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**Tomasz Śliwa*, Marek Jaszczur*, Michał Kruszewski*, **,
Aneta Sapińska-Śliwa*, Anna Bieda*,
Tomasz Kowalski*, Albert Złotkowski***

A STUDY OF HYDRAULIC CHARACTERISTIC FOR BOREHOLE HEAT EXCHANGERS

1. INTRODUCTION

Thermal Response Test is an onsite method to determine thermal properties of shallow BHE's using equipment presented in Figure 1. TRT involves measuring temperature changes of the heat carrier, circulating in closed-loop system over particular range of time.

a) b)

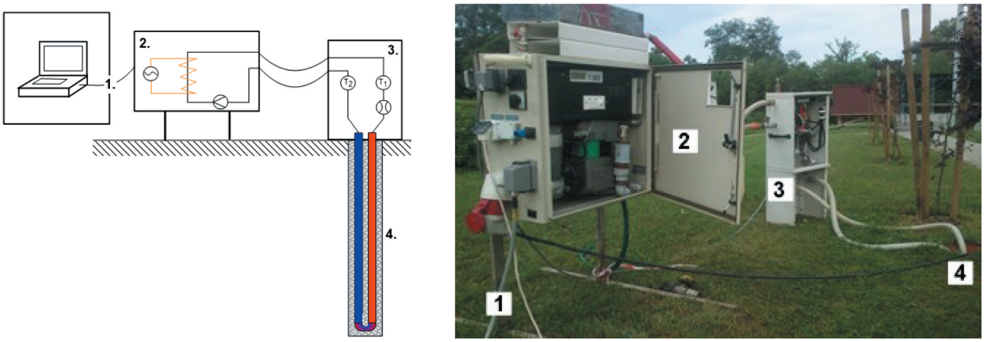

Fig. 1. Thermal Response Test measuring equipment (a and b):
1 – control unit (computer), 2 – pump and thermostat unit, 3 – valve unit,
4 – U-pipe inside borehole [10]

* AGH University of Science and Technology, Faculty of Drilling, Oil and Gas, Krakow, Poland
** Keilir Institute of Technology, Iceland

Operation is carried out with supplying or receiving thermal energy at a constant power. Following test allows selecting appropriate amount of BHE's and their distribution for given thermal properties of surrounding rock mass and heat recipient. TRT enables to calculate following parameters:

– effective thermal conductivity coefficient of surrounding rock mass,
– thermal resistivity of BHE,
– average temperature value of surrounding rock mass [7].

2. THERMAL RESPONSE TEST

TRT equipment consists of flow rate and temperature sensors, circulation pump, electrical furnace and control unit. Measurement is performed by injecting thermal energy at a constant power and constant volumetric flow rate into BHE. Temperature and flow rate sensors are assembled on inlet and outlet of BHE. Data recorded every one minute is forwarded directly to control unit and further evaluated. The average duration of TRT is around 100 hours. Thorough TRT data evaluation will enable to determine thermal characteristic of geothermal system for already established amount of BHE's [4].

Table 1 provides data for computing unit thermal power from BHE using effective thermal conductivity λ_{eff} evaluated from TRT. Correlation between effective thermal conductivity, dependent on TRT duration, and thermal conductivity coefficient is described as follows [11]:

$$\frac{1}{\lambda_{eff}} = \frac{1}{r \cdot \alpha} + \frac{1}{\lambda_n} \tag{1}$$

where:

r – radius of conduction, m,
λ_n – thermal conductivity coefficient, $W \cdot m^{-1} \cdot K^{-1}$,
α – heat transfer coefficient, $W \cdot m^{2} \cdot K^{-1}$.

Table 1

Unit thermal capacity for single U-pipe design of BHE determined using effective thermal conductivity λ_{eff} evaluated from TRT data [5]

Effective thermal conductivity calculated from TRT, $W \cdot m^{-1} \cdot K^{-1}$	BHE's unit thermal power, $W \cdot m^{-1}$
to 1.5	to 40
from 1.5 to 2.0	to 50
from 2.0 to 3.0	to 55
above 3.0	to 80

Following calculations are used to compute potential unit thermal power exchanged with underground rock mass [1]:

$$q = 20 \cdot \lambda_{eff} \tag{2}$$

or

$$q = 13 \cdot \lambda_{eff} + 10 \tag{3}$$

Average value of unit thermal power of single BHE is a calculated value of heat pump, working continuously for 2000 hours yearly in heating mode only. Empirical equations (2) and (3) are commonly used in geothermal industry. Calculated value can be only used in smaller geothermal installations, up to 20 kW [1]. For greater amount of BHE's, working continuously for longer periods of time, specialised programs have to be used. With larger number of BHE's many factors are influencing exchanged thermal power, for instance distance between boreholes. In order to establish amount of BHE's, that will work continuously for period longer than 2000 hours yearly or with heat exchanged seasonally in heating and cooling mode, geoenergetics analysis[1] is essential. This concerns especially industrial applications of heat pumps for heating purposes and domestic applications for heating and cooling using buffers in rock mass, so called Underground Thermal Energy Storage (UTES). Selection of amount, as well as distribution of BHE's should be supported by calculations in specialised programs such as EED3.16 [2] or BoHEx [4].

