Malinowski Jacek
Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland

A simple tool for evaluating the risks related to hazardous interactions between the processes realized in the Baltic Sea Region port areas

Keywords

probabilistic modeling; hazardous, harmful or catastrophic events; processes interaction; cascading effect; risk calculation.

Abstract

This paper is a continuation of [1] which presents a probabilistic model of hazard-related interactions between different operations carried out in a (generic) Baltic Sea Region port area. Each such operation, considering its hazardous aspect, is defined as a series of undesired events (emergencies and/or accidents) occurring at random instants, i.e. as a random process. An event can be primary (occurring by itself) or secondary (caused by another event in the same or another process). The processes interact in the sense that a primary event in one process can cause a cascade of events spanning multiple processes.
In [1] the formulas were derived for the cause-effect probabilities expressing the impact of a single event on the occurrence of the ensuing events in the triggered cascade. Also, the formulas for risks of undesired events, using these probabilities, were obtained. As these formulas are complicated and difficult to implement numerically, the need arose to develop a simple tool for computing the considered risks. Such a tool, in the form of an easy-to-implement algorithm, along with an illustrative example is presented in the current work.

1. Notation and definitions

1.1 General notation

$p_1,\dots,p_n$ – the individual processes as the model of operations realized in the considered port environment; n – the number of these processes

$E_1^{(i)},\dots,E_{m(i)}^{(i)}$ – different primary events that can occur in the process p_i; m(i) – the number of events in process p_i

g(i, a) – maximum strength of event $E_a^{(i)}$ in process p_i

$\lambda_1^{(i)},\dots,\lambda_{m(i)}^{(i)}$ – the intensities with which $E_1^{(i)},\dots,E_{m(i)}^{(i)}$ occur as primary events; the sequence of primary $E_a^{(i)}$ is a Poisson process with the intensity $\lambda_a^{(i)}$

$\coprod_{i=1,\dots,r} x_i$ – the “inverted pi” operation on numbers from the interval [0, 1], defined as follows:

$$\coprod_{i=1,\dots,r} x_i = 1 - \prod_{i=1,\dots,r}(1 - x_i)$$

The above operation is used to compute the probability of a sum of independent events, i.e. $\Pr(\bigcup_{i=1,\dots,r} A_i) = \coprod_{i=1,\dots,r} P(A_i)$ if the events $A_1,\dots,A_r$ are independent.

1.2 The cause-effect probabilities

$\pi_{a,b}^{(i,j)}(x, y)$ – probability that event b of strength y in process p_j is directly caused by event a of strength x in process p_i

$\pi_{a,b}^{(i,j)}(x, y, h)$ – probability that event b of strength y in process p_j occurs in step h (but not in step <h) of a cascade triggered by event a of strength x in process i; <h denotes less-than-h

$\pi_{a,b}^{(i,j)}(x, >y)$ – probability that event b of strength >y in process p_j is directly caused by event a of strength x in process p_i; note that $\pi_{a,b}^{(i,j)}(x, >y) = \sum_{z>y} \pi_{a,b}^{(i,j)}(x, z)$; >y denotes greater-than-y

$\pi_{a,b}^{(i,j)}(x, >y, h)$ – probability that event b of strength >y in process p_j occurs in step h (but not in step <h) of a cascade triggered by event a of strength x in process p_i

1.3 Risks of harmful events

$R_b^{(j)}(k, s, t, 0)$ – the probability that exactly k primary events $E_b^{(j)}$ occur in the time interval (s, t]

$R_b^{(j)}(k, s, t, h)$ – the probability that exactly k events $E_b^{(j)}$ occur in the time interval (s, t], where each $E_b^{(j)}$ is a result of a cascade of step h (but not of step <h), h≥1

$R_b^{(j)}(k, s, t)$ – the probability that exactly k events $E_b^{(j)}$ (whether primary or not) occur in the time interval (s, t]

