Scientific Journals
of the Maritime University of Szczecin

Zeszyty Naukowe
Akademii Morskiej w Szczecinie

2016, 46 (118), 141–146
ISSN 1733-8670 (Printed)
ISSN 2392-0378 (Online)
DOI: 10.17402/131

Received: 31.08.2015
Accepted: 09.05.2016
Published: 27.06.2016

Multi-step matrix game of safe ship control at various safe passing distances

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Key words: marine transport, safe navigation, optimization, game theory, computer simulation, decision support systems

Abstract

The paper introduces the process of safe ship control in collision situations using a differential game model with *m* participants. The basic model of process includes non-linear state equations and non-linear, time-varying constraints of the state variables as well as the quality game control index in the forms of game integral payment and final payment. As an approximated model of the manoeuvring process, a model of a multi-step matrix game in the form of a dual linear programming problem has been adopted here. The Game Control (*gc*) computer program has been designed in Matlab/Simulink software in order to determine the own ship safe trajectory. The considerations have been illustrated with computer simulation examples using the *gc* program for determining safe own ship trajectory in real navigation situations when passing commonly-encountered ships.

Introduction

The actual process of a ship passing other ships very often occurs in conditions of uncertainty and conflict accompanied by an inadequate cooperation of the ships with regard to the COLREG Rules. It is, therefore, reasonable to investigate, develop, and represent the methods of a ship safe handling using game theory based methods of computational intelligence. In practice, the process of handling a ship as a multidimensional dynamic object depends both on the accuracy of the details concerning the current navigational situation obtained from the ARPA anti-collision system and on the form of process model used for synthesis of the safe control algorithms. The most adequate model of the own ship control process in a situation of *m* encountered ships is the model of a differential game with *m* participants. The model of a differential game can be reduced to a model of a multi-step matrix game, which takes into account the value of collision risk with regard to determined own ship and also of strategies for *m* encountered ships (Bist, 2000; Kouemou, 2009; Zwierzewicz, 2012).

Process of safe and optimal traffic ship control

The ARPA system ensures automatic monitoring of at least $m = 20$ encountered j ships, determining their movement parameters (speed V_j and course ψ_j) and elements of approaching to the own ship (distance $D_{j\min}$ = DCPA$_j$ – Distance of the Closest Point of Approach and time $T_{j\min}$ = TCPA$_j$ – Time to the Closest Point of Approach) and also assess the risk of collision r_j (Figure 1).

While formulating the model of control process, it is essential to take into consideration both the kinematics and the dynamics of the own ship movement, the disturbances, the strategy of the encountered ships, and the formula assumed as the goal of the own ship handling. The diversity of selection of possible models directly affects the synthesis of the own ship control algorithms which

