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Non-Gaussian statistical measures of control performance*

by

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Abstract: Statistical approach to Control Performance Assessment (CPA) is of great practical importance. This is particularly visible in process industry, where there are many PID loops. They are often assessed with measures derived from the Gaussian probabilistic density function. Standard deviation, variance, skewness or kurtosis form the majority of applied indexes. The review of data originating from process industry shows, however, to the contrary, that these signals have rather non-Gaussian properties and are mostly characterized by fat-tailed distribution disable the ability. Investigations show that strong disturbances may significantly disable the capacity of proper assessment. Standard measures often fail in such cases. It is shown that non-Gaussian measures can help with this problem. Various disturbances are tested and compared. Results show that fat-tailed distributions are an interesting alternative. They are less sensitive to disturbance shadowing and still make possible loop dynamic assessment.

Keywords: Control Performance Assessment, statistical measures, PID, disturbance shadowing, non-Gaussian functions

1. Introduction

Performance of the control system affects significantly the overall operation of any industrial process or installation. It enables reaching the highest outcome, while simultaneously satisfying technology and equipment limitations. The selection of the appropriate control philosophy and its fine tuning enables reaching the expected accuracy. However, control implementation should not be considered as a single shot activity. The plant owner aims at sustainable benefits. The control system has to satisfy long-term expectations. Unfortunately, the process fluctuates, the used technologies upgrade and the equipment is subject to continuous aging and breakdowns. Thus, the control loops operate in

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a non-stationary, time-varying environment. The controllers must follow these changes. Maintenance of the I&C (Instrumentation and Control) infrastructure is crucial. The system performance must be permanently monitored. The assessment, backed up by the proper measures and followed by the rehabilitation activities, brings significant benefits, see Domański et al. (2016).

Control Performance Assessment is very often a part of an overall feasibility study. Such an activity includes various methodologies, measures and procedures that enable continuous control system monitoring, and detecting the bottlenecks, as presented by Jelali (2013). They show reasons and suggest remedies. The knowledge and benchmarking procedures are of great importance. Proper measures are crucial, as their selection affects detection efficiency. Industrial processes, especially in chemical engineering, are subject to numerous internal and external impacts, like the noises, time-varying disturbances, cross-coupled feedbacks, changeable delays, nonlinearities and equipment failures. We should not forget about frequent human interventions through manual operation, tuning and operator biasing. All of these factors greatly affect monitoring accuracy.

The above mentioned influences reveal themselves in the form of signal disturbances of various properties. Stochastic processes, often represented simply through the Probabilistic Density Function (PDF), are used to express their nature. Routine practice assumes their normal (Gaussian) character. On the other hand, the review of several hundreds of process control loops, originating from different industries, shows that only $\approx 6\%$ of real variables have normal properties, see Domański (2015). The majority of them has fat-tailed histograms. Lévy α-stable distribution seems to be the best fitted in more than 60% cases, while the rest is well described by the Cauchy function. These observations gave impact for the present work. What is the effect of loop disturbances and their properties on control measures? What kind of statistical quality indexes might be proposed and how are they affected? How the overall detection process can be improved with the new measures?

Statistical measures for control quality were investigated with Gaussian and non-Gaussian approaches, see, for instance, Choudhury et al. (2004) or Liu et al. (2008). The idea is to verify which factors of probabilistic distributions can be used effectively and robustly for the Control Performance Assessment (CPA) task. New tools have to be designed, as the industrial data display non-Gaussian properties. Unfortunately, real-time process data are difficult to analyze. It is not easy to proceed with the root-cause analysis for complex processes with incomplete information.

It is observed that similar statistical properties can be obtained with relatively simple SISO loop simulation. PID strategy is used as the control algorithm, because it is the most popular industrial strategy. Various disturbances with varying distribution functions are applied to simulate loop properties. It is observed that assessment effectiveness varies depending on disturbance properties. It is especially visible when disturbance is characterized by Lévy or Cauchy properties. The proper solution is hidden in the "*shadow*" of the rugged detection surface. This effect is called disturbance shadowing. A simple simulation example enables to find out the proper reasons. It is expected that existence of such phenomena in simple simulations should make it possible to use the reasoning for the real data as well.

The work covered in this paper fits into the general stream of research on various non-Gaussian control quality measures (statistical and fractal). First observations were obtained during the realization of the industrial projects with the review of several hundred control loops, originating from various process industries (power generation, chemical engineering, gas processing), see Domański (2015). The main observation indicates that popular approaches, using Gaussian factors are not fully appropriate. Review of the industrial examples has shown that complex installations with several cross feedbacks do not enable to confirm the assumptions that control error signal is composed of independent realizations. These data are frequently showing long-tail histograms with large number of points distant from the mean value and normal distribution. The majority of the control loops do not have Gaussian properties. These observations have brought rationale for further research.

The review of the real-time process data reveals new issues. Nonetheless, industrially available data are not good for root-cause analysis. Process complexity and many, often unknown disturbances do not enable to make any definite decisions, as shown in Domański et al. (2016). Following that path, the research switched towards the simulation. The study started with the simplest structures using the second order delayed system. The analysis has been carried out for the P, PI, PID in Domański (2016a), and for GPC predictive controllers in Domański and Ławryńczuk (2017) and Domański and Ławryńczuk (2017a). Additionally, various statistical, persistence, multi-persistence, fractal and multifractal measures have been investigated, see Domański (2017). The here presented analysis fits into the general picture of the similar analyzes. The focus of the work described in the present paper is on the comparison between different statistical indexes, not on the model types.

The paper starts with the formulation of the Control Performance Assessment task. The presentation of the common indexes is followed by the description of the analyzed statistical distributions and the introduction of new measures. The simulations for PID SISO control loop, influenced by various disturbances, form the main part. Conclusions close the paper with the discussion and the presentation of open issues.

2. Control Performance Assessment

Industrial reviews, shown in Jelali (2013), indicate that the large number of the control loops perform poorly. 60% is badly tuned, while even more (85%) loops suffer from inadequate design. Control engineers apply different methods to assess the quality of the control loops. They mostly use personal experience, which is often unique. The first reported loop assessment was done by Åstrom (1967) for paper machine. Development of CPA tools has accelerated due to the work of Harris (1989) and the introduction of Minimum Variance index.

It is a constructive index, because it enables to determine, how far the system is from the ideal solution. The research now covers many different aspects of the control processes, like nonlinear behavior in Horch and Isaksson (1998), large scale systems in Paulonis and Cox (2003), predictive control in Schäfer and Cinar (2004), valve stiction in Choudhury et al. (2008), oscillations in Srinivasan and Rengaswamy (2012), predictive control in Wei and Zhuo (2009). Besides, we may find alternative solutions using Fourier transform in Schlegel et al. (2013), nonlinear fractal data analysis in Pillay and Govender (2014), wavelets in Nesic et al. (1997), neural networks in Zhou and Wan (2008), orthogonal functions in Jelali (2013), entropy in Zhang et al. (2015), big data in Gao et al. (2016), persistence analysis in Domański and Ławryńczuk (2017), and data mining in Das et al. (2017). Resulting from the literature survey, we may enumerate six main classes:

1. time-domain indexes using the step response, like undershoot, rising, maximum, settling time, overshoot and peak amplitude, see Spinner et al. (2014),
2. time-domain approaches using normal operation data: Mean Square Error (MSE), Integral of Square Error (ISE), Integral of Absolute Error (IAE), Amplitude Index (AMP), see Spinner et al. (2014),
3. minimum variance and model-based indexes, see Harris (1989),
4. statistical indexes based on the PDF factors, like standard deviation, variance, skewness, kurtosis, scaling and shaping, see Choudhury et al. (2004),
5. alternative indexes, like fractals in Pillay and Govender (2014), multi-persistence in Domański (2016) or entropy in Zhang et al. (2015),
6. business KPIs (Key Performance Indicators) mostly expressed in currency units, see Bauer et al. (2016).

The present paper focuses on the statistical approach. PDF factors are confronted with the loops impacted by the disturbances having different probabilistic properties. They are compared with basic integral indexes (MSE, IAE). The results are also benchmarked with the undisturbed indexes calculated for the ideal step response, i.e. overshoot denoted by κ and settling time T_{set}. It is important to select appropriate loop signal for evaluations. One might use process variable (PV), control error (difference of setpoint and PV) or controller output. Various authors often select and discuss PV. Control error signal is used in this analysis as it is independent of setpoint changes and should fluctuate around zero. Thus, there is no need for detrending. Additionally, we have a clear meaning of its goodness. Any measure of the mean value should be zero.

2.1. Time domain CPA indexes

The step test measures (area index, output index, R-index, idle index), though very informative, have limited applicability. The industry hardly agrees to perform any dedicated experiments as they disturb normal operation. This is argued using the safety or economy reasons. Integral indexes are also widely used.

