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Fatigue Life Prediction of Knitted Fabrics Made of Cotton Yarns – A Statistical Approach

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Abstract

In the present paper, an assessment of the fatigue life of knitted (or knit) fabrics made of cotton yarns is discussed. Based on experimental investigations performed, the characteristics of material deterioration and damage were determined taking into account the direction of product elongation due to applied sine wave changing forces (sinusoidal loading). Aiming for a precise and adequate description of the phenomena occurring in the materials considered, a statistical approach to modelling and outcome analyses has been utilised. In the case of all knitted fabrics analysed, fatigue lives along the columns for both cases of tested yarns considered were higher than those related to the direction of rows of the knitted fabric. Fluctuated (variable) loading has a crucial influence on the deterioration of knitted fabrics, causing their damage. Within the process of fatigue damage to knitted fabrics, it was possible to observe the zone of reinforcement of threads in plait within the range of high-cycle loadings. Within the damage final phase, displacements of threads in plait are observed, which cause their separation and finally breakage of the product. Former investigations of the authors concerned similar statistical models for smooth and fancy yarns. The present work constitutes some additional investigations and assessment of fatigue life in relation to flat textile products. It was proven that the derived statistical models of fatigue life proposed are entirely suitable for the description of phenomena which happen to flat textile products during the damage process.

Key words: *fatigue strength, textile product, stress cycles, Wöhler's diagram, knitted fabric, fatigue durability.*

Introduction

The widespread implementation of modern machine design, as well as the usage of better and better materials allow for the manufacturing of modern high-quality textiles with performance parameters satisfying different groups of end-users. In particular, the new textiles are dedicated to special applications. Undertaking adequate modification of the production technologies utilised enables the manufacturing of a product of prescribed fatigue life, strength as well as thermal insulation parameters. Fatigue strength is defined as the resistance of a particular material to fatigue damage. Moreover fatigue life is defined here as the number of cycles of loading changes until the appearance of damage or complete breakage of an artifact.

The problem of calculation or evaluation of the fatigue life of an artifact subjected to variable loadings is widely considered by scientists and engineers in different branches of science. An assessment of fatigue life is frequently performed just within the design phase or in relation to machines' and devices' work regime. Fatigue life related to operation conditions is evaluated upon testing the existing prototype elements. In such cases, most frequently, some phenomenological models of the accumulation of fatigue damage are utilised. Therefore a relatively low amount of experimental data is needed. Relevant information can be obtained – among others – based on the analysis of a Wöhler chart. Such an approach to the assessment of investigated material is recommended and helps to reduce costs connected with the elimination of errors which could occur in the final product. When performing an analysis of fatigue life, we frequently utilise statistical data related to distributions of material fatigue parameters. The essentially important aspect within an assessment of fatigue life is a probabilistic approach, which is adequate due to the physical character of fatigue processes.

One type of complex textile product is knitted fabric because such materials are arranged from mutually plaited (engaged/entwined) rows and columns of yarn arrangements. Due to differing yarn types, knitted fabrics have special characteristics i.e. high elasticity and extensibility.

For practical applications, these features are desirable; however, assessment of their fatigue lives or strengths is really challenging for designers wanting performance in a proper and reliable manner.

Additionally one of the most frequently observed forms of textile material damage is fatigue damage – very dangerous considering the consequences, especially due to an unexpected failure. Textile materials are frequently subjected to damage due to loadings lower than those which had been determined during the statistical tests. Such damage (e.g. breakage) takes place without any visible plastic deformations, whereas the cause of damage is – among others – insufficient elasticity of the material.

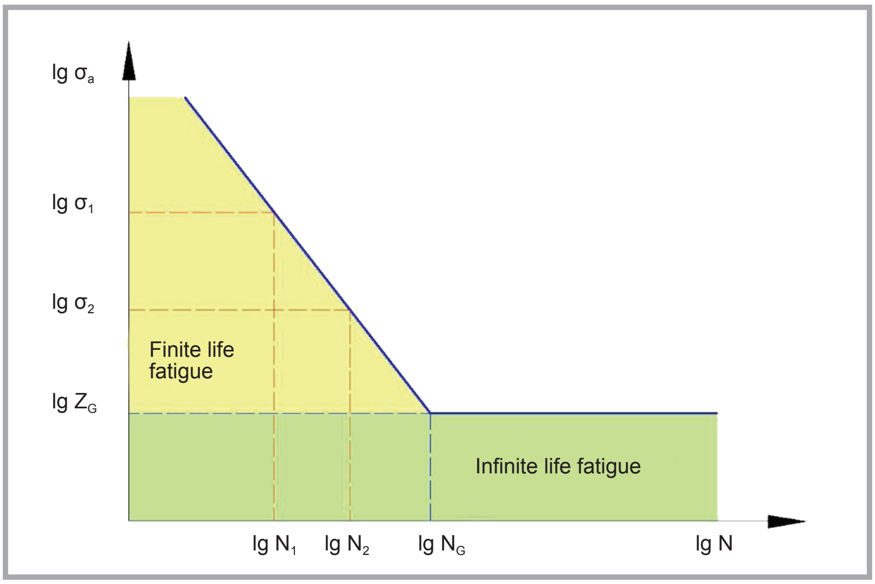

Figure 1. Exemplary Wöhler chart prepared in logarithmic co-ordinate system.

It is worth focusing attention on the commonly recognised fact that the observed development of information technologies allows the utilisation of versatile calculation systems i.e. the finite elements method (FEM) and boundary elements method (BEM) for determination of the strength of versatile technical objects. However, in the case of textile products, it is complicated because there is a lack of numerical and statistical models relevant to the spatial or even planar structure of materials as well as the way of modelling itself. Especially it is difficult to model knitted fabrics made of smooth yarns of left-right plait.

Additionally there is a requirement for determination of the material parameters of textile products for the performance of reliable computer simulations. Therefore the performance of experimental investigations is essentially important, because reliable establishment of strength parameters is possible based upon them.