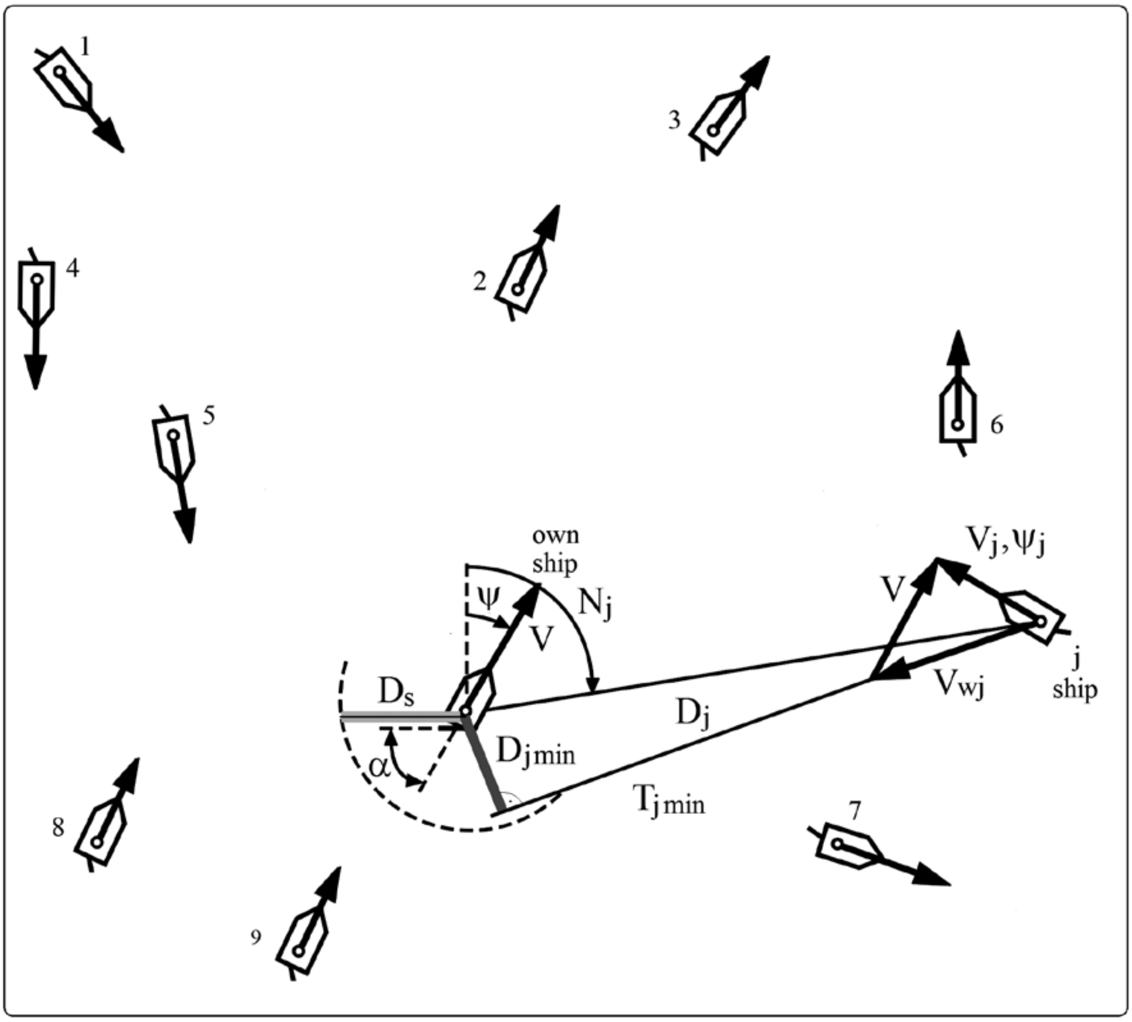

Figure 1. The process of the own ship passing *j* encountered ships

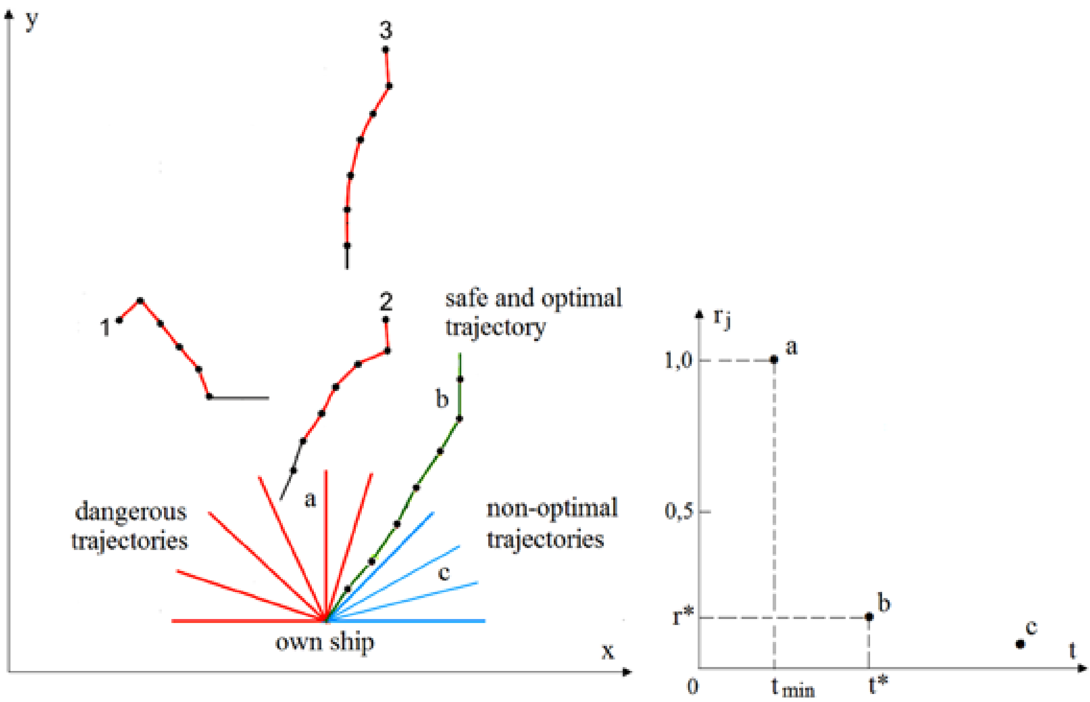

Figure 2. The possible trajectories and risk of collision of own ship in the situation of passing three encountered ships

are afterwards affected by the ship handling device, directly linked to the ARPA system and, consequently, determines the effects of safe and optimal control. Figure 2 illustrates possible trajectories with regard to their safety and effectiveness using an example of a situation where the own ship passes three other encountered ships and represents a set of compromises of own ship safe handling, measured in terms of a collision risk and time-optimal own ship strategy (Pietrzykowski, 2004; Perez, 2005; Millington & Funge, 2009).

Differential game model of the control process

The most general description of the own ship passing *j* number of other encountered ships is the model of a differential game of *j* number of objects (Figure 3).

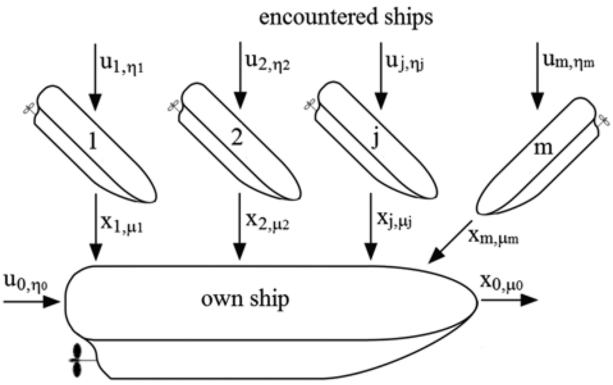

Figure 3. Block diagram of a differential game model *j* encountered ships as participants

General dynamic features of the process are described by a set of state equations in the following form:

$$\dot{x}_i = f_i[(x_{0,\mu_0}, x_{1,\mu_1}, ..., x_{j,\mu_j}, ..., x_{m,\mu_m}), (u_{0,\eta_0}, u_{1,\eta_1}, ..., u_{j,\eta_j}, ..., u_{m,\eta_m}), t]$$
$$i = 1,2,...,(j\mu_n + \mu_n),\ j = 1,2,...,m \qquad (1)$$

where:

$\vec{x}_{0,\mu_0}(t)$ – μ_0 dimensional vector of the process state of the own ship, determined in a time span $t \in [t_0, t_k]$, $\mu_0 = 1,2,...,\mu_n$;

$\vec{x}_{j,\mu_j}(t)$ – μ_j dimensional vector of the process state for the *j* encountered ship, $\mu_j = 1,2,...,\mu_m$;

