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GEODETIC PRECESSION OF THE SUN, SOLAR SYSTEM PLANETS, AND THEIR SATELLITES

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ABSTRACT. The effect of the geodetic precession is the most significant relativistic effect in the rotation of celestial bodies. In this article, the new geodetic precession values for the Sun, the Moon, and the Solar System planets have been improved over the previous version by using more accurate rotational element values. For the first time, the relativistic effect of the geodetic precession for some planetary satellites (J1–J4, S1–S6, S8–S18, U1–U15, N1, and N3–N8) with known quantities of the rotational elements was studied in this research. The calculations of the values of this relativistic effect were carried out by the method for studying any bodies of the Solar System with long-time ephemeris. As a result, the values of the geodetic precession were first determined for the Sun, planets in their rotational elements, and for the planetary satellites in the Euler angles relative to their proper coordinate systems and in their rotational elements. In this study, with respect to the previous version, additional and corrected values of the relativistic influence of Martian satellites (M1 and M2) on Mars were calculated. The largest values of the geodetic rotation of bodies in the Solar System were found in Jovian satellite system. Further, in decreasing order, these values were found in the satellite systems of Saturn, Neptune, Uranus, and Mars, for Mercury, for Venus, for the Moon, for the Earth, for Mars, for Jupiter, for Saturn, for Uranus, for Neptune, and for the Sun. First of all, these are the inner satellites of Jupiter: Metis (J16), Adrastea (J15), Amalthea (J5), and Thebe (J14) and the satellites of Saturn: Pan (S18), Atlas (S15), Prometheus (S16), Pandora (S17), Epimetheus (S11), Janus (S10), and Mimas (S1), whose values of geodetic precession are comparable to the values of their precession. The obtained numerical values for the geodetic precession for the Sun, all the Solar System planets, and their satellites (E1, M1, M2, J1–J5, J14–J16, S1–S6, S8–S18, U1–U15, N1, and N3–N8) can be used to numerically study their rotation in the relativistic approximation and can also be used to estimate the influence of relativistic effects on the orbital–rotational dynamics of bodies of exoplanetary systems.

Keywords: relativistic effects, geodetic precession, Solar System bodies, planetary satellites, exoplanetary systems bodies

1. INTRODUCTION

The relativistic effect of the geodetic precession, first considered by Willem de Sitter in 1916 (De Sitter, 1916), is a secular change in the direction of the axis of rotation of a celestial body as a result of a parallel transfer of the angular momentum vector of the body along its orbit in curved space–time.

In our previous investigation (Pashkevich and Vershkov, 2020), the main effects of the relativistic rotation for the inner satellites of Jupiter (J14, J5, J15, and J16) were studied. As results, foregoing study showed that the values of the geodetic precession can be significant not only for objects orbiting around super-massive central relativistic bodies, but also for bodies with a short distance to the central body, for example, close satellites of giant planets.

Contemporary development of cosmonautics, especially in the implementation of projects such as "Gravity Probe B" (Everitt et al. 2011) for giant planets, will make it possible, after a few years, to obtain from observations the magnitude of the geodetic precession of their inner satellites. For example, it is necessary to launch a similar artificial satellite into the orbit of one of these satellites. The range of the obtained (Pashkevich and Vershkov, 2020) theoretical values of the geodetic precession of the inner satellites of Jupiter varies from –13".37255 per year to –52".95725 per year. Thus, theoretically, within 1 year after the implementation of such a project, it will allow testing the general theory of relativity. Then, the artificial satellite can be transferred to the orbit of the next inner satellite of Jupiter and the experiment is repeated.

Thus, a more detailed study of the relativistic effects in the rotation of the planets and their satellites in the Solar System becomes relevant and interesting.

Modeling the orbital–rotational dynamics of bodies of exoplanetary systems is best done using a planetary system with well-known parameters of the motion of its bodies. New research methods have made it possible to obtain high-precision long-term ephemerides of the orbital–rotational motion of many bodies in the Solar System, thus the Solar System is a good model for studying the rotational dynamics of exoplanetary systems. Based on studies of the Solar System bodies with well-known parameters of motion, it is possible to reveal patterns in the distribution and influence of relativistic effects on the orbital–rotational dynamics of exoplanetary systems' bodies.

In this article, the geodetic precession values for Mars satellites in Euler angles are taken from our previous study (Pashkevich and Vershkov, 2019), and the geodetic precession values for the inner satellites of Jupiter are taken from our previous study (Pashkevich and Vershkov, 2020).

The main aims of this research are:

- to improve the geodetic precession values for the Sun, and the Solar System planets in the Euler angles relative to their proper coordinate systems and in the absolute value of the geodetic rotation angular velocity vector;
- to improve the geodetic precession values for the Moon in the perturbing terms of the physical libration relative to her proper coordinate systems and in the absolute value of the geodetic rotation angular velocity vector;
- to calculate, for the first time, the values of the geodetic precession for the Sun, all the Solar System planets, the Moon, and Mars satellites in their rotational elements;
- to obtain new additional and corrected values of the relativistic influence of Martian satellites (M1 and M2) on Mars;
- to calculate, for the first time, the values of the geodetic precession for the other planetary satellites with known quantities of the rotational elements (Galilean moons of Jupiter: J1–J4, satellites of Saturn: S1–S6, S8–S18, satellites of Uranus: U1–U15, and satellites of Neptune: N1, N3–N8) in the Euler angles relative to their proper coordinate systems and in their rotational elements.

The structure of this article is as follows:

- **Abstract**;
- **Section 1 Introduction;**
- **Section 2** describes the mathematical model of the problem and the applied method Pashkevich (2016) (**subsections A, B, C**) for studying relativistic effect of the geodetic rotation of any bodies of the Solar System with long-time ephemeris;
- **Section 3** is devoted to the study of the geodetic precession of the Sun and the planets of the Solar System**.** A detailed description of the novelty and improvement of the results of this article in comparison with the previous ones is given;
- **Section 4** is devoted to the study of the geodetic precession of the Solar System planetary satellites with known quantities of their rotational elements. A detailed description of the novelty and improvement of the results of this article in comparison with the previous ones is given;
- **Subsection 4.1** Geodetic precession of the Earth satellite (the Moon);
- **Subsection 4.2** Geodetic precession of Mars satellites;
- **Subsection 4.3** Geodetic precession of Jupiter's satellites;
- **Subsection 4.4** Geodetic precession of Saturn's satellites;
- **Subsection 4.5** Geodetic precession of Uranus satellites;
- **Subsection 4.6** Geodetic precession of Neptune's satellites;
- **Conclusions;**
- **Appendix** contains tables;
- **Acknowledgments; and**
- **References.**

2. MATHEMATICAL MODEL

In this investigation, we studied the most significant relativistic effect in rotational motion for the Sun, all planets and their satellites in the Solar System with known rotation parameters (Archinal et al. 2011, 2018). This effect is geodetic precession, which is the systematic or secular effect of the studied body's geodetic rotation.

The method for studying the geodetic rotation of any Solar System bodies using long-time ephemeris will be applied (Pashkevich, 2016):

A. The problem of the geodetic (relativistic) rotation of the Solar System bodies is studied with respect to the proper coordinate systems of the bodies.
B. The values of the velocities of the geodetic rotation of the Solar System bodies are determined by using the ephemeris.
C. The most essential terms of the geodetic precession are found by means of the least-squares method.

A. The problem of geodetic (relativistic) rotation of the investigated bodies was studied with respect to their proper coordinate system (Archinal et al. 2011, 2018)[1]. Calculations were made of the geodetic precession velocities of each body under study using data on the positions, velocities, and orbital elements of the bodies of the Solar System from the ephemeris at all time intervals of their existence (Appendix: Table 1).

B. For the Sun, the Moon, and planets of the Solar System, the fundamental ephemeris JPL DE431/LE431 (Folkner et al. 2014) was used. For other satellites of planets with known rotation parameters (Archinal et al. 2011, 2018), data samples were formed from the ephemeris of the satellites of Mars, Jupiter, Saturn, Uranus, and Neptune Horizons On-Line Ephemeris System (Giorgini et al. 1996). The rotation parameters for the Earth and the Moon were taken from the article by Archinal et al. (2011), and for other studied bodies, from the article by Archinal et al. (2018).

The expressions for the velocities of the geodetic rotation of bodies in the Solar System for the parameters of their orientation (α_0, δ_0, W) (see Figure 1) relative to the standard Earth equator of epoch J2000 (International Celestial Reference Frame - ICRF) (Ma et al., 1998) and the vernal equinox (for the epoch J2000.0) have been obtained (Pashkevich and Vershkov, 2020):

$$\left.\begin{aligned} \Delta\dot{\alpha}_0 &= \frac{\sigma_1 \sin \mathrm{W} + \sigma_2 \cos \mathrm{W}}{\cos \delta_0} \\ \Delta\dot{\delta}_0 &= -\sigma_1 \cos \mathrm{W} + \sigma_2 \sin \mathrm{W} \\ \Delta\dot{\mathrm{W}} &= \sigma_3 - \Delta\dot{\alpha}_0 \sin \delta_0 \end{aligned}\right\}. \quad (1)$$

Here α_0 is the right ascension of the north pole of rotation of the body; δ_0 is the declination of the north pole of rotation of the body; angle W = QB specifies the location of the prime meridian of the body[2], which is measured along the equator of the body in an easterly direction with respect to the north pole of body from the node Q (located at the right ascension $90° + \alpha_0$) of the equator of the body on the standard Earth equator of epoch J2000 to the point B, where the prime meridian crosses the equator of the body (see Figure 1) (recommended values of the constants in the expressions for α_0, δ_0 and W are given by Archinal et al. (2011, 2018)); $\Delta\dot{\alpha}_0 = \dot{\alpha}_{0r} - \dot{\alpha}_0, \Delta\dot{\delta}_0 = \dot{\delta}_{0r} - \dot{\delta}_0$, $\Delta\dot{\mathrm{W}} = \dot{\mathrm{W}}_r - \dot{\mathrm{W}}$ are the differences between the relativistic and Newtonian angles of rotation of the investigated body, respectively; the dot denotes differentiation with respect to time; $\sigma_1, \sigma_2, \sigma_3$ are reduced (Pashkevich, 2016) components of the angular velocity vector of the geodetic rotation of the body under study

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[1] Thus, in this study, the Euler angles (see Figure 1) refer to the equator of rotation of the body under investigation, as defined in Archinal et al. (2011, 2018), and may not coincide with the equator of the body figure as in Classical Mechanics (e.g., Suslov 1946), except when the equator of the body figure coincides with the equator of the body rotation.

[2] Note from Archinal et al. (2018): "The angle W specifies the ephemeris position of the prime meridian and W_0 is the value of W at J2000.0 (or occasionally, such as for comets, some other specified epoch). For planets or satellites with no accurately observable fixed surface features, the expression for W defines the prime meridian and is not subject to correction for this reason. The rotation rate (authors' note: W_1) may be redefined by some other physical property (e.g., observation of the rotation of the body's magnetic field)." Here $W = W_0 + W_1 d$, d is the time in days from standard epoch, which is JD 2451545.0, that is, 2000 January 1, 12 hours TDB (Barycentric Dynamical Time).

$$\bar{\sigma} = \frac{1}{c^2}\sum_j \frac{Gm_j}{\left|\bar{R}-\bar{R}_j\right|^3}\left(\bar{R}-\bar{R}_j\right)\times\left(\frac{3}{2}\dot{\bar{R}}-2\dot{\bar{R}}_j\right), \tag{2}$$

from the geocentric reference frame of epoch J2000 (the reference frame of DE431/LE431 ephemeris (Folkner et al. 2014)) to the body-centric reference frames (Figure 1), given by Archinal et al. (2011, 2018); *c* is the velocity of light; *G* is the gravitational constant; m_j is the mass of a perturbing body *j*; $\bar{R}$ and $\dot{\bar{R}}$ are the vectors of the barycentric position and velocity of the investigated body, respectively; $\bar{R}_j$ and $\dot{\bar{R}}_j$ are the vectors of the barycentric position and velocity of the perturbing bodies *j*, respectively. The symbol × means a vector product; the subscript *j* correspond to the perturbing bodies (the Moon, the planets, dwarf planet Pluto and the Sun, excluding the body under study from this set).