2. Introduction

This paper is a continuation of [1] which presents a probabilistic model of hazard-related interactions between different operations carried out in a (generic) Baltic Sea Region port area. Each such operation, considering its hazardous aspect, is defined as a series of undesired events (emergencies and/or accidents) occurring at random instants, i.e. as a random process. An event can be primary (occurring by itself) or secondary (caused by another event in the same or another process). The processes interact in the sense that a primary event in one process can cause a cascade of events spanning multiple processes.
In [1] the formulas were derived for the cause-effect probabilities expressing the impact of a single event on the occurrence of the ensuing events in the triggered cascade. The considered probabilities are defined in Notation section and in the "complete" version take into account the strengths of both the triggering events and their effects. In this paper the model developed in [1] is simplified in that the considered events are assumed to be one-state, i.e. their strengths are assumed irrelevant – an event just occurs or not. In consequence, the above mentioned formulas become simpler – they are presented in section 3.
The cause-effect probabilities are used in the formulas, also derived in [1], expressing the risks of undesired events. Their simplified versions are given in Section 4. Even after neglecting the events' strengths the formulas of both groups still somewhat complicated and difficult to implement numerically, thus it became necessary to develop a simple tool for computing the considered risks. Such a tool, in the form of an easy-to-implement algorithm, along with an illustrative example is presented in Sections 5 and 6. For convenience of notation and ease of numerical implementation, all the formulas and the algorithm's pseudo-code are written using the matrix calculus notation. In Section 3 a non-standard matrix operation is defined in order to express the cause-effect probabilities in a compact form that can be easily translated into a computer code.

3. Computing the cause-effect probabilities

In [1] the following key lemma has been formulated and proved, as the basis for the cause-effect probabilities computation:

Lemma 1
If both the internal impact and feedback effect are taken into consideration, then $\pi_{a,b}^{(i,j)}(x, >y, h)$, where h≥2, is given by the following recursive formula:

$$\pi_{a,b}^{(i,j)}(x, >y, h) =$$
$$= \coprod_{\substack{k=1,\dots,n \\ c=1,\dots,m(k)}} \sum_{\substack{z=0,\dots,g(k,c) \\ z\le y \ if\ (k,c)=(j,b)}} \pi_{a,c}^{(i,k)}(x,z) \times$$
$$\times \pi_{c,b}^{(k,j)}(z, >y, h-1) \qquad (1)$$

where

$$\pi_{c,b}^{(k,j)}(z, >y, 1) = \pi_{c,b}^{(k,j)}(z, >y) =$$
$$= \sum_{u>y} \pi_{c,b}^{(k,j)}(z,u) \qquad (2)$$

Remark 1:
Under the adopted assumptions, (1) also holds for (j,b)=(i,a), and (k,c)=(j,b) is in the range of the "inverted pi" operator (feedback effect). However, it should be remembered that $\pi_{a,c}^{(i,k)}(x,z) = 0$ for (k,c)=(i,a) and $\pi_{c,b}^{(k,j)}(z, >y, 1) = 0$ for (k,c)=(j,b). For explanation see formula (1) in [1].

Remark 2:
If (k,c)=(j,b), then z>y are not in the range of the summation operator, because taking such z into consideration would amount to admitting the possibility that $E_a^{(i)}$ directly causes $E_b^{(j)}$ of strength >y. This would contradict the requirement that $E_b^{(j)}$ of strength >y cannot be a <h-step cascading effect of $E_a^{(i)}$.

Our risk evaluation method in its simplified version regards the unwanted events as one-state only, i.e. the events are not graded according to their strength. For consistency, we assume that the event triggering a cascade occurs at its step 0, and the event occurring at step h directly causes one or more events occurring at step h+1, h≥0. Thus, the following cause-effect probabilities will be considered throughout the rest of the paper:
$\pi^{(i,j)}(a, b)$ – probability that event $E_a^{(i)}$ in process p_i directly causes event $E_b^{(j)}$ in process p_j (first grade C-E probability)

$\pi^{(i,j)}(a, b, h)$ – probability that event $E_b^{(j)}$ in process p_j occurs in step h (but not in step <h) of a cascade triggered by event $E_a^{(i)}$ in process p_i, h≥1; if h=1 then $\pi^{(i,j)}(a, b, 1) = \pi^{(i,j)}(a, b)$ (h-th grade C-E probability).
If strengths of events are not taken into account, Lemma 1 has the following equivalent.