$\vec{u}_{0,\eta_0}(t)$ – η_0 dimensional control vector of the own ship, $\eta_0 = 1,2,..., \eta_n$;

$\vec{u}_{j,\eta_j}$ – η_j dimensional control vector of the *j* encountered ship, $\eta_j = 1,2,..., \eta_m$.

The constraints of the control and the state of the process are connected with the basic condition for the safe passing of the ships at a safe distance D_s in compliance with COLREG Rules, generally in the following form:

$$g_j(x_{j,\mu_j}, u_{j,\eta_j}) \le 0 \quad j = 1,2,...,m \qquad (2)$$

The synthesis of the decision making pattern of the control ship leads to the determination of the optimal strategies of the players who determine the most favourable conduct of the process under given conditions. For the class of non-coalition games, often used in the control techniques, the most beneficial conduct of the own ship as a player with *j* encountered ships is the minimization of their goal function in the form of the payments – the integral payment and the final one:

$$I_{0,j} = \int_{t_0}^{t_k} \left[x_{0,\mu_0}(t)\right]^2 \mathrm{d}t + r_j(t_k) \ \rightarrow \min \qquad (3)$$

The integral payment determines the loss of way of the own ship to reach a safe passing of the encountered objects and the final one determines the risk of collision (Isaacs, 1965; Osborne, 2004; Engwerda, 2005; Wells, 2013). Generally, two types of the steering goals are taken into consideration – programmed control $u_0(t)$ and positional control $u_0[x_0(t), t]$. The basis for the decision-making is the decision-making patterns of the positional control processes, the patterns with the feedback arrangement representing the differential games.

