ &+½ * ½ * y * 1 \\ &+½ * ½ * (1 - y) * 0 \\ &¼y \end{aligned} \tag{3}$$

The first two lines of (3) describe situation where both players drew 1. In the first line player B adds to the pot (with probability x) and does not add (with probability 1-x). Third and fourth lines describe the situation where player A drew 1 and player B drew zero. Third line describes those cases where player B bluffs, and fourth line when they do not add 1 PLN to the pot. Situation where player A does not add 1 PLN does not affect their expected value.

In conclusion, player A expected value is equal or higher than zero, regardless of player's B strategy. The above game can be complicated and optimal strategies for those games can be searched.

3.2. Mathematical model of a game of chance

Game of chance is a game in terms of game theory. It consists of n players. In a game of chance we can talk about rounds. A single round in roulette will be a single spin and in poker it will be one hand. Each player takes part in k rounds (this number can be different for different players). During each round player takes m actions. Each action has its specific structure depending on the kind of game.

Taking action influences players result in the relevant round. Summary score will give us players result in the entire game. In each round a player can bring into the round some amount of money denoted w. Actions taken by a player with probability p_i will make them earn (or lose) z_i, of money, where $i=1,2,\ldots,m$ depends on what kind of game they play. Therefore, we can conclude that the expected value of their winnings in each round is given by the formula:

$$EV = \sum_{i=1}^{m} p_i z_i \tag{4}$$

Actions taken by player influence the partial benefit in a relevant round. However, expected value of an action, especially in poker, is very difficult to describe with a mathematical formula. Therefore, this problem is being left unfinished.

Restrictions are also parts of the model. In each round potential maximal loss ($min(z_i)$) cannot be higher than absolute value of w. Without this restriction it would be possible to obtain a negative balance. Games of chance are usually prepaid, therefore the above restriction is justified.

Sum of all p_i from 1 to m must be 1.

Expected value of winnings of all players is negative in most games of chance. Absolute value of this expected value will be called rake. It is the expected value of casinos winnings in a relevant round. Some players however, depending on their strategy of play, can have positive expected value of winnings. This was shown in subchapter 3.1. This phenomenon is the subject of discussion whether poker should be classified as gambling.

Analysis of the general mathematical model of a game of chance leads us to the conclusion that the casino has always a positive expected value of winnings. Players cannot win playing against the casino. The only chance for a player to achieve a positive result of the game is by playing out their opponents which requires a proper strategy.

3.3. 10 player No limit Texas Hold'em Cash Game description

Texas Hold'em is a game of chance where having a proper strategy should guarantee higher expected value of winnings than the opponents'. It is played at the table; usually ten players take part in the game. One of them is marked as the dealer. Each round the dealer mark moves to the next player clockwise. Player sitting left to the dealer is marked as small blind. The player sitting left to the small blind is marked as the big blind.

At the beginning of each round the dealer deals two cards (from a standard 52 cards deck) face down to each player (to the dealer as well). These two cards are called a players starting hand. After dealing the cards and betting blind bets the round begins. Blind bets must be bet by small and big blind. Usually small blind is equal to half of the big blind, e.g.1 PLN/2 PLN.

The round can be divided into four parts called: preflop, flop, turn, river. In each part a player can take the following action: pass (fold), call a bet (call), wait (check) or raise the bet (raise). A player takes action after all players before took action. In case of raise a player must tell the exact value of the raise (limited by the amount of money they have at the table). In the preflop phase player left to the big blinds is the first to act. In other phases first player left to the dealer who has not folded, is the first to act. The round can be finished in any case when all players except one have folded. Phase finishes when all player to act called the bets of the other players or folded.

Player wins the round when all players except them folded or their cards setup is best among other players left in the round. Before the flop, three cards are dealt cross the middle of the table face up. Before turn and river another card is dealt into the table face up bringing its total count to five. These are the community cards any player can use (combining them with their starting cards) to make the best five card poker hand. A player loses every time if at least one player has a better five card setup. If two or more players have the best combination the pot is divided between them equally.