The most popular ones are: MSE and IAE. Formally, the differences between MSE and IAE are minor. They are used alternately, without reflection [????]. However, in the literature we may find some differentiating comments. Seborg et al. (2010) showed that MSE minimization punishes large deviations from the setpoint and produces aggressive action. On the other hand, it has been suggested by Shinskey (2002) that IAE has the closest relationship to economics. In the here presented analysis both indexes are calculated and compared.

2.2. Statistical CPA measures

Gaussian normal distribution forms the basis for the most popular measures. Mean value and standard deviation are commonly used. Mean μ reflects variable average value, i.e. steady state error in case of control error analysis. Its desired value is zero. Standard deviation σ informs about signal variability, as it is directly related to th broadness of the variable histogram. Higher σ means larger variations, while small values reflect less fluctuations.

These measures are frequently followed by higher order statistics, like skewness or the kurtosis. Skewness is a measure of PDF asymmetry. Kurtosis describes the concentration of the distribution function. Statistical measures of the control error address three features of the control loop:

1. steady state error,
2. symmetricity (nonlinearity),
3. histogram broadness, i.e. dynamic response and disturbance rejection.

Control error mean value reflects the steady state error. We wish to have it equal to zero. Non-zero values may originate from the following reasons: steady state error and lack of integration, improperly set operating point due to the MV signal constraints violations (MV saturation) or operator's intervention and manual operation. Thus, identification of the above reasons may result in the solution: adding integration onto the control loop, modification of the operating points or the actuator's exchange and the determination of the reasons for the operator's intervention.

Unfortunately, it is hardly achievable to define the threshold by saying what value of the mean is good. It depends on the measurement accuracy, plant requirements or operator habits. In each case it should be adapted to the plant control and technological personnel.

Symmetricity is reflected through the skewness measures. Good control should result in the zero value of this measure. Non zero values may be the results of the process and/or actuator nonlinearities or human intervention (manual operation). The solutions are very similar to the previous case of the non zero steady state error. Additionally, the linearization of the controller output (meant to linearize the actuators nonlinearities) might be a solution to the problem. The proper values of the goodness thresholds are still relative, depending on the concrete local preferences.

PDF broadness measures are even more difficult to interpret, although they are very useful and very frequently used. In terms of the broadness measures we may use standard deviation (or variance) for the normal distribution, and scale or shaping coefficients for other PDFs. They reflect the control error histogram fatness, which is related to the intensity of the fluctuations, these being associated with the level of the process disturbances and the quality of control. This is connected with the step response measure of the overshoot.

In fact, these measures are not constructive. We never know their optimal, the best achievable, value. Single one-time evaluation does not bring any information. Thus, we may only use them in the relative way. We may say how the control quality changes in time, by, for instance, estimating what is the effect of the modifications introduced into the process and/or control system.

Similarly as in the previously discussed cases, the thresholds cannot be estimated *a priori*. One may decide upon them only after long term observations of the process or using plant operator's/engineer's expert knowledge. In fact, no discussion concerning these thresholds has been found in the literature.

The importance of the Gaussian measures and their common acceptance are unquestionable. The majority of researchers and control engineers use them. We have to be aware that they are valid only when the variable has Gaussian properties. The respective test may be simply performed graphically through visual inspection of the histogram. Further, we may calculate the mean square error between the histogram and the PDF fitted to data. Normality may be also validated through statistical tests, like the Kolmogorov-Smirnov (KS) normality test. It is possible, as well, to check the normality hypothesis through other tests, like Lilliefors, skewness or α-stability, see Lilliefors (1967).

The review of several hundred of industrial loops, originating from different process industries (power, chemical engineering, refinery), presented in Domański (2015) has shown that only a small fraction ($\approx 6\%$) holds normal properties. It was found out that the majority of them has fat-tailed properties and Lévy α-stable ($> 60\%$) or Cauchy distribution ($\approx 30\%$) are the best fitted. This is most probably caused by unknown, complex nature of real industrial loops with non-linear, time varying cross coupling, nonlinearities, variable time delays and frequent human interventions. These observations suggest the direction of research. Control loop is impacted by various types of disturbances (Gaussian, fat-tailed, others). Different statistical factors are evaluated and their relevance checked.

2.2.1. The Cauchy PDF

The Cauchy PDF is an example of a fat-tailed distribution. The shape for the values further from the mean does not decay so fast as it is with the normal PDF. It is a symmetric function. Its parameters have the meaning similar to that for the normal PDF (1). Location factor $\delta \in \mathbb{R}$ informs about distribution position, while scale $\gamma > 0$ reflects variability:

$$PDF_{\delta,\gamma}(x) = \frac{1}{\pi\gamma}\left(\frac{\gamma^2}{(x-\delta)^2+\gamma^2}\right). \tag{1}$$

2.2.2. The α-stable (Lévy) distributions

The Lévy α-stable distribution belongs to the family of stable distributions. It has more degrees of freedom, see (2), as it is parametrized by four parameters:

$$PDF_{\alpha,\beta,\delta,\gamma}(x) = exp\left\{i\delta x - |\gamma x|^{\alpha}(1 - i\beta l(x))\right\}, \tag{2}$$

where

$$l(x) = \begin{cases} sgn(x)\, tg\left(\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ -sgn(x)\,\frac{2}{\pi}\ln|x| & \text{for } \alpha = 1 \end{cases},$$

$0 < \alpha \leq 2$ is called *stability* index, $|\beta| \leq 1$ is the *skewness* parameter, $\delta \in \mathbb{R}$ is distribution *location*, and $\gamma > 0$ is the *scale* factor.

The stability parameter α is responsible for shape. Location δ provides information about function position. Additionally, we have two more shaping parameters: β informs about distribution skewness, while scaling γ has the meaning very similar to that of γ parameter of Cauchy PDF. There may be different combinations of them. In particular, $\alpha = 2$ reflects independent realizations, especially, for $\alpha = 2$, $\beta = 0$, $\gamma = 1$ and $\delta = 1$ we get the exact normal distribution equation.

2.2.3. The Laplace PDF

The Laplace distribution is called double exponential. It forms a function of differences between two independent variables with identical exponential distributions. Its probability density function is given by formula (3):

$$PDF_{\mu,b}(x) = \frac{1}{2b}e^{-\frac{|x-\mu|}{b}}, \tag{3}$$

where $\mu \in \mathbb{R}$ is the *location* factor and $b > 0$ is the scale parameter. The function shape decays exponentially and is characterized by the parameter b.

2.2.4. The Generalized Extreme Value distribution

The GEV distribution is a family of continuous PDFs, developed within the extreme value theory to combine the properties of different distributions (Gumbel, Fréchet and Weibull). The GEV parameters, see (4), do not reflect exactly the mean and standard deviation, which are calculated with (5) and (6), respectively:

$$PDF_{\mu,\sigma,\xi}(x) = \frac{1}{\sigma}\chi(x)^{\xi+1}e^{-\chi(x)}, \tag{4}$$

where

$$\chi(x) = \begin{cases} \left(1 + \left(\frac{x-\mu}{\sigma}\right)\xi\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ e^{-\frac{x-\mu}{\sigma}} & \text{if } \xi = 0 \end{cases},$$

$\mu \in \mathbb{R}$ is the PDF *location*, $\sigma > 0$ is the *scale* parameter, and $\xi \in \mathbb{R}$ is the *shape* factor.

$$mean_{GEV} = \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi} \cdot \Gamma\left(1 - \xi\right) \tag{5}$$

$$stdev_{GEV} = \sqrt{\frac{\sigma^2}{\xi^2}\left(\Gamma\left(1 - 2\xi\right) - \Gamma^2\left(1 - \xi\right)\right)}. \tag{6}$$

Thus, for the steady-state error detection one may use the location factor μ or the PDF mean value $mean_{GEV}$. Factors σ and ξ are responsible for shape. Additionally, the GEV standard deviation $stdev_{GEV}$ may be considered.

2.2.5. Fitting PDF to data

There are several methods dedicated to fitting of PDF to histograms. Cauchy parameter evaluation uses Maximum Likelihood Estimation (MLE), implemented in the Octave successive quadratic programming solver by Axensten (2006). The α-stable fitting applies regression approach proposed by Koutrouvelis (1980). The Laplace and GEV parameter estimation refers to the use of the MLE algorithm, as well.

3. Simulations

3.1. The simulation setting

The research presented in this work originates from the industrial experience. The main hypothesis of this work is that the extension of the classical Gaussian measures with the non-Gaussian approach is appropriate and useful. Unfortunately, the thorough validation in the industry is hardly achievable. There are several reasons for that: low real process data availability, rarely existing industrial processes with frequent control system tuning and the process complexity itself that does not enable clear detection of the phenomena behind the visible observation. It is especially the last issue that is decisive. Simulation performed in the simplest possible environment should enable to reflect the respective phenomena.

