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TIME-DEPENDENT SYSTEM RELIABILITY UNDER STRESS-STRENGTH SETUP

OBLICZANIE ZMIENNEJ W CZASIE NIEZAWODNOŚCI SYSTEMU NA PODSTAWIE MODELU WYTRZYMAŁOŚCI NA OBCIĄŻENIA

Consider a system which has n independent components whose time dependent strengths $Y_1(t), Y_2(t), \ldots, Y_n(t)$ are independent identically distributed random processes. Let random processes $X_1(t), X_2(t), \ldots, X_m(t)$ denote the common multiple stresses experienced by the components at time t. The reliabilities of the components in the system can chance as a result of their deterioration or in consequence of variable stresses over time. Degradation in components reliabilities in the system can lead to the degradation of the entire system reliability. In this paper, we propose a new method for determining the time dependent component reliability of the system under stress-strength setup. The proposed method provides a simple way for evaluating the reliability of the system at a certain time period. Computational results are also presented for the reliability of coherent system and consecutive k-out-of-n system.

Keywords: *time dependent reliability, stress-strength model, coherent system, consecutive k-out-of-n system.*

Rozważmy system, który posiada n niezależnych elementów, z których każdy charakteryzuje się zmienną w czasie wytrzymałością $Y_1(t), Y_2(t), \ldots, Y_n(t)$, która stanowi niezależny proces losowy o identycznym do pozostałych rozkładzie. Niech procesy losowe $X_1(t), X_2(t), \ldots, X_m(t)$ oznaczają wielorakie obciążenia powszechnie oddziałujące na elementy systemu w danym czasie t. Niezawodność elementów w systemie może ulegać zmianie w wyniku ich deterioracji lub w wyniku oddziaływania zmiennych w czasie obciążeń. Pogorszenie niezawodności komponentów systemu może prowadzić do obniżenia niezawodności całego systemu. W niniejszym artykule proponujemy nową metodę określania zmiennej w czasie niezawodności składowych systemu na podstawie modelu obciążeń i odpowiadających im wartości wytrzymałości. Zaproponowana metoda zapewnia prosty sposób oceny niezawodności systemu w pewnym okresie czasu. Przedstawiono również wyniki obliczeniowe dotyczące niezawodności systemu koherentnego oraz systemu szeregowego typu k -z- n .

Słowa kluczowe: *zmienna w czasie niezawodność, model wytrzymałości na obciążenia, system koherentny, system szeregowy k -z- n.*

1. Introduction

In engineering applications, stress-strength models are of special importance. A technical system may be subjected to several stresses such as pressure, temperature and corrosion and the survival of the system heavily depends on its strength. In the simplest terms, stress-strength model can be described as an assessment of the reliability of the component in terms of X and Y random variables where X is the random "stress" experienced by the component and Y is the random "strength" of the component available to overcome the stress. From this simplified explanation, the reliability of the component is the probability that the component is strong enough to overcome the stress applied on it. Extensive works have been done for the reliability of the component and its estimation under different choices for stress and strength distributions [4, 8, 10, 12].

Traditionally, stress and strength random variables are considered to be both static when available data on X and Y are considered not to involve the time of system operation. But in real-life reliability studies, the status of a stress-strength system clearly changes dynamically with time. This problem may be achieved by modeling at least one of the stress or strength quantities as time-dependent [2, 3, 5, 6, 7, 9, 13].

In some cases, the reliabilities of the components in the system depend on the effect of several stresses which cause degradation. Structurally, the reliability of the system depends on the reliability of its components. Thus, degradation in components reliabilities in the system can lead to the degradation of the entire system reliability. In this paper, we aim to propose a new method for computing the time-dependent reliability of the system using its time-dependent components reliabilities under stress-strength setup. A method is presented for the case in which the system consists of n independent components whose time-dependent strengths are independent identically distributed random processes and these components are subjected to m common multiple random stresses over time. We also note that the research concerns only non-renewable systems.

The rest of this paper is organized as follows. Section 2 gives some information about coherent systems and consecutive k-out-of-n systems. In section 3, we explain the proposed method for evaluation of the component's time-dependent reliability under the stress-strength setup. Section 4 contains some numerical results on the time-dependent reliability of the four components. In section 5, we compute time-dependent reliability of coherent and consecutive-k systems with four components.

2. Coherent systems and consecutive k-out-of-n systems

In this section, we analyze coherent systems and consecutive k-out-of-$n : F$ and G systems.