As can be seen from equation (2), the geodetic rotation of a body under study depends only on the masses of the disturbing bodies and on the distance to them and does not depend on the mass of the body itself. Therefore, the magnitude of the vector of the geodetic rotation of the body under study $\left|\bar{\sigma}\right| \sim \frac{M}{r^{2.5}}$ essentially depends on the proximity of the satellite to the central body $r = \left|\bar{R}-\bar{R}_{j=M}\right|$, the mass of which is dominant $M = m_{j=M}$ and around which the body rotates. It is the main property of the formula for the angular velocity vector of the geodetic rotation of a body under study.

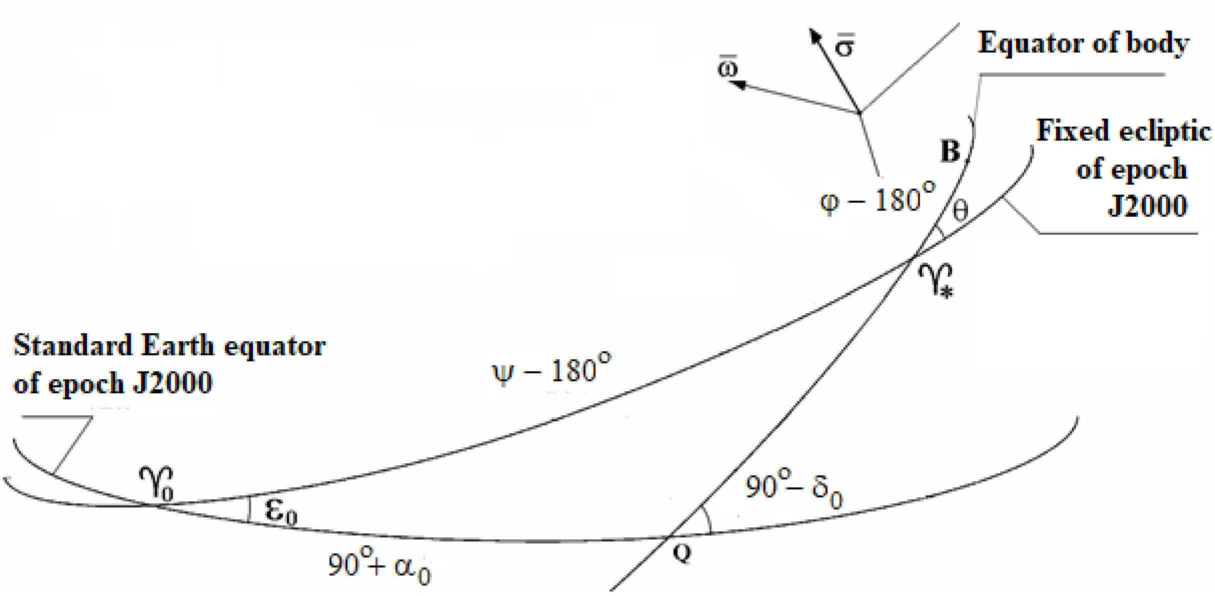

Figure 1. Triangle used to define the direction of the angular velocity vector of the geodetic rotation for any body of the Solar System

In order to eliminate the singularity $\cos^{-1}\delta_0$ in the expressions (1) were obtained expressions in the perturbing terms of the physical libration for the geodetic values of the velocity of rotation of the body by a combination of the elements of rotation:

$$\left.\begin{aligned} &\Delta\dot{\alpha}_0 + \Delta\dot{\mathrm{W}} = \sigma_3 + \left(\sigma_1 \sin\mathrm{W} + \sigma_2 \cos\mathrm{W}\right)\frac{\cos\frac{\delta_0}{2} - \sin\frac{\delta_0}{2}}{\cos\frac{\delta_0}{2} + \sin\frac{\delta_0}{2}} \\ &\Delta\dot{\delta}_0 = -\sigma_1 \cos\mathrm{W} + \sigma_2 \sin\mathrm{W} \\ &\cos\delta_0 \Delta\dot{\alpha}_0 = \sigma_1 \sin\mathrm{W} + \sigma_2 \cos\mathrm{W} \end{aligned}\right\}. \quad (3)$$

In this investigation, the expressions (3) are used to study the geodetic rotation of the Earth, for which $\cos\delta_0 = 0$ at epoch J2000. These expressions are an analog to the expressions of the velocities of the geodetic rotation for the perturbing terms of the physical libration of the Moon (4), which are a combination of the Euler angles.

The expressions of the geodetic rotation velocities are defined in the perturbing terms of the physical librations (τ, ρ, σ[3]) for the Moon (4) and in Euler angles (ψ, θ, φ) (see Figure 1) for others Solar System bodies (5) as follows (Pashkevich, 2016):

$$\left.\begin{aligned} \Delta\dot{\psi} + \Delta\dot{\varphi} &= \sigma_3 - (\sigma_1 \sin\varphi + \sigma_2 \cos\varphi)\tan\frac{\theta}{2} = \Delta\dot{\tau} \\ \Delta\dot{\theta} &= -\sigma_1 \cos\varphi + \sigma_2 \sin\varphi = \Delta\dot{\rho} \\ \sin\theta\, \Delta\dot{\psi} &= -\sigma_1 \sin\varphi - \sigma_2 \cos\varphi = \Delta(I\dot{\sigma}) \end{aligned}\right\}, \quad (4)$$

$$\left.\begin{aligned} \Delta\dot{\psi} &= -\frac{\sigma_1 \sin\varphi + \sigma_2 \cos\varphi}{\sin\theta} \\ \Delta\dot{\theta} &= -\sigma_1 \cos\varphi + \sigma_2 \sin\varphi \\ \Delta\dot{\varphi} &= \sigma_3 - \Delta\dot{\psi}\cos\theta \end{aligned}\right\}. \quad (5)$$

Here, τ, ρ, and σ are the perturbing terms of the physical librations of the Moon in the longitude, inclination, and node longitude, respectively; I is a constant angle of the inclination of the lunar equator to the fixed ecliptic J2000 ($I \sim 1^\circ\, 32'$); ψ is the longitude of the descending node of epoch J2000 of the body equator; θ is the inclination of the body equator to the fixed ecliptic J2000; φ is the proper rotation angle of the body between the descending node of epoch J2000 and point B, where the prime meridian crosses the equator of body (see Figure 1) (or for the case when the equator of the body figure coincides with the equator of the body rotation: between the descending node of epoch J2000 and the principal axis of the minimum moment of inertia); $\Delta\dot{\psi} = \dot{\psi}_r - \dot{\psi}$, $\Delta\dot{\theta} = \dot{\theta}_r - \dot{\theta}$, and $\Delta\dot{\varphi} = \dot{\varphi}_r - \dot{\varphi}$ are the differences between the relativistic and Newtonian angles of rotation of the investigated body, respectively; and the dot means time differentiation.

C. The geodetic precession velocities for each of the investigated Solar System bodies are determined over various time spans with different time spacing (Appendix: Table 1) by means of the least-squares method. The expressions for the secular terms of the body's geodetic rotation velocities can be represented as a polynomial in the degree of time:

[3] Here, σ is the perturbing term of the physical librations of the Moon in the node longitude, but $\sigma_1, \sigma_2, \sigma_3$ are reduced (Pashkevich, 2016) components of the angular velocity vector of the geodetic rotation of the body under study.

$$\Delta\dot{x} = \sum_{n=1}^{N} \Delta\dot{x}_n t^{n-1}, \qquad (6)$$

where $\Delta\dot{x}_n$ are the coefficients of the secular terms; $\dot{x} = \dot{\psi},\ \dot{\theta},\ \dot{\varphi},\ \dot{\alpha}_0,\ \dot{\delta}_0,\ \dot{\mathrm{W}}$; t is the time from standard epoch, which is JD 2451545.0, that is, 2000 January 1, 12 hours TDB (Archinal et al. 2011, 2018) (in Appendix: Table 1, the time spans and steps for the studies of the geodetic precession of the bodies are presented); and N is the degree of the approximating polynomial. As a result of calculations by the least-squares method, the value of the degree of the approximating polynomial is obtained, which provides the best approximation of the geodetic rotation $N = 2$.

After analytical integration (6), the expressions for the secular terms of the body's geodetic rotation are obtained:

$$\Delta x = \sum_{n=1}^{N} \Delta x_n t^n, \qquad (7)$$

where $x = \psi,\ \theta,\ \varphi,\ \alpha_0,\ \delta_0,\ \mathrm{W}$; and $\Delta x_n = \dfrac{\Delta\dot{x}_n}{n}$ are the coefficients of the secular terms.

The absolute value of the angular velocity vector of the geodetic rotation of the body under study is presented by the following expression:

$$|\bar{\sigma}| = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}. \qquad (8)$$

Here, $\sigma_X, \sigma_Y, \sigma_Z$ are the components of the geocentric vector of the angular velocity of the geodetic rotation of a body (Pashkevich, 2016); as defined above, $\sigma_1, \sigma_2, \sigma_3$ are reduced (Pashkevich, 2016) components of the body-centric vector of the angular velocity of the geodetic rotation of a body. Equality (8) is true, because the magnitude of the vector does not depend on the coordinate system in which its projections are considered.

The absolute value of the geodetic rotation velocity vector for the Solar System bodies in the parameters of their orientation is presented by the following expression:

$$\Delta\dot{\Omega} = \sqrt{\Delta\dot{\alpha}_0^2 + \Delta\dot{\delta}_0^2 + \Delta\dot{W}^2}. \qquad (9)$$

3. THE SUN AND ITS PLANETS

In our previous investigation (Eroshkin, and Pashkevich, 2007), the absolute geodetic precession magnitudes of the angular velocity vector $|\bar{\sigma}|$ (8) for the Sun, the Moon and the Solar System planets were calculated by using DE404/LE404 ephemeris (Standish and Newhall, 1996). In this research, the geodetic precession magnitudes $|\bar{\sigma}|$ for these bodies have been improved (Appendix: Table 2) by using more accurate DE431/LE431 ephemeris (Folkner et al. 2014). These values of the angular velocity vector of the geodetic rotation (2) for these bodies are calculated directly (8) by using the components of the geocentric vector of the angular velocity of the geodetic rotation of a body $\sigma_X, \sigma_Y, \sigma_Z$ without their reduction from the geocentric reference frame to the body-centric reference frames, as given by Archinal et al. (2011, 2018).

In the research of Klioner et al. (2009), the magnitude of the geodetic precession was obtained only for some Solar System bodies (Mercury, Venus, the Earth, the Moon and Mars). Comparison of the magnitude of the geodetic precession of our studies with those of Klioner et al. (2009) showed that the results obtained for the same bodies in our research have the same order of magnitude (Appendix: Table 2).

In our previous study (Pashkevich and Vershkov, 2019), the geodetic precession values in Euler angles for the Sun, the Moon, and the Solar System planets were calculated by using values of the rotation elements (Seidelmann et al. 2005). In this investigation, the geodetic precession values in Euler angles for these bodies have been improved (Appendix: Table 2a) by using updated values of the rotation elements (Archinal et al. 2011, 2018).

Geodetic precession of the Sun and the planets of the Solar System in Euler angles (Figure 2, left side) ranges from –870.28 μas per thousand years (for the Sun) to –425".61 per thousand years (for Mercury) (Appendix: Table 2a).

The values of geodetic precession for the Earth and the Moon are very close (Appendix: Table 2a). Therefore, in Figure 2, the point for the Moon overlaps with the point for the Earth (point of the red color). This is due to the large distance of the Moon from the Earth; as a result, the Sun has a greater influence on the Earth (comparable to its influence on the Moon) than the Moon. Thus, if we exclude the influence of the Moon on the Earth, then the value of its geodetic precession will be the same as for the Earth with the influence of the Moon – 19".19 per thousand years (Appendix: Table 2a). It is also possible to evaluate the quantity of the influence of the Moon on the geodetic rotation of the Earth; if we exclude the influence of the Sun on the Earth, then the value of its geodetic precession will be –0".005 per thousand years (Appendix: Table 2a).

In this study (subsection 4.1), the quantity of the inverse influence of the Earth on the geodetic rotation of the Moon, which is –0".30 per thousand years, was also calculated (Appendix: Table 3).

As a result of this study, the secular terms of the geodetic rotation of the planets and the Sun (Figure 2, right side) in the elements of their rotation were calculated for the first time (Appendix: Table 2b).