The aim of the present paper is an assessment of the fatigue life of knitted fabrics made of cotton yarns as well as determination of the characteristics of material damage under loadings of sine type variability at a constant amplitude in a case where the direction of elongation of the product was taken into account. The direction mentioned is related to the structure of the knitted fabric, i.e. the layout of columns of plait. Additionally statistical models for the description of phenomena occurring in the materials considered were proposed and verified.

Table 1. Characteristic of physical and strength parameters of smooth yarns used to make the knitted fabrics [6].

Lp.	Parameter analysed	Symbol	Unit	Smooth yarn	
				30 tex	40 tex
1	Linear mass	Tt_n	tex	30.90	40.94
2	Coefficient of variation of mass of yarn determined from segments of 200 m	$V(Tt_n)$	%	10.39	9.88
3	Number of twist	T_{mn}	rpm	620.10	531.40
4	Coefficient of variation of the number of twist	$V(T_{mn})$	%	3.99	3.49
5	Average value of breaking force	$\bar{R}_m$	cN	431.17	570.51
6	Coefficient of variation of breaking force	$V(R_m)$	%	9.67	6.15
7	Strength	σ	cN/tex	14.37	14.26
8	Relative mean value of breaking elongation	$\bar{\varepsilon}_r$	%	7.41	6.49
9	Coefficient of variation of breaking elongation	$V(\varepsilon_r)$	%	9.94	6.05
10	Modulus of initial elasticity	σ_0	cN/tex	2.74	2.59
11	Coefficient of variation modulus of initial elasticity	$V(\sigma_0)$	%	11.33	18.22

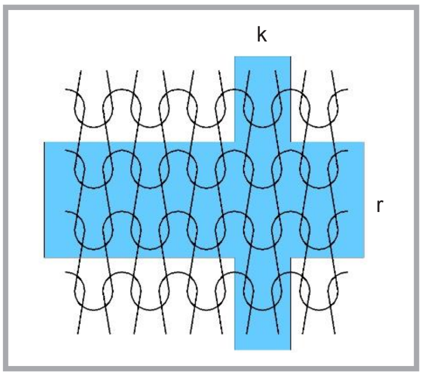

Figure 2. Knitted fabric of left-right plait (k – column, r – row).

Characteristics of the performed experiments

Textile products are frequently subjected to random, fluctuating loadings which depend on the operating conditions or utilisation manner. Therefore the loadings applied could have an arbitrary complex form and could influence the product strength in an unpredicted way [2].

In the case of knitted fabrics subjected to variable loadings, their deterioration consists in the losing of inner cohesion forces. This phenomenon occurs usually in the case of loadings lower than the ultimate strength. The basic characteristics – which are used for further analyses – are versatile charts; but mainly the Wöhler chart is utilised, also called an $S - N(\sigma - S)$ curve or Wöhler diagram (***Figure 1***) [3, 11-12]. It allows determination of the maximal loading level (σ), for which a specimen made of a particular material can work within the range of an unlimited number of cycles (N).

The left part of the Wöhler chart is related to the range of the so-called limited fatigue life, whereas the area under the fatigue limit (endurance limit) Z_G is usually called the range of the unlimited fatigue life.

Knowledge of the co-ordinates of consecutive $\sigma_a - N$ points as well as the position of $Z_G - N_G$ enables a description of the deflected branch of the chart in the $\log\sigma - \log N$ co-ordinate system, as a straight line, expressed by means of the following ***Equation (1)*** [4]:

$$m\log\sigma_a + \log N = m\log Z_G + \log N_G \quad (1)$$

where: N_G – number of cycles associated with the endurance limit, Z_G – endurance limit.

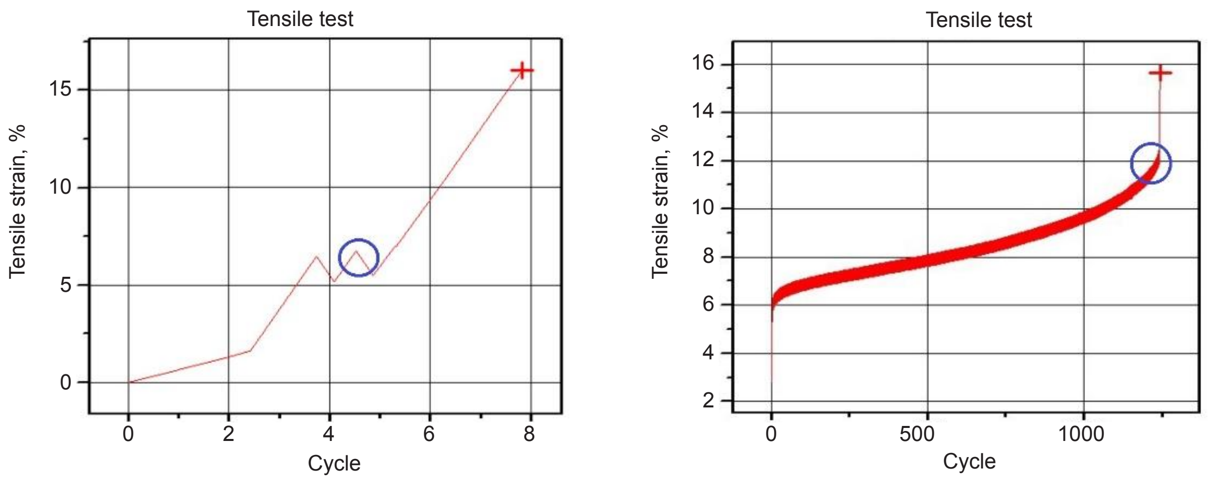

Figure 3. *Selected curves for the elongation of the knitted fabrics in the row direction – obtained during fatigue investigations on fabrics made of yarns with a line mass equal to 30 tex and cyclic loadings* $\sigma_s = 0.95$ *i* $\sigma_s = 0.55$.