There is a restriction in Texas Hold'em. A player cannot check when any other player before them raised. In practice this constrains to the situations where someone raised or the player is in the preflop phase and has not called the big blind bet. Player cannot raise more than the amount of their money at the table. If a player raises for all their money and other players have more money than them, we consider a special case where a side pot is being created (Rules, 2019). After folding no action can be taken during the round by the player who folded. Of course, they lose the money they had brought into the pot. Other special restrictions in Texas Hold'em are described in (Sklansky, 2005).

Player can bring into the pot certain amount of money using raise and call actions. Depending on taken actions during the round they increase or decrease their expected value of winnings. It is a game with incomplete probabilistic information, so the player needs to predict other player's range of starting hands, basing on action they take.

In Texas Hold'em we can very rarely be sure of winding or looping a hand. Winning or looping almost always depends on other players' cards. However, we can predict (with some probability) range of their cards according to action they took this round and knowledge about history of their play. Thanks to this we can assume some range of opponent's cards and evaluate worthwhileness of specific actions we can take. The above is a base for building Texas Hold'em strategies described in (Become, 2019).

3.4. Partial mathematical model of 10 player No Limit Texas Hold'em Cash Game

Texas Hold'em is a game of chance. Number of players, in this case is n = 10. Each player takes part in any number of rounds k. A player can be earlier forbidden to play if they lose all their money brought to the game.

The number of taken actions during a single round is limited only by the size of players stack (amount of their money at the table). Impossibility to act after folding is also an important restriction. When player calls or raises they can bring to the pot no more money than they have left at the table.

Determining the influence of a certain action to the expected value is a mathematically difficult problem. However, we can estimate expected value of winning in a round by predicting the range of starting cards of our opponents and calculating worthwhileness of each action.

For example let us imagine a situation where player holds third best starting hand QQ (pair of queens) in preflop phase. The player has got 800 PLN left (200 PLN was already invested in the pot). The only opponent raises for all their money which is 1,000 PLN. Therefore, there is 1,200 PLN in the pot. Player judged the range of opponent's cards to only better hands than theirs (pair of kings or aces). Using (Become, 2019), the player calculated that probability of winning is about 18%. Therefore the expected value of calling can be calculated:

$$EV = 0{,}18 * 1200 - 0{,}82 * 800 = -440$$

Having that said, we can see that calling that bet is absolutely unjustified. However, let us imagine the same situation but this time the player assumes wider range of cards; not only pair of aces or king but also pair of queens, pair of jacks, and AK (ace and king). This range gives them 47% probability of winning the round (Become, 2019). The opponent will still win more often (53%) (Become, 2019), but the expected value of calling this bet will be:

$$EV = 0{,}47 * 1200 - 0{,}53 * 800 = 140$$

As we can see the expected value of calling is positive. That means that although the opponent is still more likely to win this hand, calling this bet is the correct action.

It is also worth mentioning that the estimated expected value is usually measured basing on the history of the game. Main assumption here is that players change their style of play very little in long-term and the strategy of the player remains the same... It is being assumed that a good player wins equality of ten big blinds every hundred hands. Therefore expected value of such player in a single round is 0.1 of a big blind. For example, when playing 1 PLN/2 PLN blinds, the expected value is 0.2 PLN.

It is also important to mention that a player needs to play against strategies which are losing against their strategy. It was explained in subchapter 3.2. The Casino always has positive expected value. In this case the casino's expected value is the rake taken from every pot.

Concluding, No Limit Texas Hold'em is a game of chance which fits the assumptions made in subchapter 3.2. Full mathematical model of poker game is very complex. What is more, finding a winning strategy for such a complicated model is almost impossible.

3.5. Description of types of players depending on their benefits

Specific strategy of play depending on other player's strategies can result in possible winnings or loses. We can divide players into three groups basing on their benefits:

1. Winning – having positive expected value.
2. Losing – having negative expected value.
3. Breaking-even – their expected value is close to zero.

It is crucial to remember that the rake taken by the casino should be added to the expected value. Some casinos like Full Tilt Poker offer partial return of the rake known as rake-back. Thanks to this fact breaking-even players can also benefit from their play. Those players usually characterize themselves with very risky play on the border of profitability.