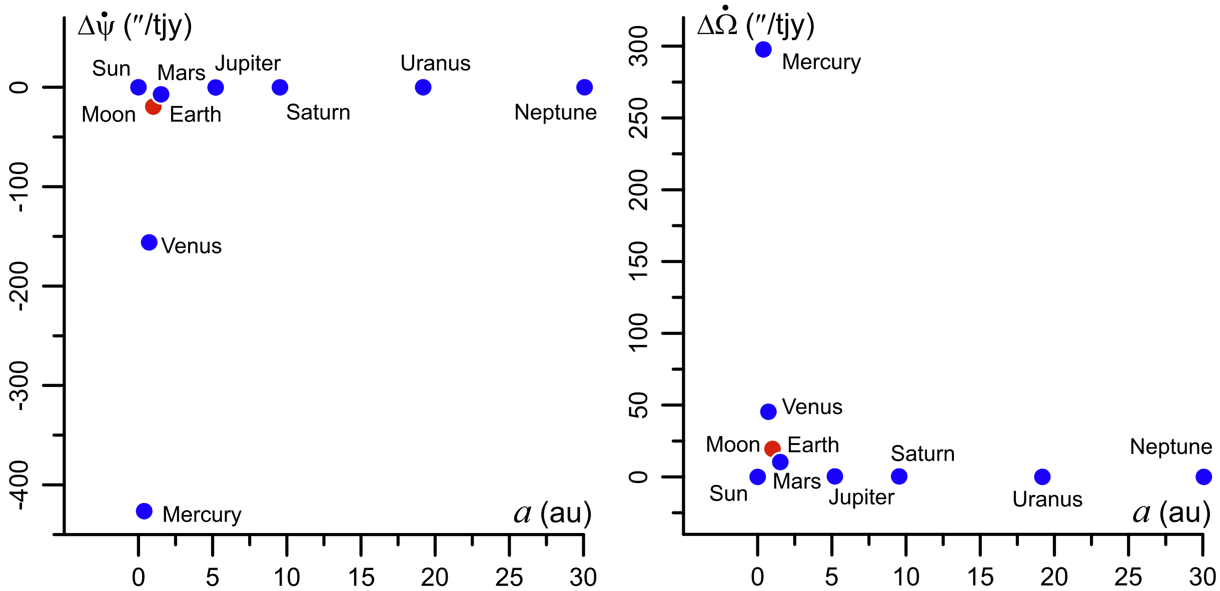

Figure 2. Geodetic precession velocity for the Sun, the Moon, and the planets of the Solar System in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (a is the length of the planetary orbit's semi-major axis)

Appendix: Table 2, 2a, 2b and Figure 2 show that all planets of the Solar System are characterized by a decrease in their absolute value of the geodetic rotation with an increase in their distance from the central body, which confirms the main property of equation (2) for the longitude of the descending node and for the absolute value of the vector of the geodetic rotation of the parameters of their orientation.

4. SATELLITES OF PLANETS

The study (Pashkevich and Vershkov, 2020) of the rotational dynamics of the inner satellites of Jupiter (Metis (J16), Adrastea (J15), Amalthea (J5), and Thebe (J14)) showed that the quantity of the relativistic geodetic rotation can be significant not only for relativistic objects, but, under certain conditions, also for ordinary satellites of planets, such as close satellites of giant planets. For all satellites of the planets of the Solar System (except for the satellites listed above, whose geodetic precession values were obtained in our previous studies), the secular terms of their geodetic rotation have been determined here for the first time.

In this research, the values of the secular terms of the geodetic rotation for the satellites of planets were first determined in the Euler angles relative to their proper coordinate systems (Appendix: Table 3 and Figures 3–7, left side) and in their rotational elements (Appendix: Table 3a and Figures 3–7, right side).

4.1. Geodetic precession of the Earth satellite (the Moon)

Geodetic precession of the Moon is –19".49 per thousand years (Appendix: Tables 2a, 3).

The value of the geodetic precession of the Moon (E1) is close to the value of the geodetic precession of the Earth (Figure 2, left side) (–19".19 per thousand years; Appendix: Table 2a). This is due to the large distance of the Moon from the Earth; as a result, the Sun has a greater influence on the Moon (comparable to its influence on the Earth) than the Earth. Thus, if we exclude the influence of the Earth on the Moon, then the value of its geodetic precession will be the same as for the Earth, that is, –19".19 per thousand years (Appendix: Table 3). It is also possible to evaluate the quantity of the influence of the Earth on the geodetic rotation of the Moon; if we exclude the influence of the Sun on the Moon, then the value of its geodetic precession will be –0".30 per thousand years (Appendix: Table 3).

In this study (Section 3), the quantity of the inverse influence of the Moon on the geodetic rotation of the Earth was also calculated, which was –0".005 per thousand years (Appendix: Table 2a).

4.2. Geodetic precession of Mars satellites

Geodetic precession of the satellites of Mars (Pashkevich and Vershkov, 2019) ranges from –27".68 per thousand years (for Deimos) to –209".31 per thousand years (for Phobos) (Figure 3, left side and Appendix: Table 3).

In the Mars satellite system, the values $\Delta\psi$ and $\Delta\Omega$ of the geodetic precession of its satellites Phobos (M1) and Deimos (M2) (Figure 3) exceed the corresponding values of the geodetic precession of Mars and the Earth (Figure 2), and additionally, the value of the geodetic precession of Phobos exceeds the value of the geodetic precession of Venus. This is because due to their close distance, Mars has a greater influence on their geodetic rotation than the Sun.

Here, for completeness, we present the values obtained in our previous studies and in this investigation. We also investigated the mutual relativistic influence of Martian satellites on each other (Pashkevich and Vershkov, 2019) and on Mars (in this study, we obtained additional values for the relativistic influence on Mars separately related to Phobos and separately related to Deimos, as well as new corrected values for the relativistic influence on Mars from both Martian satellites):

- the change in the geodetic rotation of Deimos due to the relativistic influence of Phobos is equal to –0.22 μas per thousand years in the longitude of the node, -9.3×10^{-6} μas per thousand years in the inclination, and 0.12 μas per thousand years in the proper rotation angle (Pashkevich and Vershkov, 2019);
- the change in the geodetic rotation of Phobos due to the relativistic influence of Deimos is equal to -5.3×10^{-2} μas per thousand years in the longitude of the node, 6.2×10^{-6} μas per thousand years in the inclination, and 2.9×10^{-2} μas per thousand years in the proper rotation angle (Pashkevich and Vershkov, 2019);
- the change in the geodetic rotation of Mars due to the relativistic influence of Phobos is equal to –4.48 μas per thousand years in the longitude of the node, -9.0×10^{-4} μas per thousand years in the inclination, and 2.50 μas per thousand years in the proper rotation angle;
- the change in the geodetic rotation of Mars due to the relativistic influence of Deimos is equal to -6.2×10^{-2} μas per thousand years in the longitude of the node, 3.5×10^{-5} μas per thousand years in the inclination, and 3.4×10^{-2} μas per thousand years in the proper rotation angle; and
- the change in the geodetic rotation of Mars due to relativistic influence of Phobos and Deimos is equal to –4.54 μas per thousand years in the longitude of the node, -8.6×10^{-4} μas per thousand years in the inclination, and 2.54 μas per thousand years in the proper rotation angle.

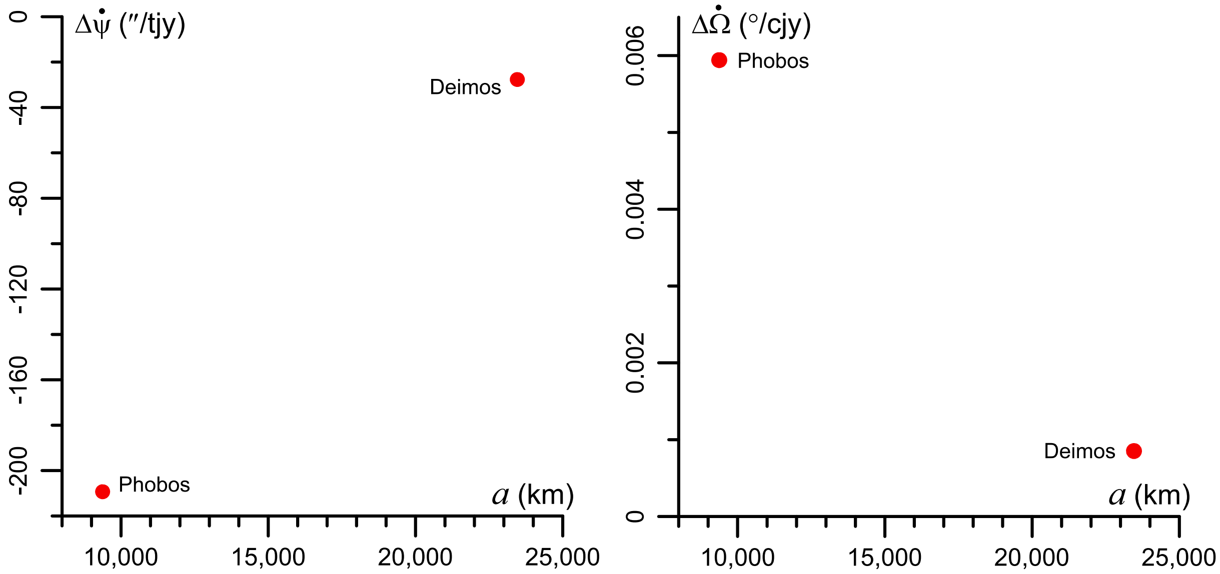

Figure 3. Geodetic precession velocity of the satellites of Mars in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (*a* is the length of the satellite orbit's semi-major axis)

4.3. Geodetic precession of Jupiter's satellites

Geodetic precession of Jupiter's moons runs from –64".00 per thousand years (for Callisto) to –52,957".25 per thousand years (for Metis) (Figure 4, left side and Appendix: Table 3).

It is found that there are objects in the Solar System with significant geodetic rotation comparable to their main rotation (Archinal et al. 2018). The values of geodetic precession of the inner satellites of Jupiter (Metis (J16), Adrastea (J15), Amalthea (J5), and Thebe (J14)) (Pashkevich and Vershkov, 2020) turned out to be comparable to the values of their precession (Appendix: Table 3a, red color) (Archinal et al. 2018). These values are, on average, 10^5 times higher than the value of geodetic precession of Jupiter itself (Figure 2, left side and Appendix: Table 2a) and 100 times higher than the value of geodetic precession of the Mercury, which is the closest planet to the Sun in the Solar System (Figure 2, left side and Appendix: Table 2a).

The next of the studied satellites in terms of distance from Jupiter is the group of Jupiter's Galilean moons (Io (J1), Europa (J2), Ganymede (J3), and Calisto (J4)). Studies have shown that Io (J1) and Europa (J2) have geodetic precession values that are 6 and 2 times higher, respectively, than the geodetic precession value for Mercury (Figure 2, left side and Appendix: Table 2a), and the geodetic precession value for Ganymede (J3) is 1.7 times higher than that for Venus (Figure 2, left side and Appendix: Table 2a). The value of the geodetic precession of Calisto (J4) is 3 times greater than that of the Earth (Figure 2, left side and Appendix: Table 2a).

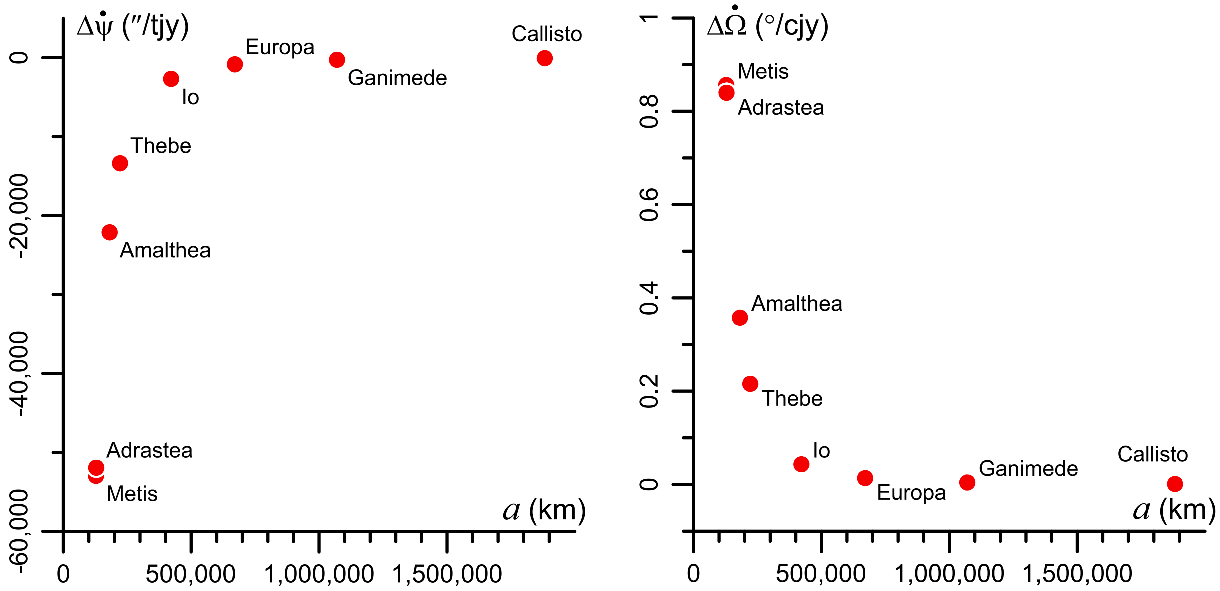

Figure 4. Geodetic precession velocity of the satellites of Jupiter in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (a is the length of the satellite orbit's semi-major axis)

4.4. Geodetic precession of Saturn's satellites

Geodetic precession of Saturn's moons runs from –0".02 per thousand years (for Phoebe) to –232".74 per thousand years (for Pan) (Figure 5, left side and Appendix: Table 3).