Therefore we can rewrite the above ***Equation (2)***:

$$\sigma_a^m N = Z_G^m N_G \tag{2}$$

Exponent *m* – which is the direction or slope coefficient – is ***Equation (3)*** to:

$$m = \frac{\log \frac{N_G}{N}}{\log \frac{\sigma_a}{Z_G}} \tag{3}$$

In investigations and design practices, other fatigue charts e.g. Weibull and/or Gumbel models are additionally used or a statistical approach to these methods is added. As another example, we could just consider the fatigue chart prepared in the probabilistic approach [5, 9-10]. Such a chart allows determination of the endurance limit or fatigue life for an arbitrary given damage probability.

The investigations were performed utilising cotton threads of linear mass equal to 30 and 40 tex. These threads were utilised for the manufacturing of pieces of fabrics. The manufacturing process was undertaken by means of a cylindrical crotchet machine (laboratory version): number of needles = 16 and number of lockings = 1. The test pieces of knitted fabrics obtained have a left-right plait (***Figure 2***). Physical parameters of the threads utilised for production of the knitted fabrics are depicted in ***Table 1*** [6].

Selected parameters of the knitted fabrics manufactured are shown in ***Table 2***.

Investigations of the fatigue life of knitted fabrics made of smooth yarns were performed using an INSTRON 5544 ripper equipped with professional software: Merlin and Test Profiler. The distance between the clamps of the strength testing machine was equal to 100 mm, whereas the width of the knitted fabric specimens was equal to 50 mm.

Investigation of the fatigue life was performed for ten consecutive levels of loading (ten different amplitudes), making 10 positive (proper) tests for each of the defined levels. The knitted fabrics tested were manufactured of smooth yarns. The specimens prepared (pieces of fabric) were subjected to fluctuating cyclic fatigue loadings, which were constantly applied until the moment of entire damage i.e. breakage of the fabric. As the criterion of damage during the tests carried out, we recognised the moment when a visible crack appeared. Test results are shown in ***Figure 3***.

The highest value of maximal loadings which does not cause damage (breakage) is defined as unlimited fatigue strength. Before starting the investigations, the value of amplitude of the loading cycle σ_a was established, set on equipment used for the performance of the fatigue life. Subsequently the minimal $\sigma_{\min}$ and maximal $\sigma_{\max}$ loadings for a particular cycle were programmed via the strength testing machine – INSTRON 5544

Table 2. *Characteristic of physical parameters and strength of knitted fabrics*

Parameter analysed	Symbol	Unit	Knitted fabric from plain yarn	
			30 tex	**40 tex**
Length of the thread in needle loop	L	mm	4.3	3.8
Number of courses PN-85/P-04787	n_r	1/10cm	108	129
Number of wales PN-85/P-04787	n_k	1/10cm	97	95
Thickness of knitted fabrics	d	mm	0.7	0.85
Relative durable elongation of knitted fabrics under tension in the wale direction PN-77/P-04883	ε_t	%	6.5	8.6
Relative total elongation of knitted fabrics under tension in the wale direction PN-77/P-04883	ε_c	%	23	14.3
Relative elastic elongation of knitted fabrics under tension in the wale direction PN-77/P-04883	ε_s	%	16.8	5.7
Mean breaking force along courses	$\bar{R}_{mr}$	N	151.82	201.78
Mean breaking force along wales	$\bar{R}_{mk}$	N	237.77	368.79
Relative mean breaking elongation along courses	$\bar{\varepsilon}_{rr}$	%	177.8	135.22
Relative mean breaking elongation along wales	$\bar{\varepsilon}_{rk}$	%	106.9	94.63
Modulus of initial elasticity along courses	σ_{0r}	MPa	0.065	0.067
Modulus of initial elasticity along wale	σ_{0k}	MPa	0.173	0.191

Table 3. *Range of elongation loadings for smooth knitted fabrics 30 tex (k – columns, r – rows) [5].*

Coefficient of current loading cycle	Loading					
	maximal σ_{max}, MPa		minimal σ_{min}, MPa		cycle amplitude σ_a, MPa	
%	k	r	k	r	k	r
0.98	5.61	3.16	0.17	0.07	5.44	3.09
0.95	5.44	3.06	0.17	0.07	5.27	3.00
0.85	4.87	2.74	0.17	0.07	4.69	2.67
0.75	4.30	2.42	0.17	0.07	4.12	2.35
0.65	3.72	2.09	0.17	0.07	3.55	2.03
0.55	3.15	1.77	0.17	0.07	2.98	1.71
0.50	2.86	1.61	0.17	0.07	2.69	1.55
0.40	2.29	1.29	0.17	0.07	2.12	1.22
0.30	1.72	0.97	0.17	0.07	1.54	0.90
0.20	1.15	0.64	0.17	0.07	0.97	0.58

Table 4. *Range of elongation loadings for smooth knitted fabrics 40 tex (k – columns, r – rows) [5].*

Coefficient of current loading cycle	Loading					
	maximal σ_{max}, MPa		minimal σ_{min}, MPa		cycle amplitude σ_a, MPa	
%	k	r	k	r	k	r
0.98	8.50	4.74	0.19	0.07	8.31	4.67
0.95	8.24	4.59	0.19	0.07	8.05	4.52
0.85	7.38	4.11	0.19	0.07	7.18	4.04
0.75	6.51	3.63	0.19	0.07	6.32	3.56
0.65	5.64	3.14	0.19	0.07	5.45	3.07
0.55	4.77	2.66	0.19	0.07	4.58	2.59
0.50	4.34	2.42	0.19	0.07	4.15	2.35
0.40	3.47	1.93	0.19	0.07	3.28	1.86
0.30	2.60	1.45	0.19	0.07	2.41	1.38
0.20	1.74	0.97	0.19	0.07	1.54	0.90

and dedicated software – Test Profiler. The minimal value of loading within the applied cycle $\sigma_{\min}$ was assumed, being equal to the mean value of the modulus of initial elasticity of the knitted fabrics made of smooth yarns σ_0 (MPa), which had been determined based on 10 measurements performed for each fabric.