The application of reductions in the description of the own ship dynamics and the dynamic of the *j* encountered ship and their movement kinematics lead to an approximated model of a matrix game.

Matrix game model

The differential game is reduced to a matrix game of *m* number of participants (Figure 4).

The state and control variables are represented by the following values:

$$x_{j,1} = D_j,\quad x_{j,2} = N_j,\quad u_{0,1} = \psi,\quad u_{0,2} = V$$
$$u_{j,1} = \psi_j,\quad u_{j,2} = V_j \qquad j = 1,2,...,m \qquad (4)$$

The matrix game includes the values determined previously on the basis of data taken from an anti-collision system ARPA: the value of collision risk, r_j, with regard to the determined strategies of the own ship and the *j* encountered ships. The form of such a game is represented by the risk matrix **R** containing the same number of columns as the

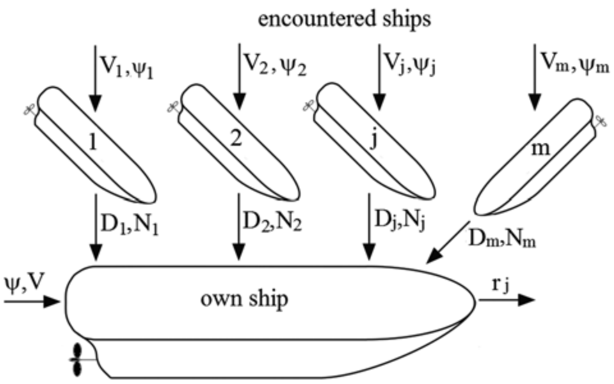

Figure 4. Block diagram of matrix game model of own ship and *j* encountered ships

number of participant OS (own ship) strategies – constant course and speed, alteration of the course 20º to starboard, 20º to port, etc. – and contains the number of lines that correspond to the joint number of participant ES (encountered ships) strategies:

$$\mathbf{R} = [r_j(\tau_j, \tau_0)] = \begin{vmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,\tau_0-1} & r_{1,\tau_n} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,\tau_0-1} & r_{2,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\tau_1,1} & r_{\tau_1,2} & \cdots & r_{\tau_1,\tau_0-1} & r_{\tau_1,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\tau_j,1} & r_{\tau_j,2} & \cdots & r_{\tau_j,\tau_0-1} & r_{\tau_j,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\tau_m,1} & r_{\tau_m,2} & \cdots & r_{\tau_m,\tau_0-1} & r_{\tau_m,\tau_n} \end{vmatrix} \tag{5}$$

The constraints affecting the choice of strategies (τ_0, τ_j) are a result of the recommendations of the COLREG way priority at sea. Player OS may use τ_0 of various pure strategies in a matrix game and player ES has τ_j of various pure strategies. As the game most often does not have a saddle point, the state of balance is not guaranteed – there is a lack of pure strategies for both players in the game. To solve this problem, dual linear programming may be used. In a dual problem, the player OS having τ_0 various strategies to be chosen tries to minimize the risk of collision:

$$\min_{\tau_0} r_j \tag{6}$$

while non-cooperating players ES having τ_j strategies to be chosen try to maximize the risk of collision:

$$\max_{\tau_j} r_j \tag{7}$$

or cooperating players ES having τ_j strategies to be chosen try to minimize the risk of collision:

$$\min_{\tau_j} r_j \tag{8}$$

The problem of determining an optimal strategy may be reduced to the task of solving a dual linear programming problem. Mixed strategy components express the probability distribution of using pure strategies by the players (Mesterton-Gibbson, 2001; Modarre, 2006; Basar & Olsder, 2013).