In the satellite system of Saturn, the geodetic precession values of the group of inner satellites closest to it (Pan (S18), Atlas (S15), Prometheus (S16), Pandora (S17), Epimetheus (S11), and Janus (S10)) exceed the geodetic precession value of Venus (Appendix: Table 2a). For the satellites of this group and Mimas (S1), the geodetic precession values turned out to be comparable to their precession values (Appendix: Table 3a in red color).

The geodetic precession values of the next largest satellite group (Mimas (S1)–Telesto (13)) exceed the geodetic precession value of the Earth (Appendix: Table 2a).

The moons of Saturn Telesto (S13) and Calypso (S14) have close orbits and synchronously rotate relative to each other. Consequently, the theoretically predicted values of their geodetic precession should be close. As follows from the main property of equation (2), this value should be slightly less than the geodetic precession of Telesto (–28″.61 per thousand years; Appendix: Table 3) and much more than that of Dione (–17″.25 per thousand years; Appendix: Table 3), between the orbits of which the orbit of Calypso is located. However, the obtained value of the geodetic precession of Calypso (S14), that is, 0″.28 per thousand years (Figure 5, left side), is two orders of magnitude less in absolute value than that of Telesto (S13) and has the opposite sign compared to similar values of other satellites of Saturn (Appendix: Table 3). The discovered feature of the geodetic rotation for this satellite is probably related to the inaccuracy of its rotation parameters (Archinal et al. 2018)[4] and their incompatibility with the used ephemeris (Giorgini et al. 1996).

Indeed, as the experiment showed, if we replace the polar rotation parameters of Calypso (α_0 and δ_0) (Archinal et al. 2018) with the corresponding polar rotation parameters of Telesto (still using the coordinates and velocities for Calypso from the ephemerides; Giorgini et al. 1996) to calculate the quantity of the Calypso geodetic precession, then the resulting geodetic precession value is in good agreement with the predicted theory (Appendix: Table 4 bold red color)[5]. Although the authors do not exclude another reason of the discovered feature of the geodetic rotation for this satellite, which may not yet be studied.

The values of the geodetic precessions of the moons of Saturn Dione (S4) and Helena (S12) are comparable in magnitude with the geodetic precession of the Earth (Figure 2, left side and Appendix: Table 2a) and the satellite of Rhea (S5) with the geodetic precession of Mars (Figure 2, left side and Appendix: Table 2a). For Titan (S6) and Iapetus (S8), the values of these quantities exceed those of Jupiter, and for Phoebe (S9), they are less than that of Saturn, but greater than that of Uranus (Figure 2, left side and Appendix: Table 2a).

[4] Note from Archinal et al. 2018: "These equations are correct for Janus, Epimetheus, Telesto, and Calypso for the period of the Voyager encounters. Because of precession these may change."

[5] Appendix: Table 4 shows the results of the experiment of varying the parameters of the polar rotation of Calypso (for the Telesto and Tethys parameters marked in red and blue colors, respectively).

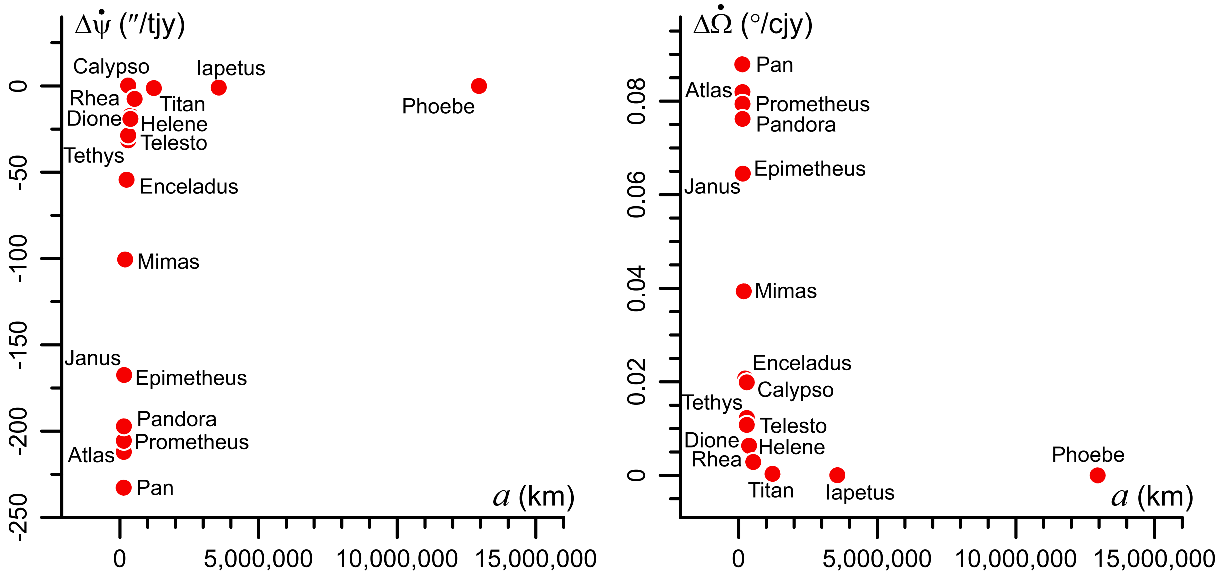

Figure 5. Geodetic precession velocity of the satellites of Saturn in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (a is the length of the satellite orbit's semi-major axis)

4.5. Geodetic precession of Uranus satellites

Geodetic precession of Uranus satellites ranges from 1".57 per thousand years (for Oberon) to 737".38 per thousand years (for Cordeli) (Figure 6, left side and Appendix: Table 3).

A distinctive feature in the system of Uranus satellites is the positive value of geodetic precession of all satellites under study (Ariel (U1), Umbriel (U2), Titania (U3), Oberon (U4), Miranda (U5), Cordelia (U6), Ophelia (U7), Bianca (U8), Cressida (U9), Desdemona (U10), Juliet (U11), Portia (U12), Rosalind (U13), Belinda (U14) and Puck (U15)) (Appendix: Table 3). This feature is due to their reverse rotation.

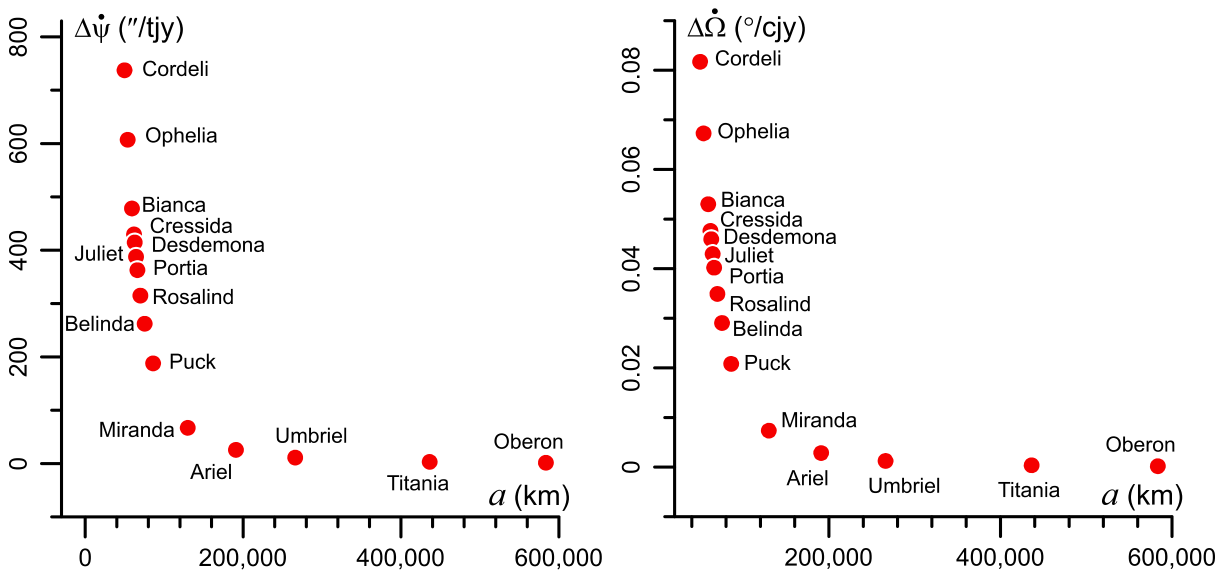

Figure 6. Geodetic precession velocity of the satellites of Uranus in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (a is the length of the satellite orbit's semi-major axis)

4.6. Geodetic precession of Neptune's satellites

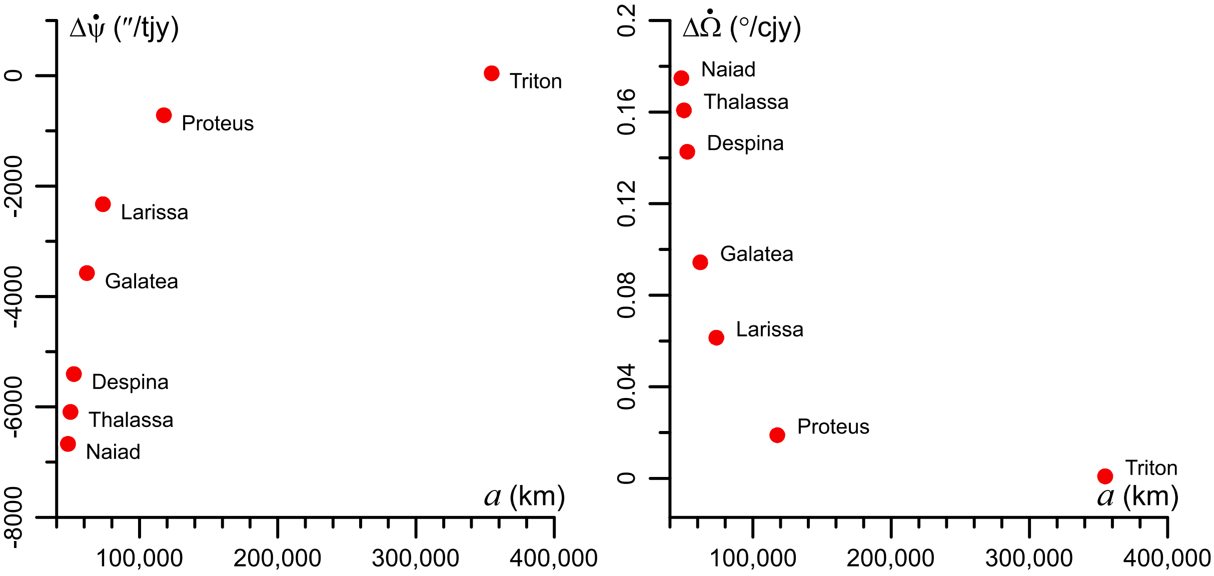

Figure 7. Geodetic precession velocity of the satellites of Neptune in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side) (a is the length of the satellite orbit's semi-major axis)

Geodetic precession of Neptune's moons runs from 43".45 per thousand years (for Triton) to –6670".30 per thousand years (for Naiad) (Figure 7, left side and Appendix: Table 3).