The maximal loading for a cycle $\sigma_{\max}$ was determined based on the average loading causing breakage of the fabric made of smooth yarns σ_{dn} (MPa). For characterisation of the maximal loading for a particular cycle, the coefficient of the current loading cycle σ_s was introduced in dependence on the loading level of the cycle. It was assumed that the fatigue life would be determined at the level of ten maximal loadings (MPa) ***Equation (4)***.

$$\sigma_s = [0{,}98\cdot\bar{\sigma}_d \quad 0{,}95\cdot\bar{\sigma}_d \quad 0{,}85\cdot\bar{\sigma}_d \quad 0{,}75\cdot\bar{\sigma}_d \quad 0{,}65\cdot\bar{\sigma}_d \quad 0{,}55\cdot\bar{\sigma}_d \quad 0{,}50\cdot\bar{\sigma}_d \quad 0{,}40\cdot\bar{\sigma}_d \quad 0{,}30\cdot\bar{\sigma}_d \quad 0{,}20\cdot\bar{\sigma}_d]^T \qquad (4)$$

Equation (4)

The coefficient of the current loading cycle was assumed based on former research [5].

An assumption of the maximal upper value of the loading cycle $\sigma_{\max} = \sigma_s = 0{,}98\cdot\bar{\sigma}_d$ (MPa) was made due to the fact that in the case of fatigue investigations applying the loading $\sigma_{\max} > \sigma_s > 0{,}98\cdot\bar{\sigma}_d$ (MPa), more than 99% of specimens of fabrics were subjected to damage for the number of fatigue cycles $N_{p,n} \leq 1$ i.e. immediately. Hence this range was essentially close to the loadings damaging the fabrics investigated.

An assumption of the maximal lower value of loading cycle $\sigma_{\max} = \sigma_s = 0{,}20\cdot\bar{\sigma}_d$ (MPa) was related to the fact that the fatigue strength was close to the endurance limit, which (in consequence) causes unlimited resistance of the fabric for such variable loadings. Investigations were performed for tension-tension loadings (one-side positive) assuming the following relationships $\sigma_a \neq 0, \sigma_m > 0, \sigma_{\min} > 0, 0 < R < +1{,}1 < \kappa < +\infty$ and f_c = 4 Hz.

Values of loadings relevant to a particular cycle applied for the smooth (in some resources 'soft fabrics') fabrics investigated are collected in ***Tables 3*** and ***4***.

Aiming for the determination of Wöhler charts for knitted fabrics in the case of the fatigue investigations viewed on the afore-described equipment and specimens, non-linear regression was utilised for outcome depiction (coefficient of current loading cycle). Namely the data achieved were approximated via logarithmic functions of the following form [7]:

$$\hat{\sigma}_{sn} = B_0 + B_1 \cdot \ln(N_n) \qquad (5)$$

$[B_0; B1]^T$ – vector of coefficients of the logarithmic function, $\hat{\sigma}_{sn}$ – value of regression function for the coefficient of the current loading cycle for the fabric investigated, N_n – number of fatigue cycles determined for the fabric investigated.

Values of the coefficient of regression function B_0 and B_1 were determined utilising the least squares method. The full procedure of calculations is presented in works [1, 6].

The compatibility of the model output to the that from the object analyzed was evaluated based on the coefficient of multiple correlation R, expressed by means of the following ***Equation (6)***:

$$R = \sqrt{\frac{\sum_{i=1}^{n}(\hat{\sigma}_{si} - \bar{\sigma}_s)^2}{\sum_{i=1}^{n}(\sigma_{si} - \bar{\sigma}_s)^2}} \qquad (6)$$

where: σ_{si} – output of an object in i-th experiment, $\hat{\sigma}_{si}$ – output of a model in i-th experiment, n – number of tests in a particular experiment, $\bar{\sigma}_s$ – average value of outputs – object and model.

The statistical significance of the regression function determined was checked based on the basic relationship between the F statistics and coefficient of multi-dimensional correlation 'R':

$$F_{0{,}05}(k\,; n-k-1) = \frac{n-k-1}{k}\cdot\frac{R^2}{1-R^2}. \qquad (7)$$

Aiming at receiving a regression equation fulfilling the significance requirement for the prescribed significance level (it was assumed $\alpha = 0.05$), the initial condition had to be held:

$$F_{obl} = F_{0.05}(k\,; n-k-1) \geq F_{kr}, \qquad (8)$$

where: F_{kr} – critical value of the Fisher-Snedecor statistics determined from the statistical tables of the Fisher-Snedecor distribution, at the confidence level $\alpha = 0.05$ and degrees of freedom k and $n - k - 1$, respectively.

Determination of the confidence intervals for the feature investigated (i.e. current loading cycle) was performed based on the following formula [8]:

$$P\left\{\bar{\sigma}_s - t_\alpha \cdot \frac{s}{\sqrt{n-1}} < m < \bar{\sigma}_s + t_\alpha \cdot \frac{s}{\sqrt{n-1}}\right\} = 1-\alpha \qquad (9)$$

where: s – standard deviation calculated using the following formula:

$$s = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (\sigma_{si} - \bar{\sigma}_s)^2}\ , \qquad (10)$$

α – confidence level, t_α – t-Student variable parameter read from the statistical tables for the t-Student distribution for n – 1 degrees of freedom, in such a way that for afore-assumed probability $1 - \alpha$ – the following inequality has to be fulfilled:

$$P\{-t_\alpha < t < t_\alpha\} = 1 - \alpha \qquad (11)$$

Graphical representations of results were arranged as Wöhler charts.

Fatigue life of smooth cotton knitted fabrics subjected to sinusoidal loading

As mentioned in the previous chapter, investigations of the fatigue life of the smooth cotton knitted fabrics (made of smooth yarns) were performed by means of an INSTRON 5544 ripper equipped with the professional software Merlin and Test Profiler. In ***Tables 5-8***, results of the experimental investigations for consecutive loading levels are shown.

