



PROPOSITION OF A STRUCTURAL HEALTH MONITORING MODEL FOR A CONCEPT OF AN INNOVATIVE VARIABLE MASS PENDULAR TUNED MASS DAMPER

Amadeusz RADOMSKI^{1,*} , Damian Jacek SIROCINŃSKI² , Bogumil Daniel CHILIŃSKI² 

¹ Faculty of Automotive and Construction Machinery Engineering, Warsaw University of Technology, Poland

² Division of Computer Techniques, Institute of Machine Design Fundamentals, Faculty of Automotive and Construction Machinery Engineering, Warsaw University of Technology, Poland

Abstract

The article focuses on the proposal of a model of a skyscraper equipped with a semi-active pendular vibration eliminator utilizing the phenomenon of fluid transfer, which could be used in monitoring the condition of slender structures. The proposed model consists of two main elements: an upper member, representing the dynamic mass of a skyscraper in the form of a trolley, and a lower member - a pendulum attached to the trolley. To consider the fluid transfer, a variable mass factor represented by an inverse tangent function was included in the equation of motion. Simulation studies in a dimensionless time domain were performed to investigate the influence of mass distribution on changes in the system's response. Three dynamic states of the system were considered, during which the system's total mass remained constant. Diagnostic parameters enabling the detection of changes in the mass of the eliminator and stiffness of a damped structure have been proposed.

Keywords: tuned mass damper, variable mass system, structure health controlling, diagnostic parameter

List of Symbols/Acronyms

μ – differentiated mass of the pendulum to the mass of the pendulum ratio [-];
 θ – dimensionless frequency [-];
 τ – dimensionless time [-];
 D – the dissipative potential of the system [J];
 Ω – excitation frequency [Hz];
 ψ – excitation frequency to the natural frequency of the trolley ratio [-];
 ρ – flow rate coefficient [1/s];
 F – forcing amplitude [N];
 δ – forcing amplitude to structure's equivalent stiffness ratio [m];
 g – gravitational constant [m/s²];
 m_{p0} – initial mass of the pendulum [kg];
 ε_0 – initial mass of the pendulum to the mass of the system ratio [-];
 T – kinetic energy of the system [J];
 L – Lagrangian of the system [J];
 l – length of the pendulum [m];
 m_p – mass of the pendulum [kg];
 ε – mass of the pendulum to the mass of the system ratio [-];
 m_f – mass of the transferred liquid [kg].
 ε_f – mass of the transferred liquid to the mass of the system ratio [-];
 m_t – mass of the trolley [kg];
 τ_0 – mass transfer activation dimensionless time [-];
 χ – members natural frequencies ratio [-];
 Ω_0 – natural frequency of the pendulum [Hz];
 ω_0 – natural frequency of the trolley [Hz];

Φ – pendulum's angular displacement [rad];
 λ – Rayleigh's damping coefficient proportional to the stiffness matrix [-];
 c – structure's equivalent damping coefficient [Ns/m];
 k – structure's equivalent stiffness [N/m];
 t – time [s];
 m_s – total mass of the system [kg];
 X – trolley's horizontal displacement [m];
 $\delta X(t)$ – virtual displacement of the trolley [m];
 δW – virtual work [J];
 DAF – Dynamic Amplification Factor;
 PTMD – Pendular Tuned Mass Damper;
 SHM – Structural Health Monitoring;
 TMD – Tuned Mass Damper;

1. INTRODUCTION

A growing demand for light and slender structures, caused by a limited area of urban agglomerations, makes modern buildings more susceptible to dynamic loads [16][26]. It is a direct reason for an increased interest in the subject of vibrations affecting buildings and, thus, publications on vibration damping using tuned mass dampers [10][11]. Vibration absorbers are devices that dampen the vibrations of a main oscillator. They usually consist of a mass attached to a main oscillating system by means of damping and elastic elements. An appropriate selection of a TMD's parameters reduces vibrations of a structure being

affected by an exciting force [7]. Passive absorbers, i.e., covering a narrow range of excitation frequencies, the damping of which is initiated only by the phenomenon of vibrations, are the most commonly used solutions. However, the number of active or semi-active systems being applied is increasing, where the damping initiation, frequency range and operation time are conditioned by control and monitoring mechatronic apparatus design [12]. Thanks to technological development, it is possible to select diagnostical parameters of a TMD by analyzing a physical model in available simulation environments, where complex phenomena can be easily mapped in a simplified way and still properly reproduce the features of a real structure. The possible utilization of the variable mass phenomenon for vibration-damping purposes was previously studied and proved the concept's validity. A theoretical model of a double pendulum was analyzed as an alternative approach for dynamics control in [17][19], continued in [18], and also independently in [20]. These separate researches conducted on chaotic systems led to the conclusion that the mass redistribution for a system in motion provides additional damping and thus reduces the vibration amplitudes. Still, it is important to mention that the character of movement remains highly chaotic and dependent on a choice of members subjected to mass variation. Another interesting example directly refers to the concept of a tuned mass damper [23][24]. Theoretical considerations upon a vertically forced mass with a TMD attached, supported by empirical research, were introduced. Besides the aspect of a type of analyzed dynamic model, the main difference lies in the approach in which mass variation was taken into account. The authors focused on the ability of retuning a passive tuned mass damper to proper operating conditions by providing discrete changes in its mass, utilizing a control algorithm analyzing subsequent acceleration ratios. Simulations and experiments both delivered satisfying results. Comparison of the stated works raised a supposition that it is possible to introduce a solution of a variable mass TMD not only designed to sustain its characteristics in typical operating conditions but also reactive to sudden changes in them by allowing a rapid fluid flow when in motion. The aforementioned observation led to the preparation of a combined trolley-pendulum model in this article. The proposed model is more general than the system presented in [24], as it includes more physical phenomena occurring in the typical operation of objects damped by PTMDs. It can be concluded that the operation of structures with pendular tuned mass damper enables linearizing the system and applying further simplifications. So, if the model is linearized, then the entire analysis will be performed in the neighborhood of a stable critical point of the structure. It allows to exclude the

analysis of the chaotic behavior of the PTMD, which is beyond the scope of the article.