3. PRESSURE LOSS AND HYDRAULIC POWER CALCULATIONS

During turbulent flow of Newtonian fluids in a long, straight pipe pressure losses can be calculated using Fanning equation for linear hydraulic pressure (head) losses for single-phase flow of drilling fluid. Fanning equation is modified version of Darcy–Weisbach equation, which is used commonly to calculate pressure drop along the pipe per unit length [3]. During pressure loss and hydraulic power calculations, equations (4) to (14) were applied.

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[1] Geoenergetics analysis – result of mathematical modelling that provides information about the amount, depth and distribution of BHE's.

$$p_l = \frac{32 \cdot f \cdot l \cdot \rho \cdot \dot{V}^2}{\pi^2 \cdot d_{in}^5} \tag{4}$$

where:

- p_l – linear pressure drop of circulating heat carrier, Pa,
- l – U-tube length, m,
- d_{in} – U-tube inner diameter, m,
- $\dot{V}$ – volumetric flow rate of heat carrier, $m^3 \cdot s^{-1}$,
- ρ – density of Newtonian fluid, $kg \cdot m^{-3}$,
- f – Fanning's friction coefficient.

Fanning's friction coefficient is calculated [3] using following equation:

$$f = \frac{\lambda}{4} \tag{5}$$

where λ – friction loss coefficient (Darcy–Weisbach friction factor).

Friction loss coefficient equations for laminar (6) and turbulent (7) flow are presented below [3]:

$$\lambda = \frac{64}{\mathrm{Re}} \quad \text{for} \quad \mathrm{Re} \le 2100 \tag{6}$$

Blazius equation:

$$\lambda = \frac{0.316}{\mathrm{Re}^{0.25}} \quad \text{for} \quad \mathrm{Re} > 2100 \tag{7}$$

To determine local pressure losses, caused by the arc at the bottom of U-tube, distance between axes of U-tube has to be calculated. For further analysis case A (Fig. 2) with longest possible distance between U-tube's axes was chosen.

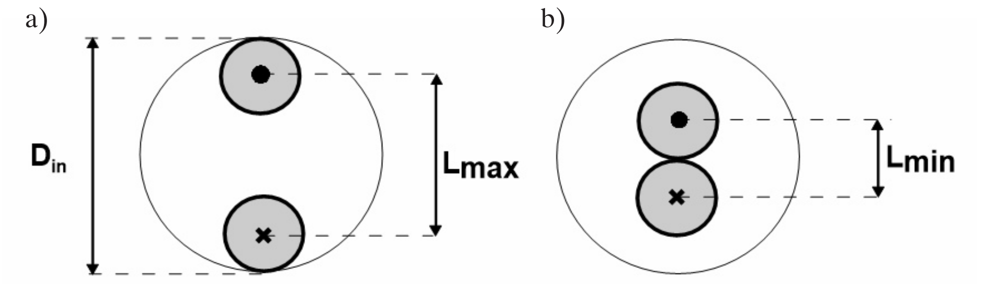

Fig. 2. Extreme position cases of U-tube inside borehole:
a) maximum distance between pipes (L_{max}); b) minimum distance between pipes (L_{min})

Distance between U-tube's axes should be as large as possible. Outer diameter of U-tube and distance between axes should meet following technical requirements [4]:

$$L < \frac{D_{in}}{2} \tag{8}$$

$$\text{range}\begin{cases} L_{\min} = d_{out} \\ L_{\max} = D_{in} - d_{out} \end{cases} \tag{9}$$

where:

D_{in} – borehole diameter, m,
d_{out} – outer diameter of single U-tube pipe, m.

Local pressure losses caused by friction during flow of heat carrier through the arc at the bottom of U-tube (Fig. 3) are computed using dynamic pressure [6]:

$$p_f = \xi \cdot \frac{v^2 \cdot \rho}{2} \tag{10}$$

where:

p_f – local pressure losses, Pa,
v – velocity of heat carrier, $m^2 \cdot s^{-1}$,
ξ – local pressure loss coefficient.