Lemma 2
For h≥2 and (j,b) ≠ (i,a) the following recursive formula holds:

$$\pi^{(i,j)}(a,b,h) = \left[1-\pi^{(i,j)}(a,b)\right] \times \times \coprod_{\substack{k=1,\dots,n \\ c=1,\dots,m(k)}} \begin{bmatrix} \pi^{(i,k)}(a,c) \times \\ \pi^{(k,j)}(c,b,h-1) \end{bmatrix} \quad (3)$$

For h≥1 we have:

$$\pi^{(i,i)}(a,a,h) = 0 \quad (4)$$

Proof: Let $E_{a,i,c,k}$ and $E_{c,k,b,j}^{(h)}$, h≥1, be events defined as follows:

$E_{a,i,c,k}$ = {an instance of $E_b^{(j)}$ is directly caused by an instance of $E_a^{(i)}$}
$E_{c,k,b,j}^{(h)}$ = {an instance of $E_b^{(j)}$ occurs in step h (but not in step <h) of the cascade triggered by an instance of $E_c^{(k)}$}; clearly, $E_{a,i,c,k} = E_{a,i,c,k}^{(1)}$

The probabilities $\pi^{(i,i)}(a, a, h)$, h≥1, i=1,…,m, a=1,…,m(i), are equal to zero, because the underlying events $E_{a,i,a,i}^{(h)}$ are impossible. Indeed, for $E_{a,i,a,i}^{(h)}$ to take place the triggering event $E_a^{(i)}$ should occur at step 0 of the cascade, but the definition of $E_{a,i,a,i}^{(h)}$ yields that $E_a^{(i)}$ cannot occur at step <h, hence $E_{a,i,a,i}^{(h)}$ is an impossible event.
Let now h≥2 and (j,b)≠(i,a). The event $E_{a,i,b,j}^{(h)}$ takes place if (1) a pair of consecutive events $E_{a,i,c,k}^{(1)}$ and $E_{c,k,b,j}^{(h-1)}$ occurs, where k=1,…,n, c=1,…,m(k), (k,c)≠(j,b), and (2) the event $E_{a,i,b,j}^{(1)}$ does not occur. The condition (k,c)≠(j,b) and the second requirement are in place to ensure that $E_b^{(j)}$ does not occur at step one, according to the definition of $E_{a,i,b,j}^{(h)}$. We thus have:

$$\pi^{(i,j)}(a,b,h) = = \Pr\left[\left(\begin{matrix} (\neg E_{a,i,b,j}) \cap \\ \bigcup_{\substack{k=1,\dots,n \\ c=1,\dots,m(k) \\ (k,c)\neq(j,b)}} \left[E_{a,i,c,k} \cap E_{c,k,b,j}^{(h-1)}\right] \end{matrix}\right)\right] \quad (5)$$

As assumed in [1], the triggering events along with the triggered cascades are mutually independent, hence (3) is a direct consequence of (5); (k,c)=(j,b) can be included in the range of the operator ∐ in (3), because, in view of (4), if (k,c)=(j,b) then $\pi^{(i,k)}(a, c)\cdot\pi^{(k,j)}(c, b, h-1) = \; = \pi^{(i,j)}(a, b)\cdot\pi^{(j,j)}(b, b, h-1) = 0$.

Remark: When applying (3) it should be taken into account that $\pi^{(i,k)}(a, c) = 0$ for (k,c)=(i,a), according to (4).