A coherent system consists of n components such that the system and each component may only be in one of two possible states, working or failed, can be described as follows.

Suppose that x_i, $i=1,2,\ldots,n$, the state of component i, is a random variable such that:

$$x_i = \begin{cases} 1 & \text{if component } i \text{ works,} \\ 0 & \text{if component } i \text{ is failed.} \end{cases}$$

Let $\phi(\mathbf{x})$, be the structure function of the system. Then,

$$\phi(\mathbf{x}) = \begin{cases} 1 & \text{if the system works,} \\ 0 & \text{if the system is failed,} \end{cases}$$

where $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ which represents the states of all components and is called the component state vector.

A system is said to be coherent if each component is relevant and the structure function $\phi(\mathbf{x})$ is non-decreasing in each argument. For more details, we refer to [1].

The reliability of a coherent system can be computed from the structure function $\phi(\mathbf{x})$ as $R_\phi(\boldsymbol{p}) = P(\phi(\mathbf{x})=1) = E\phi(\mathbf{x})$, where $\boldsymbol{p}=(p_1,\ldots,p_n) = P(x_1=1,\ldots,x_n=1)$.

Example 1. Consider the four-component serial-parallel system whose structure is illustrated by Figure 1. Its structure function can be expressed as:

$$\phi(x_1,x_2,x_3,x_4) = \min(x_1, \max(x_2,x_3,x_4)).$$

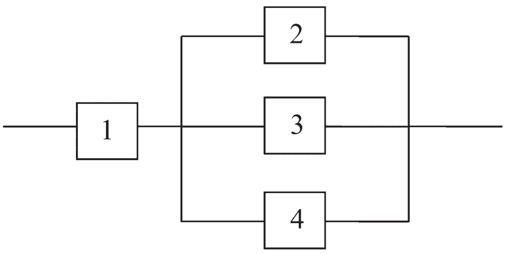

Fig. 1. Four-component serial-parallel system

The system reliability which is the stochastic independence of components is given by:

$$R_\phi(p_1,p_2,p_3,p_4) = E(\min(x_1,\max(x_2,x_3,x_4)))$$

$$= p_1(p_2+p_3+p_4-p_2p_3-p_2p_4-p_3p_4+p_2p_3p_4) \quad (1)$$

A consecutive k-out-of-$n:F$ system consists of n ordered components such that the system fails if and only if at least k consecutive components fail. Another special type of system related to the consecutive k-out-of-$n:F$ system is the consecutive k-out-of-$n:G$ system. A consecutive k-out-of-$n:G$ system is an ordered sequence of n components such that the system works if and only if at least k consecutive components work. Consecutive k-out-of-n: F and G systems are divided into linear and circular systems correspond to the components arranged along a line or circle.

Lambiris and Papastavridis [14] developed an exact formulas for the reliability of a consecutive k-out-of-$n:F$ system with n linearly or circularly arranged independent and identically distributed components. But in some cases components are not necessarily identical. Therefore, Zuo and Kuo [15] obtained the following closed-form equations for the reliability of a linear consecutive k-out-of-$n:F$ system, say $R_{LF}(k,n,p_i)$, and circular consecutive k-out-of-$n:F$ system, say $R_{CF}(k,n,p_i)$, respectively. Here, the value p_i, $i=1,2,\ldots,n$ is called the reliability of the i th component.

$$R_{LF}(k,n,p_i) = 1 - \sum_{i=1}^{n-k+1}\left(p_{i+k}\prod_{j=i}^{i+k-1}(1-p_j)\right), \quad p_{n+1}\equiv 1, \quad k\le n\le 2k, \quad (2)$$

where $i=1,2,\ldots,n$ and $R_{LF}(k,n,p_i)=1$ for $0\le n\le k$, and

$$R_{CF}(k,n,p_i) = 1 - \sum_{i=1}^{n}\left(p_{i+k}\prod_{j=i}^{i+k-1}(1-p_j)\right) - \prod_{i=1}^{n}(1-p_i), \quad k<n\le 2k+1, \quad (3)$$

where $i=1,2,\ldots,n$, $p_j\equiv p_{j-n}$ for $j>n$, $R_{CF}(k,n,p_i)=1$ for $0\le n<k$ and $R_{CF}(k,n,p_i) = 1-\prod_{i=1}^{n}(1-p_i)$ for $n=k$.