For non-cooperative matrix game the optimal quality game control index has the form:

$$\left(I_0^j\right)^* = \min_{\tau_0} \max_{\tau_j} r_j \tag{9}$$

while for cooperative matrix game the optimal quality game control index has the form:

$$\left(I_0^j\right)^* = \min_{\tau_0} \min_{\tau_j} r_j \tag{10}$$

The following probability matrix **P** of using particular pure strategies may be obtained:

$$\mathbf{P} = [p_j(\tau_j, \tau_0)] = \\ = \begin{vmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,\tau_0-1} & p_{1,\tau_n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,\tau_0-1} & p_{2,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\tau_1,1} & p_{\tau_1,2} & \cdots & p_{\tau_1,\tau_0-1} & p_{\tau_1,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\tau_j,1} & p_{\tau_j,2} & \cdots & p_{\tau_j,\tau_0-1} & p_{\tau_j,\tau_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\tau_m,1} & p_{\tau_m,2} & \cdots & p_{\tau_m,\tau_0-1} & p_{\tau_m,\tau_n} \end{vmatrix} \tag{11}$$

The solution for the control goal is the strategy of the highest probability; this will also be the optimal value approximated to the pure strategy:

$$\left(u_0^{\tau_0}\right)^* = u_o^{\tau_0} \left\{ \left[p_j\left(\tau_j, \tau_o\right) \right]_{\max} \right\} \tag{12}$$

Algorithm *gc* of game control

The safe trajectory of the own ship has been treated here as a sequence of subsequent changes of its course and speed in time. The values established are as follows: safe passing distances among the ships under given visibility conditions at sea D_s, time of the calculation advance t_a, and the duration of one stage of the trajectory t_c as one calculation step. At each step, the most dangerous object is determined with regard to the value of the collision risk r_j. Consequently, on the basis of the semantic interpretation of the regulations of the COLREG Rules, the direction of a turn of the own ship is selected with respect to the most dangerous encountered ships. The collision matrix risk **R** is determined for the admissible strategies of the own ship τ_0 and those