The behavior of the considered multi-process environment can be described by the collection of matrices $\pi^{(i,j)}(h)$, i,j=1,…,n, h≥1, where $\pi^{(i,j)}(a, b, h)$ is the element in row a and column b of matrix $\pi^{(j,i)}(h)$, a=1,…,m(i), b=1,…,m(j). Thus, matrix $\pi^{(i,j)}(h)$ expresses the impact of events occurring in process i on the events in process j, where the latter events are h-step (but not <h-step) cascading effects of the former ones. For a fixed h, matrices $\pi^{(i,j)}(h)$, i,j=1,…,n can be arranged in matrix π(h) as in *Figure 1*.

$$\pi(h) = \begin{matrix} \pi^{(1,1)}(h) & \pi^{(1,2)}(h) & \cdots & \pi^{(1,n)}(h) \\ \pi^{(2,1)}(h) & \pi^{(2,2)}(h) & \cdots & \pi^{(2,n)}(h) \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{(n,1)}(h) & \pi^{(n,2)}(h) & \cdots & \pi^{(n,n)}(h) \end{matrix}$$

Figure 1. Matrix π(h) composed of matrices $\pi^{(i,j)}(h)$, i,j=1,…,n.

Let us note that π(h) is a square matrix, because $\pi^{(i,j)}(h)$ has m(i) rows and m(j) columns, hence π(h) has $\sum_{i=1,\dots,n} m(i)$ rows and $\sum_{j=1,\dots,n} m(j)$ columns, thus the same number of rows and columns. Also, according to (3), $\pi^{(i,j)}(a, b, h)$ is computed using, for each successive k=1,…,n, the elements from the row a of $\pi^{(i,k)}(1)$ and the column b of $\pi^{(k,j)}(h-1)$, which means that in order to obtain π(m(1)+…+m(i–1)+a, m(1)+…+m(j–1)+b) we use the elements from the row m(1)+…+m(i–1)+a of π(1) and the column m(1)+…+m(j–1)+b of π(h–1).
By analogy to the usual multiplication operation on matrices, where

$$(A \times B)(p,q) = \prod_{r=1,\dots,\kappa(A)} A(p,r)B(r,q) \quad (6)$$

let us define an operation ⊗ in the following way:

$$(A \otimes B)(p,q) = = [1 - A(p,q)] \coprod_{r=1,\dots,\kappa(A)} A(p,r)B(r,q) \quad (7)$$

for p≠q, and

$$(A \otimes B)(p,p) = 0 \quad (8)$$

where κ(A) is the number of A's columns and ∐ is the "inverted pi" operation defined in Notation section. Let us note that (7) and (8) convert to (3) and (4) if we put A=π(1) and B=π(h–1). We can thus put (3) and (4) in a much simpler form:

$$\pi(h) = \pi(1) \otimes \pi(h-1) \quad (9)$$

As follows from the previous paragraph, the element in row p and column q of π(h) is obtained using the elements in row p of π(1) and column q of π(h–1), similarly as in matrix multiplication. However, comparing (6) with (7) and (8) we see that ⊗ is not the matrix multiplication operation. The formula (9), being a shortened version of (3) and (4), is used to obtain the cause effect probabilities needed for computing the risks of unwanted events. The formulas for computing these risks are presented in the next section.

4. Computing the risks of unwanted events

This section starts with one assertion and two theorems. They were already given in [1], but here they are presented in simplified versions, strength of events not being taken into account.

Assertion 1
Primary events $E_b^{(j)}$ constitute a Poisson process with the intensity $\lambda_b^{(j)}$.
This assertion repeats of one of the general assumptions.

Theorem 1 (direct impact)
The events $E_b^{(j)}$ directly caused by primary events $E_a^{(i)}$ constitute a Poisson process with the intensity $\lambda_{a,b}^{(i,j)} = \lambda_a^{(i)} \cdot \pi^{(i,j)}(a,b)$.
Further, the occurrences of $E_b^{(j)}$ directly caused by any primary event in any process (excluding $E_b^{(j)}$), constitute a Poisson process with the intensity given by the following formula:

$$\Lambda_b^{(j)}(1) = = \sum_{\substack{i=1,\dots,n \\ a=1,\dots,m(i)}} \lambda_a^{(i)} \cdot \pi^{(i,j)}(a,b) \quad (10)$$

It should be remembered that $\pi^{(i,j)}(a, b)=0$ for (a,i)=(b,j).