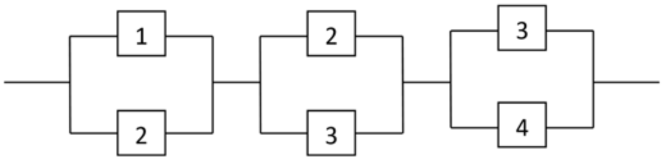

Fig. 2. Linear consecutive 2-out-of-$4:F$ system

Example 2. Consider the linear consecutive 2-out-of-$4:F$ system in Figure 2. It consists of a sequence of four components along a line such that the system is failed if and only if at least two consecutive components in the system are failed. Using Equation (2), one obtains the system reliability as:

$$R_{LF}(2,4,p_1,p_2,p_3,p_4)$$

$$= 1-[p_3(1-p_1)(1-p_2)+p_4(1-p_2)(1-p_3)+(1-p_3)(1-p_4)] \quad (4)$$

Example 3. Consider the circular consecutive 2-out-of-$4:F$ system. It consists of a sequence of four components along a circle such that the system is failed if and only if at least two consecutive components in the system are failed. Using Equation (3), one obtains the system reliability as:

$$R_{CF}(2,4,p_1,p_2,p_3,p_4) = 1-[p_3(1-p_1)(1-p_2)+p_4(1-p_2)(1-p_3)$$

$$+p_1(1-p_3)(1-p_4)+p_2(1-p_1)(1-p_4)]-(1-p_1)(1-p_2)(1-p_3)(1-p_4) \quad (5)$$

Kuo et al.[11] state that the consecutive k-out-of-$n:G$ and F systems are the duals of each other. Therefore, the reliability of the linear consecutive k-out-of-$n:G$ system, denoted by $R_{LG}(k,n,p_i)$, is then equal to $1-R_{LF}(k,n,1-p_i)$ and the reliability of the circular consecutive k-out-of-$n:G$ system, denoted by $R_{CG}(k,n,p_i)$, is then equal to $1-R_{CF}(k,n,1-p_i)$.

Example 4. Consider the linear consecutive 2-out-of-4 : G system. It consists of a sequence of four components along a line such that the system is good if and only if at least two consecutive components in the system are good. Using Equation (4) for $p_i = 1-p_i$, $i=1,2,3,4$, one obtains the system reliability as:

$$R_{LG}(2,4,p_1,p_2,p_3,p_4) = p_1p_2(1-p_3) + p_2p_3(1-p_4) + p_3p_4 \quad (6)$$

Example 5. Consider the circular consecutive 2-out-of-4 : G system. It consists of a sequence of four components along a circle such that the system is good if and only if at least two consecutive components in the system are good. Using Equation (5) for $p_i = 1-p_i$, $i=1,2,3,4$, one obtains the system reliability as:

$$R_{CG}(2,4,p_1,p_2,p_3,p_4)$$

$$= p_1p_2(1-p_3) + p_2p_3(1-p_4) + p_3p_4(1-p_1) + p_1p_4(1-p_2) + p_1p_2p_3p_4 \quad (7)$$

From the equations for system reliability given above, we can see that the reliability of a system with components whose reliabilities are not necessarily identical is a function of p_i and the component reliabilities are constant. But, the reliabilities of the components in the system can change as a result of their deterioration or in consequence of variable stresses over time. The components may not fail completely, but can degrade. Degradation in components reliability can lead to the degradation of the entire system reliability. Consideration of dynamic component reliability under stress-strength setup offers realistic application to real life applications.

In the following section, we propose a new approach for determining the dynamic components reliabilities of the system when it is subjected to a common multiple stresses over time.

3. Proposed method

Consider a system that consists of n independent components whose deteriorating strengths $Y_i(t) \geq 0$, $i=1,2,\ldots,n$ are independent identically distributed random processes with continuous cumulative distribution function $G_{i(t)}(y) = P\{Y_i(t) \leq y\}$. Assume that these components are subjected to common multiple stresses $X_j(t) \geq 0$, $j=1,2,\ldots,m$. Let the stresses are independent random processes having cumulative distribution function $F_{j(t)}(x) = P\{X_j(t) \leq x\}$.

Let us defined the random lifetime, T_i, of the component i as:

$$T_i(t) = \inf\{t : t \geq 0, X_{1:m}(t) < Y_i(t)\}, \quad (8)$$

where:

$$X_{1:m}(t) = \min(X_1(t),\ldots,X_m(t)).$$

In this framework, the component i is fail if $Y_i(t) < X_{1:m}(t)$. Because the i th component's strength remains weak in all stresses.