In the system of the investigated satellites of Neptune (Triton (N1), Naiad (N3), Thalassa (N4), Despina (N5), Galatea (N6), Larisa (N7), Proteus (N8)), Triton is the most interesting. This satellite, like the satellites of Uranus, has a positive value of the geodetic precession and reverse rotation (Appendix: Table 3). The value of the geodetic precession of other satellites of Neptune turned out to be on average an order of magnitude higher than the value of the geodetic precession of Mercury, which is the closest planet to the Sun in the Solar System (Figure 2, left side and Appendix: Table 2a). This is due to the greater influence of Neptune on them as the central body than the influence on Mercury from the more massive central body of the Sun.

The obtained analytical expressions for the parameters of the geodetic rotation of all satellites of the planets of the Solar System can be used to numerically study their rotation in the relativistic approximation.

Appendix: Tables 2–3a and Figures 3–7, right side show that the absolute value of the geodetic rotation of the central body is always less than that of its satellites.

5. CONCLUSIONS

The theoretical investigations of the relativistic effects in the rotational motions for the Sun, all planets of the Solar System, and their satellites with known quantities of their rotational elements (E1, M1, M2, J1–J5, J14–J16, S1–S6, S8–S18, U1–U15, N1, N3–N8) were carried out.

As a result, the most significant secular terms of the geodetic rotation have been improved

- for the Sun and the Solar System planets, in the Euler angles relative to their proper coordinate systems and in the absolute value of the geodetic rotation angular velocity vector and

- for the Moon (E1) in the perturbing terms of the physical libration relative to her proper coordinate systems and in the absolute value of the geodetic rotation angular velocity vector.

The values of the geodetic precession were first calculated

- for the Sun, all the Solar System planets, the Moon (E1), and satellites of Mars (M1, M2) in their rotational elements and
- for Galilean moons of Jupiter (J1–J4), satellites of Saturn (S1–S6, S8–S18), satellites of Uranus (U1–U15), and satellites of Neptune (N1, N3–N8), in the Euler angles relative to their proper coordinate systems and in their rotational elements.

The values of geodetic rotation were determined

- for the Earth and for the Moon (E1) without taking into account the perturbations from the Sun;
- for the Earth without taking into account the perturbations from the Moon (E1); and
- for the Moon (E1) without taking into account the perturbations from the Earth.

Additional values were determined for the relativistic influence on Mars separately related to Phobos (M1) and separately related to Deimos (M2), as well as new corrected values for the relativistic influence on Mars from both Martian satellites.

The largest values of the geodetic rotation of bodies in the Solar System were found in Jovian satellites system (Appendix: Tables 2, 3, and 3a). Further, in decreasing order, these values were found in the satellite systems of Saturn, Neptune, Uranus, and Mars, for Mercury, for Venus, for the Moon, for the Earth, for Mars, for Jupiter, for Saturn, for Uranus, for Neptune, and for the Sun (Appendix: Table 2). First of all, these are the inner satellites of Jupiter: Metis (J16), Adrastea (J15), Amalthea (J5), and Thebe (J14) and satellites of Saturn: Pan (S18), Atlas (S15), Prometheus (S16), Pandora (S17), Epimetheus (S11), Janus (S10), and Mimas (S1), whose values of geodetic precession are comparable to the values of their precession (Appendix: Table 3a, red color).

Such an arrangement of the geodetic precession values of the angular velocity vector differs somewhat from the location of the geodetic precession values in the longitudes of the descending nodes of the bodies under study. Thus, the largest values of geodetic precession in longitude of the descending node (Appendix: Tables 2a and 3) were found in the satellite system of Jupiter, then, in descending order of these values, follow the satellites of the Neptune system, the satellites of the Uranus system, the planet of Mercury, the Saturn satellite system, the Mars satellite system, the planet of Venus, the Moon, and the planets of the Earth, Mars, Jupiter, Saturn, Uranus, Neptune and the Sun.

For all studied objects of the Solar System, a characteristic pattern has been revealed:

1) a decrease in their absolute value of the geodetic precession with an increase in their distance from the central body (it is the main property of the formula for the angular velocity vector of the geodetic rotation of a body under study, which confirms for the longitude of the descending node and for the absolute value of the vector of the geodetic rotation of the parameters of their orientation) and
2) an absolute value of the geodetic rotation of the central body is always less than that of its satellites.

The obtained analytical values for the geodetic precession for the Sun, all the Solar System planets and their satellites can be used to numerically study their rotation in the relativistic approximation and as an estimate of the influence of relativistic effects on the orbital–rotational dynamics of bodies of exoplanetary systems.

The results of this study can also be used to test the general theory of relativity in the implementation of space projects like "Gravity Probe B" (Everitt et al. 2011).

In the future, it is planned to expand our studies of the relativistic effect of geodetic rotation for other bodies of the Solar System (dwarf planets and asteroids). Also, our studies will be expanded for all investigated bodies of the Solar System to obtain the values of the most significant periodic terms of their geodetic nutation.

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REFERENCES

Archinal B. A., A'Hearn M. F., Bowell E., Conrad A., Consolmagno G. J., Courtin R., Fukushima T., Hestroffer D., Hilton J. L., Krasinsky G. A., Neumann G., Oberst J., Seidelmann P. K., Stooke P., Tholen D. J., Thomas P. C., Williams I. P. (2011) Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009. Celest Mech Dyn Astr 109, 101–135; (https://doi.org/10.1007/s10569-010-9320-4).

Archinal B.A., Acton C.H., A'Hearn M.F., Conrad A., Consolmagno G.J., Duxbury T., Hestroffer D., Hilton J. L., Kirk R. L., Klioner S. A., McCarthy D., Meech K., Oberst J., Ping J., Seidelmann P. K., Tholen D. J., Thomas P. C., Williams I. P. (2018) Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2015, *Celest. Mech. Dyn. Astron*., Vol. 130, No. 22, 21-46; (https://doi.org/10.1007/s10569-017-9805-5).

De Sitter W. (1916) On Einstein's theory of Gravitation and its Astronomical Consequences, *Monthly Notices of the Royal Astronomical Society*, Vol. 76, No. 9, 699-728; (https://doi.org/10.1093/mnras/76.9.699).

Eroshkin G.I., Pashkevich V.V. (2007) Geodetic rotation of the Solar system bodies, Artificial Satellites, Vol. 42, No. 1, pp. 59–70; (https://doi.org/10.2478/v10018-007-0017-1).

Everitt C. W. F., DeBra D. B., Parkinson B. W., Turneaure J. P., Conklin J. W., Heifetz M. I.,. Keiser G. M, Silbergleit A. S., Holmes T., Kolodziejczak J., Al-Meshari M., Mester J. C., Muhlfelder B., Solomonik V., Stahl K., Worden P., Bencze W., Buchman S., Clarke B., Al-Jadaan A., Al-Jibreen H., Li J., Lipa J. A., Lockhart J. M., Al-Suwaidan B., Taber M., Wang S. (2011). "Gravity Probe B: Final Results of a Space Experiment to Test General Relativity". *Physical Review Letters*. **106** (22): 221101.doi:10.1103/PhysRevLett.106.221101.

arXiv:1105.3456. Bibcode:2011PhRvL.106v1101E. PMID 21702590. S2CID 11878715.

Folkner W.F., Williams J.G., Boggs D.H., Park R.S., and Kuchynka P. (2014) The Planetary and Lunar Ephemerides DE430 and DE431, *IPN Progress Report 42-196*, February 15, 2014.

Giorgini J.D., Yeomans D.K., Chamberlin A.B., Chodas P.W., Jacobson R.A., Keesey M.S., Lieske J.H., Ostro S.J., Standish E.M., Wimberly R.N. (1996) "JPL's On-Line Solar System Data Service", *Bulletin of the American Astronomical Society*, Vol. 28, No. 3, 1158.

Klioner S.A., Gerlach E., and Soffel M.H. (2009) "Relativistic aspects of rotational motion of celestial bodies", Proceedings IAU Symposium No. 261, 2009, 112-123; (https://doi.org/10.1017/S174392130999024X).

Ma C., Arias E.F., Eubanks T.M., Fey A.L., Gontier A.-M., Jacobs C.S., Sovers O.J., Archinal B.A., Charlot P. (1998) The international celestial reference frame as realized by very long baseline interferometry, *Astron. J.*, Vol. 116, No. 1, 516–546; (https://doi.org/10.1086/300408).

Pashkevich V.V. (2016) New high-precision values of the geodetic rotation of the major planets, Pluto, the Moon and the Sun, *Artificial Satellites*, *Journal of Planetary Geodesy*, Vol. 51, No. 2, 61-73; (https://doi.org/10.1515/arsa-2016-0006).

Pashkevich V.V., Vershkov A.N. (2019) New High-Precision Values of the Geodetic Rotation of the Mars Satellites System, Major Planets, Pluto, the Moon and the Sun, *Artificial Satellites*, *Journal of Planetary Geodesy*, Vol. 54, No. 2, 31-42; (https://doi.org/10.2478/arsa-2019-0004).

Pashkevich V.V., Vershkov A.N. (2020) Relativistic effects in the rotation of Jupiter's inner satellites, *Artificial Satellites, Journal of Planetary Geodesy,* Vol. 55, No. 3, 118-129; (https://doi.org/10.2478/arsa-2020-0009).

Seidelmann P.K., Archinal B.A., A'Hearn M.F., Cruikshank D.P., Hilton J.L., Keller H.U., Oberst J., Simon J.L., Stooke P., Tholen D.J., and Thomas P.C. (2005) Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements: 2003, Celestial Mechanics and Dynamical Astronomy, 91, 203–215; (https://doi.org/10.1007/s10569-004-3115-4).

Standish, E., Newhall, X. (1996). New accuracy levels for solar system ephemerides. Symposium - International Astronomical Union, 172, 29-36. (https://doi.org/10.1017/S0074180900127081)

Suslov G.K. (1946): Theoretical mechanics. OGIZ, Moscow, (in Russian).

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APPENDIX

Table 1. The time spans and steps for the studies of the geodetic precession of the bodies *(part 1/2)*

Satellites	Time span (years)	Spacing
The Earth		
The Moon (E1)	2000 (from AD1000 01 Jan. to AD3000 01 Jan.)	1 day 00 h 00 min
Mars		
Phobos (M1) Deimos (M2)	900 (from AD1600 01 Jan. to AD2499 14 Oct.)	09 h 30 min
Jupiter		
Metis (J16) Adrastea (J15)	400 (from AD1799 19 Dec. to AD2200 13 Jan.)	42 min
Amalthea (J5)	1000 (from AD1600 07 Feb. to AD2599 06 Dec.)	01 h 00 min
Thebe (J14)	400 (from AD1799 19 Dec. to AD2200 13 Jan.)	01 h 30 min
Io (J1) Europa (J2) Ganimede (J3) Callisto (J4)	1000 (from AD1600 07 Feb. to AD2599 07 Dec.)	04 h 15 min
Saturn		
Pan (S18) Atlas (S15)	100 (from AD1949 27 Dec. to AD2050 09 Jan.)	01 h 20 min
Prometheus (S16)	100 (from AD1949 27 Dec. to AD2050 09 Jan.)	01 h 00 min
Pandora (S17)	100 (from AD1949 27 Dec. to AD2050 09 Jan.)	01 h 20 min
Epimetheus (S11) Janus (S10)	100 (from AD1949 27 Dec. to AD2050 09 Jan.)	01 h 40 min
Mimas (S1)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	02 h 00 min
Enceladus (S2)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	03 h 00 min
Tethys (S3) Telesto (S13) Calypso (S14)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	04 h 30 min
Dione (S4) Helene (S12)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	06 h 30 min
Rhea (S5)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	10 h 50 min
Titan (S6)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	1 day 14 h 20 min
Iapetus (S8)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	7 days 22 h 00 min
Phoebe (S9)	300 (from AD1849 29 Dec. to AD2150 07 Jan.)	5 days 10 h 00 min

Table 1. The time spans and steps for the studies of the geodetic precession of the bodies *(part 2/2)*