Based on the data depicted in ***Tables 5-8***, Wöhler charts (***Figures 4-7***) were determined. These charts are relevant to the knitted fabrics made of yarns of linear masses 30 tex and 40 tex in the directions of rows and columns, respectively. As mentioned above, an approximation

Table 5. *Results of determined fatigue lives of smooth cotton knitted fabrics made of yarn of linear mass 30 tex in the direction of rows [5].*

	Coefficient of current loading cycle σ_{si}, MPa									
Lp.	**N_n, cycle**									
	$\sigma_{s0.98}$	$\sigma_{s0.95}$	$\sigma_{s0.85}$	$\sigma_{s0.75}$	$\sigma_{s0.65}$	$\sigma_{s0.55}$	$\sigma_{s0.50}$	$\sigma_{s0.40}$	$\sigma_{s0.30}$	$\sigma_{s0.20}$
1	1	5	90	546	2368	4767	8843	13658	36952	98324
2	1	8	65	698	4321	6987	12364	23654	36586	26548
3	1	11	12	1143	2365	2543	3251	36549	69752	116543
4	1	14	45	236	365	2364	6953	9631	110238	169872
5	1	7	21	297	3547	8453	9431	16325	69816	98632
6	1	9	79	236	254	5621	16394	6382	36597	69875
7	1	16	112	1176	471	2364	2365	3214	48762	69874
8	1	13	91	94	3654	1697	6329	7543	98723	116354
9	1	8	132	197	1698	1241	3397	5439	3654	69811
10	1	9	36	553	2140	365	17346	18736	94376	36987
$\bar{N}_n$	1	10	68	518	2118	3640	8667	14113	60546	87282
S	–	3.4	39.5	387.1	1445.9	2672.4	5318.1	10208.5	33865.4	42035.3
V	–	34.3	57.9	74.8	68.3	73.4	61.4	72.3	55.9	48.2

Tabela 6. *Results of determined fatigue lives of smooth cotton knitted fabrics made of yarn of linear mass 30 tex in the direction of columns [5].*

	Coefficient of current loading cycle σ_{si}, MPa									
Lp.	**N_n, cycle**									
	$\sigma_{s0.98}$	$\sigma_{s0.95}$	$\sigma_{s0.85}$	$\sigma_{s0.75}$	$\sigma_{s0.65}$	$\sigma_{s0.55}$	$\sigma_{s0.50}$	$\sigma_{s0.40}$	$\sigma_{s0.30}$	$\sigma_{s0.20}$
1	1	1	2	4	24	174	336	1020	10789	83437
2	1	1	2	9	36	369	632	2369	6897	98741
3	1	1	1	5	25	469	321	659	794	123654
4	1	1	3	1	25	135	121	889	13698	36984
5	1	1	2	2	21	69	187	164	5972	15469
6	1	1	2	3	43	431	364	1648	12364	23654
7	1	1	1	3	8	211	97	364	16321	98754
8	1	1	2	5	23	109	521	941	9874	111456
9	1	1	2	5	4	63	63	311	3654	62943
10	1	1	1	6	63	55	171	1369	1364	9685
$\bar{N}_n$	1	1	2	4	27	209	281	973	8173	66478
S	–	–	0.6	2.3	17.0	157.7	188.6	677.7	5290.7	42434.3
V	–	–	35.1	52.6	62.4	75.6	67.1	69.6	64.7	63.8

Table 7. *Results of determined fatigue lives of smooth cotton knitted fabrics made of yarn of linear mass 40 tex in direction of rows [5].*

	Coefficient of current loading cycle σ_{si}, MPa									
Lp.	**N_n, cycle**									
	$\sigma_{s0.98}$	$\sigma_{s0.95}$	$\sigma_{s0.85}$	$\sigma_{s0.75}$	$\sigma_{s0.65}$	$\sigma_{s0.55}$	$\sigma_{s0.50}$	$\sigma_{s0.40}$	$\sigma_{s0.30}$	$\sigma_{s0.20}$
1	1	5	27	79	237	3759	9943	17843	69524	110361
2	1	12	36	108	697	6523	16354	36254	36587	154631
3	1	4	25	33	265	7831	9472	19875	35978	98235
4	1	8	41	56	487	2537	6581	9397	26971	66351
5	1	5	35	98	364	2634	5534	22597	54698	169422
6	1	7	21	66	269	1297	6971	31973	3698	66891
7	1	5	19	171	664	2697	3349	13452	11697	6524
8	1	5	38	34	123	3642	8235	9187	36971	36541
9	1	3	12	63	231	2934	7638	6523	42156	165871
10	1	1	19	76	251	4931	19874	3542	38268	163254
$\bar{N}_n$	1	6	27	78	359	3879	9395	17064	35655	103808
S	–	3.0	9.7	40.5	194.1	2002.6	5035.8	10839.7	18950.8	58786.6
V	–	54.4	35.7	51.7	54.1	51.6	53.6	63.5	53.2	56.6

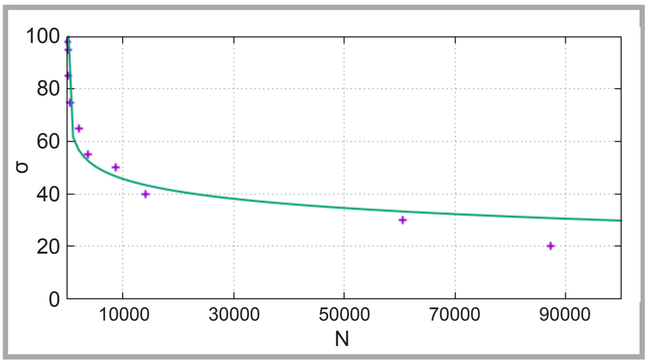

__Figure 4.__ Wöhler chart for smooth cotton knitted fabrics made of yarns of linear mass of 30 tex in the row direction, approximated by means of the logarithmic function described by the equation: $y = -6.92 \ln(N) + 109.49$, where: $R^2 = 0.932$ and $F_{cal} = 106 > F_{crit,\alpha=0.05} = 7.71$.

