Effect of hall currents on thermal instability of dusty couple stress fluid

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Abstract   In this paper, effect of Hall currents on the thermal instability of couple-stress fluid permeated with dust particles has been considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, dust particles and Hall currents are found to have destabilizing effect while couple stresses have stabilizing effect on the system. Magnetic field induced by Hall currents has stabilizing/destabilizing effect under certain conditions. It is found that due to the presence of Hall currents (hence magnetic field), oscillatory modes are produced which were non-existent in their absence.

Keywords: Couple-stress fluid; Dust particles; Hall currents; Stationary convection; Oscillatory modes; Magnetic field

Nomenclature

C	–	concentration of the fluid
C_{pt}	–	heat capacity of particles, $\mathrm{J\,kg^{-1}K^{-1}}$
C_v	–	heat capacity of fluid at constant volume, $\mathrm{J\,kg^{-1}K^{-1}}$
D	–	derivative with respect to z $(= d/dz)$
d	–	depth of layer, m
e	–	charge of an electron
F	–	couple-stress parameter $(= \mu'/\rho d^2)$
i	–	imaginary unit
$\vec{g}(0,0,-g)$	–	gravity field, $\mathrm{m\,s^{-2}}$
g	–	gravitational acceleration, $\mathrm{m\,s^{-2}}$

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$\vec{H}\,(0,0,H)$ –   magnetic field, G

$\vec{h}(h_x, h_y, h_z)$ –   perturbation in magnetic field

K –   Stokes' drag coefficient, $\mathrm{kg\,s^{-1}}$

k –   wave number, $\mathrm{m^{-1}}$

k_T –   thermal diffusivity, $\mathrm{m^2 s^{-1}}$

k_x, k_y –   components of wave number k along x-axis, y-axis, $\mathrm{m^{-1}}$

M –   Hall current parameter $= (H/4\pi N'e\eta)$

m –   mass of single particle, kg

N –   perturbation in suspended particle number density N_0

N_0 –   suspended particles number density

N'_0 –   electron number density

n –   growth rate, $\mathrm{s^{-1}}$

p –   pressure, Pa

$\mathrm{Pr_1}$ –   Prandtl number

Pr_m –   magnetic Prandtl number

Q –   Chandrasekhar number $(= \mu_e H^2 d^2/4\pi\rho_0\nu\eta)$

$\vec{q}\,(u,v,w)$ –   velocity of fluid with components u, v, w, $\mathrm{m\,s^{-1}}$

$\vec{q}_d\,(l,r,s)$ –   velocity of suspended particles with components l, r, s, $\mathrm{m\,s^{-1}}$

R –   Rayleigh number $(= g\alpha\beta d^4/\nu\kappa)$

R_1 –   modified Rayleigh number

T –   temperature, K

t –   time, s

x –   dimensionless wave number

$\vec{x}\,(x,y,z)$ –   space coordinate

x, y, z –   Cartesian coordinates

Greek symbols

α –   coefficient of thermal expansion, $\mathrm{K^{-1}}$

β –   uniform temperature gradient, $\mathrm{K\,m^{-1}}$

η –   electrical resistivity, $\mathrm{m^2 s^{-1}}$

η' –   suspended particle radius, m

δp –   perturbation in pressure

$\delta\rho$ –   perturbation in density

θ –   perturbation in temperature, K

μ –   dynamic viscosity, $\mathrm{kg\,m^{-1}s^{-1}}$

μ' –   couple-stress viscosity

μ_e –   magnetic permeability

ν –   kinematic viscosity, $\mathrm{m^2 s^{-1}}$

ν' –   kinematic viscoelasticity, $\mathrm{m^2 s^{-1}}$

ρ –   fluid density, $\mathrm{kg\,m^{-3}}$

ζ – z component of vorticity

η – z electrical resistivity

ξ – z component of current density

∇ –   nabla operator

Subscripts and superscripts

d	–	upper surface
0	–	botom surface
$*$	–	complex conjugate

1   Introduction

Chandrasekhar [1] has given the theory of Benard convection in a viscous, Newtonian fluid layer heated from below. Chandra [2] observed that in an air layer, convection occur at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, 'Columnar instability'. However, for layers deeper than 10 mm, a Benard type cellular convection was observed. Thus, there is a contradiction between the theory and the experiment. The use of Boussinesq approximation has been made through out which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma [3] has considered the effect of rotation and magnetic field on the thermal instability in compressible fluids. The fluid has been considered to be Newtonian in all the above studies while Scanlon and Segel [4] have considered the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced solely because of the heat capacity of the pure fluid.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The theory of couple-stress fluids is proposed by Stokes' [5]. One of the applications of the couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid. According to the theory of Stokes [5], couple stresses are found to appear in noticeable magnitude in fluids with very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [6] modeled synovial fluid as a couple-stress fluids in human joints. Goel *et. al.* [7] has studied the hydromagnetic stability of