Dedicated simulation experiments are run to check the hypothetical effect of disturbance properties on control quality assessment. Experiments are run in the environment presented in Fig. 1. It is a control template common in the process industry, represented by a single-element SISO loop. The controlled process is linear and is described by the second order transfer function with delay. Such a selection is intentional. This type of model is very popular in process industry. One may say that it even accounts for the majority of cases.

There are two disturbances, representing industrially existing situations: the random measurement noise *d(t)* with relatively low magnitude 0.05 and the disturbance added before the process. The impact of the measurement noise is not addressed, as its effect is minor, see Domański and Ławryńczuk (2017a).

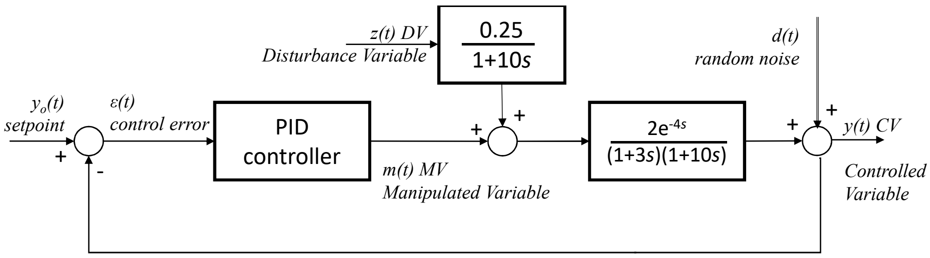


Figure 1: The closed loop simulation environment

The persistence properties of the simulated control error are similar to the ones observed in the industrial data. The idea behind simulations is to find out how factors of different functions behave and how they may be used in the detection of poor control in the case of significant non-Gaussian disturbances.

The simulation investigation is conducted to verify the adopted hypothesis, i.e. to see if non-Gaussian PDF factors may be used as the control quality measures and how they are affected by different types of disturbances. Thus, each measure will be used for certain disturbance scenario. Each scenario consists of many loop simulations for different PID controller settings. On the other hand, the optimal setting is known. The analysis of the 2D surface of the selected measure value depending of the PID setting will enable to verify whether the best measure value points out the known optimal setting.

The whole analysis is done according to the following procedure. The variable histograms are plotted to check the statistical properties of the loop. Next, each scenario is run and for each measure the 2D surface is evaluated. The analysis of the results obtained is performed and the observations are formulated. Six scenarios (properties of *z(t)* variable) are considered:

1. **SC_Ndist**: no disturbance added, i.e. $z(t) = 0$.
   The goal is to check how detection works with different statistical factors, while the loop is not impeded by any disturbance.
2. **SC_Gauss**: Gaussian process: $\mu = 0$, $\sigma = 1.5$ and gain $K = 0.75$.
   The goal is to check how detection works with different statistical factors, while the loop is impeded by disturbance of Gaussian character.
3. **SC_Cauchy**: Cauchy PDF: $\mu = 0$, $\gamma = 0.15$ and gain $K = 0.2$.
   The goal is to check how detection works with different statistical factors, while the loop is impeded by disturbance of Cauchy character.
4. **SC_Levy**: α-stable PDF: $\alpha = 1.35$, $\beta = -0.05$, $\gamma = 0.35$, $\delta = 0$ and $K = 0.75$.
   The goal is to check how detection works with different statistical factors, while the loop is impeded by disturbance of α-stable character.
5. **SC_Lap**: Laplace PDF: $\mu = 0$, $b = 1.5$ and gain $K = 0.75$.
   The goal is to check how detection works with different statistical factors, while the loop is impeded by disturbance of Laplace character.

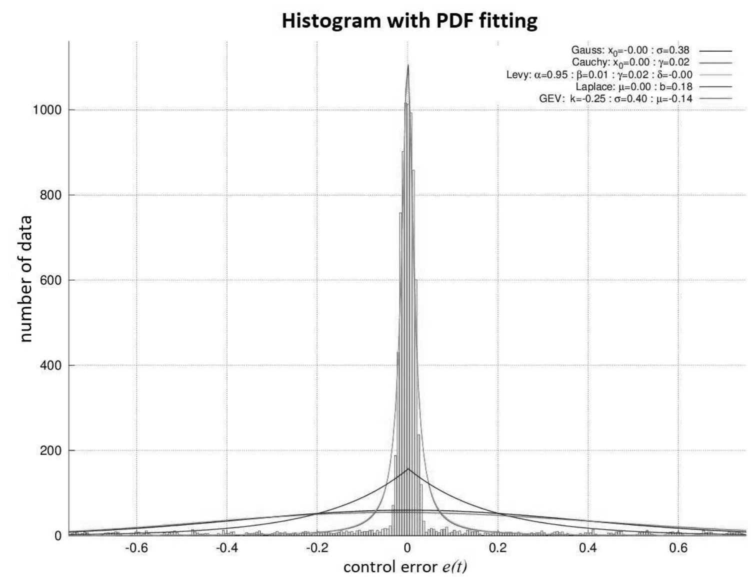


Figure 2: Histograms for optimal PID tuning with various disturbances: no disturbance

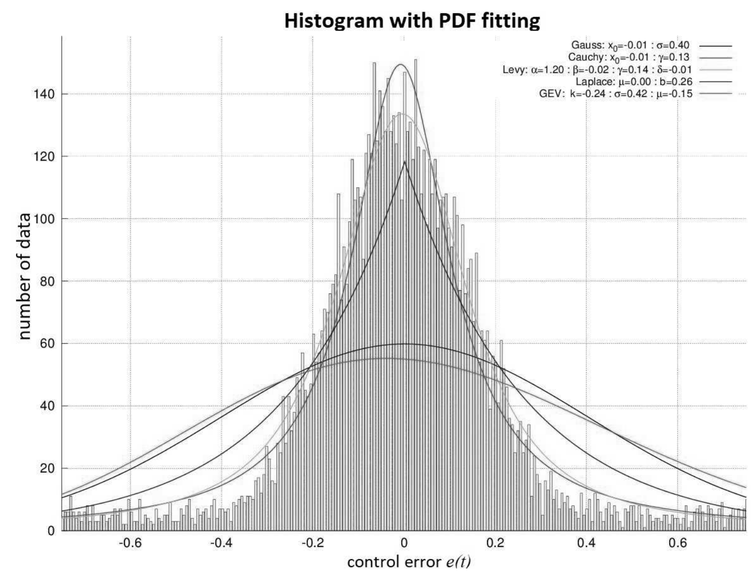


Figure 3: Histograms for optimal PID tuning with various disturbances: Gauss

6. **SC_GEV**: GEV function: $\mu = 0,\ \sigma = 1.5,\ \xi = 1.0$ and $K = 0.75$.
   The goal is to check how detection works with different statistical factors,

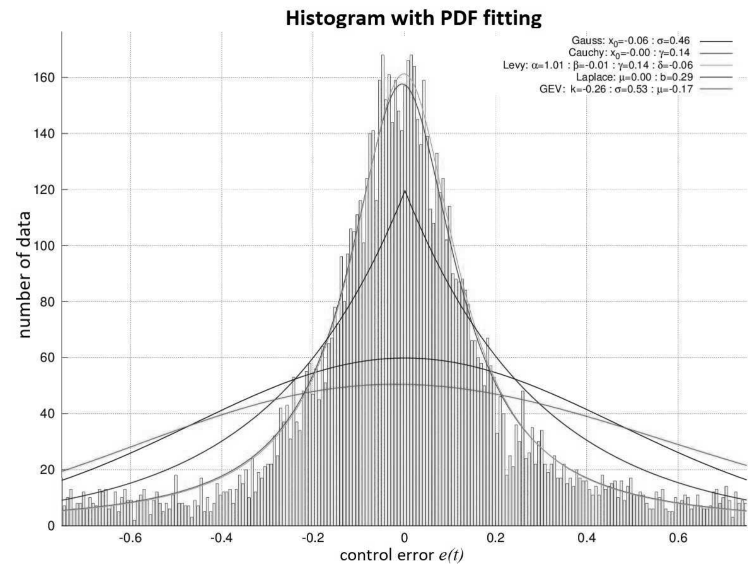


Figure 4: Histograms for optimal PID tuning with various disturbances: α-stable

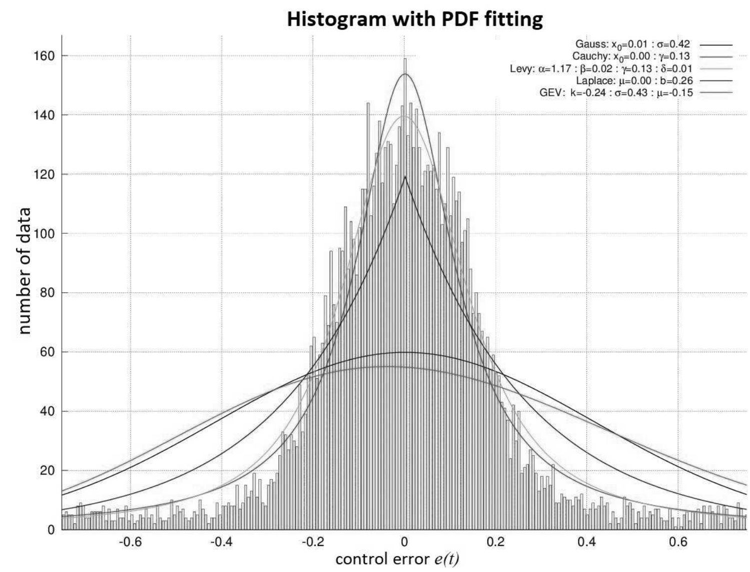


Figure 5: Histograms for optimal PID tuning with various disturbances: Laplace

while the loop is impeded by disturbance of GEV character.