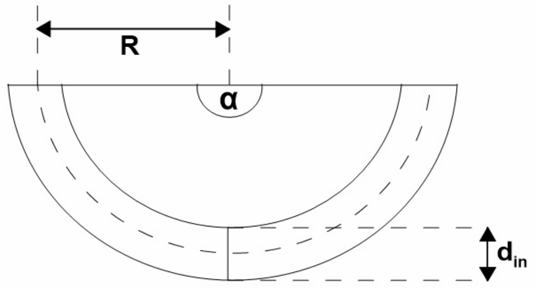

Fig. 3. Technical parameters of arc at the bottom of U-tube ($\alpha = 180^o$)

Two cases of calculating local pressure loss coefficient are described in equations below [6]:

a) for $0.5 < R/d_{in} < 1.5$

$$\xi = 0.7 + 0.35 \cdot \frac{\alpha}{90} \cdot 0.21 \cdot \left(\frac{R}{d_{in}}\right)^{-0.5} \tag{11}$$

b) for $R/d_{in} > 1.5$

$$\xi = 0.7 + 0.35 \cdot \frac{\alpha}{90} \cdot 0.21 \cdot \sqrt{\frac{d_{in}}{R}} \tag{12}$$

where:

d_{in} – inner diameter of U-tube, m,

R – radius of arc curvature, m,

α = 180°.

Adding local pressure losses caused by the arc at the bottom of U-tube and linear pressure losses along U-tube length, total pressure loss of heat carrier circulating in closed-loop system is calculated [6]:

$$p = p_l + p_f \tag{13}$$

Hydraulic power is calculated as follows:

$$N_h = p \cdot \dot{V} \tag{14}$$

4. HYDRAULIC TESTS

In order to determine proper hydraulic characteristic of BHE, it is advised to perform hydraulic tests in average static temperature of rock mass (Fig. 4) and during heating mode (Fig. 5). After thorough data evaluation, correlation between pressure losses and volumetric flow rate of heat carrier can be obtained (Fig. 6), as well as correlation between hydraulic power and volumetric flow rate of heat carrier (Fig. 7). As it is presented in Figures 4 and 5, hydraulic test lasts up to 10 minutes.

Example of hydraulic test in average static temperature of surrounding rock mass was carried out for following parameters:

- average temperature of heat carrier during hydraulic test: 11.92°C,
- average density of heat carrier: 1024 $kg \cdot m^{-3}$,
- average dynamic viscosity of heat carrier: 0.0052 Pa·s,
- Re = 2100 appears in volumetric flow rate: 32.8 $dm^3 \cdot min^{-1}$ (for double U-tube).

Example of hydraulic test during heating mode of BHE was carried out for following parameters:

- average temperature of heat carrier during hydraulic test: 22.26°C,
- average density of heat carrier: 1022.5 $kg \cdot m^{-3}$,
- average dynamic viscosity of heat carrier: 0.0032 Pa·s,
- Re = 2100 appears in volumetric flow rate: 20.2 $dm^3 \cdot min^{-1}$ (for double U-tube).

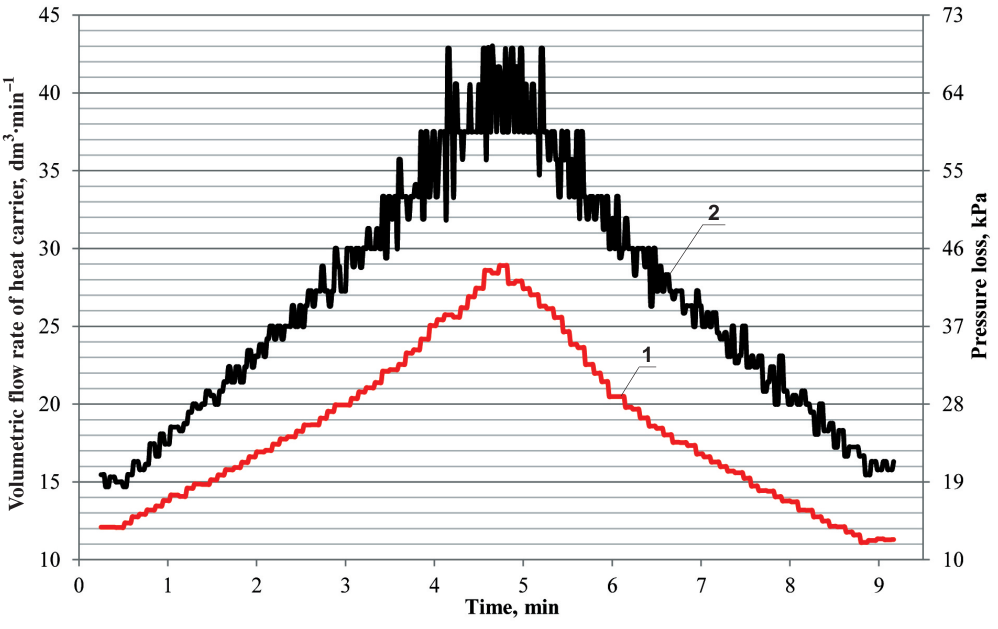

Fig. 4. Hydraulic test in average static temperature (11.92°C) of surrounding rock mass in double U-tube (red line (1) – pressure loss, black line (2) – flow rate)