To judge to which group specific player belongs we can do mathematical analysis of their and their opponents' strategies. However, remembering that the opponents switch very often and mathematical model of the game is very complex, it appears almost impossible to conduct such strict mathematical analysis. Example of the solution to this problem is the assumption that opponents use one of the popular strategies (described in (Become, 2019)). Next we should implement poker Simulator and input players strategy and their opponents' strategies. Then after running the Simulator we can see how our strategy copes with popular strategies. This approach can be named empirical estimation of the expected value.

The approach described above has got few important disadvantages. It completely ignores opponents' alternation or the fact that they can make different decisions according to so-called table image of the player. More commonly used approach is the analysis of historical data about the hands that a player already played. Analysing respectively big sample of such data we draw the graph of winnings of the player. Then we analyse the trend of the player; if it is ascending then we can tell that it is a winning player. If the trend is descending then, of course, we observe a losing player.

Examples of graphs of two players; winning player and breaking-even player (as described above) are shown in figures 3 and 4.

The strategy is being entered into the program which plays instead of the player, often on many tables simultaneously. We need to mention here that usage of such programs is forbidden in many casinos. To achieve a successful strategy we need to model behaviour of our opponents. Proposal of modelling such parameters as probability of folding and probability of bluffing was described in detail in (Salim, and Rohwer, 2005). Finally, basing on conducted strategy tests, we can reason if our strategy is winning after long time of play by the robot.

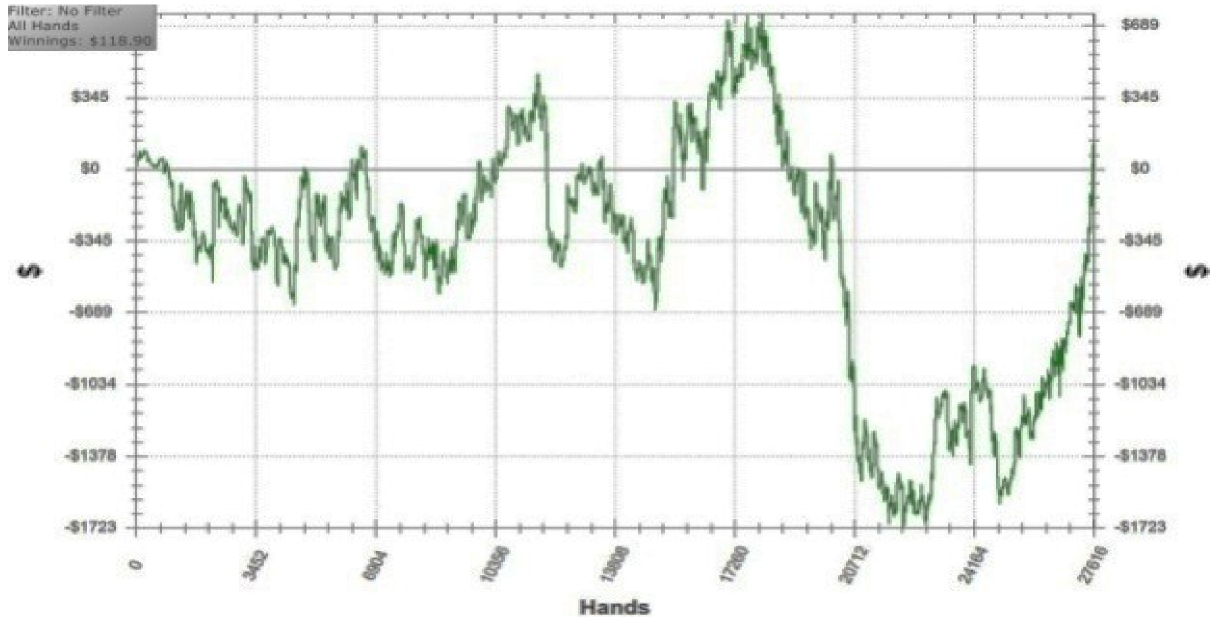

Figure 3. Winnings graph of breaking-even player (Forum, 2019).