In all the cases considered the setpoint is simulated as a rectangular wave.

The length of datasets is 20000. The standard parallel PID algorithm is used. At first, optimal tuning is found with the standard Octave function: $k_p = 0.809,\ T_i = 13.767,\ T_d = 2.861$.

Loop analysis starts with the review of the control error histogram. Its properties should enable choosing the proper factors, reflecting tuning quality. Selection of wrong measures can result in shadowing of real control performance. Control error histograms for four major scenarios are evaluated (Figs. 2 through 5).

The diagrams present the fitted probabilistic density functions of different properties. We notice that none of the histograms is Gaussian. It is due to long tails associated with two separate operating regimes of the control loop: transient states occurring after setpoint changes and steady state operation covering the majority of time instances. Such a character of control errors often happens in real situations with steady setpoint control.

Selected statistical indexes are evaluated as loop assessment measures. They are based on the factors of the probabilistic functions. Both integral indexes are calculated for reference. Overshoot κ and settling time T_{set} are evaluated to benchmark the real loop tuning quality for the ideal step test.

Performance assessment is investigated through consecutive simulations of each disturbance scenario for different PID parameter tuning. Time derivative is fixed at the optimal tuning value $T_d = 2.861$ in all cases. The other two parameters vary.

The proportional parameter changes within the interval $k_p = 0.20 \ldots 2.00$ with increment of 0.1, while integration time in $T_i = 1.0 \ldots 40.00$ with increment of 1.0. Altogether, 760 simulations are run for each disturbance variant. The experiment results are analyzed and compared in two ways: in quantitative way by the comparison of best solutions and in qualitative way through observation of contour plots for the obtained assessment results.

3.2. Quantitative comparison

Table 1 presents all the results gathered from all simulations. Each measure is grouped into columns. Rows show scenarios. For each scenario the following numbers are presented: values for optimal tuning of k_p and T_i, value of the selected measure (KPI) and values of the overshoot and the settling time.

We may see that disturbance scenarios have strong influence on the controller selected by each measure. This impact also varies with the applied measure. However, there in no single one, general measure effective in any conditions. Each of the indexes has different preference and is biased towards some specific performance. Gaussian standard deviation prefers sluggish control with relatively minor overshoot. Proportional gain is almost the same for all scenarios, $k_p = 0.9 \ldots 1.2$. Simultaneously, the integration time is relatively large $T_i > 33$ resulting in long loop settling.

To the contrary, both of the fat-tailed distributions select a more aggressive tuning. They end up with smaller gains $k_p = 0.8 \ldots 1.1$, while integration time

disturbance		Gauss.sig	Cauchy.gam	Levy.gam	Levy.alf	Lap.b	GEV.sig	GEV.stdev	IAE	MSE
	kp	1.1	**0.8**	**0.9**	0.7	0.9	2.0	2.0	**0.9**	1.1
	Ti	40	**15**	**15**	35	15	40	40	**15**	40
SC_Nist	KPI	0.351	**0.02**	**0.021**	0.356	0.177	0.363	0.13	**0.177**	0.123
	κ	5.3%	**0.0%**	**4.9%**	0.0%	4.9%	69.7%	69.7%	**4.9%**	5.3%
	Tset	130.5	**70**	**33.7**	300	33.7	299.5	299.5	**33.7**	130.5
	kp	1.1	**0.8**	1.1	1.3	1.1	0.9	1.9	1.1	1.1
	Ti	40	**15**	16	9	16	31	36	16	40
SC_Gauss	KPI	0.369	**0.02**	0.129	1.128	0.232	0.376	0.146	0.232	0.133
	κ	5.3%	**0.0%**	17.6%	50.0%	17.6%	0.0%	63.6%	17.6%	5.3%
	Tset	130.5	**70**	32.6	18.1	32.6	200	50.7	32.6	130.5
	kp	0.9	**0.8**	0.8	**0.6**	1.0	0.9	0.9	1.0	0.9
	Ti	40	**15**	15	**14**	19	40	40	19	40
SC_Cauchy	KPI	0.373	**0.086**	0.085	**0.703**	0.217	0.378	0.149	0.217	0.136
	κ	0.0%	**0.0%**	0.0%	**0.0%**	69.6%	0.0%	0.0%	69.6%	0.0%
	Tset	300	**70**	70	**50**	48.7	300	300	48.7	300
	kp	1.1	1.0	1.0	0.8	1.1	1.1	1.1	1.1	1.1
	Ti	33	16	16	18	19	33	33	19	33
SC_Levy	KPI	0.966	0.127	0.126	0.906	0.245	0.391	0.165	0.245	0.147
	κ	7.2%	10.2%	10.2%	0.0%	14.1%	7.2%	7.2%	14.1%	7.2%
	Tset	96.2	34.2	34.2	100	47.7	96.2	96.2	47.7	96.2
	kp	1.2	1.1	1.1	**0.7**	1.1	1.2	2.0	1.1	1.2
	Ti	39	21	21	**14**	18	39	37	18	39
SC_Lap	KPI	0.371	0.121	0.13	**1.024**	0.228	0.374	0.144	0.228	0.132
	κ	12.7%	12.0%	12.0%	**0.0%**	16.2%	12.7%	70.3%	16.2%	12.7%
	Tset	119.5	56.3	56.3	**50**	22.1	119.5	172.5	22.1	119.5
	kp	1.2	1.0	1.0	1.3	1.0	2.0	2.0	1.0	1.2
	Ti	34	17	17	10	17	39	39	17	34
SC_GEV	KPI	0.376	0.12	0.13	1.094	0.227	0.374	0.141	0.227	0.131
	κ	13.6%	9.1%	9.1%	44.8%	9.1%	69.9%	69.9%	9.1%	13.6%
	Tset	104.2	35.6	35.6	24.9	35.6	232	232	35.6	104.2

Table 1: Loop quality measures for all simulations

Table 2: Overshoot for selected loop quality measures

	Gauss. sig	**Cauchy. gam**	**Levy. gam**	**Levy. alf**	**Lap. b**	**GEV. sig**	**GEV. stdev**	**IAE**	**MSE**	**mean**
SC_Ndist	5.3%	**0.0%**	4.9%	**0.0%**	4.9%	69.7%	69.7%	4.9%	5.3%	**18.3%**
SC_Gauss	5.3%	**0.0%**	17.6%	50.0%	17.6%	**0.0%**	63.6%	17.6%	5.3%	**19.6%**
SC_Cauchy	**0.0%**	**0.0%**	**0.0%**	**0.0%**	69.6%	**0.0%**	**0.0%**	69.6%	**0.0%**	**15.5%**
SC_Levy	7.2%	10.2%	10.2%	**0.0%**	14.1%	7.2%	7.2%	14.1%	7.2%	*8.6%*
SC_Lap	12.7%	12.0%	12.0%	**0.0%**	16.2%	12.7%	70.3%	16.2%	12.7%	**18.3%**
SC_GEV	13.6%	9.1%	9.1%	44.8%	9.1%	69.9%	69.9%	9.1%	13.6%	**27.6%**
mean	**7.3%**	*5.2%*	**9.0%**	**15.8%**	**21.9%**	**26.6%**	**46.8%**	**21.9%**	**7.3%**	

is almost twice smaller, $T_i = 15 \dots 21$, than previously. Resulting performance is faster with large overshoots. Laplace lies somewhere in between with detected smaller integration time value. GEV achieves extremely sluggish, poorest performance numbers. Integral indexes (MSE and IAE) are of different character. Square error is biased towards sluggish control with high settling times and minimal overshoot. The results are very similar to those for the Gaussian case. IAE favors aggressive tuning, closer to Laplace and fat-tailed distributions.