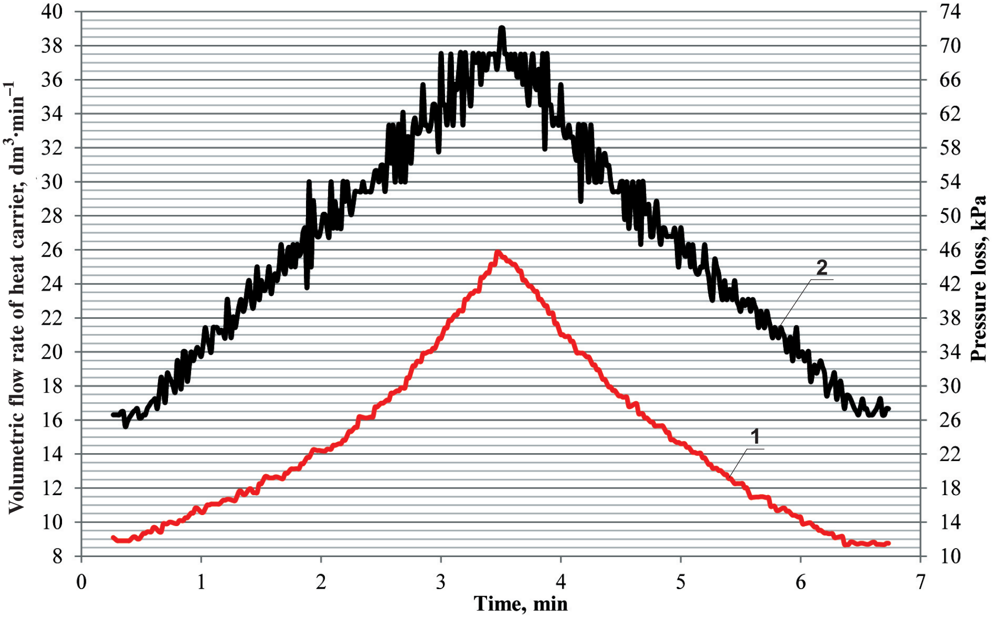

Fig. 5. Hydraulic test during heating mode of double U-tube BHE in temperature of heat carrier of 22.26°C (red line (1) – pressure loss, black line (2) – flow rate)

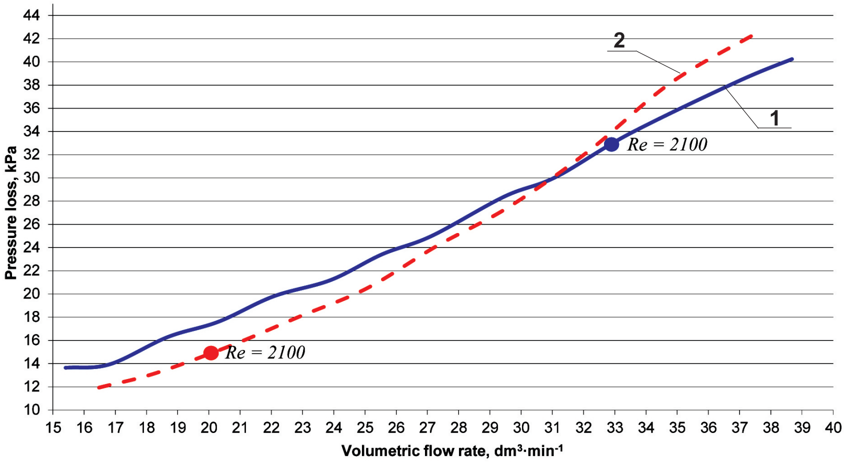

Fig. 6. Correlation between pressure losses and volumetric flow rate of heat carrier (blue line (1) – pressure loss during hydraulic test in average static temperature of rock mass, red line (2) – pressure loss during TRT, • – Reynolds number equals to 2100)

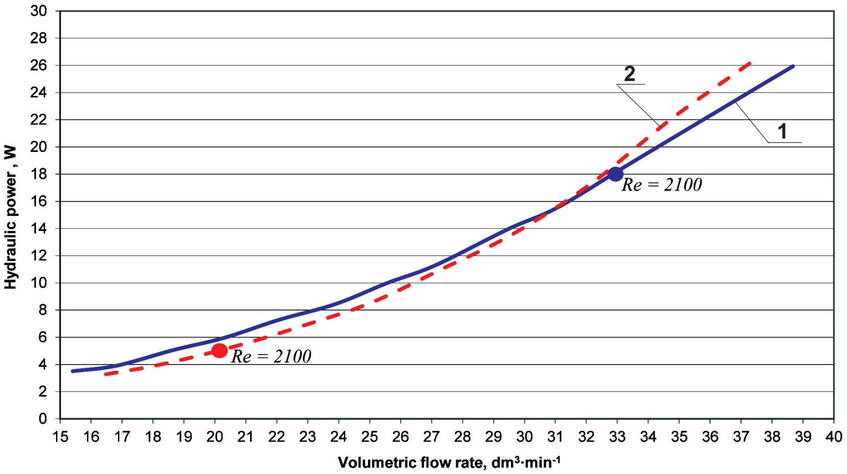

Fig. 7. Correlation between hydraulic power and volumetric flow rate of heat carrier (blue line (1) – hydraulic power during hydraulic test in average static temperature of rock mass, red line (2) – hydraulic power during TRT, • – Reynolds number equals to 2100)

5. ANALYSIS

Table 2 shows characteristics of eight analysed BHE's, i.e. date of drilling, diameter of the borehole, U-tube wall thickness, diameter, material and type of heat carrier circulating in the system together with its volumetric flow rate during TRT. Table 2 is complemented with effective thermal conductivity evaluated from TRT and calculated using available literature sources [2].