In order to shorten the notation and facilitate the numerical implementation, (10) can be converted to the following form:

$$\Lambda(1) = \lambda \times \pi(1) \quad (11)$$

where × is the usual matrix multiplication operation, π(1) is defined in Fig. 1, while λ and Λ(1) are one-row matrices composed of the intensities $\lambda_b^{(j)}$ and $\Lambda_b^{(j)}(1)$ respectively, and defined as follows:

$$\lambda = [\lambda_1^{(1)},\dots, \lambda_{m(1)}^{(1)},\dots, \lambda_1^{(n)},\dots, \lambda_{m(n)}^{(n)}] \quad (12)$$

and

$$\Lambda(1) = [\Lambda_1^{(1)}(1),\dots, \Lambda_{m(1)}^{(1)}(1),\dots, \Lambda_1^{(n)}(1),\dots, \Lambda_{m(n)}^{(n)}(1)] \quad (13)$$

Theorem 2 (cascading effect of step h≥2)
Let h≥2. The events $E_b^{(j)}$ each of which is a h-step (but not <h-step) cascading effect of a primary event $E_a^{(i)}$ constitute a Poisson process with the intensity $\lambda_{a,b}^{(i,j)}(h) = \lambda_a^{(i)} \cdot \pi^{(i,j)}(a, b, h)$, where the probabilities $\pi^{(i,j)}(a, b, h)$ are given by (3) or (8).
Further, the events $E_b^{(j)}$ each of which is a h-step (but not <h-step) cascading effect of any primary event in any process (excluding $E_b^{(j)}$), constitute a Poisson process with the intensity given by the following formula:

$$\Lambda_b^{(j)}(h) = = \sum_{\substack{i=1,\dots,n \\ a=1,\dots,m(i)}} \lambda_a^{(i)} \cdot \pi^{(i,j)}(a,b,h) \quad (14)$$

It should be remembered that $\pi^{(i,j)}(a, b, h)=0$ for (a,i)=(b,j).

Corollary from Assertion 1 and Theorems 1 and 2:
The risk $R_b^{(j)}(k, s, t, h)$, h≥0, i.e. the probability that exactly k events $E_b^{(j)}$ occur in the time interval (s, t], each event being a result of h-step (but not less-than-h-step) cascade triggered by any primary event (different than $E_b^{(j)}$), is found from the following formula:

$$R_b^j(k,s,t,h) = \frac{\left[\Lambda_b^{(j)}(h)\cdot(t-s)\right]^k}{k!} \times \times \exp\left[-\Lambda_b^{(j)}(h)\cdot(t-s)\right] \quad (15)$$

Let us note that $R_b^{(j)}(0,s,t,h)$ is the probability that none of the above specified events $E_b^{(j)}$ occurs in the (s,t] interval, thus $1–R_b^{(j)}(0,s,t,h)$ is the probability that at least one such event occurs in that interval.
Formula (14), in the same way as (10), can be converted to the following form:

$$\Lambda(h) = \lambda \times \pi(h) \quad (16)$$

where λ, $\times$ and $\pi(h)$ have been already defined, and $\Lambda(h)$ is a one-row matrix composed of the intensities $\Lambda_b^{(j)}$, and defined as follows:

$$\Lambda(h) = [\Lambda_1^{(1)}(h),\ldots, \Lambda_{m(1)}^{(1)}(h),\ldots,$$

$$\Lambda_1^{(n)}(h),\ldots, \Lambda_{m(n)}^{(n)}(h)] \quad (17)$$

Let us now define the stochastic process $X_t^{(b,j)}$ as the number of events $E_b^{(j)}$, whether primary or not, in the time interval (0, t]. The following theorem holds:

Theorem 3
$X_t^{(b,j)}$ is a Poisson process with the following intensity:

$$\Lambda_b^{(j)} = \lambda_b^{(j)} + \sum_{h\geq 1} \Lambda_b^{(j)}(h) \quad (18)$$

In practice, the sum in (18) is only computed for several values of h, i.e. for $h \leq h_{max}$, where h_{max} is such that $\Lambda_b^{(j)}(h)$ is close to 0 for $h > h_{max}$.