an unbounded couple stress binary fluid mixture having vertical temperature and concentration gradients with rotation. An electrically conducting couple-stress fluid heated from below in a porous medium in the presence of uniform horizontal magnetic field has also been submitted by Sharma and Sharma [8]. The use of magnetic field is being made for the clinically purposes in detection and cure of certain diseases with the help of magnetic field devices/instruments. The problem of a couple-stress fluid heated from below in a porous medium is considered by Sharma and Sharma [9] and Sharma and Thakur [10].

Recent space craft observations have confirmed that the dust particles play a significant role in the dynamics of the atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. Further, environmental pollution is the main cause of dust to enter the human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromosomes and genetic mutation so observed are likely to breed cancer as malformations in the coming progeny. Therefore, it is very essential to study the blood flow with dust particles. Considering blood as couple-stress fluid and dust particles as microorganisms, Rathod and Thippeswamy [11] have studied the gravity flow of pulsatile blood through closed regular inclined channel with microorganisms. Sunil *et. al.* [12] have studied the effect of suspended particles on couple-stress fluid heated and soluted from below in a porous medium and found that suspended particles have destabilizing effect on the system.

If an electric field is applied at right angles to the magnetic field, the whole currents will not flow along the electric field. The tendency of electric current to flow across an electric field in the presence of magnetic field is called Hall effect. The Hall currents are likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sharma and Sharma [13] have studied the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field. Gupta [14] has seen the effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of uniform vertical magnetic field.

Singh and Dixit [15] have studied the effect of Hall currents on the thermal instability of a compressible couple-stress fluid with suspended particles while Kumar and Kumar [16] have seen the combined effect of dust particles, magnetic field and rotation on a couple-stress fluid heated from below.

Aggarwal and Makhija [17] have studied the combined effect of magnetic field and rotation on couple-stress fluid heated from below in the presence of suspended particles. The effect of suspended particles, magnetic field and rotation on the thermal stability of a ferromagnetic fluid has been studied by Aggarwal and Verma [18]. The effect of compressibility and suspended particles on thermal convection in Walters' B' elastico-viscous fluid in hydromagnetics has been considered by Sharma and Aggarwal [19]. Postrzednik [20] has studied the influence of the heat transfer on the specific thermal capacity of the flowing compressible fluid. The unsteady natural convection in micropolar nanofluids has been studied by Rup and Nering [21].

In this paper, the effect of Hall currents on thermal instability of couple-stress fluid in the presence of dust particles is considered.

2 Mathematical formulation

Consider a static state in which an incompressible, Stokes' [5] couple-stress fluid layer of thickness d heated from below so that a uniform temperature and density at the bottom surface $z = 0$, are T_0, ρ_0, respectively, and at the upper surface $z = d$, are T_d, ρ_d, and a uniform adverse temperature gradient $\beta = \left| \frac{dT}{dz} \right|$ is maintained and the layer is acted upon by the gravity field $\vec{g}(0, 0, -g)$ and uniform magnetic field $\vec{H}(0, 0, H)$.
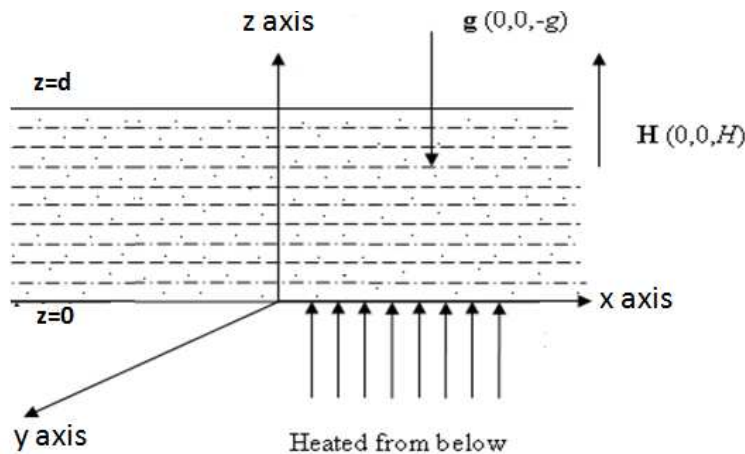

Figure 1: Schematic sketch of the problem studied.