An assumption of a periodically constant mass makes it possible to obtain an analytical solution that, despite the simplifications, maintains high compliance with the simulation results for the nonlinear model and can be used to create a dynamic amplification factor spectrum. In order to define proper diagnostic criteria, dimensionless time and parameters were used, as in [17], since the dimensionless approach allows for simpler investigation of influence factors and easy comparison between engineering cases at different scales. An operating principle of a tuned mass damper is based on its adjustment with regard to a main structure, vibrations of which have to be suppressed.

The tuning process requires proper identification of the natural frequency of a main system [3][14]. Focusing on buildings, it is necessary to note the continuous degradation of materials taking place from the very beginning of a structure's work life [4][25]. Regardless of causes (environmental, design flaws, poor maintenance, etc.), the general term of degradation could be divided into corrosion (relevant to metals, reinforcement and polymers) and deterioration (concrete, plasters, mortars). Both factors may result in a change of a construction's stiffness [1][5][8], which affects its natural frequency and leads to non-optimal operating conditions, and thus the performance of a passive tuned mass damper [2][6][21][28]. Semi-active TMDs, as proposed in the article, similarly to active mass dampers, provide the ability to overcome an issue of detuning but with a less complex design.

Control algorithms are fundamental for properly operating active/semi-active TMD mechatronic systems and SHM environments [27]. An aspect of optimization between the effectiveness of a damping system, its overall application cost, and complexity results in various structural solutions and mathematical approaches to implementing a control loop [13][24]. Especially in the case of active mass dampers, one of the most crucial problems lies in a proper assessment of activation/delay time [9][22]. Nonetheless, semi-active systems are also sensitive to changes in activation time parameter [15]. The problems of control algorithms, as well as analyzing and selecting a suitable damping delay time, do not lay in the scope of this article. However, familiarization with the topic was necessary due to the unavoidable consideration of such a parameter when simulating the system's behavior in the transient state connected with a fluid transfer.

2. PHYSICAL MODEL

A simplified physical model consisting of basic elements imitating considered parts of a structure

was developed to analyze the operation of a pendular dynamic vibration eliminator with a variable mass. In general, the degree of complexity of the model depends on a studied phenomenon. To simplify calculations, a model with two degrees of freedom was used to describe the dynamics of the considered system. The generalized coordinate X was used to represent the displacement of the dynamic mass of a skyscraper modeled as a trolley with respect to the ground. The coordinate Φ describes an angular displacement of the TMD modeled as a pendulum. The generalized excitation force F models wind interacting with the skyscraper. The harmonic force allows for the investigation of dynamics under resonance condition by selecting an appropriate frequency of excitation Ω . The model (Figure 1) is based in the gravitational field, which affects the pendulum and the liquid transfer between the members. During operation, the system's total mass remains constant and transferred liquid does not alter the length of the pendulum eliminator but affects the damping of both members. It was assumed that three states will be considered during simulations:

- whole liquid volume in the upper member,
- transitional state, liquid transferred between the upper and lower members,
- liquid has been completely transferred into the lower member.

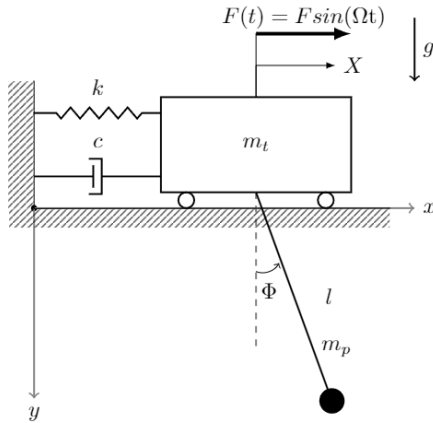


Fig. 1. Physical model of the considered system

Equations of motion of the variable-mass system were derived using the Euler-Lagrange equations. After linearization, the Rayleigh damping model was adopted. Further, a dimensionless form of equations of motion with dimensionless time and parameters was presented, enabling the study of dependencies for easy comparison between cases at different scales, as they allow to establish a condition of similarity between a model and a full-scale object. Calculations were carried out using proprietary libraries in the Python programming language.

3. DIMENSIONLESS EQUATIONS OF MOTION

Based on the proposed model, kinetic energy (1), potential energy (2) and dissipative potential (3)

were stated, then the Lagrangian of the system (4) was determined. Moreover, generalized forces were derived from the virtual work method (5):

$$T = \frac{l^2 m_p \dot{\Phi}^2}{2} + l m_p \cos(\Phi) \dot{\Phi} \dot{X} + \frac{m_p \dot{X}^2}{2} + \frac{m_t \dot{X}^2}{2} \quad (1)$$

$$V = \frac{kX^2}{2} - g l m_p \cos(\Phi) \quad (2)$$

$$D = \frac{c\dot{\Phi}^2}{2} + \frac{c\dot{X}^2}{2} \quad (3)$$

$$L = g l m_p \cos(\Phi) - \frac{kX^2}{2} + \frac{l^2 m_p \dot{\Phi}^2}{2} + l m_p \cos(\Phi) \dot{\Phi} \dot{X} + \frac{m_p \dot{X}^2}{2} + \frac{m_t \dot{X}^2}{2} \quad (4)$$

$$\delta W = F \sin(\Omega t) \delta(X(t)) \quad (5)$$

Lagrangian and dissipative potential were taken for calculations of appropriate derivatives of Lagrange method. (Non-potential forces come from virtual work method.) All of obtained terms (3)-(5) were used to form the system's equations of motion:

$$(m_p + m_t) \ddot{X} + (m_p + m_t + c) \dot{X} + kX + l m_p \cos(\Phi) \ddot{\Phi} + l m_p \cos(\Phi) \dot{\Phi} - l m_p \sin(\Phi) \dot{\Phi}^2 = F \sin(\Omega t) \quad (6)$$

$$l^2 m_p \ddot{\Phi} + (l^2 m_p + c) \dot{\Phi} + g l m_p \sin(\Phi) + l m_p \cos(\Phi) \ddot{X} + l m_p \cos(\Phi) \dot{X} = 0 \quad (7)$$