Table 2

Technical characteristic of analysed BHE's during TRT with theoretical thermal conductivity values [8, 9]

BHE	Year	Location	BHE depth, m	Borehole diameter, m	BHE design	U-tube OD, mm	Wall thickness, mm	Pipe material	Heat carrier	Volumetric flow rate during TRT, $dm^3 \cdot min^{-1}$	$\lambda_{eff\,TRT}$, $W \cdot m^{-1} \cdot K^{-1}$	$\lambda_{theoretical}$, $W \cdot m^{-1} \cdot K^{-1}$ [2]
1	2009	Polkowice	153	0.149	1×U Mouvitech's turbocolektor	40	3.7	PE	Methanol	24	2.08	1.84
2	2009	Rawa Mazowiecka	140	0.323 mm (to 29.5 m depth), 0.169 mm (in 29.5–140 m interval)	1×U	40	3.4	PE	Water	20	2.33	2.20
3	2009	Rawa Mazowiecka	48	0.323	1×U	40	3.4	PE	Water	20	3.36	2.03
4	2010	Kraków	150	0.168 mm (to 20.0 m depth), 0.140 mm (to 150 m depth)	1×U	40	3.0	PE	Propylene glycol (28.5% conc.)	20	2.33	2.59
5	2013	Lublin	100	0.143	1×U	40	2.0	PE 100	Propylene glycol (30% conc.)	20	3.48	1.87
6	2014	Szczecin	100	0.143	1×U	40	2.0	PE 100	Water	20	2.15	2.17
7	2014	Niepołomnice	140	0.125	1×U	40	3.7	PEHD	Propylene glycol (30% conc.)	20	2.11	2.10
8	2016	Żukowo	100	0.143	1×U	40	3.7	PE 100	Ethylene glycol (30% conc.)	20	2.46	2.10

In Table 4, analysis of pressure losses, as well as hydraulic power for eight different BHE's was presented. Specific construction type of U-tube used in BHE No. 1 (turbocollector by Mouvitech) is presented in Figure 8. As an example, results of all theoretical calculations and measured values of BHE No. 4, where propylene glycol with 28.5% concentration was used as a heat carrier are presented in Table 3, together with graphs in Figures 9 and 10. Significant decrease of pressure losses with temperature increase during TRT is noticed. Analysis of power, exchanged with surrounding rock mass and energy during TRT is presented in Table 5.

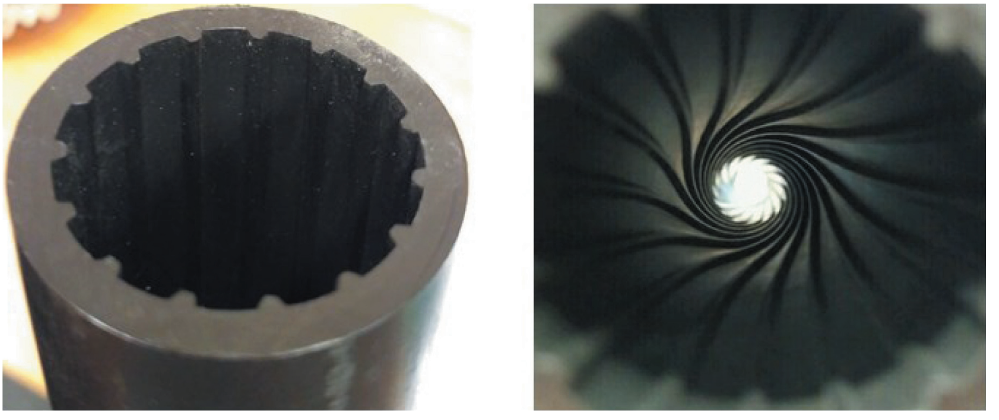

Fig. 8. Specific construction design of turbocollector by Mouvitech used in BHE No. 1 with methanol as a heat carrier circulating in closed-loop system

Table 3

Calculated and measured values for BHE No. 4 during TRT (U-tube inner diameter, d_{in} = 34 mm, BHE's depth, H = 150 m)