Let p, ρ, T, α, ν, μ', μ_e, k_T, and $\vec{q}(u, v, w)$ denote respectively pressure, density, temperature, coefficient of thermal expansion, kinematic viscosity, couple-stress viscosity, magnetic permeability, thermal diffusivity and fluid velocity. $\vec{q}_d(l, r, s)$ and N_0 denote the velocity and number density of suspended particles, respectively. $K = 6\pi\mu\eta'$, where η' is the particle radius, is a constant and $\vec{x} = (x, y, z)$. Then the equation of motion and continuity of couple-stress fluid are

$$
\begin{aligned}
\frac{\partial \vec{q}}{\partial t} &= -\frac{1}{\rho_0}\nabla p + \vec{g}\alpha\theta - \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\vec{q} \\
&\quad + \frac{KN_0}{\rho_0}(\vec{q}_d - \vec{q}) + \frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla \times \vec{h}\right) \times \vec{H}\right],
\end{aligned} \tag{1}
$$

$$
\nabla \cdot \vec{q} = 0 . \tag{2}
$$

Assuming uniform particles size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equation of motion (1), proportional to the velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The distances between particles are assumed to be so large compared with their diameter that interparticle reactions need not be accounted for. The effect of pressure, gravity and magnetic field on suspended particles, assuming large distances apart, are negligibly small and therefore ignored. If mN is the mass of the particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$
mN_0\frac{\partial \vec{q}_d}{\partial t} = KN_0(\vec{q} - \vec{q}_d) , \tag{3}
$$

$$
\frac{\partial N}{\partial t} + \nabla \cdot (N\vec{q}_d) = 0 . \tag{4}
$$

Let C_v and C_{pt} denote the heat capacity of fluid at constant volume and heat capacity of the particles, respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of energy yields

$$
\frac{\partial T}{\partial t} + \frac{mNC_{pt}}{\rho_0 C_v}\left(\frac{\partial}{\partial t} + \vec{q}_d \cdot \nabla\right)T = k_T\nabla^2 T . \tag{5}
$$

The kinematic viscosity, ν, couple-stress viscosity, μ', thermal diffusivity, k_T, and coefficient of thermal expansion, α, are all assumed to be constants. The Maxwell's equations in the presence of Hall currents yield

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{H} \right) + \eta \nabla^2 \vec{h} - \frac{C}{4\pi N' e} \nabla \times \left[\left(\nabla \times \vec{h} \right) \times \vec{H} \right] \ , \qquad (6)$$

$$\nabla \cdot \vec{h} = 0 \ , \qquad (7)$$

where η, N', and e stands for the electrical resistivity, electron number density, and the charge of an electron, respectively.

The equation of state for the fluid can be express as

$$\rho = \rho_0 \big[1 - \alpha \left(T - T_0 \right) \big] \ . \qquad (8)$$

The basic motionless solution is

$$\vec{q} = (0, 0, 0) \ , \qquad \vec{q}_d = (0, 0, 0) \ , \qquad T = T_0 - \beta z \ ,$$
$$\rho = \rho_0 \left(1 + \alpha \beta \, z \right) \ , \quad N = N_0 = \text{const.} \qquad (9)$$

Assume small perturbations around the basic solution and let δp, $\delta \rho$, θ, $\vec{h}(h_x, h_y, h_z)$, $\vec{q}(u, v, w)$, and $\vec{q}_d(l, r, s)$ denote, respectively, the perturbations in fluid pressure, p, density, ρ, temperature, T, magnetic field, $\vec{H}(0, 0, H)$, fluid velocity, $(0, 0, 0)$, and suspended particles velocity, $(0, 0, 0)$. The change in density $\delta \rho$ caused mainly by the perturbation θ in temperature is given by

$$\delta \rho = -\alpha \rho_0 \theta \ . \qquad (10)$$

Then the linearized perturbation equations of couple-stress fluid become

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p + \overrightarrow{g} \alpha \theta - \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \vec{q}$$
$$+ \frac{K N_0}{\rho_0} \left(\vec{q}_d - \vec{q} \right) + \frac{\mu_e}{4\pi \rho_0} \left[\left(\nabla \times \vec{h} \right) \times \vec{H} \right] \ , \qquad (11)$$