By approximating individual derivatives with a Taylor series around the equilibrium point of the system, the linearized equations of motion for the system were derived:

$$(m_p + m_t) \ddot{X} + (m_p + m_t + c) \dot{X} + kX + l m_p \ddot{\Phi} + l m_p \dot{\Phi} = F \sin(\Omega t) \quad (8)$$

$$l^2 m_p \ddot{\Phi} + (l^2 m_p + c) \dot{\Phi} + g l m_p \Phi + l m_p \ddot{X} + l m_p \dot{X} = 0 \quad (9)$$

Based on linearized equations of motion (8)-(9), the Rayleigh damping model was adopted with a damping coefficient λ proportional to the system's stiffness matrix. The damping coefficient proportional to the inertia matrix is equal to 0:

$$(m_p + m_t) \ddot{X} + \lambda k \dot{X} + kX + l m_p \ddot{\Phi} + l m_p \dot{\Phi} = F \sin(\Omega t) \quad (10)$$

$$l^2 m_p \ddot{\Phi} + (l^2 m_p + \lambda g l m_p) \dot{\Phi} + g l m_p \Phi + l m_p \ddot{X} + l m_p \dot{X} = 0 \quad (11)$$

Subsequently, transformations were conducted by introducing dimensionless time with respect to the trolley's natural frequency and by dividing the coefficients at the accelerations of individual coordinates, which resulted in dimensionless parameters:

$$m_s = m_p + m_t \quad (12)$$

$$\epsilon = \frac{m_p}{m_s} \quad (13)$$

$$\dot{\epsilon} = \frac{\dot{m}_p}{m_s} \quad (14)$$

$$\epsilon_0 = \frac{m_{p0}}{m_s} \quad (15)$$

$$\epsilon_f = \frac{m_f}{m_s} \quad (16)$$

$$\mu = \frac{m_p}{m_p} \quad (17)$$

$$\chi = \frac{\Omega_0}{\omega_0} \quad (18)$$

$$\psi = \frac{\Omega}{\omega_0} \quad (19)$$

$$\delta = \frac{F}{k} \quad (20)$$

$$\tau = \omega_0 t \quad (21)$$

By substitution of the obtained formulas (12)-(21) to linearized equations of motion with assumed Rayleigh's damping model, their dimensionless versions were obtained:

$$\ddot{X} + \lambda \omega_0 \dot{X} + X + \epsilon l \ddot{\Phi} + \frac{l \dot{\epsilon}}{\omega_0} \dot{\Phi} = \delta \sin(\psi \tau) \quad (22)$$

$$\ddot{\Phi} + \left(\chi^2 \lambda \omega_0 + \frac{\mu}{\omega_0} \right) \dot{\Phi} + \chi^2 \Phi + \frac{\ddot{\chi}}{l} + \frac{\mu}{l \omega_0} \dot{X} = 0 \quad (23)$$

The transfer of liquid was modeled using the arctan function. Flow rate coefficient ρ and mass transfer activation dimensionless time τ_0 were considered:

$$m_p = \epsilon_0 m_s + \epsilon_f m_s \left(\frac{\text{atan}\left(\frac{\rho \tau}{\omega_0} - \frac{\rho \tau_0}{\omega_0}\right)}{\pi} + 0.5 \right) \quad (24)$$

$$m_t = m_s (1 - \epsilon_f - \epsilon_0) + \epsilon_f m_s \left(-\frac{\text{atan}\left(\frac{\rho \tau}{\omega_0} - \frac{\rho \tau_0}{\omega_0}\right)}{\pi} + 0.5 \right) \quad (25)$$

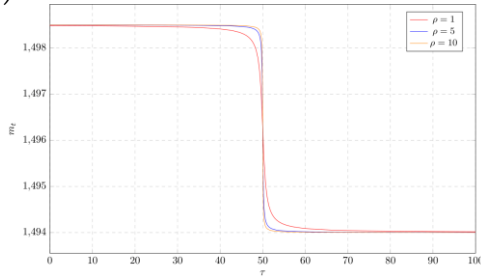


Fig. 2. Liquid flow chart for various flow rate coefficients and sample data

Figure 2 explains the influence of parameter ρ on mass flow change in the transfer function. With a higher value of the coefficient, the rate of liquid transfer increases and it is easier to prevent mass changes beyond the damping activation dimensionless time. Thus, the parameter τ_0 becomes more precise.

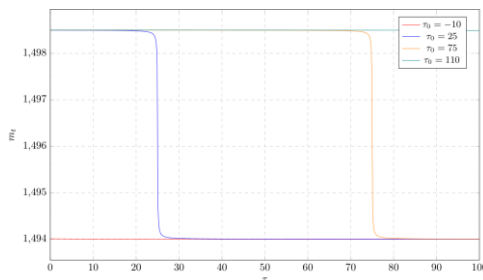


Fig. 3. Liquid flow chart for various mass transfer activation dimensionless time and sample data

Figure 3 shows the influence of parameter τ_0 on mass transfer function. The parameter allows for control of the activation time of the liquid transfer.

The mass transfer function for the pendulum operates similarly to that of the trolley, with the difference being that the pendulum's mass increases.

At the same time, the influence of the parameters τ_0 and ρ remains the same.

Expressions (22)-(25) form the basis for describing the motion of the model and serve to conduct research on the influence of individual parameters on the system's dynamics. Given the intricate nature of the motion equations, it becomes feasible to introduce simplifications, resulting in an analytical solution applicable to a model with constant mass.