	Minimum value	Average value	Maximum value
Velocity, v, $m \cdot s^{-1}$	0.354	0.367	0.368
Dynamic viscosity of heat carrier, η, mPa·s	2.31	2.62	4.78
Density of heat carrier, ρ, $kg \cdot m^{-3}$	1 019	1 020	1 025
Reynolds number, Re	2584	4949	5508
Friction loss coefficient, λ (7)	0.0367	0.0378	0.0443
Fanning friction coefficient, f (5)	0.0092	0.0094	0.0111
Total theoretical pressure loss, p, kPa (13)	22.29	22.94	25.39
Local pressure loss, p_m, Pa (10)	55.00	58.24	59.00
Theoretical hydraulic power, N_h, W (14)	7.43	7.64	8.47
Total measured pressure loss, p_{TRT}, kPa	36.83	38.49	41.44
Measured hydraulic power, N_{TRT}, W (14)	12.29	12.82	13.82

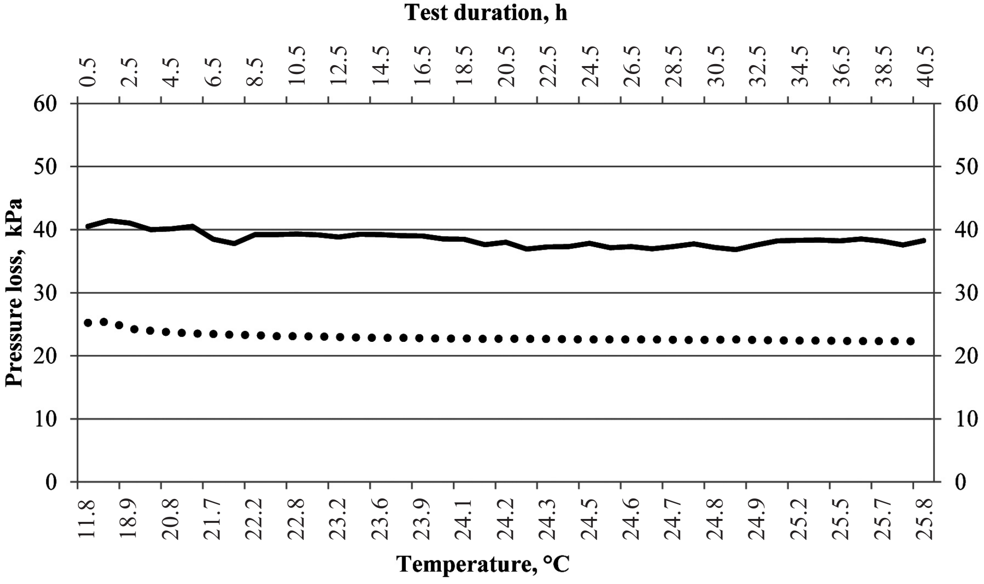

Fig. 9. Correlation between total pressure loss, test duration and temperature during TRT for BHE No. 4 (dotted line – theoretical values, continuous line – measured values)

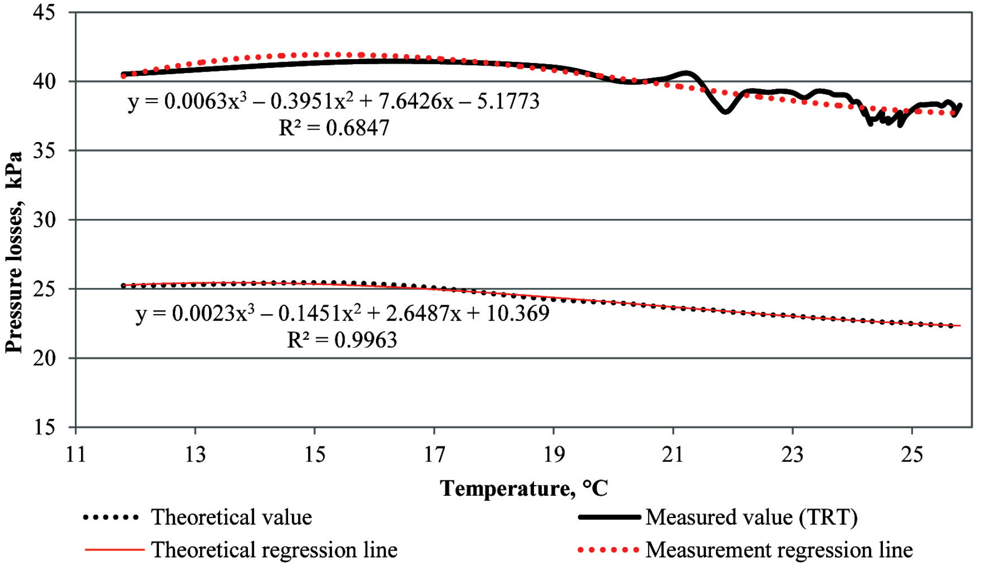

Fig. 10. Correlation between total pressure losses (theoretical and measured) and temperature during TRT with regression lines for BHE No. 4

Table 4

Summary of measured and calculated pressure loss and hydraulic power values for analysed BHE's