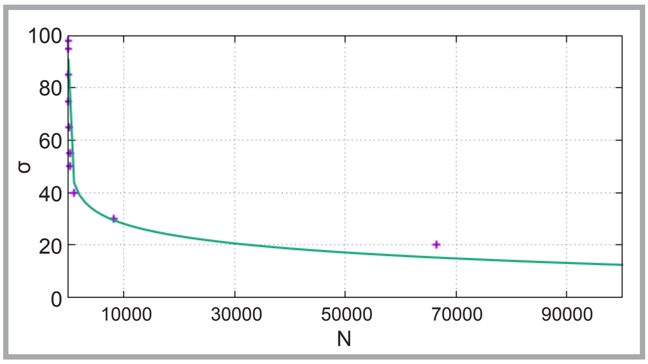

__Figure 5.__ Wöhler chart for the smooth cotton knitted fabrics made of yarns of linear mass of 30 tex in the column direction , approximated by means of the logarithmic function described by the equation: $y = -6.808 \ln(N) + 90.796$, where: $R^2 = 0.975$ and $F_{cal} = 313.28 > F_{crit,\alpha=0.05} = 7.71$.

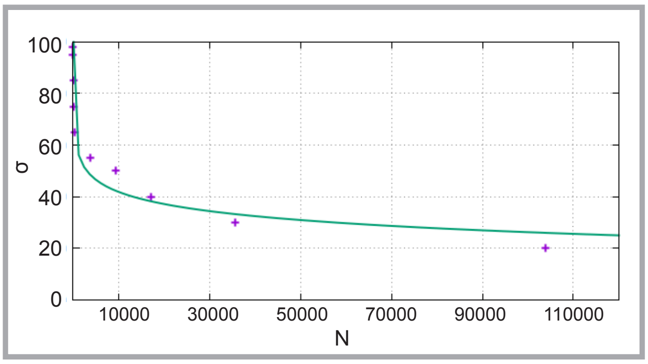

__Figure 6.__ Wöhler chart for smooth cotton knitted fabrics made of yarns of linear mass of 40 tex with elongation in the row direction, approximated by the logarithmic function of the following equation: $y = -6.78 \ln(N) + 104.34$, where: $R^2 = 0.969$ and $F_{cal} = 250 > F_{crit,\alpha=0.05} = 7.71$.

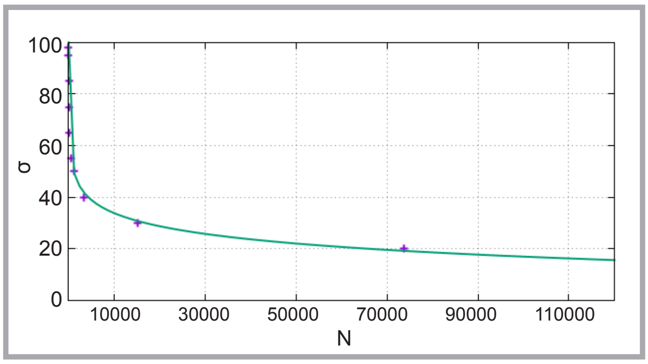

__Figure 7.__ Wöhler chart for smooth cotton knitted fabrics made of yarns of linear mass of 40 tex with elongation in the column direction, approximated by the logarithmic function of the following equation: $y = -7.33 \ln(N) + 101.39$, where: $R^2 = 0.992$ and $F_{cal} = 1017 > F_{crit,\alpha=0.05} = 7.71$.

of the Wöhler charts was made taking into account curvilinear regression with the logarithmic function. The functions determined for all knitted fabrics considered are statistically significant and useful for a description of the phenomenon of fatigue life representation.

Analysing the course of variability of the number of cycles until damage for the consecutive fatigue experiments presented in ***Tables 5-6*** and ***Figures 4-5***, one can conclude that knitted fabrics made of yarns of linear mass of 30 tex have higher fatigue life in the row direction within the entire range of loadings, especially in the low-cycle and high-cycle ranges [5], in comparison to the column direction.

The highest differences among fatigue lives were registered in relation to the direction of acting loading on the knitted fabrics tested taking into account the direction of rows. There variations can be observed within the range of low-cycle loadings i.e. $\sigma_{s0,65} \div \sigma_{s0,85}$. It means that the elongation action of the knitted fabrics in the direction of rows gives higher fatigue lives to them in comparison to loading action in the direction of columns. These phenomena are typical for knitted fabrics of left-right plait.

In the case of a high-cycle loading area i.e. $\sigma_{s0,40} \div \sigma_{s0,20}$, otherwise than for loadings in the direction of columns for fabrics of linear mass 30 tex, fatigue lives do not reach the limit values from the range considered. This is caused by the type of knit plait applied. Further investigations of fatigue lives under the level $\sigma_{s0,20}$ for the row direction is purposeless

__Table 8.__ Results of determined fatigue lives of smooth cotton knitted fabrics made of yarn of linear mass 40 tex in the direction of columns [5].

Lp.	Coefficient of current loading cycle σ_{si}, MPa									
	N_n, cycle									
	$\sigma_{s0.98}$	$\sigma_{s0.95}$	$\sigma_{s0.85}$	$\sigma_{s0.75}$	$\sigma_{s0.65}$	$\sigma_{s0.55}$	$\sigma_{s0.50}$	$\sigma_{s0.40}$	$\sigma_{s0.30}$	$\sigma_{s0.20}$
1	1	3	20	33	189	487	1178	3552	12542	132554
2	1	5	36	39	236	597	2698	6537	29831	76542
3	1	2	25	28	241	976	1584	5783	23649	26487
4	1	3	14	43	132	214	567	2397	9354	39754
5	1	1	8	66	34	294	479	1643	4531	72649
6	1	1	36	12	79	372	1136	1648	11697	22698
7	1	2	14	34	267	354	1357	3549	9634	93456
8	1	3	11	17	311	294	1397	4834	19861	111456
9	1	3	6	12	65	697	973	2113	12647	63547
10	1	3	25	34	129	329	854	956	17324	98457
$\bar{N}_n$	1	3	20	32	168	461	1222	3301	15107	73760
S	–	1.2	10.8	16.2	94.2	234.2	627.7	1894.9	7575.1	36495.6
V	–	45.1	55.5	51.1	56.0	50.8	51.4	57.4	50.1	49.5