Given that component i is of age t, the remaining life after time t is random. For a specific time period $[t, t+h)$, the reliability of the component i, $p_i(t)$, which is defined as the probability of surviving at time $t+h$, follows from (8) that:

$$p_i(t) = P\{T_i(t) > t+h | T_i(t) > t\}.$$

The above conditional probability function depends on h and event $T_i(t) > t$. Thus, from the definition of conditional probability we have:

$$p_i(t) = \frac{P\{T_i(t) > t+h\}}{P\{T_i(t) > t\}} = \frac{1-H_i(t+h)}{1-H_i(t)} \quad (9)$$

where $H_i(t) = P\{T_i(t) \leq t\}$, $i=1,2,\ldots,n$. Clearly, as $H_i(t)$ is absolutely continuous, the probability distribution function $h_i(t) = \frac{d}{dt}H_i(t)$ exists almost everywhere. Then, we have main exponential formula:

$$H_i(t) = 1 - \exp\left(-\int_0^t \lambda_i(u)du\right) \quad (10)$$

where $\lambda_i(u)$, $i=1,2,\ldots,n$ is the corresponding failure rate.

Then, using Equation (10) in (9), we can obtain the i th component's reliability as follows:

$$p_i(t) = \exp\left(-\int_t^{t+h} \lambda_i(u)du\right). \quad (11)$$

An important tool for system reliability in our proposed method is the failure rate that describes the component reliability. Therefore, the shape of the failure rate plays an important role under stress-strength setup of the system.

Assume that the failure rate, $\lambda_i(t)$, is increasing as a result of their deterioration and common multiple stresses. As the finite mean of $Y_i(t)$ and $X_j(t)$; say $EY_i(t)$ and $EX_j(t)$ respectively, exists the $\lambda_i(t)$ can be expressed as:

$$\lambda_i(t) = \left(\sum_{j=1}^{m}\left(EY_i(t) - EX_j(t)\right)\psi_{EY_i\left(t_i^j\right) - EX_j\left(t_i^j\right) > 0}\right)^{-1}, \quad (12)$$

where ψ is an indicator function such that:

$$\psi_A = \begin{cases} 1, & \text{if event } A \text{ occurs at time } t \\ 0, & \text{if event } A \text{ does not occurs at time } t. \end{cases}$$

Using Equation (12) in (11), when running into time t, the $p_i(t)$ can be obtained under stress-strength setup as:

$$p_i(t) = \exp\left(-\int_t^{t+h}\left(\sum_{j=1}^{m}\left(EY_i(u) - EX_j(u)\right)\psi_{EY_i\left(t_i^j\right) - EX_j\left(t_i^j\right) > 0}\right)^{-1} du\right). \quad (13)$$

It should be noted that:

1) If the failure rate $\lambda i(t)$ is increasing (decreasing) in t, then the reliability $p_i(t)$ is decreasing (increasing) in $[0,1]$. Therefore, the system is decreasing (increasing).

2) The range of t_i^j is $0 < t_i^j < \infty$, which represents the time that j th stress is equal i th component strength.

Thus, the dynamic system reliability under stress-strength setup can be computed by substituting the Equation (13) in the equations for system reliability given in Section 2.

In the following section, we use parametric statistical model to compute the dynamic reliability of the components under stress-strength setup.

4. Dynamic reliability for four components

In this section, we compute reliability for four components under the stress-strength setup. We assume that all components are subjected to three stresses, which are stochastically increasing in time, whereas its strengths are independent identically distributed random variables, which are stochastically decreasing in time. We use Weibull process for stress and strength variates, because it is a useful model for events that are changing over time.

Let, $Y_i(t)$, $i = 1,2,3,4$ be a Weibull processes whose distribution is:

$$G_{i(t)}(y) = 1 - \exp\left\{-\left(\frac{y}{\alpha_i(t)}\right)^{\beta_i}\right\},\ y > 0,$$

where $\alpha_i(t) > 0$, $\beta_i > 0$. Because of $\alpha_i(t)$ decreases over time, let $\alpha_i(t) = 1/\alpha_i t$ then we have the finite mean of $Y_i(t)$ as:

$$EY_i(t) = \frac{1}{\alpha_i t}\Gamma\left(1 + \frac{1}{\beta_i}\right). \tag{14}$$