BHE	Heat carrier	Total U-tube length, m	U-tube ID, d_w, mm	Pressure loss per meter of U-tube (measured), Pa·m^{-1}	Average measured pressure loss, p_{TRT}, kPa	Average measured hydraulic power, N_{TRT}, W	Pressure loss per meter of U-tube (theoretical), Pa·m^{-1}	Average theoretical pressure loss, p, kPa	Average theoretical hydraulic power, N_h, W	Pressure loss difference between measured and theoretical value, kPa	Hydraulic power difference between measured and theoretical value, W	Approximation error using Fanning method, %
1	Methanol	306	32.6	166.54	50.96	20.38	70.92	21.70	8.68	29.26	11.70	135
2	Water	280	33.2	97.14	27.20	9.06	66.07	18.50	6.17	8.70	2.89	47
3	Water	96	33.2	145.31	13.95	4.64	66.98	6.43	2.14	7.52	2.50	117
4	Propylene glycol	300	34.0	128.30	38.49	12.82	76.47	22.94	7.64	15.55	5.18	68
5	Propylene glycol	200	36.0	106.45	21.29	7.09	58.00	11.60	3.86	9.69	3.23	84
6	Water	200	36.0	86.65	17.33	5.78	44.70	8.94	2.98	8.39	2.80	94
7	Propylene glycol	280	32.6	122.93	34.42	11.47	100.46	28.13	9.37	6.29	2.10	22
8	Ethylene glycol	200	32.6	154.15	30.83	11.82	89.93	17.99	6.00	12.84	5.82	71

Table 5

Summary of calculations of thermal energy during TRT and power exchanged with underground rock mass in analysed BHE's

BHE	$\lambda_{eff\,\mathrm{TRT}}$, $W \cdot m^{-1} \cdot K^{-1}$	Average potential unit power exchanged with rock mass, P_j, $W \cdot m^{-1}$	Average potential total power exchanged with rock mass, P_{TOTAL}, kW	$N_{\mathrm{TRT}}/P_{\mathrm{TOTAL}}$, %	Duration of TRT, h	Thermal energy during TRT, kWh
1	2.08	39.32	6.02	0.34	100.50	605
2	2.33	43.45	6.08	0.15	118.50	720
3	3.36	60.44	2.90	0.16	139.50	405
4	2.33	43.45	6.52	0.20	40.50	264
5	3.48	62.42	6.24	0.11	108.50	677
6	2.15	40.48	4.05	0.14	87.50	354
7	2.11	39.82	5.57	0.21	96.50	538
8	2.46	45.59	4.56	0.26	102.00	465

Figure 11 represents pressure loss per one meter of heat exchanger's pipe length. It can be clearly noticed that regardless of heat carrier type and BHE's construction (Tab. 2), pressure losses measured during TRT are much higher than these using theoretical calculations.

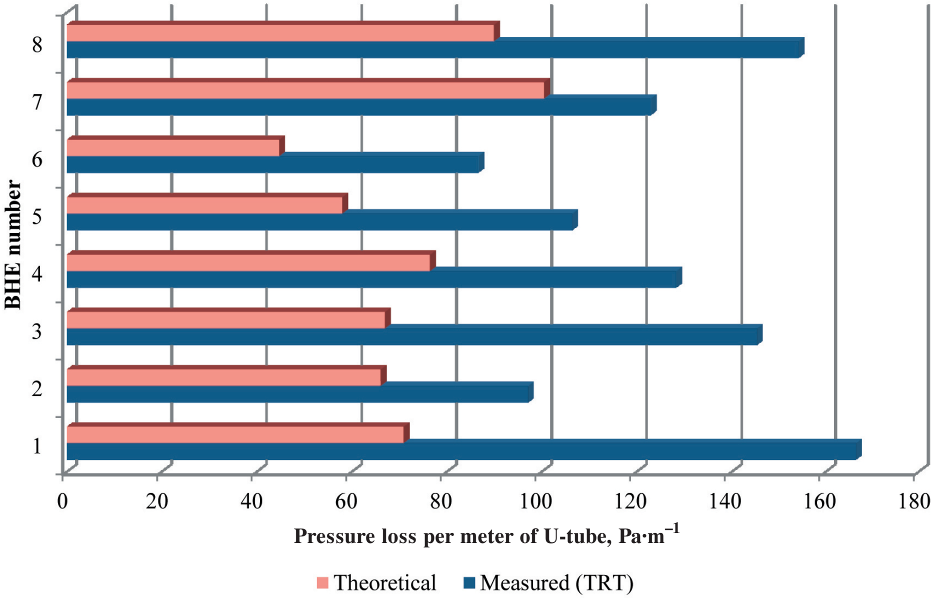

Fig. 11. Pressure loss per one meter of pipe for BHE's with following heat carriers: 1 – methanol, 2, 3, 6 – water, 4, 5, 7 – propylene glycol, 8 – ethylene glycol

6. DISCUSSION

Attempt to analyse eight BHE's with six different U-tube lengths, four different inner diameters of U-tube, four different types of heat carriers, two different volumetric flow rates and six different borehole diameters was carried out in following paper.