The results in the table, which are the closest to the optimal tuning, are highlighted with bold numbers. None of them is selected with Gaussian standard deviation. They are mostly found out by fat-tailed PDFs.

The preferred results, i.e. selected by the overshoot measure (Table 2) and the settling time (Table 3) are presented as the reference. We see that the fat-tailed stable distribution does not deteriorate detection and does not alter the assessment. Once such type of disturbance affects the loop, assessment is possible. Proper selection is still possible. Detection is not efficient for normal distribution disturbance.

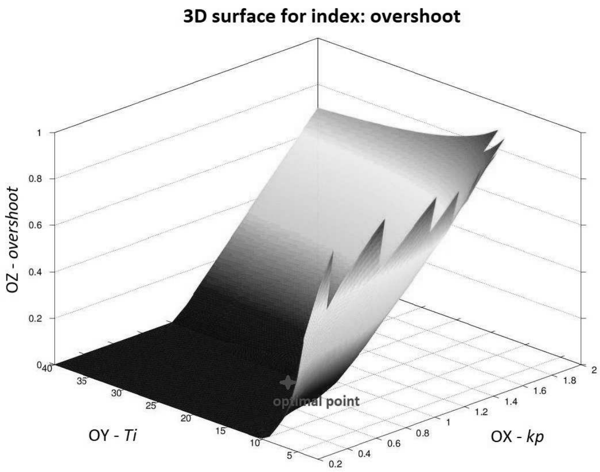
Unfortunately, the numbers do not disclose all of the relevant phenomena. They do not inform us clearly about the causes of the obtained results. It seems that each of the measures delivers something. However, visual inspection of the resulting index 2D contour surfaces, evaluated in the next section, shows that in many cases the selected minimum value is inappropriate. Real performance is hidden (shadowed) by the character of the disturbance.

3.3. Qualitative inspection

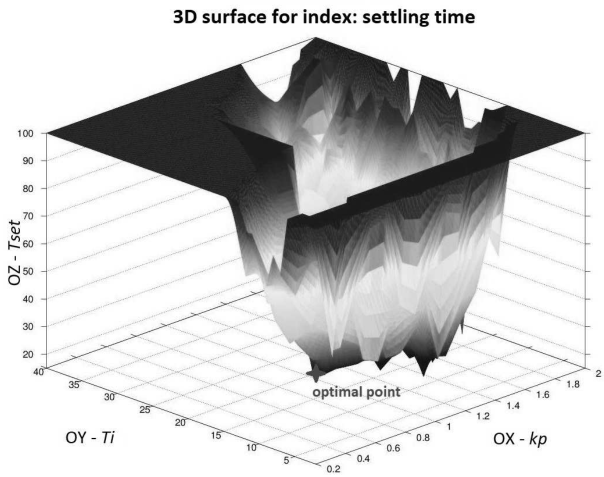
Selected scenarios are visualized by surface plots, showing calculated measures versus controller parameters (k_p nd T_i) for all settings. Such an approach enables visual inspection of how detection behaves and how the optimal solution is achieved. Similar approach is used in other studies of that subject, like, for instance, in Spinner et al. (2014). Only selected most relevant surfaces are presented. The work is carried out from the perspective of each considered measure, showing four most representative surfaces. The results are commented from the perspective of the disturbance impact.

The review of control performance contours starts with the presentation of two surfaces, representing reference indexes of overshoot and settling time (Fig. 6) calculated for step response. The optimal PID setting is shown in both plots for reference. One may see that the surface is in the form of the single smooth slope from the perspective of overshoot with optimal solutions lying on the border of zero value bottom. To the contrary, the settling time represents a relatively narrow valley, limited by the large values of sluggish or oscillating controls.

Figure 7 shows four representative cases for normal standard deviation as the quality measure. Shape of the *valley* is well visible in undisturbed situation with optimal point at the large value of the integration time. The impact of Gaussian or Laplace disturbance disrupts the smoothness of the surface, but detection of



Overshoot

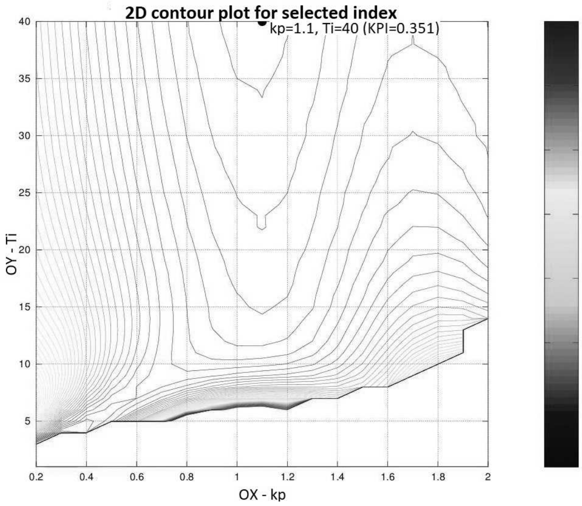


Settling time

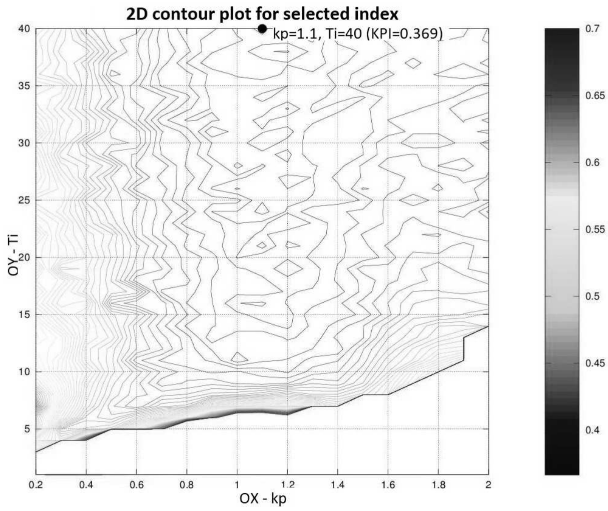
Figure 6: Surfaces for classical indexes with highlighted optimal tuning

the minimum measure value is still possible. To the contrary, disturbance with long-tailed properties effectively screens the original shape and prevents any reasonable detection. Results do constitute, in fact, a random selection.

Cauchy scaling factor γ represents signal variability with outliers. The obtained results are promising. No disturbance shadowing is visible (Fig. 8). All surfaces are smooth and enable detection. Furthermore, surface minimum lies close to the optimal PID settings. Assessment with Cauchy parameter γ



(a) Scenario SC_Ndist

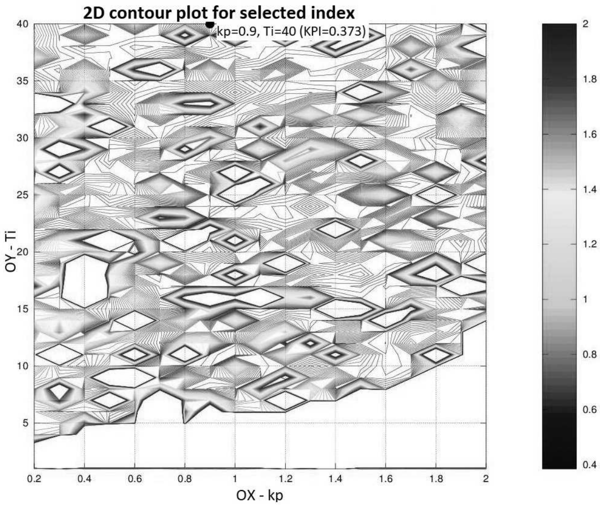


(b) Scenario SC_Gauss

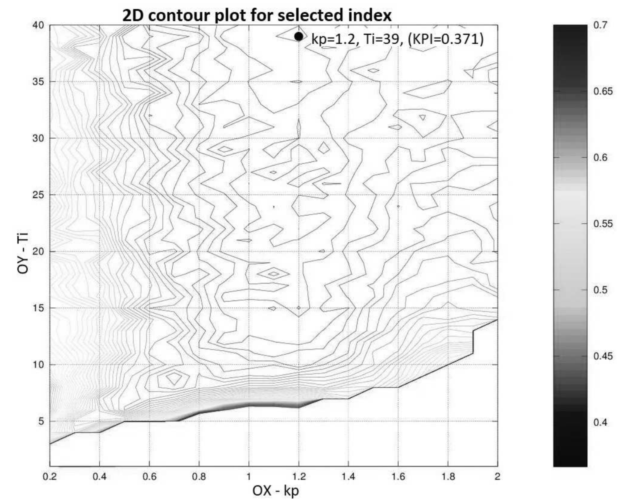
Figure 7: Gaussian standard deviation σ for various disturbances

is robust and effective. It is expected that the scaling of Lévy α-stable PDF should behave similarly. Simulations prove this, as shown in Fig. 8. Normal and fat-tailed disturbances give relatively smooth surfaces. Minimum (i.e. optimal) values are clearly detected. The assessed tuning for both disturbances is the same.