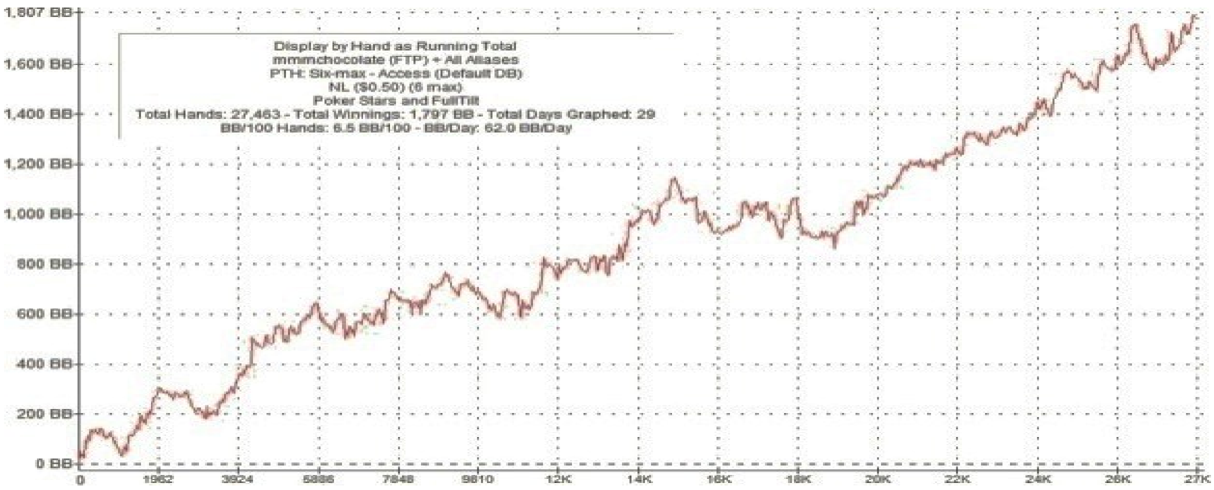

Figure 4. Winnings graph of winning player (Forum, 2019).

Other popular method of testing strategies is using programs which play poker instead of the player. It is possible because there are many casinos where anybody with a computer with access to Internet can play.

Concluding, we can use different meanings of expected value of winnings in poker. There are many ways to designate it empirically.

3.6. Risk in poker

Risk in poker is a term which is ambiguous in respect to economy or mathematics. Therefore, measuring risk in poker is not an easy task. Every action in each round is saddled with risk. Even possessing better hand in one phase of the round, the situation may turn around in the very next phase. Risk in poker can be measured in a sense of a single action, single round or whole set of rounds which were played.

Risk in context of a single action from scenarios shown in subchapter 3.4 can be calculated by the formula for the standard aberration Std(X):

$$Std(X) = \sqrt{\sum \left(x_i - E(X)\right)^2 p(x_i)} \quad (5)$$

where:
E(X) – expected value of random variables X,
$p(x_i)$ – probability of x_i random variable, for i = 1, 2, 3, …, n.

Therefore risk of the first action described in subchapter 3.4 is 768 PLN. The risk of the second action is 998 PLN.

Speaking about risk in poker it is impossible to ignore the sill of proper bankroll management which enables players to minimize the risk. Let us imagine a situation shown in subchapter 3.4 but with ten times smaller stakes. Then we can win 120 PLN or lose 80 PLN. Let us assume that we came cross that situation ten times. Then, the expected value of the first case is still -440 PLN and 140 PLN in the second case. However, the standard aberration changes noticeably. After calculations from formula given before, standard aberration is about 243 PLN in the first case and about 316 PLN in the second case. We conclude that proper bankroll management can give us the same expected value while the risk is noticeably minimized. Unfortunately it affects time of gaining the same profits which in this case is ten times longer.

Risk analysis basing on historical data of player hands is an interesting task to consider. Now, we mainly come across empirical analyses based on other players' experiences like in (Become, 2019) and (Forum, 2019). Risk analysis there is often based on intuitive understanding of the problem and it is common that advice given there has no support in mathematics. Therefore, application of new methods of risk analysis to achieve results is mathematically correct.

4. Risk management in a game of luck

4.1. Transforming graphs into images which allow fractal risk analysis

Graphs given by the poker analytic programs usually contain other data which disables instant fractal analysis. Poker Tracker is an example of such program (Poker tracker, 2019). Therefore we need to erase that data to prevent them from disturbing the results of fractal analysis. Our goal is having black graph on a white background.