Proof: Due to the assumption that primary events in all the processes are mutually independent, and cascades of events occur instantaneously, $X_t^{(b,j)}$ is a superposition of independent processes $X_t^{(b,j,h)}$, $h \geq 0$, where $X_t^{(b,j,h)}$ is the number of events $E_b^{(j)}$ in the interval (0, t], such that each $E_b^{(j)}$ is the result of a h-step cascade triggered by a primary $E_a^{(i)}$, $(a,i) \neq (b,j)$ for $h \geq 1$. From Assertion 1 and Theorems 1 and 2 it follows that $X_t^{(b,j)}$ is a Poisson process with the intensity given by (18), Q.E.D.

Corollary:
The risk $R_b^{(j)}(k, s, t)$, i.e. the probability that exactly k events $E_b^{(j)}$ (whether primary or not) occur in the time interval (s, t] is found from the following formula:

$$R_b^j(k,s,t) = \sum_{h\geq 0} R_b^j(k,s,t,h) =$$

$$= \frac{\left[\Lambda_b^{(j)} \cdot (t-s)\right]^k}{k!} \exp\left[-\Lambda_b^{(j)} \cdot (t-s)\right] \quad (19)$$

where $\Lambda_b^{(j)}$ is given by (18).

Let us note that $R_b^{(j)}(0,s,t)$ is the probability that no event $E_b^{(j)}$ (whether primary or not) occurs in the (s,t] interval, thus $1–R_b^{(j)}(0,s,t)$ is the probability that at least one event $E_b^{(j)}$ occurs in that interval.

Theorem 4
Formula (19) can be put in the following simpler form:

$$\Lambda = \lambda + \lambda \times \sum_{h\geq 1} \pi(h) \quad (20)$$

where + and $\sum$ are the usual addition operations on matrices, and Λ is a one-row matrix composed of the intensities $\Lambda_b^{(j)}$, and defined as follows:

$$\Lambda = [\Lambda_1^{(1)},\ldots, \Lambda_{m(1)}^{(1)},\ldots, \Lambda_1^{(n)},\ldots, \Lambda_{m(n)}^{(n)}] \quad (21)$$

In practice, the sum in (20) is only computed for several values of h, i.e. for $h \leq h_{max}$, where h_{max} is such that the elements of $\pi(h)$ are close to 0 for $h > h_{max}$.

Proof: The definition of Λ and formula (18) yield:

$$\Lambda = \lambda + \sum_{h\geq 1} \Lambda(h) \quad (22)$$

In view of (16) and distributivity of matrix multiplication w.r.t. addition the above equality converts to:

$$\Lambda = \lambda + \sum_{h\geq 1} \lambda \times \pi(h) = \lambda + \lambda \times \sum_{h\geq 1} \pi(h) \quad (23)$$

Q.E.D.

Corollary:
With use of (20) the elements of Λ are computed much faster than by using (18) or (22). If (22) along with (16) is applied, then each $\Lambda(h)$, $h \geq 1$, is computed individually, i.e. h_{max} matrix multiplications are executed. In turn, (20) requires only one matrix multiplication and $h_{max}–1$ additions to be executed, and adding $\pi(h)$ to $\pi(h+1)$ is numerically less complex than multiplying λ by $\pi(h)$. However, we need $\Lambda(h)$ to compute the risk $R_b^{(j)}(k, s, t, h)$.