The highest total losses (local and linear) are observed for BHE No. 1 (methanol as heat carrier), Nos 4 and 7 (propylene glycol) and also for No. 8 (ethylene glycol). As for pressure loss per one meter of U-tube BHE No. 1 has also the highest values and is followed by BHE No. 8 and BHE No. 3 (water). BHE No. 1 has much higher pressure losses, caused by specific design of U-tube (Fig. 8) and as a result of that, heat transfer coefficient α from heat carrier (methanol) into pipe's material is increased. It means that more thermal energy will be retrieved from BHE system, but simultaneously higher pressure losses will be observed. Turbocollector presented in Figure 8 produces higher turbulence of heat carrier flow and thus higher Reynolds numbers were observed. Specific design of U-tube in BHE No. 1 was not taken into consideration during theoretical calculations thus such high approximation error was achieved.

High discrepancies between measured and calculated values can result from:

- accuracy of volumetric flow rates of heat carrier measurements;
- discrepancies between temperatures of heat carrier during measurements and temperature applied for theoretical calculations (one average temperature value was applied for calculations, whereas in reality temperatures along BHE are changing significantly);
- possible local decrease in pipe diameter (external pressure can exceed the nominal pressure values for BHE's pipes, due to application of sealants with density of 1500 kg·m^{-3}; as a result, in 100 m depth BHE's, bottom hole pressure can exceed 15 bars);
- possible decrease in diameter of pipes caused by pipe connection (in older types of BHE's, pipes were connected onsite using thermal polyfusion methods; using such butt welds, there is a high possibility of creating so called "collars" on the internal and external of BHE's pipe);
- pressure losses in TRT measurement equipment.

Values of thermal conductivity coefficient λ_{eff} obtained during TRT and values calculated from available literature can be varied (Tab. 2). These differences are caused by geological and hydrogeological parameters of rock mass (e.g. presence of underground waters flow). For energy and thermal power calculations, thermal conductivity efficient obtained from TRT was taken into consideration. Highest total potential thermal power exchanged with rock mass is achieved for BHE's with higher effective thermal conductivity and greater borehole depth.

More research should be done in the future with main focus on TRT data analysis with fewer variables (e.g. diameters of U-tube or heat carriers). High amount of variables, as presented in following paper, can significantly limit making a validate conclusions.

7. SUMMARY

1. Many different factors are affecting pressure losses during circulation of heat carrier inside closed-loop BHE system, while performing TRT. Some of these factors are: U-tube diameter and length, density and viscosity of heat carrier and volumetric flow rate. Comparative analysis of BHE's with different construction designs, heat carriers and volumetric flow rates is a difficult task. Results presented in this paper conclude that a lowest discrepancy between measured and theoretical values (lowest approximation error) was achieved in BHE No. 7 (22%), 2 (47%) and 4 (68%), whereas in BHE No. 1 and 3 approximation error was higher than 100%.

2. Special attention should be paid to BHE no 1 with new generation U-tube design, methanol as a heat carrier and higher volumetric flow rate (24 $dm^3 \cdot min^{-1}$). Such construction design creates more turbulent flow and as a result produces higher total pressure losses (the highest amongst other BHE's). Specific construction was not included in theoretical calculations, thus high discrepancies between measured and calculated values were observed.
3. In BHE Nos 4, 5 and 7, propylene glycol was used as a heat carrier during TRT. High difference between theoretical and measured values was noted for BHE No. 4 and 5. Total pressure loss and pressure loss per meter of pipe are higher from BHE with water-based heat carrier (BHE Nos 2, 3 and 6) and are lower than with BHE where methanol and ethylene glycol were used during TRT (BHE Nos 1 and 8).
4. Before any investment consisting of greater amount of BHE's, it is advised to perform onsite Thermal Response Test. Recorded data not only will enable to determine effective thermal conductivity and average temperature of surrounding rock mass and thermal resistance values, but also will allow to adjust the project to its real requirements without oversizing intake of low-enthalpy geothermal energy.
5. From correlation between pressure losses and volumetric flow rate in double U-tube BHE, it can be clearly seen that initially, pressure losses in average temperature of rock mass were higher that pressure losses during TRT. This phenomenon continues to volumetric flow rate of 30 $dm^3 \cdot min^{-1}$. After achieving this value, pressure losses during TRT are considerably higher than these during hydraulic test in average static temperature. Very similar situation is noticed in correlation between hydraulic power and volumetric flow rate, where after achieving 30 dm^3/min of flow, hydraulic power starts to increase (in TRT).
6. Analysis of thermal power exchanged with rock mass concludes that the highest potential unit power is seen in BHE Nos 3 and 5, and the lowest (below 40 $W \cdot m^{-1}$) in BHE Nos 1 and 7. Total potential thermal power exchanged with rock mass higher than 6 kW is achieved in BHE Nos 1, 2, 4 and 5. Following thermal power is closely connected to depth of BHE and geological conditions. Hydraulic power represents approximately 0.3% of total power exchanged with rock mass.

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