Each of the graphs contains data about axis markings. In case of winnings graphs on one axis there are winnings measured with set currency or big blinds multiplications. On the other axis is the number of hands played till the time of generating the graph. Additionally, some programs add special net on the graph which helps to read values from particular rounds. A text box with information about the graph is also very common. All the information is illustrated on images 3 and 4.

We usually come across graphs which colour is different than the colour of the axes. If we have uncompressed image (e.g. *.png or *.gif) then it is enough to change every colour to white (except the graph colour, of course) and then change the graph colour to white. If we have a compressed image (e.g. *.jpg) then the colour of the graph is not flat. Sometimes it even gets too similar to axes colour. Then we need to use more advanced software like Gimp or Adobe Photoshop. We use the „magic wand" tool (Adobe Creative Team, 2005) to select and cut the graph of our interest.

This way we obtain black graph on white background without any redundant hypes.

4.2. Using program 'Fractalyse' to measure fractal dimension

The program Fractalyse (Fractalyse, 2019) is used for broadly understood image fractal analysis. It lets us, inter alia, calculate the fractal dimension of an image. It uses linear or exponential increase of the size of the box. It also generates the graph mentioned in subchapter 2.3. From the angle of the regression line it obtains the fractal dimension.

It is also worth mentioning that the program can calculate the fractal dimension in a way similar to Bouligand-Minkowski method described in (Weisstein, 2019). The main difference is that it uses squares, not circles, to calculate the fractal dimension. The program searches for the lowest number of squares of an even size which can cover the examined object. According to (Fractalyse, 2019) it is more general method of calculating the fractal dimension.

We have chosen five graphs of winning players and five graphs of breaking even players to conduct our research. Firstly the graphs were prepared as explained in subchapter 4.1. Graphs were found on the Internet on sites covering poker subjects (Forum, 2019). Most of them were published with a request to evaluate them by other players. We can conclude that they are authentic graphs of players.

4.3. Research process

Research was conducted in the following way: Firstly the selection was done. Players with obvious winning tendency were selected. Players with results close to zero were selected to the second group. Then, using Fractalyse (Fractalyse, 2019), the calculation of fractal dimension was performed. Additional measures were performed on different numbers of played hands to check if length of playing period has influence to the measures. Results are shown in Table 1.

4.4. Research results

The results were put together in table 1. Columns mean respectively: Lp. – number of measure, L – number of hands used to generalise the measure (in thousands), Q – quality of the player 1 – winning, 0 – breaking even, D – fractal dimension of the players graph.

Table 1.
Research results

Lp.	L	Q	D
1	20	0	1.259
2	73	0	1.427
3	100	0	1.396
4	220	0	1.354
5	380	0	1.377
6	40	1	1.044
7	50	1	1.087
8	100	1	1.132
9	275	1	1.198
10	400	1	1.033

The research was conducted with a minimal size of the box equal to one pixel with linear increase (Weisstein, 2019) equal to four pixels and stop condition when a single box covers the entire graph.