4. STUDY ON SYSTEM DYNAMICS

The lack of a closed analytical solution of equations of motion with a variable mass resulted in selecting the numerical approach as a method of solution for further works, as it allows to study the influence of the phenomenon of mass transfer on the dynamics of individual members. The main set of simulations was run, where each time, a single parameter was analyzed for multiple value ranges while others remained constant. The proposed default constant values of the parameters are shown in Table 1. The dimensionless damping time activation τ_0 was set to 10π (five periods of the main oscillator in a dimensionless time domain) as such delay enables observation of the trolley's dynamics under resonance vibrations with low damping but also ensures that the amplitude of occurring oscillations potentially does not exceed values that could be considered threatening to the real structure.

Table 1. Default values of the system's parameters

	Parameter	Value
1	ϵ_0	0.001
2	ψ	1
3	ω_0	0.6264
4	δ	0.0051
5	l	25
6	λ	0.03
7	τ_0	10π
8	ρ	10
9	ϵ_f	0.003
10	χ	1

A study was conducted to analyze how the amplitudes of individual members are influenced by the ratio of the mass of the transferred liquid to the overall system mass. Simulations were performed for the parameter ranging from 0.0001 to 0.005.

Figure 4 shows that the amount of mass of the liquid transferred into the lower member positively impacts the damping of vibrations of the main oscillator. The greater mass of the eliminator after the liquid transfer leads to faster damping of vibrations of the main oscillator and lowers the vibration amplitude of the trolley displacement. The

effective mass of the TMD is equal to 0.004-0.006 dynamic mass of the main oscillator modeled as a trolley.

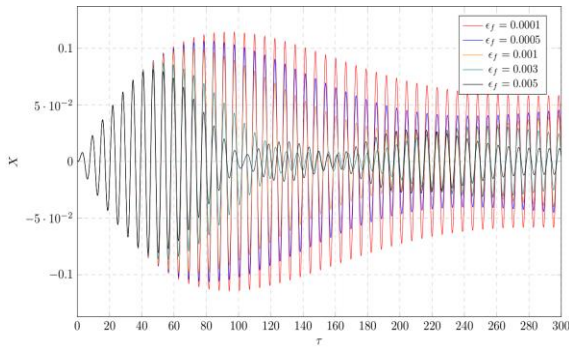


Fig. 4. Trolley displacement depending on the transferred mass to the mass of the system ratio

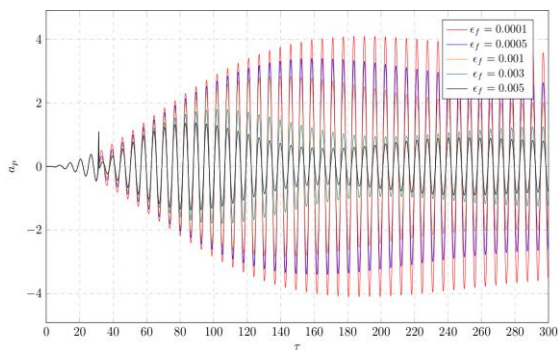


Fig. 5. Linear acceleration of the pendular tuned mass damper

Figure 5 presents how the linear acceleration of the pendulum is affected by the ratio of the transferred liquid's mass to the overall system mass. As the mass increases, the linear acceleration decreases. Greater TMD weight results in more effective energy dissipation from main oscillator vibrations due to the inertia forces of the pendulum. The sudden mass transfer results in a temporary change in acceleration due to the momentum of the transferred liquid, but the peaks do not exceed maximum amplitude values in the range of the simulation. Further research on damping time activation should be performed to determine better the influence of a mass momentum on the behavior of the system's dynamics.

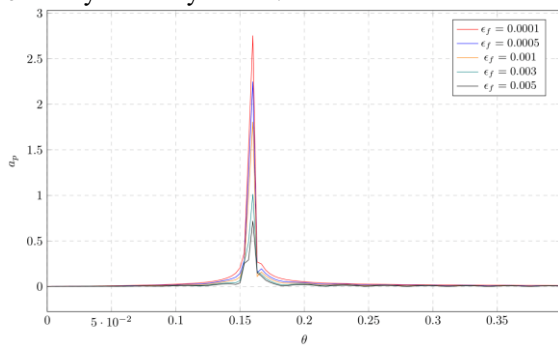


Fig. 6. Linear acceleration of the pendulum in the dimensionless frequency domain

The frequency spectrum of the pendulum's linear accelerations for different values of the masses ratio of the transferred liquid is shown in Figure 6. The amplitudes of the pendulum vary depending on the amount of the transferred mass. The vibration amplitude of a linear acceleration may serve as a diagnostic parameter that could be used to determine the pendulum's mass in the case of a known tune ratio.

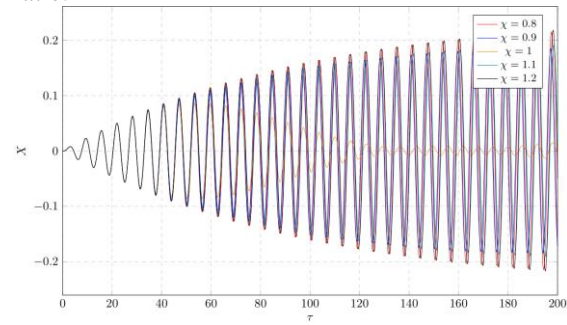


Fig. 7. Trolley displacement as a function of the members' natural frequency ratio

The chart in Figure 7 shows the influence of the tune ratio on the damping of the main member's oscillations. Ideally tuned eliminator effectively absorbs energy from the trolley, which results in faster damping of vibrations, while for other values of parameters, the main oscillator tends to stay in the resonance zone. The closer the members' natural frequencies are to themselves, the lower the amplitude and faster damping of vibrations.