$$\nabla \cdot \vec{q} = 0 \ , \qquad (12)$$

$$m N_0 \frac{\partial \vec{q}_d}{\partial t} = K N_0 \left(\vec{q} - \vec{q}_d \right) \ , \qquad (13)$$

$$(1 + h_1) \frac{\partial \theta}{\partial t} = \beta \left(w + h_1 s \right) + \kappa \nabla^2 \theta \ , \qquad (14)$$

$$\nabla \cdot \vec{h} = 0 \ , \qquad (15)$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{H} \right) + \eta \nabla^2 \vec{h} - \frac{C}{4\pi N'e} \nabla \times \left[\left(\nabla \times \vec{h} \right) \times \vec{H} \right] , \qquad (16)$$

where $\kappa = \frac{q}{\rho_0 C_v}$ and $h_1 = \frac{m N_0 C_{pt}}{\rho_0 C_v}$.

Eliminating $\vec{q}_d$ in Eq. (11) using Eq. (13) as dusty particles velocity is not very slow ($(q - q_d)$ is not very large) and writing the scalar components of resulting equations and eliminating u, v, h_x, h_y, and δp between them, by using Eqs. (12) and (15), we obtain

$$\left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \left[\frac{\partial}{\partial t} \nabla^2 w - g\alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{\mu_e H}{4\pi \rho_0} \frac{\partial}{\partial z} \nabla^2 h_z \right] + \frac{m N_0}{\rho_0} \frac{\partial}{\partial t} \nabla^2 w$$

$$= \left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 w , \qquad (17)$$

$$\left\{ \frac{\partial}{\partial t} \left[1 + \frac{m N_0}{\rho_0 \left(\frac{m}{k} \frac{\partial}{\partial t} + 1 \right)} \right] - \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \right\} \zeta = \frac{\mu_e H}{4\pi \rho_0} \frac{\partial \xi}{\partial z} , \qquad (18)$$

$$\left(H_1 \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \theta = \beta \left(\frac{m}{k} \frac{\partial}{\partial t} + H_1 \right) w , \qquad (19)$$

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2 \right) h_z = H \frac{\partial w}{\partial z} - \frac{CH}{4\pi N'e} \frac{\partial \xi}{\partial z} , \qquad (20)$$

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2 \right) \xi = H \frac{\partial}{\partial z} \zeta + \frac{CH}{4\pi N'e} \frac{\partial}{\partial z} h_z , \qquad (21)$$

where $H_1 = 1 + h_1$, $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is z-component of vorticity and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is z-component of current density.

3   Normal mode analysis

Analyzing the disturbances into normal modes and assume that the perturbation quantities are of the form

$$[w, h_z, \theta, \zeta, \xi] = \left[W(z), K(z), \Theta(z), Z(z), X(z) \right] \exp \left(i k_x x + i k_y y + n t \right) , \tag{22}$$

where k_x, k_y are wave numbers along x and y directions, respectively, $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number of the disturbances and n

is the growth rate, $W(z)$, $K(z)$, $\Theta(z)$, $Z(z)$, $X(z)$ are defined by Chandrasekhar [1].

Using expression (22), Eqs. (17)–(20) in nondimensional form become

$$\left[\sigma' + F\left(D^2 - a^2\right) - 1\right]\left(D^2 - a^2\right)W$$

$$= -\frac{g\alpha a^2 d^2}{\nu}\Theta + \frac{\mu_e H d}{4\pi\rho_0\nu}\left(D^2 - a^2\right)DK \, , \qquad (23)$$

$$\left\{\sigma' - d^2\left[1 - F\left(D^2 - a^2\right)\right]\right\}Z = \frac{\mu_e H d}{4\pi\rho_0\nu}DX \, , \qquad (24)$$

$$\left(D^2 - a^2 - H_1\mathrm{Pr}_1\sigma\right)\Theta = \frac{\beta d^2}{\kappa}\left(H_1 + \frac{\tau_1\nu}{\kappa}\sigma\right)W \, , \qquad (25)$$

$$\left(D^2 - a^2 - \mathrm{Pr}_m\sigma\right)K = -\frac{Hd}{\eta}DW + \frac{cHd}{4\pi N'e\eta}DX \, , \qquad (26)$$