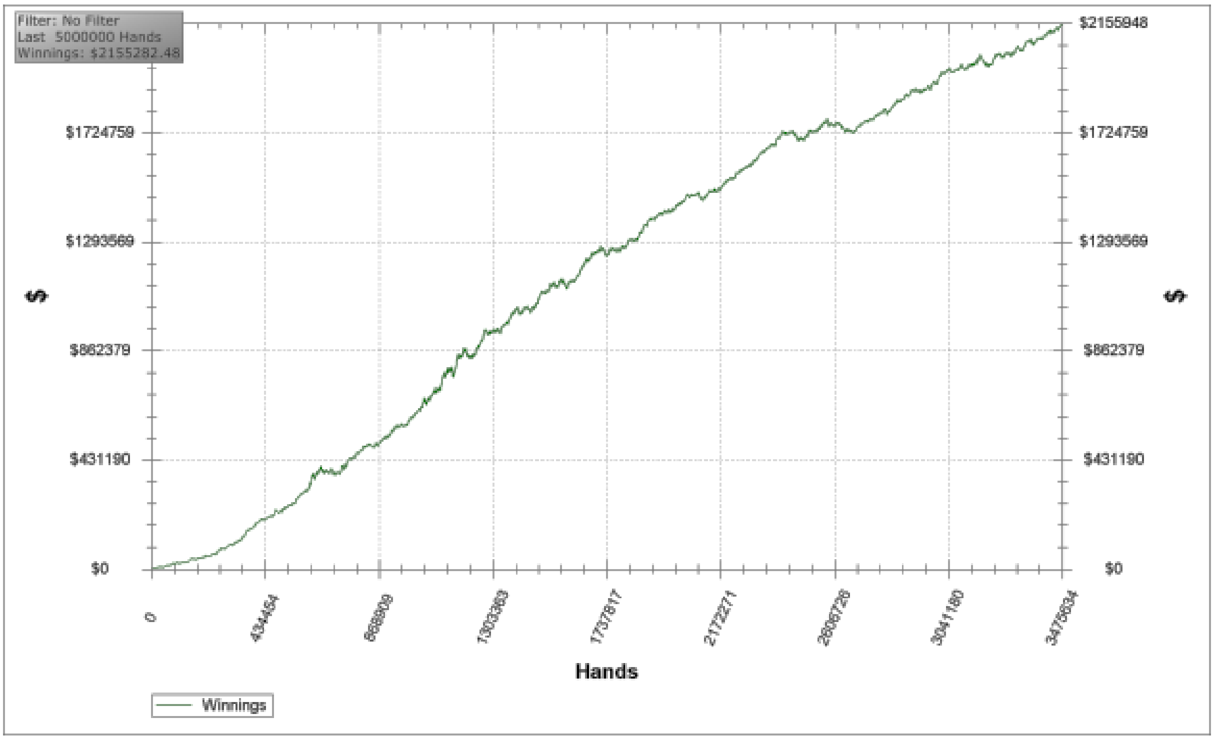

Figure 5. Graph Lp. 10; low fractal dimension and almost straight graph are worth noticing.

4.5. Observations

The results are shown in table 1. They show a strict relation between expected value of a player and fractal dimension of their winnings graph generated in time of their play. Let us notice that for winning players fractal dimension was in any case no higher than 1.2. On the other hand, players who break even find their graphs fractal dimension between 1.2 and 1.5. The fact confirms that the fractal dimension is not dependent on quantity of played hands.

Analysing graphs and table results we can also conclude that the lower the fractal dimension is the less ragged the graph is. Examples of graphs with lowest and highest fractal dimensions are shown in figures 5 and 6.