Satellites	Time span (years)	Spacing
	U r a n u s	
Cordelia (U6)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	48 min
Ophelia (U7)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	50 min
Bianca (U8)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 00 min
Cressida (U9) Desdemona (U10)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 05 min
Juliet (U11) Portia (U12)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 10 min
Rosalind (U13)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 20 min
Belinda (U14)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 30 min
Puck (U15)	80 (from AD1980 02 Jan. to AD2059 31 Dec.)	01 h 50 min
Miranda (U5)	1000 (from AD1599 08 Dec. to AD2600 12 Jan.)	03 h 20 min
Ariel (U1)	1000 (from AD1599 08 Dec. to AD2600 12 Jan.)	06 h 00 min
Umbriel (U2)	1000 (from AD1599 08 Dec. to AD2600 12 Jan.)	10 h 00 min
Titania (U3)	1000 (from AD1599 08 Dec. to AD2600 12 Jan.)	20 h 00 min
Oberon (U4)	1000 (from AD1599 08 Dec. to AD2600 12 Jan.)	1 day 07 h 40 min
	N e p t u n e	
Naiad (N3)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	42 min
Thalassa (N4)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	45 min
Despina (N5)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	48 min
Galatea (N6)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	01 h 00 min
Larissa (N7)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	01 h 20 min
Proteus (N8)	100 (from AD1950 02 Jan. to AD2049 30 Dec.)	02 h 40 min
Triton (N1)	1000 (from AD1599 04 Dec. to AD2599 31 Dec.)	13 h 20 min
The bodies	**Time span (years)**	**Spacing**
The Sun and the planets	2000 (from AD1000 01 Jan. to AD3000 01 Jan.)	1 day 00 h 00 min

Table 2. Magnitudes of the geodetic precession for the Sun, the Solar System planets and for each planet system, its satellite with the largest geodetic precession, calculated for the angular velocity vector $|\bar{\sigma}|$ of the geodetic rotation of the body under study

Name	Eroshkin and Pashkevich (2007)	Klioner et al. (2009)	In this paper	a (au)
	″ per century	″ per century	″ per century	
The Sun	0.0001		0.0000692	0
Mercury	21.4905	21.43	21.4902924	0.387098
Venus	4.3124	4.32	4.3123523	0.723330
The Earth	1.9199	1.92	1.9198805	1.000001
The Moon (E1)	1.9495	1.95	1.9494951	
Mars	0.6756	0.68	0.6754500	1.523679
Phobos (M1)			30.6419590	
Jupiter	0.0312		0.0311851	5.202603
Metis (J16)			2653.6443645	
Saturn	0.0069		0.0068507	9.554910
Pan (S18)			390.9201274	
Uranus	0.0012		0.0011949	19.218446
Cordelia (U6)			276.4934392	
Neptune	0.0004		0.0003876	30.110387
Naiad (N3)			381.1538211	

1 astronomical unit (au) = 149,597,870.7 km
(taken from Horizons On-Line Ephemeris System; Giorgini et al. 1996);
a is the length of the planetary orbit's semi-major axis.

Table 2a. Secular terms of the geodetic rotation for the Sun and the Solar System planets, calculated for the Euler angles, and for the Moon, calculated for the perturbing terms of the physical libration

	The Sun	**Mercury** *a* **(au) = 0.387098**	**Venus** *a* **(au) = 0.723330**	**The Moon** *a* **(km) = 384400**
	Δψ (μas)	Δψ (μas)	Δψ (μas)	Δτ (μas)
t	−870.2788	−425,606,984.4341	−155,952,178.4711	−19,494,198.9139
t^2	1.3770	−33,155.9302	−687,024.3196	−77.7041
	Δθ (μas)	Δθ (μas)	Δθ (μas)	Δρ (μas)
t	−1.8970	−43,920.9632	−740,253.4678	−413.2193
t^2	0.0809	504.4556	60,179.7955	−1436.3972
	Δφ (μas)	Δφ (μas)	Δφ (μas)	Δ(Iσ) (μas)
t	179.6136	213,919,825.1563	112,930,676.1063	511,726.8500
t^2	−1.3915	−3798.8818	687,231.8895	−14,383.0938
	The Earth[6] *a* **(au) = 1.000001**	**without the Moon**[7]	**without the Sun**[8]	**Mars** *a* **(au) = 1.523679**
	Δψ (μas)	Δψ (μas)	Δψ (μas)	Δψ (μas)
t	−19,199,865.4438	−19,194,966.2971	−5289.2214	−7,125,692.1811
t^2	49,150.8059	49,136.4217	11.7867	10,109.0014
	Δθ (μas)	Δθ (μas)	Δθ (μas)	Δθ (μas)
t	−4127.7653	−4127.7520	−9.5398	127,569.2300
t^2	−1878.6778	−1878.5492	−0.2078	−1098.6657
	Δφ (μas)	Δφ (μas)	Δφ (μas)	Δφ (μas)
t	1174.6090	1172.9236	−1.3267	414,234.7545
t^2	−53,414.8819	−53,399.9048	−12.2192	−11,846.4356
	Jupiter *a* **(au) = 5.202603**	**Saturn** *a* **(au) = 9.554910**	**Uranus** *a* **(au) = 19.218446**	**Neptune** *a* **(au) = 30.110387**
	Δψ (μas)	Δψ (μas)	Δψ (μas)	Δψ (μas)
t	−212,778.4891	−67,171.5760	−11,949.3883	−3902.8771
t^2	3097.9909	−54.6002	−21.3019	4.3541
	Δθ (μas)	Δθ (μas)	Δθ (μas)	Δθ (μas)
t	−5974.5301	−2892.9323	−161.0625	−118.6838
t^2	133.7664	−27.8319	1.4159	0.1154
	Δφ (μas)	Δφ (μas)	Δφ (μas)	Δφ (μas)
t	−99,066.0037	−1440.3592	10.3345	32.9359
t^2	−3118.0679	137.5508	−1.0611	0.7553

a is the length of the planetary orbit's semi-major axis;
1 au = 149,597,870.7 km (from Horizons On-Line Ephemeris System; Giorgini et al. 1996);
t is the time in Julian thousand years.

[6] Geodetic Earth rotation taking into account the perturbations from the planets, dwarf planet Pluto, the Moon, and the Sun. The rotation parameters for the Earth were taken from the article by Archinal et al. (2011), and for other studied planets, from the article by Archinal et al. (2018).

[7] Geodetic Earth rotation without taking into account the perturbations from the Moon.

[8] Geodetic Earth rotation without taking into account the perturbations from the Sun.

Table 2b. The rotational elements of the Sun and its planets
and their secular terms of the geodetic rotation

Name, *a* (au)	**Archinal et al. (2018)**		**Present paper**	***T***	***T***^{2}
The Sun 0	α_0 (°)	286.13	$\Delta\alpha_0$ (″)	1×10^{-5}	−3×10^{-10}
	δ_0 (°)	63.87	$\Delta\delta_0$ (″)	1×10^{-5}	−2×10^{-9}
	W(°)	84.176+14.1844000d	ΔW (″)	−0.0001	1×10^{-11}
Mercury 0.387098	α_0 (°)	281.0103− 0.0328T	$\Delta\alpha_0$ (″)	8.5439	0.0015
	δ_0 (°)	61.4155− 0.0049T	$\Delta\delta_0$ (″)	3.2367	−0.0047
	W(°)	329.5988+6.1385108d	ΔW (″)	−28.3505	−0.0018
Venus 0.723330	α_0 (°)	272.76	$\Delta\alpha_0$ (″)	0.2342	0.0016
	δ_0 (°)	67.16	$\Delta\delta_0$ (″)	0.3331	−0.0001
	W(°)	160.20−1.4813688d	ΔW (″)	−4.5144	−0.0014
Name	**Archinal et al. (2011)**[9]		**Present paper**	***T***	***T***^{2}
The Earth 1.000001	α_0 (°)	0.00−0.641T	cos $\delta_0\Delta\alpha_0$ (″)	0.0426	−3×10^{-5}
	δ_0 (°)	90.00−0.557T	$\Delta\delta_0$ (″)	0.7622	−0.0002
	W(°)	190.147+360.9856235d	$\Delta\alpha_0$+ ΔW$_0$ (″)	−1.7614	0.0001
Name	**Archinal et al. (2018)**		**Present paper**	***T***	***T***^{2}
Mars 1.523679	α_0 (°)	317.269202−0.10927547T	$\Delta\alpha_0$ (″)	0.3972	−0.0001
	δ_0 (°)	54.432516−0.058271056T	$\Delta\delta_0$ (″)	0.1991	−0.0002
	W(°)	176.049863+350.891982443297d	ΔW (″)	−0.9273	0.0001
Jupiter 5.202603	α_0 (°)	268.056595−0.006499T	$\Delta\alpha_0$ (″)	0.0023	−4×10^{-6}
	δ_0 (°)	64.495303+0.002413T	$\Delta\delta_0$ (″)	0.0003	−1×10^{-6}
	W(°)	284.95+870.5360000d	ΔW (″)	−0.0332	3×10^{-6}
Saturn 9.554910	α_0 (°)	40.589−0.036T	$\Delta\alpha_0$ (″)	0.0199	−1×10^{-5}
	δ_0 (°)	83.537−0.004T	$\Delta\delta_0$ (″)	0.0023	1×10^{-6}
	W(°)	38.90+810.7939024d	ΔW (″)	−0.0258	1×10^{-5}
Uranus 19.218446	α_0 (°)	257.311	$\Delta\alpha_0$ (″)	0.0012	2×10^{-7}
	δ_0 (°)	−15.175	$\Delta\delta_0$ (″)	−0.0001	−3×10^{-8}
	W(°)	203.81−501.1600928d	ΔW (″)	0.0002	2×10^{-8}
Neptune 30.110387	α_0 (°)	299.36	$\Delta\alpha_0$ (″)	0.0002	−1×10^{-8}
	δ_0 (°)	43.46	$\Delta\delta_0$ (″)	0.0001	2×10^{-8}
	W(°)	249.978+541.1397757d	ΔW (″)	−0.0005	1×10^{-7}

a is the length of the satellite orbit's semi-major axis;
1 au = 149,597,870.7 km (from Horizons On-Line Ephemeris System; Giorgini et al. 1996);
T is the time in Julian centuries years;
d is the time in days from standard epoch is JD 2451545.0, that is, 2000 January 1, 12 h TDB.

[9] The rotational elements for the Earth are taken from Archinal et al. (2011).

Table 3. Secular terms of the geodetic rotation for the satellites of the Solar System planets, calculated for the Euler angles *(part 1/4)*

The Earth							
Name	**$\Delta\tau$ (")**		**$\Delta\rho$ (")**		**$\Delta(I\sigma)$ (")**		***a* (km)**
	t	***t*²**	***t***	***t*²**	***t***	***t*²**	
The Moon (E1)[10]	−19.4942	−0.0001	−0.0004	−0.0014	0.5117	−0.0144	384,400
without the Earth[11]	−19.1932	-3×10^{-5}	−0.0005	−0.0014	0.5171	−0.0144	149,597,870
without the Sun[12]	−0.3014	-4×10^{-5}	3×10^{-5}	−0.0001	−0.0054	-1×10^{-5}	384,400
Mars							
Name	**$\Delta\psi$ (")**		**$\Delta\theta$ (")**		**$\Delta\varphi$ (")**		***a* (km)**
	t	***t*²**	***t***	***t*²**	***t***	***t*²**	
Phobos (M1)[13]	−209.3145	0.0411	0.1096	−0.0800	113.6015	−0.0202	9376
Deimos (M2)[13]	−27.6800	0.0145	0.1189	−0.0057	11.8433	−0.0124	23,458
Jupiter							
Name	**$\Delta\psi$ (")**		**$\Delta\theta$ (")**		**$\Delta\varphi$ (")**		***a* (km)**
	t	***t*²**	***t***	***t*²**	***t***	***t*²**	
Metis (J16)[14]	−52,957.2516	−20.0929	−0.4232	−3.9838	26,460.9380	19.8859	128,000
Adrastea (J15)[14]	−51,932.8456	−19.7509	−0.4151	−3.9067	25,949.0709	19.5347	129,000
Amalthea (J5)[14]	−22,118.2274	−0.7460	−0.0923	4.7351	11,055.1784	0.5755	181,400
Thebe (J14)[14]	−13,372.5500	−2.8287	−2.4703	37.7619	6693.8317	2.8902	221,900
Io (J1)	−2682.6602	−0.2122	−0.1196	−0.1392	1342.6373	0.2016	421,800

[10] Geodetic Moon rotation (Pashkevich et al. 2019) taking into account the perturbations from the planets, dwarf planet Pluto, and the Sun. The rotation parameters for the Moon were taken from the article by Archinal et al. (2011), and for other studied satellites, from the article by Archinal et al. (2018).

[11] Geodetic Moon rotation without taking into account the perturbations from the Earth.

[12] Geodetic Moon rotation without taking into account the perturbations from the Sun.

[13] The values of geodetic rotation for the satellites of Mars were obtained by us earlier (Pashkevich et al. 2019).