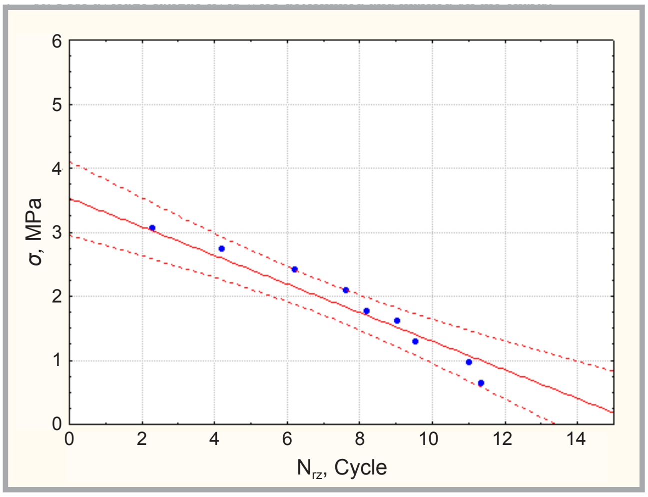

Figure 8. *Statistically elaborated charts of the fatigue lives for smooth cotton knitted fabrics made of yarns of linear mass 30 tex, in the direction of rows.*

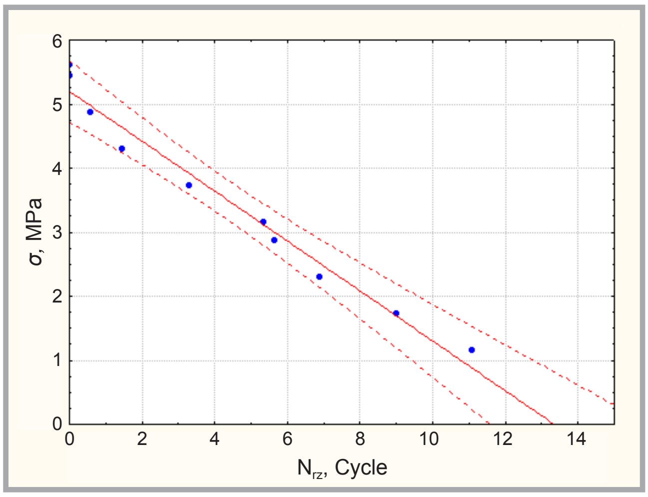

Figure 9. *Statistically elaborated charts of fatigue lives for smooth cotton knitted fabrics made of yarns of linear mass 30 tex, in the direction of columns.*

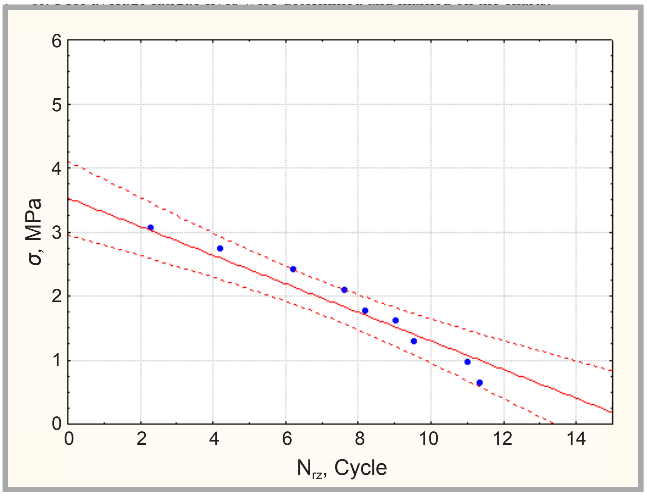

Figure 10. *Statistically elaborated charts of fatigue lives for smooth cotton knitted fabrics made of yarns of linear mass 40 tex, in the direction of rows.*

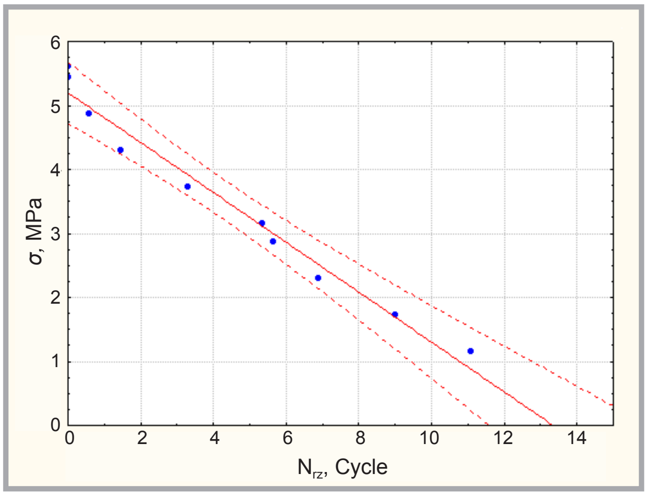

Figure 11. *Statistically elaborated charts of fatigue lives for smooth cotton knitted fabrics made of yarns of linear mass 40 tex, in the direction of columns.*

because an assessment would be performed within the range near the limit of elastic deformations. A similar tendency was registered for knitted fabrics made of yarn of linear mass 40 tex.

Similarly in the case of an assessment of the fatigue life in the direction of columns for fabrics made of yarns of linear mass 40 tex elongated in the row direction, they were characterised by higher fatigue lives within the entire range of loads, especially within the low-cycle and high-cycle ranges [6].

The highest differences of fatigue lives were noted in relations to the direction of loading action on the fabrics in the direction of rows. These variations can be observed within low-cycle loadings i.e. $\sigma_{s0,50} \div \sigma_{s0,85}$. It means that the elongation of knitted fabrics in the direction of rows is characterised by higher fatigue lives in comparison to loadings in the direction of columns. These phenomena are typical for knitted fabrics of left-right plait.