5. The risk evaluation algorithm

Based on the results of Sections 3 and 4 the following algorithm for computing the risk of occurrence of one or more instances of $E_b^{(j)}$ in the time interval (s, t] is constructed:

1. Arrange the input data into the matrixes λ and $\pi(1)$ defined respectively by (12) and Fig. 1
2. Using (9) determine the matrices $\pi(h)$, $h \geq 2$
3. Using (11) and (16) determine the matrices $\Lambda(h)$, $h \geq 1$ defined by (13) and (17)
4. Using (18) determine the matrix Λ defined by (21)

5. Compute the risks $R_b^{(j)}(k, s, t)$, j=1,…,n, b=1,…,m(j) for given k, s and t

In practice, the computations are carried out only for the first several values of h for which the elements of $\pi(h)$ differ significantly from 0. The author has implemented the algorithm as a computer program whose example results are presented in the next section.

6. A real-life example

Let us consider three processes being realized in a port area which incorporates oil and container terminals: p_1 – vessel traffic to or from the harbor, p_2 – crude oil transfer to or from tankers in the oil terminal, p_3 – truck traffic to and from the container terminal. The following events can occur in the individual processes:
In p_1:
$E_1^{(1)}$ – vessel collision with another vessel or pier,
$E_2^{(1)}$ – spill of burning oil in the port waters,
$E_3^{(1)}$ – vessel on fire;
In p_2:
$E_1^{(2)}$ – pipeline or hose damage and/or ignition,
$E_2^{(2)}$ – onshore tank on fire;
In p_3:
$E_1^{(3)}$ – truck accident.
Let us assume that the following cause-effect relations hold between the above events:
$E_1^{(1)} \rightarrow E_2^{(1)}$, $E_2^{(1)} \rightarrow E_3^{(1)}$, $E_2^{(1)} \rightarrow E_1^{(2)}$
$E_1^{(2)} \rightarrow E_2^{(2)}$, $E_2^{(2)} \rightarrow E_1^{(2)}$, $E_1^{(2)} \rightarrow E_2^{(1)}$
$E_1^{(3)} \rightarrow E_1^{(2)}$
which means that $\pi^{(1,1)}(1,2)$, $\pi^{(1,1)}(2,3)$, $\pi^{(1,2)}(3,1)$, $\pi^{(2,2)}(1,2)$, $\pi^{(2,2)}(2,1)$, $\pi^{(2,1)}(1,2)$ and $\pi^{(3,2)}(1,1)$ have non-zero values, and all other 1-st grade C-E probabilities are equal to zero. We also assume that only the events $E_1^{(1)}$ (vessel collision), $E_3^{(1)}$ (vessel on fire), $E_2^{(2)}$ (tank on fire) and $E_1^{(3)}$ (truck accident) can occur as primary ones, i.e. only the respective intensities $\lambda_b^{(j)}$ are greater than 0. The unit of each $\lambda_b^{(j)}$, j=1,…,n, b=1,…,m(j) is 1/year, thus, on average, one instance of $E_b^{(j)}$ as a primary event occurs in a period of $1/\lambda_b^{(j)}$ years.
The author-developed computer program based on the algorithm from Section 5, applied to the above described case, produced the following output:

INPUT DATA PRINTOUT:

Number of processes: 3
Number of events in process 1: 3
Number of events in process 2: 2
Number of events in process 3: 1

Matrix lambda[j,b], j=1,...,n, b=1,...,m(j):
0.5000 0.0000 0.5000
0.0000 0.5000
0.5000

Matrix pi_1:
0.0000 0.5000 0.0000   0.0000 0.0000   0.0000
0.0000 0.0000 0.9000   0.0000 0.0000   0.0000
0.0000 0.0000 0.0000   0.4000 0.0000   0.0000

0.0000 0.9000 0.0000   0.0000 0.9000   0.0000
0.0000 0.0000 0.0000   0.5000 0.0000   0.0000

0.0000 0.0000 0.0000   0.8000 0.0000   0.0000

RESULTS PRINTOUT:

Matrix Lambda_1[j][b]:
0.0000 0.2500 0.0000
0.8500 0.0000
0.0000

Matrix Lambda_2[j][b]:
0.0000 0.7650 0.2250
0.0000 0.5400
0.0000

Matrix Lambda_3[j][b]:
0.0000 0.0000 0.5265
0.0900 0.0000
0.0000

Matrix Lambda_4[j][b]:
0.0000 0.0344 0.0000
0.0000 0.0810
0.0000

Matrix Lambda_5[j][b]:
0.0000 0.0000 0.2369
0.0000 0.0175
0.0000

Cascades of step >5 are neglected.
RISK MATRICES FOR DIFFERENT TIME AND QUANTITY PARAMETERS

Matrix R[j][b](2.00 years, 0 events):
0.3679 0.1226 0.0510
0.1526 0.1026
0.3679

Each of the above values subtracted from 1 is the probability that at least one respective $E_b^{(j)}$ (whether primary or not) occurs in a 2-year period.

Matrix R[j][b](2.00 years, 1 event):
0.3679 0.2573 0.1517
0.2869 0.2336

0.3679

Matrix R[j][b](2.00 years, 2 events):
0.1839 0.2700 0.2258
0.2697 0.2660
0.1839

Matrix R[j][b](2.00 years, 3 events):
0.0613 0.1889 0.2240
0.1690 0.2019
0.0613

Matrix R[j][b](2.00 years, 4 events):
0.0153 0.0991 0.1667
0.0794 0.1149
0.0153

Matrix R[j][b](2.00 years, 5 events):
0.0031 0.0416 0.0993
0.0299 0.0523
0.0031

Matrix R[j][b](2.00 years, 6 events):
0.0005 0.0146 0.0492
0.0094 0.0199
0.0005

Matrix R[j][b](2.00 years, 7 events):
0.0001 0.0044 0.0209
0.0025 0.0065
0.0001

Matrix R[j][b](2.00 years, 8 events):
0.0000 0.0011 0.0078
0.0006 0.0018
0.0000

Matrix R[j][b](2.00 years, 9 events):
0.0000 0.0003 0.0026
0.0001 0.0005
0.0000

Matrix R[j][b](2.00 years, 10 events):
0.0000 0.0001 0.0008
0.0000 0.0001
0.0000

Clearly, the presented model is a simplified representation of a complex conglomerate of technical facilities and operations that constitute a real port environment. Also, the intensities $\lambda_b^{(j)}$ and probabilities $\pi_{a,b}^{(i,j)}$ are example values rather than expert elicited ones or statistically processed operational data.

Acknowledgements

The paper presents the results developed in the scope of the HAZARD project titled "Mitigating the Effects of Emergencies in Baltic Sea Region Ports" that has received funding from the Interreg Baltic Sea Region Programme 2014-2020 under grant agreement No #R023. https://blogit.utu.fi/hazard/

References

[1] Malinowski, J. (2017). Modeling hazard-related Interactions between the processes realized in and around the Baltic Sea Region ports. *Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, Vol. 8, No. 1, 2017*, 87-96.

[2] Swift, A. W. (2008). Stochastic models of cascading failures. *Journal of Applied Probability* 45, 907-921.

[3] Iyer, S. M., Nakayama, M.K. & Gerbessiotis A. V. (2009). A Markovian Dependability Model with Cascading Failures. *IEEE Transactions On Computers* 58, 1238-1249.

[4] Dong, H. & Cui, L. (2016). System Reliability Under Cascading Failure Models. *IEEE Transactions on Reliability* 65, 929-940.

[5] Bielecki, T.R., Vidozzi, A., Vidozzi, L. & Jakubowski, J. (2008). Study of dependence for some stochastic processes. *Stochastic Analysis and Applications* 26, 1-16.

[6] Pescaroli, G. & Alexander, D. (2015). A definition of cascading disasters and cascading effects: Going beyond the "toppling dominos" metaphor. In: Planet@Risk 2(3), 58-67, Davos: Global Risk Forum GRF Davos.