$$\left(D^2 - a^2 - \mathrm{Pr}_m\sigma\right)X = -\frac{Hd}{\eta}DZ + \frac{cH}{4\pi N'e\eta d}\left(D^2 - a^2\right)DK \, , \qquad (27)$$

where we have nondimensionalized various parameters as follows:

$$a = kd, \quad \sigma = \frac{nd^2}{\nu}, \quad \tau_1 = \frac{\tau\nu}{d^2}, \quad \mathrm{Pr}_1 = \frac{\nu}{\kappa}, \quad \mathrm{Pr}_m = \frac{\nu}{\eta}, \quad D = \frac{d}{dz},$$

$$F = \frac{\mu'}{\nu\rho_0 d^2}, \quad \sigma' = \frac{n'd^2}{\nu}, \quad H_1 = 1 + h_1, \quad n' = n\left(1 + \frac{mN_0 K/\rho_0}{mn + K}\right).$$

After eliminating Θ, Z, X, and K from Eqs. (23)–(27), we obtain

$$\left\{[\sigma' - d^2\left[1 - F(D^2 - a^2)\right]\right\}(D^2 - a^2)W + \frac{\mathrm{R}a^2}{(D^2 - a^2 - H_1\mathrm{Pr}_1\sigma)}\left(H_1 + \frac{\tau_1\nu}{\kappa}\sigma\right)W$$

$$+ \mathrm{Q}\left[\frac{(D^2 - a^2 - \mathrm{Pr}_m\sigma)\left\{[\sigma' - d^21 - F(D^2 - a^2)]\right\} + \mathrm{Q}D^2}{(D^2 - a^2 - \mathrm{Pr}_m\sigma)^2\left\{\sigma' - d^2\left[1 - F(D^2 - a^2)\right]\right\} + \mathrm{Q}(D^2 - a^2 - \mathrm{Pr}_m\sigma)D^2}\right.$$

$$\left. - \mathrm{M}\left\{\sigma' - d^2\left[1 - F(D^2 - a^2)\right]\right\}(D^2 - a^2)D^2\right]DW = 0 \, , \qquad (28)$$

where $\mathrm{Q} = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is Chandrasekhar number, $\mathrm{R} = \frac{g\alpha\beta d^4}{\nu\kappa}$ is thermal Rayleigh number, and $\mathrm{M} = \left(\frac{H}{4\pi N'e\eta}\right)$ is the nondimensional number accounting for Hall currents.

Consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The boundary conditions, appropriate for the problem, are

$$W = 0\,, Z = 0\,, \Theta = 0 \text{ and } D^2 W = 0\,, D^4 W = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (29)$$

The proper solution of Eq. (29) characterizing the lowest mode is

$$W = W_0 \sin \pi z \,, \quad (30)$$

where W_0 is a constant. Substituting the proper solution, Eq. (30), in Eq. (28), we obtain the dispersion relation

$$\frac{R_1 x}{(1 + x + iH_1 \mathrm{Pr}_1 \sigma_1)} \left(H_1 + \frac{\tau_1 v}{\kappa} \sigma \right)$$
$$= Q_1 \left[\frac{(1 + x + i\mathrm{Pr}_m \sigma_1)\{i\sigma' + [1 + F_1(1+x)]\} + Q_1}{(1 + x + i\mathrm{Pr}_m \sigma)^2 \{i\sigma' + [1 + F_1(1+x)]\} + Q_1(1 + x + i\mathrm{Pr}_m \sigma_1)} \right.$$
$$\left. - M\{i\sigma' + [1 + F_1(1+x)]\}(1+x) - \{i\sigma' + [1 + F_1(1+x)]\}(1+x) \right], \quad (31)$$

where: $R_1 = \frac{\mathrm{R}}{\pi^4}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $Q_1 = \frac{Q}{\pi^2}$, and $F_1 = \pi^2 F$.