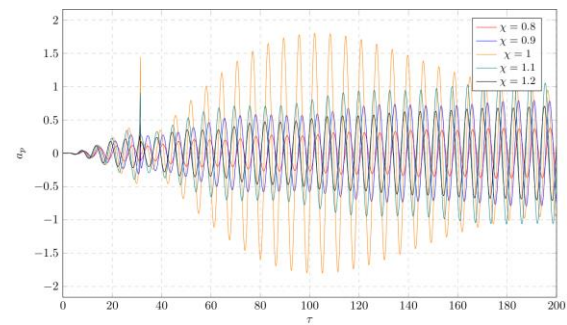


Fig. 8. Linear acceleration of the pendulum as a function of the members' natural frequency ratio

Figure 8 shows the linear accelerations obtained during the pendulum operation for different members' natural frequency ratio values. In the case of TMD tuned for the natural frequency of the main system, amplitudes of linear acceleration achieve high values very quickly right after mass transfer initiation. For the mistuned eliminator, amplitudes do not reach high values and the pendulum is less sensitive to the influence of the transferred mass. Similarly to the studies of the previous parameter, peaks of accelerations occur during the mass transfer. However, in the case of a mistuned eliminator, it is impossible to predict the PTMD's behavior unambiguously at this stage of dynamics

analysis, whether the pendulum will accelerate or decelerate.

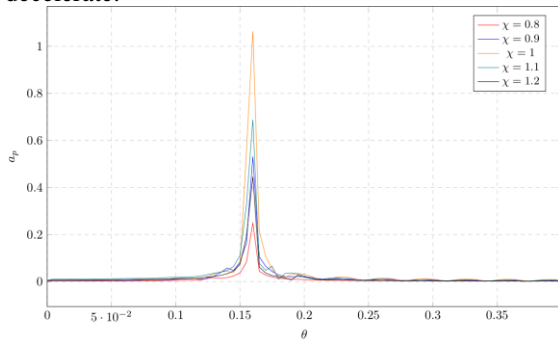
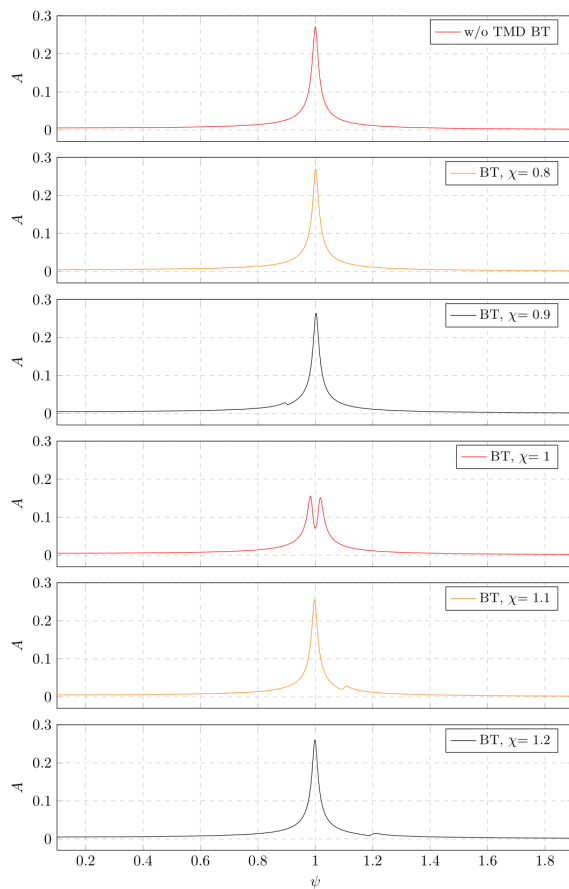


Fig. 9. Linear acceleration of a pendulum in the dimensionless frequency domain in the function of various tune ratios

The plot presented in Figure 9 shows the frequency spectrum of linear accelerations obtained during the operation of the pendulum for different values of the tune ratios. The highest amplitudes can be seen for TMD tuned to the main system's natural frequency, which ensures the best damping of the main oscillator. A direct read-out from an acceleration sensor alone would not provide accurate information about whether the frequency of a pendulum is lower or higher than the natural frequency of a main oscillator. Still, it would help to find a range in which the tuning parameter is located.



Narrowing this range to a minimum ensures a proper tuning of a TMD. The smaller the total mass of a pendulum after a liquid transfer, the more sensitive a pendulum is to high amplitudes of linear accelerations during ideal tuning. Thus, it is easier to tune a TMD. This information might be used in controlling and monitoring the health of structures. With the progress of concrete cracking, a structure's stiffness decreases, which leads to a change in the natural frequency of the damped member, amplitudes of a linear acceleration of the eliminator drop and amplitudes of a structure's displacement increase during resonance. Using this diagnostic parameter, it is possible to detect changes in TMD's operation and retune the eliminator to a proper frequency, which results in extending the working life of a structure.

Figure 10 shows the dynamic amplification factor of the trolley's displacement in the case of various tuning ratios. The charts were divided into those before and after the transfer of the mass and calculated using an analytical solution with the assumption of constant mass.

A comparison of maximum DAF values and indices is provided in Table 2. The highest displacement amplitude before liquid transfer equals 0.271 m, which happens during resonance when TMD is not mounted on an excited structure.

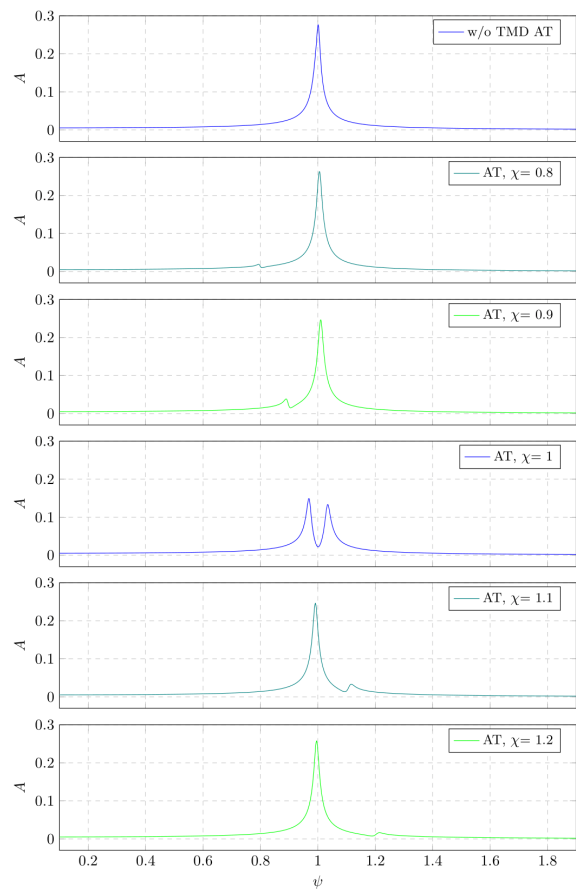


Fig. 10. DAF of trolley's displacement, BT - before mass transfer, AT - after mass transfer