Let, $X_j(t)$, $j = 1,2,3$ be a Weibull processes whose distribution is:

$$F_{j(t)}(x) = 1 - \exp\left\{-\left(\frac{x}{\theta_j(t)}\right)^{\lambda_j}\right\},\ x > 0,$$

where $\theta_j(t) > 0$, $\lambda_j > 0$. Because of $\theta_j(t)$ increasing over time, let $\theta_j(t) = \theta_j t$ then we have the finite mean of $X_j(t)$ as:

$$EX_j(t) = (\theta_j t)\Gamma\left(1 + \frac{1}{\lambda_j}\right). \tag{15}$$

Then, using (14) and (15) in (13) we have:

$$p_i(t) = \exp\left(-\int_t^{t+h}\left(\sum_{j=1}^{3}\left(\frac{1}{\alpha_i t}\Gamma\left(1+\frac{1}{\beta_i}\right) - (\theta_j t)\Gamma\left(1+\frac{1}{\lambda_j}\right)\right)\psi_{EY_i(t_i^j) - EX_j(t_i^j) > 0}\right)^{-1} du\right). \tag{16}$$

where $i = 1,2,3,4$.

Using Equation (16), we can obtain dynamic reliability for four components under stress-strength model. The reliability of the components decreases over time, depending on the various selection of the parameters α_i, β_j, θ_j and λ_j. For an illustration, let α_1=0.02, β_1=0.2 and θ_1=0.8, λ_1=0.6; θ_2=1.2, λ_2=0.6; θ_3=1.1, λ_3=1.6 . Then $t_1^1 = 70.6031$, $t_1^2 = 57.6472$ and $t_1^3 = 77.9985$. This implies that after the time t_1^2 the stress $X_2(t)$ is above of the strength $Y_1(t)$, after the time t_1^1 the stress $X_1(t)$ is above of the strength $Y_1(t)$ and after the time t_1^3 the stress $X_3(t)$ is above of the strength $Y_1(t)$. As a result, we observe that the 1th component strength cannot resist all of the stresses after the time t_1^3 and component 1 is assigned to be completely failed.

In Table 1, we give the t_i^j, $i = 1,2,3,4$; $j = 1,2,3$, time for various selection of α_i, β_j, θ_j and λ_j. From the table we observe that the component 4 has the highest reliability compared to other component and component 3 has the lowest reliability compared to other components.

Table 1. The t_i^j time that j th stress is equal i th component strength (θ_1=0.8, λ_1=0.6; θ_2=1.2, λ_2=0.6; θ_3=1.1, λ_3=1.6)

α_1=0.02, β_1=0.2	α_2=0.03, β_2=0.2	α_3=0.01, β_3=0.3	α_4=0.01, β_4=0.2
$t_1^1 = 70.6031$	$t_2^1 = 57.6472$	$t_3^1 = 27.7374$	$t_4^1 = 99.8478$
$t_1^2 = 57.6472$	$t_2^2 = 47.0687$	$t_3^2 = 22.6475$	$t_4^2 = 81.5254$
$t_1^3 = 77.9985$	$t_2^3 = 63.6855$	$t_3^3 = 30.6428$	$t_4^3 = 110.307$

In Table 2, we compute dynamic reliability for four components for the parameters given in Table 1. From the table we observe that component 1 can resist all three stresses until the time period $[72,74)$, component 2 can resist all three stresses until the time period $[60,62)$, component 3 can resist all three stresses until the time period $[24,26)$, and component 4 can resist all three stresses until the time period $[108,110)$, after the these time periods all four components are assigned to be completely failed component.

We have everything in place now for obtaining the dynamic reliability of the components under stress-strength setup, which is crucial for reliability of the system. In the following section, we compute dynamic reliability of the system with four components under stress-strength setup.

Table 2. Dynamic reliability for four components for selected time period $[t, t+h)$

$[t, t+h)$	$p_1(t)$	$p_2(t)$	$p_3(t)$	$p_4(t)$
0-2	0.9998	0.9998	0.9992	0.9999
12-14	0.9985	0.9977	0.9876	0.9992
24-26	0.9967	0.9947	0.8996	0.9985
36-38	0.9941	0.9887	0	0.9975
48-50	0.9883	0.9647	0	0.9962
60-62	0.9687	0.6780	0	0.9942
72-74	0.8196	0	0	0.9901
84-86	0	0	0	0.9794
96-98	0	0	0	0.9441
108-110	0	0	0	0.3611

5. Reliability for coherent and consecutive k-out-of-n systems with four components

In this section, using dynamic reliability for four components given in the previous section, we compute dynamic reliability of coherent systems and consecutive k-out-of-$n:F(G)$ systems with four components. In Table 3, we compute the dynamic reliability for coherent system and consecutive k-out-of-$n:F(G)$ systems given in Section 2, using the $p_i(t)$, $i=1,2,3,4$ values given in Table 2.