4   Stationary convection

At stationary convection, when the instability sets, the marginal state will be characterized by $\sigma = 0$. Thus, putting $\sigma = 0$ in Eq. (31), we get

$$R_1 = \frac{Q_1}{xH_1} \left[\frac{(1+x)[1 + F_1(1+x)] + Q_1}{(1+x)[1 + F_1(1+x)] + Q_1 - \mathrm{M}[1 + F_1(1+x)]} \right]$$
$$+ \frac{(1+x)^2[1 + F_1(1+x)]}{xH_1}. \quad (32)$$

The above relation expresses the modified Rayleigh number R_1 as a function of the parameters Q_1, H_1, F_1, M, and dimensionless wave number x. To study the effect of magnetic field, dust particles, couple stresses and Hall currents, we examine the nature of $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dH_1}$, $\frac{dR_1}{dF_1}$, and $\frac{dR_1}{dM}$ analytically. Equation (32) gives:

$$\frac{dR_1}{dQ_1} = \frac{1}{xH_1} \left[\frac{(1+x)[1 + F_1(1+x)] + Q_1}{(1+x)[1 + F_1(1+x)] + Q_1 - \mathrm{M}[1 + F_1(1+x)]} \right]$$
$$- \frac{1}{xH_1} \frac{\mathrm{M}Q_1[1 + F_1(1+x)]}{\{(1+x)[1 + F_1(1+x)] + Q_1 - \mathrm{M}[1 + F_1(1+x)]\}^2}, \quad (33)$$
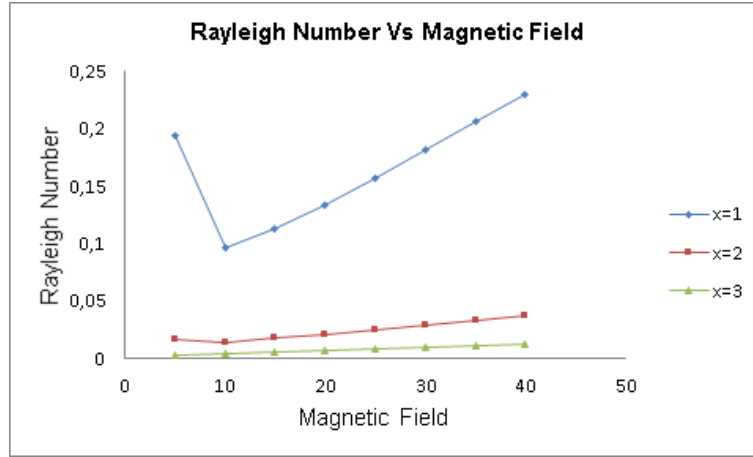
Figure 2: Variation of Rayleigh number, R_1, and magnetic field, Q_1, for a fixed $H_1 = 0.1$, $F_1 = 0.1$, M $= 6$ for different values of wave numbers and magnetic field parameter.

which shows that magnetic field has a stabilizing or destabilizing effect according to $(1+x)[1+F_1(1+x)]+Q_1$ is greater or less than M $[1+F_1(1+x)]$. This result, illustrated in Fig. 2, is in agreement with the result obtained by Aggarwal and Makhija [17], Kumar and Kumar [16].

$$\frac{dR_1}{dH_1} = -\frac{Q_1}{xH_1^2}\left\{\frac{(1+x)[1+F_1(1+x)]+Q_1}{(1+x)[1+F_1(1+x)]+Q_1 - \mathrm{M}[1+F_1(1+x)]}\right\}$$
$$-\frac{(1+x)^2[1+F_1(1+x)]}{xH_1^2}, \tag{34}$$

which clearly shows that dust particles have destabilizing effect on thermal instability in a couple-stress fluid. This result is also evident from Fig. 3 and is same as observed by Aggarwal and Makhija [17], Kumar and Kumar [16].

$$\frac{dR_1}{dF_1} = \frac{Q_1(1+x)}{xH_1}$$
$$\times\left\{(1+x)\left[(1+x)[1+F_1(1+x)]+Q_1-\mathrm{M}[1+F_1(1+x)]\right]\right.$$
$$\left.-\frac{\left[(1+x)[1+F_1(1+x)]+Q_1\right][1+x-\mathrm{M}]}{\left[(1+x)[1+F_1(1+x)]+Q_1-\mathrm{M}[1+F_1(1+x)]\right]^2}+1\right\}. \tag{35}$$