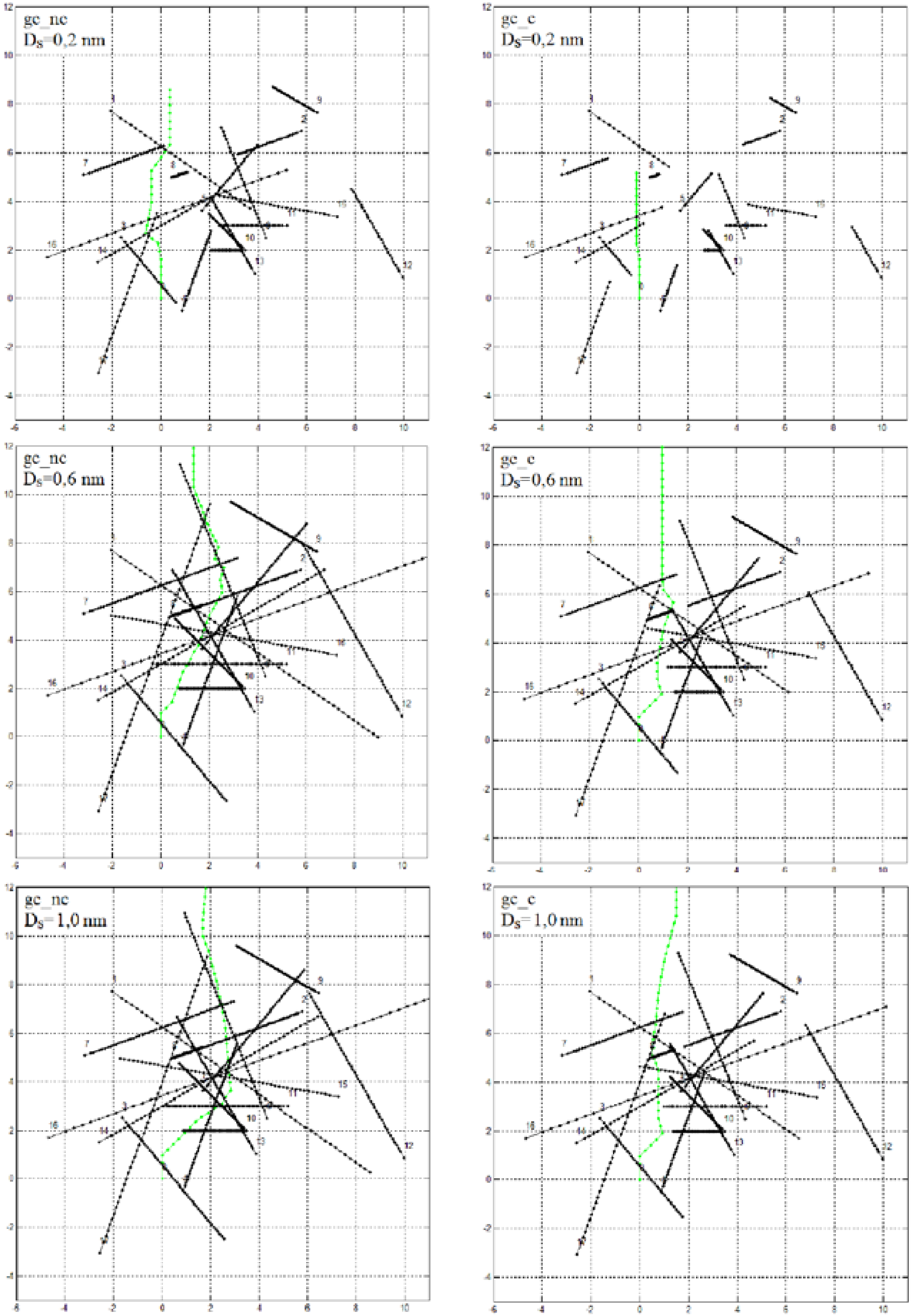

Figure 5. The results of the computer simulation for the safe manoeuvring of the own ship in a situation of passing 17 encountered ships for various values of safe distances D_s passing ships

for j encountered ship. By applying dual linear programming, in order to solve the matrix game, you obtain the optimal values of the own course and that of the j ship at the smallest deviation from their initial values. If, at a given step, no solution can be found at a speed of the own ship V, the calculations are repeated at the speed reduced by 25% until the game has been solved. The calculations are repeated step by step until the moment when all elements of the matrix **R** become equal to zero and the own ship, after having passed the encountered ships, returns to her initial course and speed. In this manner, optimal safe trajectory of the own ship is obtained in a collision situation.

The value of the risk of the collision is defined as the reference of the current situation of the approach described by the parameters $D_{j\min}$ and $T_{j\min}$, to the assumed assessment of the situation defined as safe and determined by the safe distance of approach D_s and the safe time T_s, which are necessary to execute a manoeuvre avoiding a collision:

$$r_j = \left(1 - 0.6 \log \frac{D_{j\min}}{D_s}\right)\left(\frac{\frac{T_{j\min}}{T_s} + 5}{4}\right) + \left(\frac{D_s}{D_j}\right)^2 \quad (13)$$

Using the function of *lp – linear programming* from the Optimization Toolbox contained in MATLAB software the Game Control *gc* program has been designed for the determination of the safe own ship trajectory in a collision situation (Straffin, 2001; Nisan et al., 2007).

Computer simulation

Simulation tests of the *gc* program have been carried out with reference to a real situation at sea. The situations have been recorded on the basis of the ARPA screen. For the basic version of the *gc* program, the following values for the strategies have been adopted:

$\tau_0 = 13 \rightarrow |0° \div 60°|$ for each of the 5°,

$\tau_j = 25 \rightarrow |-60° \div +60°|$ for each of the 5°.

Figure 5 shows the computer simulation, performed on *gc_nc* program of non-cooperational matrix game (left) and *gc_c* program of cooperational matrix game (right), for determination of the own ship trajectory in a situation passing 17 encountered ships in the Kattegat Strait for various safe distances D_s passing ships.

Conclusions

Application of the model of a multi-step matrix game for the synthesis of an optimal manoeuvring makes it possible to determine the safe game trajectory of the own ship in situations when it passes a greater number of the encountered ships. The trajectory is treated as certain sequence of manoeuvres with the course and speed. The Game Control *gc* computer program designed in MATLAB takes into consideration the following: degree of co-operation with its own ship have encountered ships, COLREG Rules, advance time for a manoeuvre calculated with regard to the own ship dynamic features, and the assessment of the final deviation between the real and reference trajectories. For especially dangerous situations, the program does not precisely follow the assumed values for safe distance D_s and its work is limited to the selection of a manoeuvre that guarantees minimization of the risk of collision in relation to every ship encountered.

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