Table 3. Dynamic reliability for systems for selected time period $[t,t+h)$

$[t,t+h)$	$R(p_i(t))$	$R_{LF}(2,4,p_i(t))$	$R_{CF}(2,4,p_i(t))$	$R_{LG}(2,4,p_i(t))$	$R_{CG}(2,4,p_i(t))$
0-2	0.9998	1.	1.	1.	1.
12-14	0.9985	0.9999	0.9999	0.9999	0.9999
24-26	0.9966	0.9993	0.9992	0.9991	0.9996
36-38	0.9940	0.9862	0.9862	0.9828	0.9940
48-50	0.9881	0.9610	0.9610	0.9534	0.9881
60-62	0.9668	0.6740	0.6740	0.6567	0.9668
72-74	0.8114	0	0	0	0.8114
84-86	0	0	0	0	0
96-98	0	0	0	0	0
108-110	0	0	0	0	0

6. Conclusion

In this paper, it is theoretically assumed that a system which consists of n components operates under m common multiple stresses. We provide a new method for computing the time-dependent reliability of the system using its time-dependent components reliabilities under stress-strength setup. The proposed method described here is a simple and clearly show the chance of component and system reliability depending on time.

References

1. Barlow R E, Proschan F. Statistical Theory of Reliability and Life Testing: Probability Models, To Begin With, 1981.
2. Basu A P, Ebrahimi N. On the reliability of stochastic systems. Statist. Probab. Lett. 1983; 1: 265-267, https://doi.org/10.1016/0167-7152(83)90042-1.
3. Basu S, Lingham R T. Bayesian estimation of system reliability in Brownian stress-strength models. Ann. Inst. Statist. Math. 2003; 55: 7-19, https://doi.org/10.1007/BF02530482.
4. Chandra S, Owen D B. On estimating the reliability of a component subject to several different stresses (strengths). Naval Res. Log. Quart. 1975; 22: 31-40, https://doi.org/10.1002/nav.3800220104.
5. Ebrahimi N. Two suggestions of how to define a stochastic stress-strength system. Statist. Probab. Lett. 1985; 3: 295-297, https://doi.org/10.1016/0167-7152(85)90058-6.
6. Ebrahimi N, Ramallingam T. Estimation of system reliability in Brownian stress-strength models based on sample paths. Ann. Inst. Statist. Math. 1993; 45: 9-19, https://doi.org/10.1007/BF00773665.
7. Eryılmaz S. On stress-strength reliability with a time-dependent strength, J. Qual. Reliab. Eng. 2013, 2013: 1-6.
8. Eryılmaz S, İşçioğlu F. Reliability evaluation for a multi-state system under stress-strength setup. Commun. Statist. Theor. Meth. 2011; 40: 547-558, https://doi.org/10.1080/03610920903411242.
9. Gokdere G, Gurcan M. New Reliability Score for Component Strength Using Kullback-Leibler Divergence. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2016; 18: 367-372, https://doi.org/10.17531/ein.2016.3.7.
10. Kotz S, Lumelskii Y, Pensky M. The Stress-Strength Model and its Generalizations. Theory and Applications. Singapore: World Scientific, 2003, https://doi.org/10.1142/9789812564511.
11. Kuo W, Zhang W, Zuo M. A consecutive-k-out-of-n: G system: The mirror image of a consecutive-k-out-of-n: F system. IEEE Transactions on Reliability 1990; R-39(2): 244-253, https://doi.org/10.1109/24.55888.
12. Nadarajah S, Kotz S. Reliability for some bivariate exponential distributions. Mathematical Problems in Engineering 2006; 2006: 1-14.
13. Whitmore G A. On the reliability of stochastic systems: a comment. Statist. Probab. Lett. 1990; 10: 65-67, https://doi.org/10.1016/0167-7152(90)90113-L.
14. Lambiris M, Papastavridis S. Exact reliability formulas for linear & circular consecutive-k-out-of-n: F systems. IEEE Transactions on Reliability 1985; R-34(2): 124-126, https://doi.org/10.1109/TR.1985.5221969.
15. Zuo M J, Kuo W. Design and performance analysis of consecutive-k-out-of-n structure. Naval Research Logistics 1990; 37: 203-230, https://doi.org/10.1002/1520-6750(199004)37:2<203::AID-NAV3220370203>3.0.CO; 2-X.

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