One of the most interesting results is represented by the Lévy stability factor



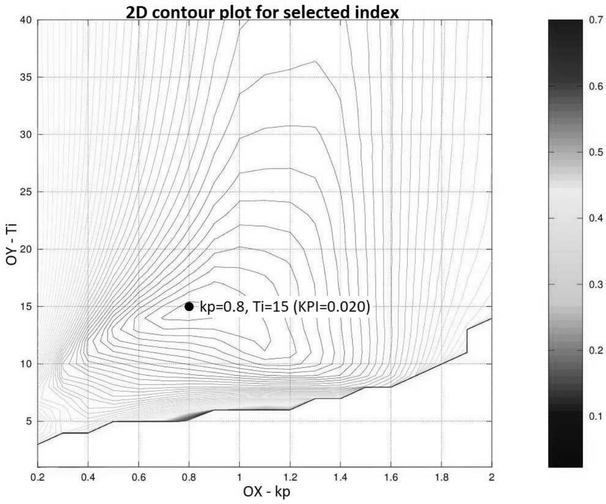
(c) Scenario SC_Cauchy



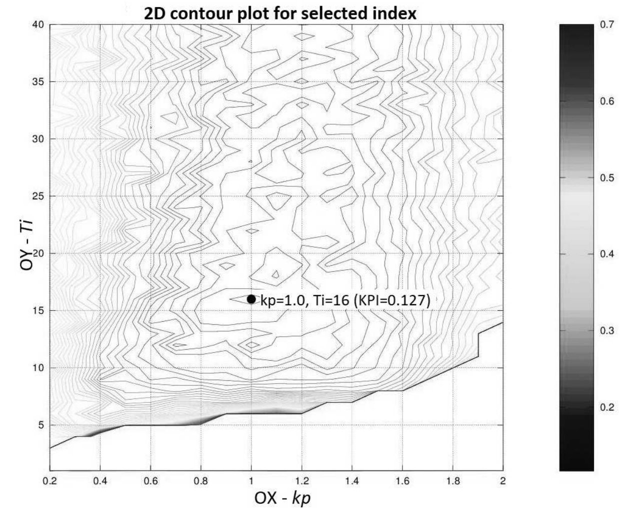
(d) Scenario SC_Lap

Figure 7: Gaussian standard deviation σ for various disturbances

α. Detection surfaces (Fig. 9) resemble those for the fat-tailed functions and there is no clear shadowing. Detection is achievable. The worst assessment happens in undisturbed case. Existence of strong noises improves the situation. The meaning of stability differs from that of scaling. The measures γ and σ reflect the strength of signal fluctuations, while α of the α-stable function reflects the fatness of tails. Peters (1996) states that it is equivalent to the



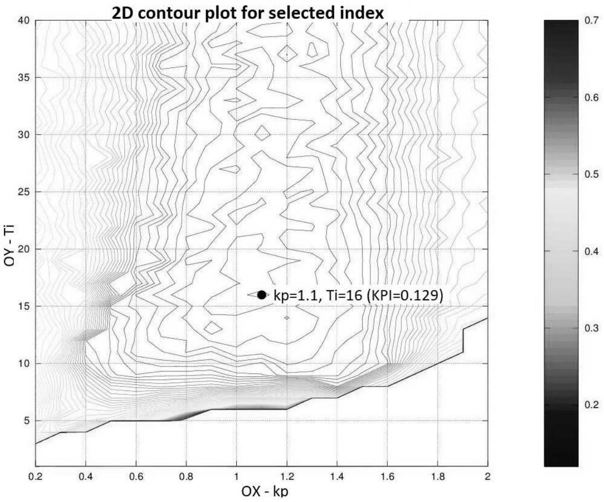
(a) Cauchy scaling: scenario SC_Ndist



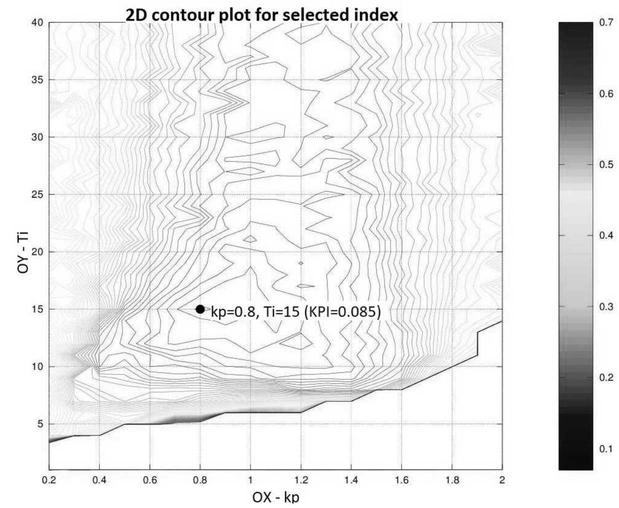
(b) Cauchy scaling: scenario SC_Levy

Figure 8: Cauchy and α-stable PDFs with scaling γ for various disturbances

inverse of Hurst exponent ($H = 1/\alpha$). This relation holds when there are no correlations. Visible differences suggest that the correlations are the main reasons for inconsistency. Stability factor α incorporates more information than signal variability. Comparison of Lévy measures with Hurst exponent, presented in Domański (2016), may reveal the nature of the phenomenon. This subject surely requires further investigation, as it may enable deeper assessment.



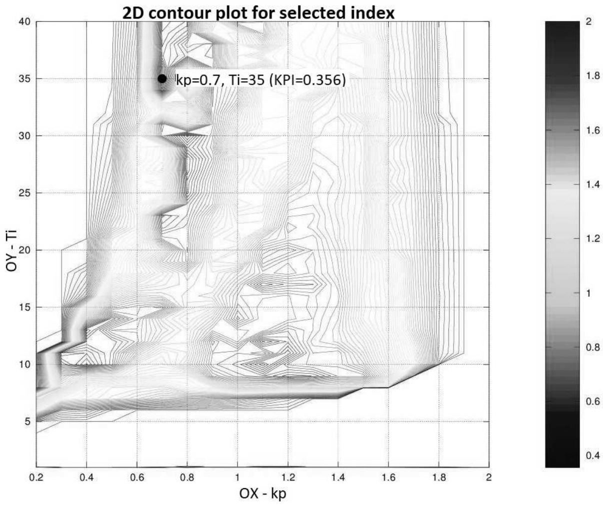
(c) α-stable scaling: scenario SC_Gauss



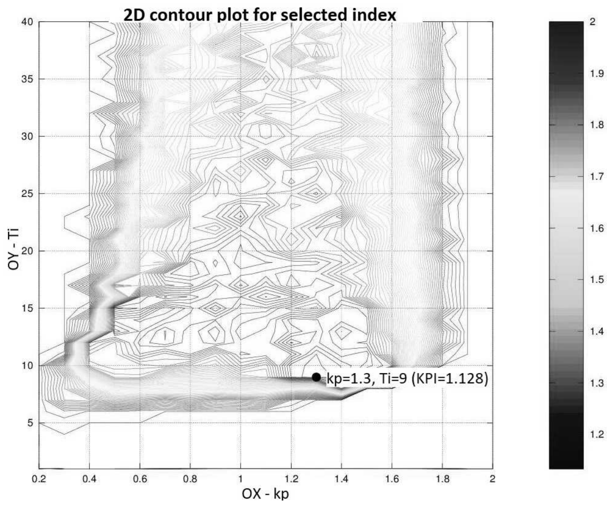
(d) α-stable scaling: scenario SC_Cauchy

Figure 8: Cauchy and α-stable PDFs with scaling γ for various disturbances

Detection with Laplace measures (Fig. 10) is characterized by unclear observations. There is a shadowing effect of the fat-tailed disturbances. It is not as strong as in the Gaussian case (scenario SC_Gauss), but still significant. On the other hand, discovered solutions (2D surface shapes) are closer to the ones of Cauchy (scenario SC_Cauchy) and Lévy (scenario SC_Levy). This detection occurs to be more robust than with normal distribution, but more sensitive to