[14] The values of geodetic rotation for the inner satellites of Jupiter were obtained by us earlier (Pashkevich et al. 2020).

Table 3. Secular terms of the geodetic rotation for the satellites of the Solar System planets, calculated for the Euler angles *(part 2/4)*

Jupiter (continue)							
Name	***Δψ* ('')**		***Δθ* ('')**		***Δφ* ('')**		***a* (km)**
	t	***t²***	***t***	***t²***	***t***	***t²***	
Europa (J2)	−840.1721	0.0710	−0.0242	−0.0184	420.3792	−0.0663	671,100
Ganimede (J3)	−261.5694	−0.0066	−0.0112	−0.0131	130.7141	0.0084	1,070,400
Callisto (J4)	−63.9972	0.0399	−0.0102	−0.0022	31.8543	−0.0360	1,882,700
Saturn							
Name	***Δψ* ('')**		***Δθ* ('')**		***Δφ* ('')**		***a* (km)**
	t	***t²***	***t***	***t²***	***t***	***t²***	
Pan (S18)	−232.7364	−1.9587	−657.4148	−3.6720	−3639.3938	2.4248	133,585
Atlas (S15)	−212.0393	−1.8264	−608.6535	−3.4048	−3378.7169	2.1959	137,774
Prometheus (S16)	−205.5740	−1.8422	−590.0954	−3.3490	−3275.6980	2.1078	139,429
Pandora (S17)	−197.1983	−1.6079	−566.0443	−3.1597	−3142.2080	2.2945	141,810
Epimetheus (S11)	−167.4579	−1.7650	−479.4434	−2.3182	−2660.0806	2.2733	151,422
Janus (S10)	−167.8324	−1.8772	−479.5517	−2.6082	−2659.7131	2.0168	151,472
Mimas (S1)	−100.5028	−1.0302	−285.8875	−1.9636	−1600.2855	1.2301	185,539
Enceladus (S2)	−54.3148	−0.4696	−154.5777	−0.9223	−857.7077	0.5695	238,042
Tethys (S3)	−31.5081	−0.8095	−90.1979	−0.9998	−503.0932	0.8855	294,672
Telesto (S13)	−28.6117	−0.4386	−80.2283	−0.9827	−507.3979	0.5456	294,720
Calypso (S14)[15]	0.2767	0.9812	−84.1557	−0.6733	−532.3406	−0.7741	294,721

[15] Calypso (S14) has the opposite sign of the velocity of geodetic precession compared to similar values of other satellites of Saturn. The discovered feature of the geodetic rotation for this satellite is most likely associated with the inaccuracy or inconsistency of the ephemeris used, which determine the parameters of its rotation.

Table 3. Secular terms of the geodetic rotation for the satellites of the Solar System planets, calculated for the Euler angles *(part 3/4)*

Saturn (continue)							
Name	***Δψ*** **('')**		***Δθ*** **('')**		***Δφ*** **('')**		***a*** **(km)**
	t	t^2	***t***	t^2	***t***	t^2	
Dione (S4)	−17.2533	−0.1941	−48.8066	−0.3387	−270.7718	0.2360	377,415
Helene (S12)	−19.1048	−0.3180	−49.2847	−0.3515	−269.0649	0.3216	377,444
Rhea (S5)	−7.4990	−0.0869	−21.2180	−0.2158	−117.4761	0.1224	527,068
Titan (S6)	−1.2430	−0.3765	−2.6297	−0.2605	−14.1293	0.3897	1,221,865
Iapetus (S8)	−0.9239	−0.6512	−0.3113	0.0624	−0.1668	0.6569	3,560,854
Phoebe (S9)	−0.0214	−0.0148	0.0005	0.0104	−0.0046	0.0181	12,947,918

Uranus [16]							
Name	***Δψ*** **('')**		***Δθ*** **('')**		***Δφ*** **('')**		***a*** **(km)**
	t	t^2	***t***	t^2	***t***	t^2	
Cordelia (U6)	737.3755	−5.7294	−0.0321	−4.5750	−2743.4732	−0.2103	49,800
Ophelia (U7)	607.0945	−3.4941	−0.0225	−3.1162	−2258.9890	−0.3097	53,800
Bianca (U8)	478.2575	−3.0865	−0.0291	−2.1307	−1779.4917	−0.1653	59,200
Cressida (U9)	429.6241	−2.5533	−0.0104	−2.3131	−1598.6213	−0.2149	61,800
Desdemona (U10)	414.5513	−2.0570	−0.0060	−1.5043	−1542.6289	−0.2530	62,700
Juliet (U11)	387.6458	−2.2473	−0.0075	−1.8160	−1442.4253	−0.1466	64,400
Portia (U12)	362.6995	−2.2701	0.0121	−2.2402	−1349.5799	−0.1988	66,100
Rosalind (U13)	314.9000	0.0858	0.0312	−2.2876	−1172.0590	−0.3380	69,900
Belinda (U14)	262.1423	−1.4551	0.0062	−1.2547	−975.4948	−0.1107	75,300

[16] The satellites of Uranus have positive values of the velocity of geodetic precession and reverse rotation.

Table 3. Secular terms of the geodetic rotation for the satellites of the Solar System planets, calculated for the Euler angles *(part 4/4)*

Uranus (continue)							
Name	***Δψ* (")**		***Δθ* (")**		***Δφ* (")**		***a* (km)**
	t	***t*²**	***t***	***t*²**	***t***	***t*²**	
Puck (U15)	187.8308	−4.1306	−0.0069	−0.4330	−698.6046	−0.0686	86,000
Miranda (U5)	67.0376	0.3657	−0.4716	−0.0004	−249.4464	0.0003	129,900
Ariel (U1)	25.7442	0.0158	−0.2705	−0.0105	−95.1556	−0.0012	190,900
Umbriel (U2)	11.2105	−0.0032	−0.0603	−0.0418	−41.5373	0.0025	266,000
Titania (U3)	3.2515	0.0270	−0.0165	−0.0204	−12.0532	−0.0030	436,300
Oberon (U4)	1.5706	−0.0019	−0.0084	−0.0129	−5.8282	0.0000	583,500
Neptune							
Name	***Δψ* (")**		***Δθ* (")**		***Δφ* (")**		***a* (km)**
	t	***t*²**	***t***	***t*²**	***t***	***t*²**	
Naiad (N3)	−6670.3047	−14.7325	−5.1794	−55.2350	3809.5037	64.7022	48,227
Thalassa (N4)	−6092.0806	−8.2229	−4.2733	−17.4608	3467.5926	58.1521	50,074
Despina (N5)	−5405.6116	−6.2192	−3.6995	−15.2733	3076.8866	50.3420	52,526
Galatea (N6)	−3576.1301	−0.6725	−2.4523	−9.8477	2035.4328	30.7942	61,953
Larissa (N7)	−2328.1449	1.1397	−1.4266	−7.7510	1325.4696	20.5667	73,548
Proteus (N8)	−716.2634	−5.9168	−0.7619	−16.4520	409.4543	17.8405	117,646
Triton (N1) [17]	43.4450	0.0594	0.8107	−0.1928	−25.3711	−0.2680	354,759

t is the time in Julian thousand years.

[17] Triton (N1), like the satellites of Uranus, has a positive value of the velocity of geodetic precession and reverse rotation.

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 1/6)*

The Earth					
Name, *a* (km)	**Archinal et al. (2011)**		**Present paper**	***T***	***T*²**
The Moon (E1) 384,400	α_0 (°)	$269.9949 + 0.0031T$	$\Delta\alpha_0$ (°)	2×10^{-5}	-2×10^{-7}
	δ_0 (°)	$66.5392 + 0.0130T$	$\Delta\delta_0$ (°)	3×10^{-8}	1×10^{-8}
	W (°)	$38.3213{+}13.17635815d{-}1.4\times10^{-12}d^2$	ΔW (°)	−0.0006	1×10^{-7}
Mars					
Name, *a* (km)	**Archinal et al. (2011)**		**Present paper**	***T***	***T*²**
Phobos (M1) 9376	α_0 (°)	317.67071657 − 0.10844326T	$\Delta\alpha_0$ (°)	0.0033	-2×10^{-7}
	δ_0 (°)	52.88627266 − 0.06134706T	$\Delta\delta_0$ (°)	0.0017	-1×10^{-6}
	W (°)	$34.9964842535 + 1128.8447592d$	ΔW (°)	−0.0047	1×10^{-6}
Deimos (M2) 23,458	α_0 (°)	316.65705808 − 0.10518014T	$\Delta\alpha_0$ (°)	0.0004	-1×10^{-8}
	δ_0 (°)	53.50992033 − 0.05979094T	$\Delta\delta_0$ (°)	0.0002	-2×10^{-7}
	W (°)	$79.39932954 + 285.16188899d$	ΔW (°)	−0.0007	2×10^{-6}
Jupiter					
Name, *a* (km)	**Archinal et al. (2018)**		**Pashkevich et al. (2020)**	***T***	***T*²**
Metis (J16) 128,000	α_0 (°)	$268.05 - 0.009T$	$\Delta\alpha_0$ (°)	0.1241	-7×10^{-5}
	δ_0 (°)	$64.49 + 0.003T$	$\Delta\delta_0$ (°)	−0.0199	-4×10^{-5}
	W (°)	$346.09 + 1221.2547301d$	ΔW (°)	−0.8469	6×10^{-5}
Adrastea (J15) 129,000	α_0 (°)	$268.05 - 0.009T$	$\Delta\alpha_0$ (°)	0.1217	-6×10^{-5}
	δ_0 (°)	$64.49 + 0.003T$	$\Delta\delta_0$ (°)	−0.0195	-4×10^{-5}
	W (°)	$33.29 + 1206.9986602d$	ΔW (°)	−0.8306	6×10^{-5}
Amalthea (J5) 181,400	α_0 (°)	$268.05 - 0.009T$	$\Delta\alpha_0$ (°)	0.0518	-3×10^{-5}
	δ_0 (°)	$64.49 + 0.003T$	$\Delta\delta_0$ (°)	−0.0083	-2×10^{-5}
	W (°)	$231.67 + 722.6314560d$	ΔW (°)	−0.3536	3×10^{-5}
Thebe (J14) 221,900	α_0 (°)	$268.05 - 0.009T$	$\Delta\alpha_0$ (°)	0.0312	-2×10^{-5}
	δ_0 (°)	$64.49 + 0.003T$	$\Delta\delta_0$ (°)	−0.0050	-2×10^{-5}
	W (°)	$8.56 + 533.7004100d$	ΔW (°)	−0.2133	1×10^{-5}
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T*²**
Io (J1) 421,800	α_0 (°)	$268.05 - 0.009T$	$\Delta\alpha_0$ (°)	0.0063	-4×10^{-6}
	δ_0 (°)	$64.50 + 0.003T$	$\Delta\delta_0$ (°)	−0.0010	-2×10^{-6}
	W (°)	$200.39 + 203.4889538d$	ΔW (°)	−0.0428	3×10^{-6}
Europa (J2) 671,100	α_0 (°)	$268.08 - 0.009T$	$\Delta\alpha_0$ (°)	0.0019	2×10^{-7}
	δ_0 (°)	$64.51 + 0.003T$	$\Delta\delta_0$ (°)	−0.0003	-7×10^{-7}
	W (°)	$36.022 + 101.3747235d$	ΔW (°)	−0.0134	-2×10^{-7}