Table 2. A comparison of dynamic amplification factor amplitude values and indices for different cases, BT - before mass transfer, AT - after mass transfer

Case	Max. value [m]	ψ at max. value	Amplitude for $\psi = 1$ [m]
w/o TMD	0.271	1	0.271
BT			
w/o TMD	0.276	1.001	0.274
AT			
BT $\chi = 0.8$	0.269	1.001	0.267
AT $\chi = 0.8$	0.263	1.005	0.229
BT $\chi = 0.9$	0.264	1.003	0.255
AT $\chi = 0.9$	0.246	1.01	0.172
BT $\chi = 1$	0.155	0.983	0.071
AT $\chi = 1$	0.149	0.969	0.022
BT $\chi = 1.1$	0.255	0.998	0.248
AT $\chi = 1.1$	0.246	0.991	0.178
BT $\chi = 1.2$	0.259	0.999	0.257
AT $\chi = 1.2$	0.257	0.995	0.232

Transferring the same ratio of fluid mass out of the trolley without an eliminator barely changes its natural frequency and even negatively affects the achieved vibration amplitudes, increasing them due to the lower mass of the structure. The addition of TMD changes the resulting frequency of the trolley and pendulum, moving them away from the resonance region and thereby reducing the vibration amplitude during its occurrence. Parameter χ equal to 1 provides the best results by effectively decreasing the amplitude of the trolley in the resonance zone, especially after mass transfer, achieving displacement of 0.022 m for $\psi = 1$ and around 0.15 m for values of ψ near 1. In other cases of tuning parameters, the transferred mass is not so critical because the resultant natural frequency after transfer does not change much. The most significant impact on the change in the trolley's frequency compared to the system without a dynamic vibration absorber is the proper tuning of TMD to the natural frequency of the structure and the mass of TMD.

5. CONCEPT OF A METHOD FOR STRUCTURE HEALTH CONTROL

Performed investigation indicates a sensitivity of the system with PTMD on specified changes in its natural frequencies. It affects the dynamic response of the entire damped object (strictly subjected to the influence of a vibrational absorber) and results in higher amplitudes in most cases. On the one hand, shifts in the spectral structure of the system are caused by changes in the system's health (due to a decrease of, e.g., stiffness as presented in the introduction), which indicates that the performance of object members is lower (in the sense of measurable parameters). On the other hand, the structure's health can be treated as the dynamic behavior of the entire system, not as a state of the isolated members, since the load capacity of single parts is not too low to ensure proper maintenance of the monitored structure. Basically, the system's

general (dynamic) state results in certain amplitudes and frequencies. It can be recalculated directly to operational time due to fatigue strength, which is one of many ways to estimate a structure's health. It can be claimed that if the parameters of the system's members change, but the dynamical response of an entire system is the same, then it will not significantly affect the structure's health. The proposed concept assumes this approach to control (keep at the same level) the health of the entire structure, even if the parameters of members are worsening. It is needed to highlight that it is useful since the changes in parts structure do not significantly impact its performance and possibility to carry the loads. In such case, lower amplitudes ensure an extended operation period despite the worse health of the selected elements. It can be concluded that the properly tuned TMD (as commonly used attenuating device) and the possibility of online control of the mass of the members can be efficiently used to increase system durability. Moreover, the concept assumes that the commonly used PTMD is equipped or enhanced with an additional feature of transferring a small amount of mass.

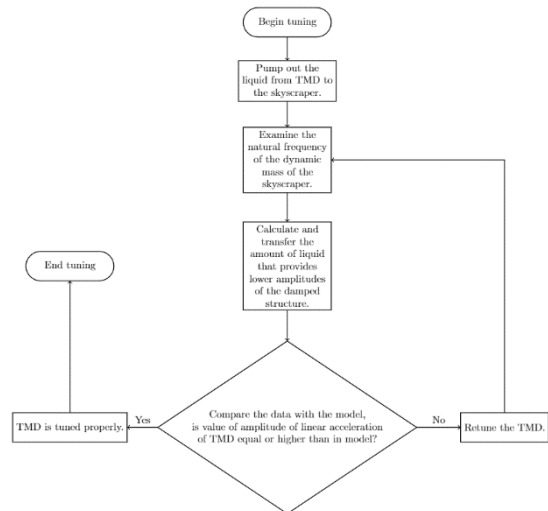


Fig. 11. Diagram describing a method of tuning the TMD to proper operation

The diagram in Figure 11 illustrates the TMD's tuning process to the skyscraper's natural frequency, which is needed to utilize the presented proposal correctly. Based on the model calculations, it is possible to obtain the desired skyscraper's natural frequency. Subsequently, the pendulum should be tuned to the main oscillator. Application of subsequent mass changes of both members (due to small transfers of liquid) allows to estimate the influence of mass changes on structure dynamics. This procedure should be repeated until TMD linear accelerations achieve values similar to the model. Implementing a mechatronic diagnostic system that monitors the pendulum's performance during various excitation frequencies becomes feasible.