(a) Scenario SC_Ndist

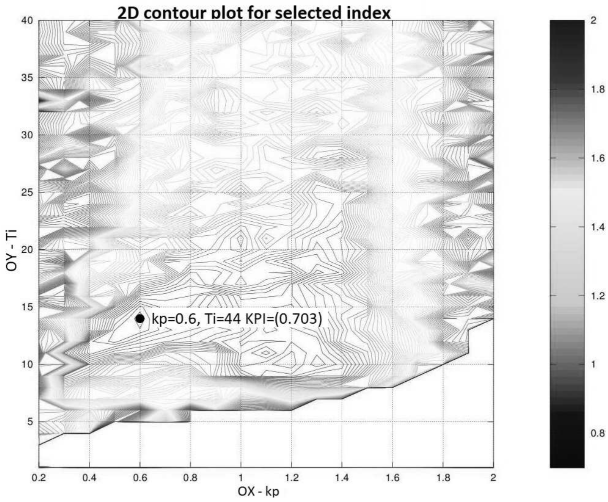


(b) Scenario SC_Gauss

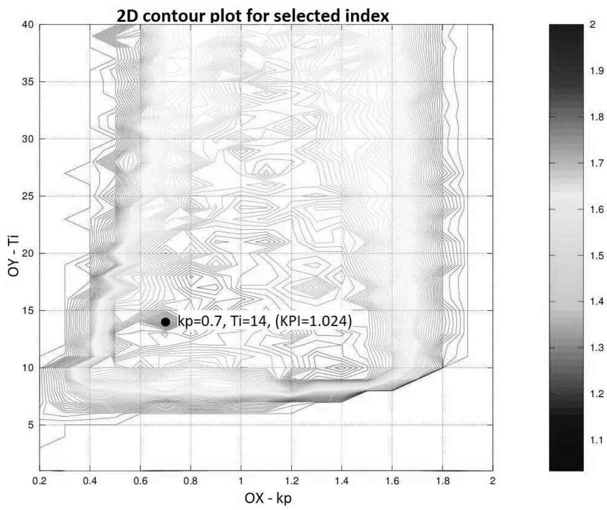
Figure 9: The α-stable stability factor α for various disturbances

strong disturbances than those of the fat-tail scaling factor γ.

Detection with GEV is not effective at all. None of the considered factors (Fig. 11), like scaling σ and standard deviation is robust with respect to the fat-tail shadowing effect. Detection surfaces are highly ragged, disabling any reasonable assessment. Moreover, disturbance shadowing effect happens (to a smaller extent) also in the Gaussian case. The detected points (tuning) are far



(c) Scenario SC_Cauchy

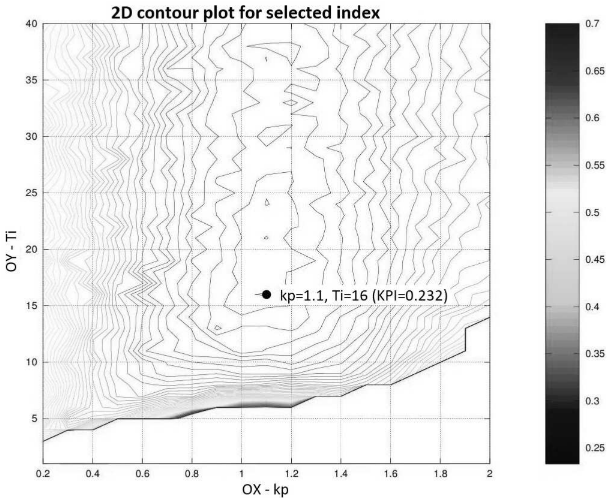


(d) Scenario SC_Lap

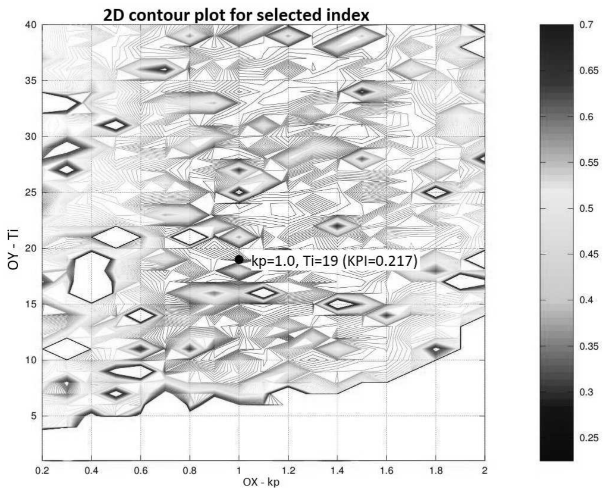
Figure 9: The α-stable stability factor α for various disturbances

from the optimal ones. Results are biased in the direction of slow or oscillating control. GEV factors do not suit the CPA task.

Figure 12 shows the surfaces for IAE and MSE. The IAE behaviour lies between robust Cauchy / Lévy cases and the sensitive Gaussian one. The identified controllers are close to the optimal ones. Detected tuning is aggressive, with limited overshoot. MSE surfaces are similar to Gaussian σ. Solution is slug-



(a) Scenario SC_Gauss

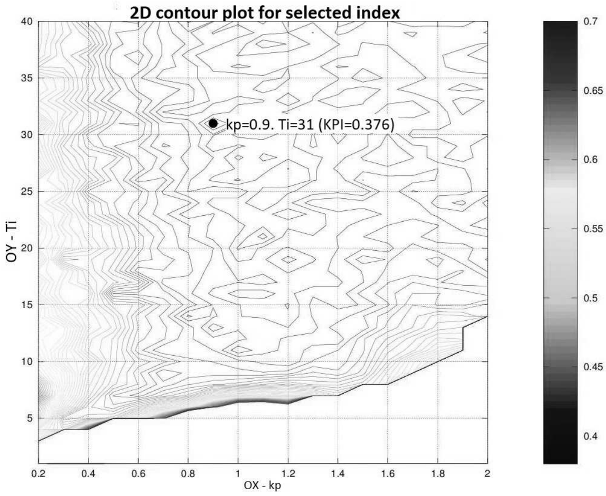


(b) Scenario SC_Cauchy

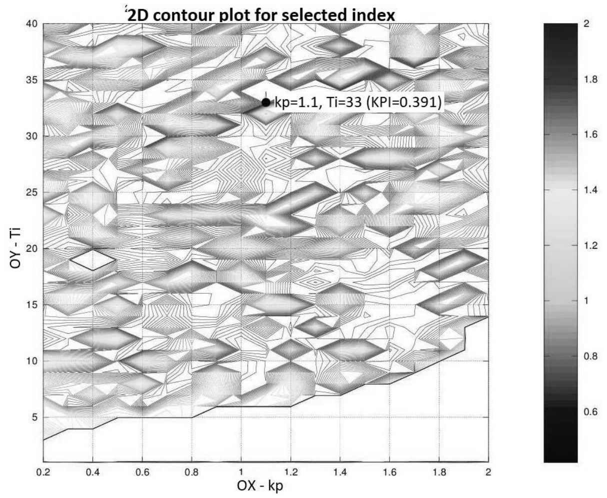
Figure 10: Laplace b for various disturbances

gish with large settling time and no oscillations. Disturbances alter smoothness, particularly the ones with fat tails. No assessment is possible in the shadowed cases.

Visual inspection expands quantitative analysis. It shows the detection sensitivity. Though some parameters may seem to be good, the shape of the detection surface is ragged, so that detection is just random. Simple observation of the numbers may be misleading. Results must be always verified against



(a) σ: scenario SC_Gauss



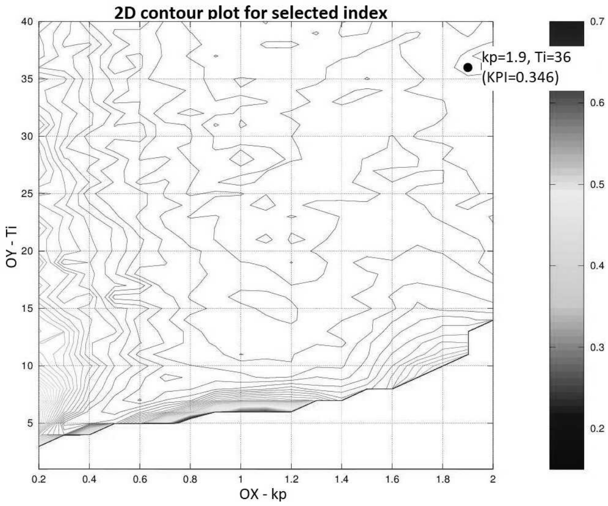
(b) σ: scenario SC_Levy

Figure 11: GEV factors for selected disturbances

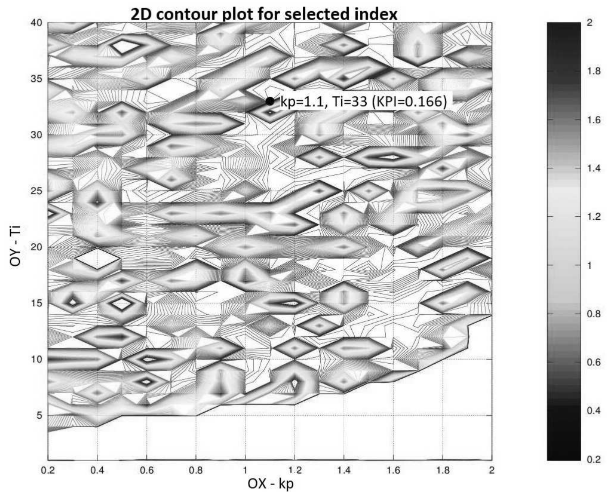
disturbances.