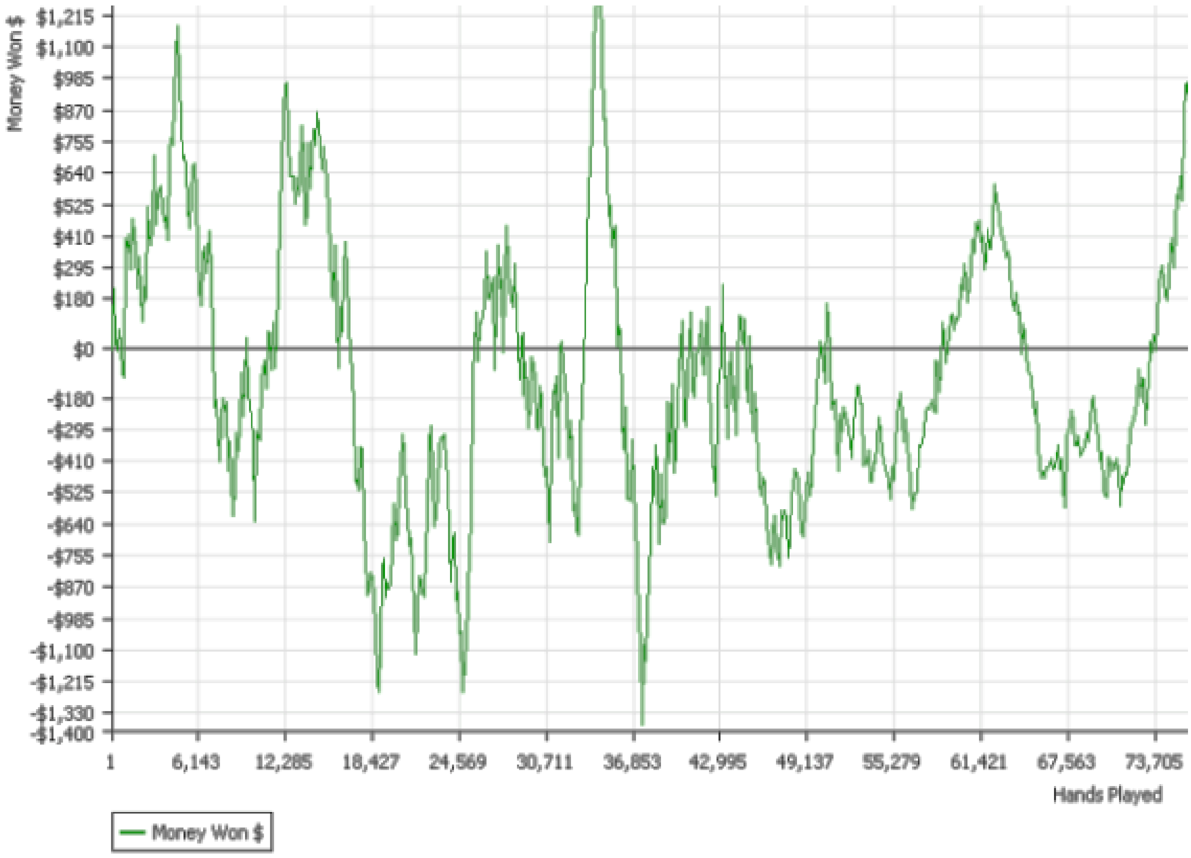

Figure 6. Graph Lp. ,2; high fractal dimension and very ragged graph are worth noticing.

4.6. Conclusion

According to Markowitz's theory (Markowitz, 1952) the higher the alternation is the higher the risk is. In our case poker table is the market and risk is measured with fractal dimension. We can conclude that breaking even players play much in a more risky way than winning players, because the fractal dimension of winning players is much lower.

Taking into consideration the fact that the fractal dimension is not dependent on number of played hands we can conclude that these graphs are self-similar. It satisfies the basic fractal condition described in (Kudrewicz, 2007). Having that said, we can consider fractal analysis of poker players' graphs as manageable and faultless.

The fact that all values of fractal dimensions are shown in figures, 1.5 gives a strong base to claim that poker is not a random result game. According to (Peters, 1994) when the Hurst exponent belongs to interval [0.5, 1] we may consider a reliable strategy. The higher the Hurst exponent is the less probable the randomness is. Therefore it is claimed that, considering winning players, there exists a relation between consecutive observations of the graph and possibility of highly probable prediction of further part of the graph. According to Hurst theory the same phenomenon can be observed but in a limited degree. Having that said, we can claim that breaking even players are more expose to randomness and their play is more similar to random stray. However, as long as their fractal dimension is 1.5, their play does not show vital traits of random stray according to fractal analysis, so from a purely mathematical point of view we can consider their play as following some kind of strategy.

Graph analysis lets us confirm the assumptions about the relation between fractal dimension and how much ragged the graph is. The higher the fractal dimension is, the more ragged the graph is.

Summarizing, we can reason that fractal dimension describes the risk in compliance with intuitional understanding of risk. Fractal analysis leads us to a conclusion that players who use winning strategies make less risky decisions in comparison to other players.

5. Performance review and further research

In this paper we have shown both basics of chaos theory and basics of games of chance, especially No Limit Texas Hold'em. Mathematical model of a game of chance was analysed and detailed to partial model of Texas Hold'em poker. Research, which purpose was to analyse the risk in poker, was conducted.

Risk management in poker is not an easy task. Fractal analysis is surely a new approach to this problem. Nowadays, strictly probabilistic approach is used to analyse the risk of poker games and on its base proper bankroll management is determined. Similar situation is observed on capital markets, however, in this case we do not invest in well prospering player. The payers invest in themselves and they are the ones who need tools which will lead them to increasing their benefits and effectiveness of their game.

When developing work covering risk management in poker it is worth. above all, to pay attention to players who win and want to win even more as well as those who have just began their journey to the top. It is crucial to consider minimization of the risk of decisions being made as well as increasing expected value. Another important aspect is using the influence of psychology in a poker game. There are plenty directions in which this subject can be developed.

The main conclusion of our deliberations about risk in games of chance is that there is a strong need of further development of this inscrutable and often underrated branch of science.

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