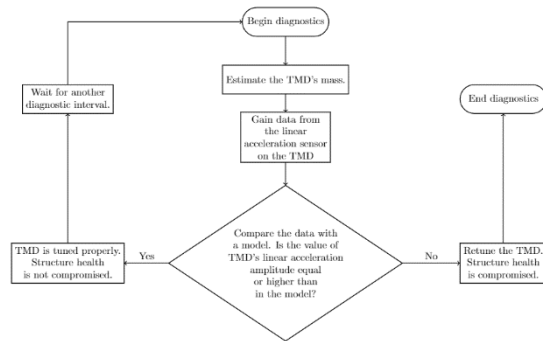


Fig. 12. Diagram describing a method of controlling structural health

Figure 12 presents a simple diagram describing a structure health control method. An appropriate tuning of a TMD causes a structure to work in a range of vibrations assumed by a designer, which significantly extends its service life and increases safety. The proposed concept assumes that the algorithm operates automatically using a specially developed mechatronic system combining several sensors, including a linear acceleration sensor located on the TMD and a computer program performing signal analysis comparing the received accelerations with the data collected during the model analysis. In case of deviations exceeding the predefined threshold, the mechatronic system signals the need for readjusting the pendulum to the changed natural frequency of the skyscraper.

6. CONCLUSIONS

The article proposes a model with two degrees of freedom, consisting of a trolley and a pendulum, which may be successfully used in diagnostics to control health and attenuate resonant vibrations of large and slender structures. Performed simulations show that the eliminator allows for vibration amplitude reduction of about 89% in the resonance zone and 47% near the resonance zone. In the rest of the frequency spectrum, the TMD barely works, as it is not tuned to excitation frequency and the trolley itself does not reach amplitudes higher than 0.03 m outside the resonance area. The paper shows two exemplary simulations, which were done in order to show diagnostic possibilities, but the scope of the investigation also included the influence of Rayleigh's proportional to the stiffness matrix damping coefficient, excitation frequency to natural frequency of upper member ratio, and members' initial mass ratio. The proposed model provides an interesting alternative to systems that consider only linear displacements of a TMD. The carried-out simulations provide useful information on the concept of a dynamic vibration eliminator with a variable mass and prove the positive effect of the mass of a transferred liquid on the damping of vibrations of a main oscillator. The effective range of a total pendulum mass equal to the dynamic mass of an oscillating structure was assessed to 0.004-

0.006. It was proven that the pendulum's linear acceleration amplitudes decreased with an increasing mass of a transferred liquid. Further research showed that the considered TMD is sensitive to a tune ratio parameter, which results in reducing its linear acceleration amplitudes the more the parameter deviates from the value equal to 1. Based on the provided information, a diagnostic parameter defined by measuring a linear acceleration of the pendulum was proposed. By using a simple acceleration sensor mounted on an eliminator, it is possible to monitor the mass of a pendulum or control the health of a structure by retuning the TMD to a proper frequency, dependent on structure's changing stiffness and consequently natural frequency due to reinforced concrete corrosion, steel beams fatigue or other factors. An analytical solution can be used for successful model-based diagnosis for states before and after liquid transfer in an implemented mechatronic system for controlling the health of a structure, as described in the suggested control loop. In the future, a more detailed study on damping time activation and flow rate is needed to provide useful information about how it affects the system's dynamics due to the momentum of a transferred mass. In addition, the solution to the problem can be improved by modeling transients, mass transfer process and considering the system's stochastic properties.

Source of funding: *The article did not have external source of funding and was not written as part of a research project.*

Author contributions: *research concept and design, A.P.R., B.D.C; Collection and/or assembly of data, A.P.R.; Data analysis and interpretation, A.P.R., D.J.S.; Writing the article, A.P.R., D.J.S.; Critical revision of the article, D.J.S., B.D.C; Final approval of the article, B.D.C.*

Declaration of competing interest: *The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.*

REFERENCES

1. Torres-Acosta AA, Fabela-Gallegos MJ, Muñoz-Noval A, Vázquez-Vega D, Hernandez-Jimenez JR, Martínez-Madrid M. Influence of Corrosion on the Structural Stiffness of Reinforced Concrete Beams. *Corrosion* 2004; 60(9): 862–72. <https://doi.org/10.5006/1.3287868>.
2. Bathaei A, Zahrai SM, Ramezani M. Semi-active seismic control of an 11-DOF building model with TMD+MR damper using type-1 and -2 fuzzy algorithms. *Journal of Vibration and Control* 2018; 2938–2953. <https://doi.org/10.1177/1077546317696369>.
3. Sadeqi A, Esfandiari A, Sanayei M, Rashvand M. Automated operational modal analysis based on long-term records: A case study of Milad Tower structural health monitoring. *Structural Control and Health Monitoring* 2022; 29(10): e3037. <https://doi.org/10.1002/stc.3037>.