In the case of a high-cycle range i.e. $\sigma_{s0,40} \div \sigma_{s0,20}$, for loadings in the direction of columns for yarns of linear mass 40 tex, the fatigue lives do not reach the limit values from this range. This is caused by the applied plait of the knitted fabrics investigated. Further performance of an assessment of fatigue lives under the level $\sigma_{s0,20}$ in the row direction was purposeless, because it would be carried out within the range near the limit of elastic deformations.

The charts presented in ***Figures 4-7*** represent the traditional manner of presentation and analysis of outcomes of an assessment of fatigue lives. The results of the assessment of fatigue lives presented above are subjected to high dispersions. Dispersion around the mean value is higher the greater the loading level σ_m Is. Such behavior was also observed in former investigations [5] and could be recognised for all linear masses of yarns from which the fabrics were manufactured. The main causes of this phenomenon are differences resulting from the characteristics of the material investigated, variability of the structure of the material investigated and the direction of fabric elongation. Aiming for achievement of the complete characteristics of the fatigue lives for the materials investigated, the traditional Wöhler charts have to be complemented by statistical analyses. Wöhler charts representing fatigue lives could be elaborated postulating a linear relationship in semi-logarithmic or logarithmic coordinate systems [5, 6].

The significance of the model was checked by means of ***Equation (8)***. ***In Figures 8-11***, there are fatigue life charts for smooth, cotton knitted fabrics made of yarns of linear masses 30 and 40 tex in the row and column directions of loading , respectively. The confidence intervals for confidence level (1 – α) = 0.95 for average fatigue lives were determined and marked on the charts.

The regression equation of fatigue life elaborated for smooth cotton knitted fabrics of linear mass 30 tex in the direction of rows is expressed by the function.: $\hat{\sigma}_{sn} = 3.529 - 0.223 \cdot N_{rz}$. The coefficient of multiple correlation was equal to R = 0.93; Fisher-Snedecor statistics – $F_{cal} = F_8^1 = 106 > F_{crit} = 17.602$, whereas the confidence interval is as follows: $2.706 < N_n < 6.287$. However, the regression equation determined for knitted fabrics of 30 tex elongated in the direction of columns has the following form: $\hat{\sigma}_{sn} = 5.199 - 0.389 \cdot N_{rz}$. The coefficient of multiple correlation was equal to R = 0.98; Fisher-Snedecor statistics $> F_{cal} = F_8^1 = 313.227 > F_{crit} = 24.389$, whereas the confidence interval is: $2.161 < N_p < 9.466$.

Analysing the data shown in ***Figures 8*** and ***9***, one can observe the essential differences the fatigue lives of the knitted fabrics, taking into account the direction of loading action i.e. in the row and column directions, respectively. The fatigue chart is approximated by a straight line, whereas the points of the chart are placed within the ranges of the confidence intervals for the parameter (1 – α) = 0.95. This confirms the correctness of the choice of levels of the coefficient of the current cycle loading within the ranges of fatigue lives, i.e.: at all ten levels of loadings, for 10 positive (correct) tests at each level (4).

The proposed functions utilised in the above statistical analyses are perfectly suitable for the presentation of results in the elaborated method of assessment of fatigue lives for the knitted fabrics investigated. For the knitted fabrics made of yarns of linear mass 40 tex, the relationship observed also has linear versions, which are presented in ***Figures 10*** and ***11***.

The regression equation of fatigue life determined for smooth cotton knitted fabrics made of yarns of linear mass 40 tex, in the direction of rows is described via a function in the following form: $\hat{\sigma}_{sn} = 5.044 - 0.579 \cdot N_{rz}$.

The coefficient of multiple correlation was equal to R = 0,97; Fisher-Snedecor statistics – $F_{cal} = F_8^1 = 252.655 > F_{crit} = 25.286$, and the confidence interval: $2.706 < N_n < 6.287$. In turn, the regression function elaborated for the knitted fabrics made of yarns 40 tex elongated in the column direction has the form: $\hat{\sigma}_{sn} = 8.796 - 0.635 \cdot N_{rz}$. The coefficient of multiple correlation was equal to R = 0,99, Fisher-Snedecor statistics $> F_{cal} = F_8^1 = 996.220 > F_{crit} = 48.321$, whereas the confidence interval: $2.161 < N_p < 9.466$.

Analysing the data presented in ***Figures 10*** and ***11***, one can observe the essential differences between the fatigue lives of the fabrics in dependence on the direction of applying loadings i.e. in the row or column direction. The fatigue chart is approximated by means of a straight line, and the points on the chart (experimental outcomes) are placed within the limits of confidence intervals established for (1 – α) = 0.95. This fact confirms the correct choice of the levels of the coefficient of the current cycle loading within the range of fatigue life, i.e. for all ten levels, for ten positive (correct) tests at each level (4).

Recapitulation and conclusions

Based upon an analysis of the Wöhler charts (***Figures 4-7***) as well as other charts (***Figures 8-11***), damage to the knitted fabrics tested was observed within the area of low-cyclic loadings I [1], which takes place in the case of maximal plastic deformation. For all the knitted fabrics investigated, fatigue life in the column direction for both cases of yarn enumeration considered was higher than the fatigue life in the row direction of fabrics.

The fatigue life of the knitted fabrics considered was influenced by the fluctuating loading applied, which caused damage to those fabrics. The damage process runs without visible plastic deformations on the outer surface of the product. Within the process of fatigue deterioration of the knitted fabric tested, it is possible to observe strengthening of the trestle of threads in the plait within the range of high cycle fatigue life. Within the final phase of knitted fabric damage, it was possible to observe the displacement of particular threads in the plait, which – in consequence – causes the moving apart and finally breakage of the product.

The statistical models of fatigue life determined are highly adequate for the description of phenomena occurring during the damage of plain textile products.

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