3.4. Resulting methodology

The simulations provide the observations that may be useful in the practical control quality assessment. First of all, the automatic evaluation of the indexes without reflection about the loop environment may be misleading. Visual



(c) standard deviation: scenario SC_Gauss

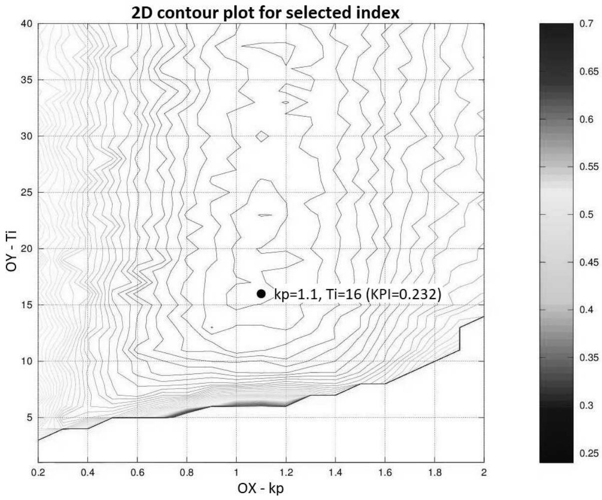


(d) standard deviation: scenario SC_Levy

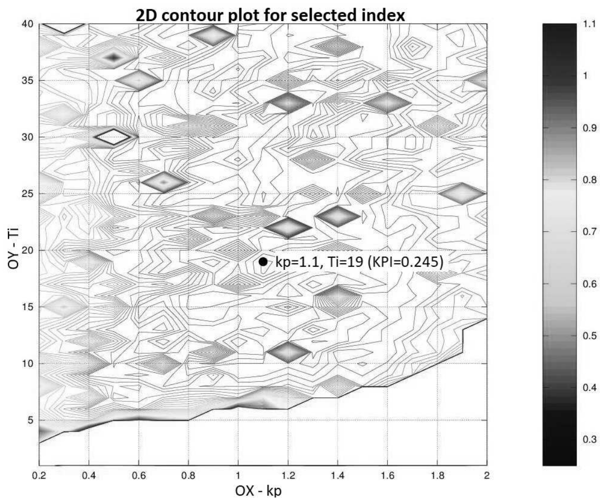
Figure 11: GEV factors for selected disturbances

inspection of the process data, i.e. of the trends and histograms, is strongly suggested. Although such an observation is discouraging, it seems to be reasonable. The following assessment procedure is proposed:

1. Review the time trends of process variables. Look for the artifacts in the data and manually eliminate them. It should be also reminded that process data should be real, i.e. without compression effects of the SCADA



(a) IAE: scenario SC_Gauss



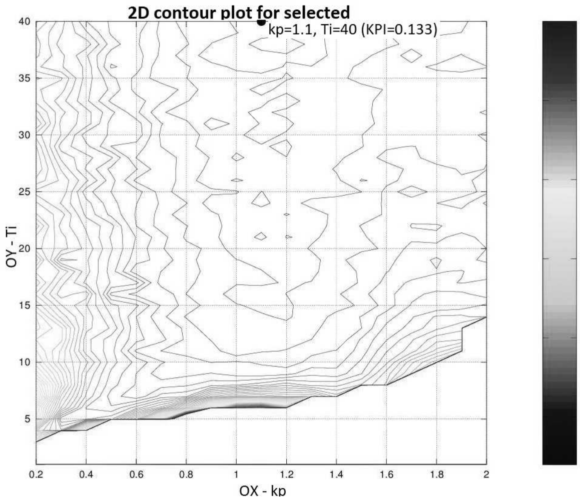
(b) IAE: scenario SC_Levy

Figure 12: IAE and MSE indexes for selected disturbances

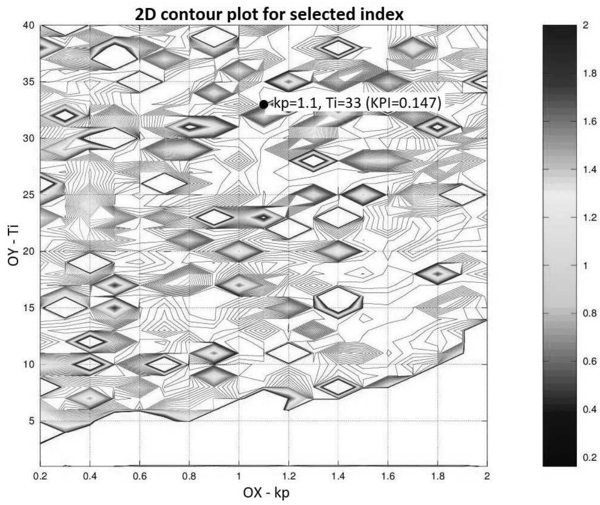
system and should come from the normal operating regimes of the installations.

2. Use control error signal as the assessment variable.
3. Draw the control error histogram to evaluate the best PDF fit.

   (a) In the Gaussian case, normal standard deviation should be used as the control measure.



(c) MSE: scenario SC_Gauss



(d) MSE: scenario SC_Levy

Figure 12: IAE and MSE indexes for selected disturbances

(b) Once the fat tails are detected, the factors of Cauchy or stable PDF should be used. Scaling γ is the suggested choice, as it is the most robust against disturbances.

(c) Otherwise, select the factors for specific PDF.

4. Selection of appropriate measure protects against the biasing by shadowing.

5. MSE and IAE should be used with caution, although the IAE seems to be a better solution.

4. Conclusions and further research

The paper presents the results of the research on statistical control performance measures applied to the control loop with PID algorithm. Process industry data analysis shows frequently appearing non-Gaussian properties. In the majority of cases we encounter the fat-tailed distribution (Cauchy or Lévy). Dedicated simulations have been performed in order to investigate the subject. Six scenarios have been considered with various loop disturbance properties: Gauss, Cauchy, α-stable, Laplace and GEV. Control quality assessment has been done with indexes originating from factors of these selected PDFs. They are compared with popular integral indexes, MSE and IAE. The analysis reveals the following:

(a) Quantitative comparison is not sufficient. It may give random results. Disturbance shadowing has strong impact on detection. An in-depth process insight is required.

(b) Control quality assessment depends on the disturbance properties. Selection of the indexes should be based on well fitted distribution.

(c) Control error histogram helps to select appropriate density function.

(d) Gaussian approach seems to be appropriate in case of no disturbances or while they are close to normal.

(e) Two types of the surface characters are observed: the one similar to the standard normal deviation and the MSE, and the other one, obtained with the IAE and the fat tailed distribution scaling parameter γ.

(f) Laplace function constitutes a compromise between these two cases.

(g) GEV does not meet the assessment goals and should not be used.

(h) Comparison of the MSE and IAE confirms that the MSE selects aggressive solutions and seems to be closer to normal standard deviation. The IAE is biased towards solutions associated with fat-tailed distribution scaling and selects tuning with the lower overshoot.

(i) Stability factor α of the Lévy PDF should be investigated as it keeps characteristic features associated with the persistence properties.

Observations made on the basis of investigations enable to derive a procedure for the statistical control performance assessment. Results are obtained with the simulations, though the rationale came from the process industry projects. The methodology should be confronted with the extensive industrial validation. The combination of real phenomena with statistical measures ends up with constructive approach for the detection and the root-cause analysis. The procedure consists of the fusion of different approaches with an inevitable human expertise.

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Table 3: Settling time for selected loop quality measures

	Gauss.sig	Cauchy.gam	Levy.gam	Levy.alf	Lap.b	GEV.sig	GEV.stdev	IAE	MSE	**mean**
SC_Ndist	130.5	70.0	33.7	300.0	33.7	299.5	299.5	33.7	130.5	**147.9**
SC_Gauss	130.5	70.0	32.6	18.1	32.6	200.0	50.7	32.6	130.5	**77.5**
SC_Cauchy	300.0	70.0	70.0	50.0	48.7	300.0	300.0	48.7	300.0	**165.3**
SC_Levy	96.2	34.2	34.2	100.0	47.7	96.2	96.2	47.7	96.2	*72.1*
SC_Lap	119.5	56.3	56.3	50.0	22.1	119.5	172.5	22.1	119.5	**82.0**
SC_GEV	104.2	35.6	35.6	24.9	35.6	232.0	232.0	35.6	104.2	**93.3**
mean	**146.8**	**56.0**	**43.7**	**90.5**	*36.7*	**207.9**	**191.8**	*36.7*	**146.8**	