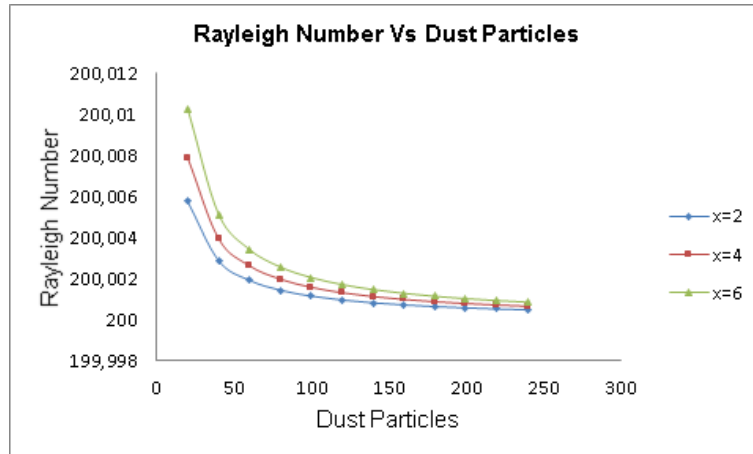From Eq. (35), it follows that couple stresses have stabilizing effect on the system which is clear from Fig. 4.

Figure 3: Variation of Rayleigh number, R_1, and dust particles, H_1, for a fixed $Q_1 = 1$, $F_1 = 1$, M = 10 for different values of wave numbers and dust particle parameters.
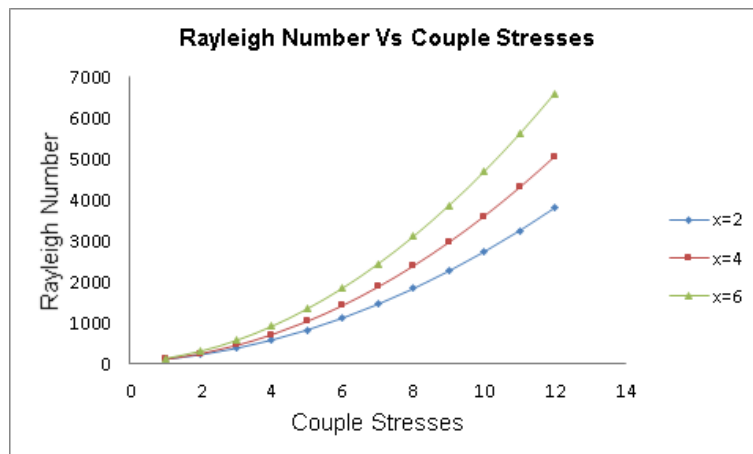

Figure 4: Variation of Rayleigh number, R_1, and couple stresses, F_1, for a fixed $Q_1 = 5$, $H_1 = 1$, M = 1 for different values of wave number and couple stresses.

$$\frac{dR_1}{dM} = -\frac{Q_1}{xH_1} \left\{ \frac{\left[\,(1+x)\,[1+F_1\,(1+x)] + Q_1\right]\left[1 + F_1\,(1+x)\,\right]}{\left[\,(1+x)\,[1+F_1\,(1+x)] + Q_1 - \mathrm{M}\,[1 + F_1\,(1+x)]\,\right]^2} \right\}.$$
(36)

From Eq. (36) can be observed that Hall currents have destabilizing effect on the system which is in agreement with Fig. 5. This result is same as obtained by Singh and Dixit [15]. The reason for destabilizing effect of Hall currents is accounted by Chandrasekhar [1].
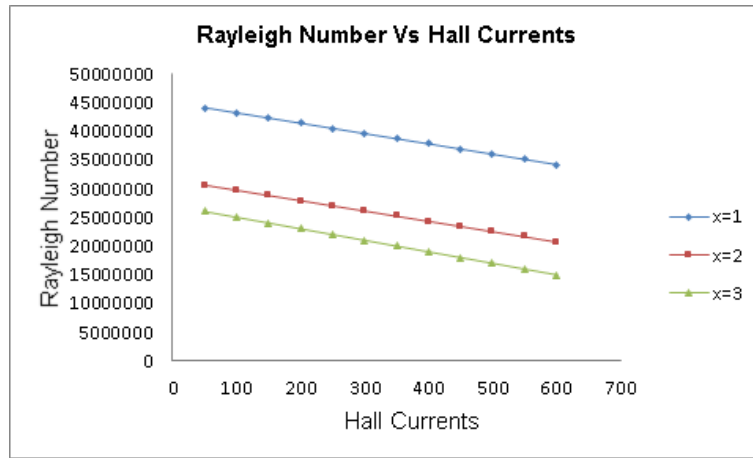

Figure 5: Variation of Rayleigh number, R_1 and Hall currents, M for a fixed $Q_1 = 3000$, $H_1 = 1$, $F_1 = 1$ for different values of wave number and Hall currents.