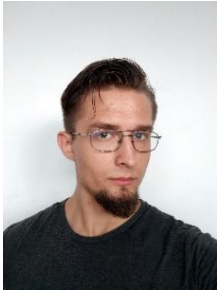
4. Basu B. Identification of stiffness degradation in structures using wavelet analysis. *Construction and Building Materials* 2005; 19(9): 713–21. <https://doi.org/10.1016/j.conbuildmat.2005.02.018>.
5. Liu F, Zhou J, Yan L. Study of stiffness and bearing capacity degradation of reinforced concrete beams under constant-amplitude fatigue. *PLOS ONE* 2018; 13(3): e0192797. <https://doi.org/10.1371/journal.pone.0192797>.
6. Yang F, Sedaghati R, Esmailzadeh E. Vibration suppression of structures using tuned mass damper technology: A state-of-the-art review. *Journal of Vibration and Control* 2022; 28(7–8): 812–36. <https://doi.org/10.1177/1077546320984305>.
7. Fang H, Liu L, Zhang D, Wen M. Tuned mass damper on a damped structure. *Structural Control and Health Monitoring* 2019; 26(3): e2324. <https://doi.org/10.1002/stc.2324>.
8. Zhong J, Gardoni P, Rosowsky D. Stiffness Degradation and Time to Cracking of Cover Concrete in Reinforced Concrete Structures Subject to Corrosion. *Journal of Engineering Mechanics* 2010; 136(2): 209–19. [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000074](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000074).
9. Teng J, Xing HB, Lu W, Li ZH, Chen CJ. Influence analysis of time delay to active mass damper control system using pole assignment method. *Mechanical Systems and Signal Processing* 2016; 80: 99–116. <https://doi.org/10.1016/j.ymssp.2016.04.008>.
10. Wang L, Zhou Y, Nagarajaiah S, Shi W. Bi-directional semi-active tuned mass damper for torsional asymmetric structural seismic response control. *Engineering Structures* 2023; 294: 116744. <https://doi.org/10.1016/j.engstruct.2023.116744>.
11. Wang L, Nagarajaiah S, Shi W, Zhou Y. Seismic performance improvement of base-isolated structures using a semi-active tuned mass damper. *Engineering Structures* 2022; 271: 114963. <https://doi.org/10.1016/j.engstruct.2022.114963>.
12. Wang L, Shi W, Zhang Q, Zhou Y. Study on adaptive-passive multiple tuned mass damper with variable mass for a large-span floor structure. *Engineering Structures* 2020; 209: 110010. <https://doi.org/10.1016/j.engstruct.2019.110010>.
13. Wang L, Shi W, Zhou Y. Study on self-adjustable variable pendulum tuned mass damper. *The Structural Design of Tall and Special Buildings* 2019; 28(1): e1561. <https://doi.org/10.1002/tal.1561>.
14. Sun M, Li Q, Li Y. Investigation of time-varying natural frequencies of high-rise buildings under harsh excitations using a high-resolution combined scheme. *Journal of Building Engineering* 2022; 57: 104859. <https://doi.org/10.1016/j.jobeb.2022.104859>.
15. Lin PY, Chung LL, Loh CH. Semiactive Control of Building Structures with Semiactive Tuned Mass Damper. *Computer-Aided Civil and Infrastructure Engineering* 2005; 20(1): 35–51. <https://doi.org/10.1111/j.1467-8667.2005.00375.x>.
16. Li QS, Zhi LH, Yi J, To A, Xie J. Monitoring of typhoon effects on a super-tall building in Hong Kong. *Structural Control and Health Monitoring* 2014; 21(6): 926–49. <https://doi.org/10.1002/stc.1622>.
17. Kwiatkowski R, Hoffmann TJ, Kołodziej A. Dynamics of a Double Mathematical Pendulum with Variable Mass in Dimensionless Coordinates. *Procedia Engineering* 2017; 177: 439–43. <https://doi.org/10.1016/j.proeng.2017.02.242>.
18. Kwiatkowski R. The concept of vibration damping of the variable mass assembly. *MATEC Web of Conferences* 2019; 254: 03003. <https://doi.org/10.1051/mateconf/201925403003>.
19. Kwiatkowski R. Vibration Damping in the Double Mathematical Pendulum with Variable Mass. *Machine Dynamics Research* 2014; Vol. 38, No. 4: 23–32.
20. Espíndola R, Valle GD, Hernández G, Pineda I, Muciño D, Díaz P. The Double Pendulum of Variable Mass: Numerical Study for different cases. *Journal of Physics: Conference Series* 2019; 1221(1): 012049. <https://doi.org/10.1088/1742-6596/1221/1/012049>.
21. Bakre SV, Jangid RS. Optimum parameters of tuned mass damper for damped main system. *Structural Control and Health Monitoring* 2007; 14(3): 448–70. <https://doi.org/10.1002/stc.166>.
22. Chu SY, Soong TT, Lin CC, Chen YZ. Time-delay effect and compensation on direct output feedback controlled mass damper systems. *Earthquake Engineering & Structural Dynamics* 2002; 31(1): 121–37. <https://doi.org/10.1002/eqe.101>.
23. Shi W, Wang L, Lu Z, Wang H. Experimental and numerical study on adaptive-passive variable mass tuned mass damper. *Journal of Sound and Vibration* 2019; 452: 97–111. <https://doi.org/10.1016/j.jsv.2019.04.008>.
24. Shi W, Wang L, Lu Z. Study on self-adjustable tuned mass damper with variable mass. *Structural Control and Health Monitoring* 2018; 25(3): e2114. <https://doi.org/10.1002/stc.2114>.
25. Lei Y, Zhou H, Lai ZL. A Computationally Efficient Algorithm for Real-Time Tracking the Abrupt Stiffness Degradations of Structural Elements. *Computer-Aided Civil and Infrastructure Engineering* 2016; 31(6): 465–80. <https://doi.org/10.1111/micc.12217>.
26. Wang YW, Ni YQ. Full-scale monitoring of wind effects on a supertall structure during six tropical cyclones. *Journal of Building Engineering* 2022; 45: 103507. <https://doi.org/10.1016/j.jobeb.2021.103507>.
27. Zhao Y. Structural Health Monitoring Applications in Tall Buildings. *E3S Web of Conferences* 2020; 198: 02020. <https://doi.org/10.1051/e3sconf/202019802020>.
28. Z Chen. Influence of bridge-based designed TMD on running trains. *Journal of Vibration and Control* 2019; 25: 182–193. <https://doi.org/10.1177/1077546318773022>.



Amadeusz RADOMSKI

was born in Warsaw, Poland in the 18th of October 2001. He studies the Engineering of Hybrid and Electric Vehicles in Warsaw University of Technology, Poland at the Faculty of Automotive and Construction Machinery Engineering. He obtained his engineering degree in February

2024. Amadeusz is interested in mechanical vibrations, dynamic systems, programming and autonomous vehicles. He is a member of the students scientific association, where he writes articles together with Ph.D. students and lecturers.



Damian Jacek SIEROCIŃSKI graduated with M.Sc. in Engineering degree in the field of Mechanics of Vehicles and Construction Machinery at the Faculty of Automotive and Construction Machinery Engineering of Warsaw University of Technology in 2021, where currently resides as an associate and conducts work

on Ph.D. dissertation. Main scientific interests focus on mechanical vibrations, especially in rail vehicles, and utilization of programming languages for engineering purposes.



Bogumil Daniel CHILIŃSKI received Ph.D. degree at the Faculty of Automotive and Construction Machinery Engineering from Warsaw University of Technology, Poland, in 2016. Now he works at the same place, author of more than 70 papers. His current research interests include dynamics, theory of differential

equations and numerical simulations.