5 Stability of the system and oscillatory modes

To determine under what conditions the principle of exchange of stabilities (PES) is satisfied (i.e., σ is real and the marginal states are characterized by $\sigma = 0$) and oscillations come into play, we multiply Eq. (23) with W^* (complex conjugate of W) and integrate over the range of z and making use of Eqs. (24)–(27) together with the boundary conditions (29) and then we get

$$(1 - \sigma')\,I_1 - FI_2 + \frac{g\alpha\kappa a^2}{\beta\nu}\left(H_1 + \frac{\tau_1\nu}{\kappa}\sigma^*\right)^{-1}(I_3 + H_1\mathrm{Pr}_1\sigma^*I_4)$$
$$-\frac{\mu_e\eta}{4\pi\rho_0\nu}\,(I_5 + \mathrm{Pr}_m\sigma^*I_6) + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu}\,(I_7 + \mathrm{Pr}_m\sigma^*I_8)$$
$$+\,d^2\big[\,(\sigma' - 1)\,I_9 - FI_{10}\big] \,=\, 0\,,\quad (37)$$

where:

$$I_1 = \int \left(|DW|^2 + a^2 |W|^2 \right) dz , \qquad I_2 = \int \left(\left|D^2W\right|^2 + 2a^2 |DW|^2 + a^4 |W|^2 \right) dz ,$$

$$I_3 = \int \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz , \qquad\qquad\qquad I_4 = \int |\Theta|^2 dz ,$$

$$I_5 = \int \left(\left|D^2K\right|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \right) dz , \qquad I_6 = \int \left(|DK|^2 + a^2 |K|^2 \right) dz ,$$

$$I_7 = \int \left(|DX|^2 + a^2 |X|^2 \right) dz , \qquad\qquad\qquad I_8 = \int |X|^2 dz ,$$

$$I_9 = \int |Z|^2 dz , \qquad\qquad\qquad\qquad I_{10} = \int \left(|DZ|^2 + a^2 |Z|^2 \right) dz ,$$

and σ^* is complex conjugate of σ. The integrals $I_1 - I_{10}$ are all positive definite. Putting $\sigma = i\sigma_i \, (\sigma^* = -i\sigma_i)$ in Eq. (37) and equating imaginary parts, we obtain

$$\sigma_i \left[I_1 + \frac{g\alpha\kappa^2 a^2}{\beta\tau_1\nu^2} I_3 + \frac{g\alpha\kappa a^2}{\beta\nu} \mathrm{Pr}_1 I_4 - \frac{\mu_e\eta}{4\pi\rho_0\nu} \mathrm{Pr}_m I_6 + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu} \mathrm{Pr}_m I_8 - d^2 I_9 \right] = 0 . \tag{38}$$

It is clear from Eq. (38) that σ_i (growth rate parameter) may be zero or nonzero, which implies that modes may be nonoscillatory or oscillatory. In the absence of magnetic field (hence Hall currents) and dust particles, Eq.(38) reduces to

$$\sigma_i \left[I_1 + \frac{g\alpha\kappa^2 a^2}{\beta\tau_1\nu^2} I_3 + \frac{g\alpha\kappa a^2}{\beta\nu} \mathrm{Pr}_1 I_4 + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu} \mathrm{Pr}_m I_8 \right] = 0 . \tag{39}$$

The terms in the bracket are positive definite. Thus $\sigma_i = 0$ which means that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of magnetic field (hence Hall currents) and dust particles.

6   Conclusions

In this paper, the effect of Hall currents and dust particles has been considered on the thermal instability of a couple-stress fluid. The effect of various parameters such as magnetic field, dust particles, couple-stresses and Hall currents has been investigated analytically as well as graphically. The principal conclusions from the analysis are:

1. In order to investigate the effects of magnetic field, dust particles, couple-stresses and Hall currents, we examine the behaviour of $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dH_1}$, $\frac{dR_1}{dF_1}$, and $\frac{dR_1}{dM}$ analytically.

2. It is found that magnetic field has a stabilizing or destabilizing effect according to $(1+x)[1+F_1(1+x)]+Q_1$ is greater or less than $M[1+F_1(1+x)]$. This result is also verified from Fig. 2.

3. The dust particles and Hall currents have destabilizing effect on the system. The reason for destabilizing effect of Hall currents is accounted by Chandrasekhar [1].

4. The couple-stresses have stabilizing effect on the system.

5. The principle of exchange of stabilities is satisfied in the absence of magnetic field (hence Hall currents) and dust particles.

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