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 2/6)*

Jupiter (continue)					
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T²***
Ganimede (J3) 1,070,400	α_0 (°)	268.20 − 0.009*T*	$\Delta\alpha_0$ (°)	0.0006	2×10^{-7}
	δ_0 (°)	64.57 + 0.003*T*	$\Delta\delta_0$ (°)	−0.0001	-1×10^{-7}
	W (°)	44.064 + 50.3176081*d*	ΔW (°)	−0.0042	-2×10^{-7}
Callisto (J4) 1,882,700	α_0 (°)	268.72 − 0.009*T*	$\Delta\alpha_0$ (°)	0.0001	7×10^{-7}
	δ_0 (°)	64.83 + 0.003*T*	$\Delta\delta_0$ (°)	-1×10^{-5}	-2×10^{-7}
	W (°)	259.51 + 21.5710715*d*	ΔW (°)	−0.0010	-6×10^{-7}
Saturn					
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T²***
Pan (S18) 133,585	α_0 (°)	40.6 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0829	-5×10^{-5}
	δ_0 (°)	83.5 − 0.004*T*	$\Delta\delta_0$ (°)	0.0160	7×10^{-6}
	W (°)	48.8 + 626.0440000*d*	ΔW (°)	−0.0244	5×10^{-5}
Atlas (S15) 137,774	α_0 (°)	40.58 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0775	-4×10^{-5}
	δ_0 (°)	83.53 − 0.004*T*	$\Delta\delta_0$ (°)	0.0147	7×10^{-6}
	W (°)	137.88 + 598.3060000*d*	ΔW (°)	−0.0220	4×10^{-5}
Prometheus (S16) 139,429	α_0 (°)	40.58 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0752	-4×10^{-5}
	δ_0 (°)	83.53 − 0.004*T*	$\Delta\delta_0$ (°)	0.0143	7×10^{-6}
	W (°)	296.14 + 587.289000*d*	ΔW (°)	−0.0213	4×10^{-5}
Pandora (S17) 141,810	α_0 (°)	40.58 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0721	-4×10^{-5}
	δ_0 (°)	83.53 − 0.004*T*	$\Delta\delta_0$ (°)	0.0137	6×10^{-6}
	W (°)	162.92 + 572.7891000*d*	ΔW (°)	−0.0205	4×10^{-5}
Epimetheus (S11) 151,422	α_0 (°)	40.58 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0610	-1×10^{-5}
	δ_0 (°)	83.52 − 0.004*T*	$\Delta\delta_0$ (°)	0.0116	8×10^{-6}
	W (°)	293.87 + 518.4907239*d*	ΔW (°)	−0.0173	1×10^{-5}
Janus (S10) 151,472	α_0 (°)	40.58 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0609	-2×10^{-5}
	δ_0 (°)	83.52 − 0.004*T*	$\Delta\delta_0$ (°)	0.0116	7×10^{-6}
	W (°)	58.83 + 518.2359876*d*	ΔW (°)	−0.0175	2×10^{-5}
Mimas (S1) 185,539	α_0 (°)	40.66 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0376	-3×10^{-5}
	δ_0 (°)	83.52 − 0.004*T*	$\Delta\delta_0$ (°)	0.0068	5×10^{-6}
	W (°)	333.46 + 381.9945550*d*	ΔW (°)	−0.0096	3×10^{-5}
Enceladus (S2) 238,042	α_0 (°)	40.66 − 0.036*T*	$\Delta\alpha_0$ (°)	−0.0196	-1×10^{-5}
	δ_0 (°)	83.52 − 0.004*T*	$\Delta\delta_0$ (°)	0.0038	2×10^{-6}
	W (°)	6.32 + 262.7318996*d*	ΔW (°)	−0.0057	1×10^{-5}

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 3/6)*

S a t u r n (continue)					
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T*²**
Tethys (S3) 294,672	α_0 (°)	$40.66 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0116	5×10^{-6}
	δ_0 (°)	$83.52 - 0.004T$	$\Delta\delta_0$ (°)	0.0022	3×10^{-6}
	W (°)	$8.95 + 190.6979085d$	ΔW (°)	−0.0032	-5×10^{-6}
Telesto (S13) 294,720	α_0 (°)	$50.51 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0084	-1×10^{-5}
	δ_0 (°)	$84.06 - 0.004T$	$\Delta\delta_0$ (°)	0.0021	2×10^{-6}
	W (°)	$56.88 + 190.6979332d$	ΔW (°)	−0.0064	1×10^{-5}
Calypso (S14)[18] 294,721	α_0 (°)	$36.41 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0193	-2×10^{-5}
	δ_0 (°)	$85.04 - 0.004T$	$\Delta\delta_0$ (°)	0.0016	6×10^{-8}
	W (°)	$153.51 + 190.6742373d$	ΔW (°)	0.0045	2×10^{-5}
Dione (S4) 377,415	α_0 (°)	$40.66 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0062	-4×10^{-6}
	δ_0 (°)	$83.52 - 0.004T$	$\Delta\delta_0$ (°)	0.0012	7×10^{-7}
	W (°)	$357.6 + 131.5349316d$	ΔW (°)	−0.0018	4×10^{-6}
Helene (S12) 377,444	α_0 (°)	$40.85 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0058	-3×10^{-6}
	δ_0 (°)	$83.34 - 0.004T$	$\Delta\delta_0$ (°)	0.0012	9×10^{-7}
	W (°)	$245.12 + 131.6174056d$	ΔW (°)	−0.0021	3×10^{-6}
Rhea (S5) 527,068	α_0 (°)	$40.38 - 0.036T$	$\Delta\alpha_0$ (°)	−0.0027	-2×10^{-6}
	δ_0 (°)	$83.55 - 0.004T$	$\Delta\delta_0$ (°)	0.0005	4×10^{-7}
	W (°)	$235.16 + 79.6900478d$	ΔW (°)	−0.0007	2×10^{-6}
Titan (S6) 1,221,865	α_0 (°)	39.4827	$\Delta\alpha_0$ (°)	−0.0003	-9×10^{-7}
	δ_0 (°)	83.4279	$\Delta\delta_0$ (°)	0.0001	9×10^{-7}
	W (°)	$186.5855 + 22.5769768d$	ΔW (°)	−0.0001	1×10^{-6}
Iapetus (S8) 3,560,854	α_0 (°)	$318.16 - 3.949T$	$\Delta\alpha_0$ (°)	-3×10^{-5}	2×10^{-6}
	δ_0 (°)	$75.03 - 1.143T$	$\Delta\delta_0$ (°)	8×10^{-6}	3×10^{-7}
	W (°)	$355.2 + 4.5379572d$	ΔW (°)	2×10^{-6}	-2×10^{-6}
Phoebe (S9) 12,947,918	α_0 (°)	356.90	$\Delta\alpha_0$ (°)	6×10^{-7}	2×10^{-7}
	δ_0 (°)	77.80	$\Delta\delta_0$ (°)	2×10^{-7}	5×10^{-9}
	W (°)	$178.58 + 931.639d$	ΔW (°)	-1×10^{-6}	-1×10^{-7}

[18] Calypso (S14) has the opposite sign for the values ΔW compared to similar values of other satellites of Saturn. (see comment 6 for Table 3).

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 4/6)*

Uranus					
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T*²**
Cordelia (U6) 49,800	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0209	2×10^{-5}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0018	1×10^{-5}
	W (°)	$127.69 - 1074.5205730d$	ΔW (°)	−0.0789	5×10^{-7}
Ophelia (U7) 53,800	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0172	1×10^{-5}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0015	8×10^{-6}
	W (°)	$130.35 - 956.4068150d$	ΔW (°)	−0.0650	1×10^{-6}
Bianca (U8) 59,200	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0136	9×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0012	5×10^{-6}
	W (°)	$105.46 - 828.3914760d$	ΔW (°)	−0.0512	5×10^{-7}
Cressida (U9) 61,800	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0122	8×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0010	6×10^{-6}
	W (°)	$59.16 - 776.5816320d$	ΔW (°)	−0.0460	4×10^{-7}
Desdemona (U10) 62,700	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0118	6×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0010	4×10^{-6}
	W (°)	$95.08 - 760.0531690d$	ΔW (°)	−0.0444	4×10^{-7}
Juliet (U11) 64,400	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0110	7×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0009	4×10^{-6}
	W (°)	$302.56 - 730.1253660d$	ΔW (°)	−0.0415	5×10^{-7}
Portia (U12) 66,100	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0103	7×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0009	5×10^{-6}
	W (°)	$25.03 - 701.4865870d$	ΔW (°)	−0.0388	4×10^{-7}
Rosalind (U13) 69,900	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0089	8×10^{-7}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0008	6×10^{-6}
	W (°)	$314.90 - 644.6311260d$	ΔW (°)	−0.0337	2×10^{-6}
Belinda (U14) 75,300	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0074	4×10^{-6}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0006	3×10^{-6}
	W (°)	$297.46 - 577.3628170d$	ΔW (°)	−0.0281	3×10^{-7}
Puck (U15) 86,000	α_0 (°)	257.31	$\Delta\alpha_0$ (°)	−0.0053	1×10^{-5}
	δ_0 (°)	−15.18	$\Delta\delta_0$ (°)	0.0005	-3×10^{-7}
	W (°)	$91.24 - 472.5450690d$	ΔW (°)	−0.0201	-1×10^{-6}
Miranda (U5) 129,900	α_0 (°)	257.43	$\Delta\alpha_0$ (°)	−0.0019	-1×10^{-6}
	δ_0 (°)	−15.08	$\Delta\delta_0$ (°)	0.0002	9×10^{-8}
	W (°)	$30.70 - 254.6906892d$	ΔW (°)	−0.0071	1×10^{-7}

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 5/6)*

U r a n u s (continue)					
Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***T***	***T*²**
Ariel (U1) 190,900	α_0 (°)	257.43	$\Delta\alpha_0$ (°)	−0.0007	-4×10^{-8}
	δ_0 (°)	−15.10	$\Delta\delta_0$ (°)	0.0001	3×10^{-8}
	W (°)	$156.22 - 142.8356681d$	ΔW (°)	−0.0027	1×10^{-8}
Umbriel (U2) 266,000	α_0 (°)	257.43	$\Delta\alpha_0$ (°)	−0.0003	2×10^{-8}
	δ_0 (°)	−15.10	$\Delta\delta_0$ (°)	3×10^{-5}	1×10^{-7}
	W (°)	$108.05 - 86.8688923d$	ΔW (°)	−0.0012	-7×10^{-9}
Titania (U3) 436,300	α_0 (°)	257.43	$\Delta\alpha_0$ (°)	−0.0001	-6×10^{-8}
	δ_0 (°)	−15.10	$\Delta\delta_0$ (°)	8×10^{-6}	6×10^{-8}
	W (°)	$77.74 - 41.3514316d$	ΔW (°)	−0.0003	2×10^{-8}
Oberon (U4) 583,500	α_0 (°)	257.43	$\Delta\alpha_0$ (°)	-4×10^{-5}	8×10^{-9}
	δ_0 (°)	−15.10	$\Delta\delta_0$ (°)	4×10^{-6}	4×10^{-8}
	W (°)	$6.77 - 26.7394932d$	ΔW (°)	−0.0002	-2×10^{-11}

Table 3a. The rotational elements of the satellites of the Solar System planets and their secular terms of the geodetic rotation *(part 6/6)*

Neptune					
Name, *a* (km)		**Archinal et al. (2018)**	**Present paper**	***T***	***T*^{2}**
Naiad (N3) 48,227	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	0.1098	0.0002
	δ_0 (°)	43.36	$\Delta\delta_0$ (°)	0.0361	0.0005
	W (°)	254.06 + 1222.8441209d	ΔW (°)	−0.1311	0.0001
Thalassa (N4) 50,074	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	0.1006	0.0002
	δ_0 (°)	43.45	$\Delta\delta_0$ (°)	0.0331	0.0004
	W (°)	102.06 + 1155.7555612d	ΔW (°)	−0.1209	0.0002
Despina (N5) 52,526	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	0.0892	0.0001
	δ_0 (°)	43.45	$\Delta\delta_0$ (°)	0.0294	0.0003
	W (°)	306.51 + 1075.7341562d	ΔW (°)	−0.1073	0.0002
Galatea (N6) 61,953	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	0.0590	0.0001
	δ_0 (°)	43.43	$\Delta\delta_0$ (°)	0.0194	0.0002
	W (°)	258.09 + 839.6597686d	ΔW (°)	−0.0710	0.0001
Larissa (N7) 73,548	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	0.0385	0.0001
	δ_0 (°)	43.41	$\Delta\delta_0$ (°)	0.0126	0.0001
	W (°)	179.41 + 649.0534470d	ΔW (°)	−0.0462	0.0001
Proteus (N8) 117,646	α_0 (°)	299.27	$\Delta\alpha_0$ (°)	0.0119	6×10^{-6}
	δ_0 (°)	42.91	$\Delta\delta_0$ (°)	0.0039	0.0001
	W (°)	93.38 + 320.7654228d	ΔW (°)	−0.0141	5×10^{-5}
Triton (N1) 354,759	α_0 (°)	299.36	$\Delta\alpha_0$ (°)	−0.0005	-8×10^{-6}
	δ_0 (°)	41.17	$\Delta\delta_0$ (°)	−0.0002	1×10^{-6}
	W (°)	296.53 − 61.2572637d	ΔW (°)	0.0007	5×10^{-6}

T is the time in Julian centuries years;
d is the time in days from standard epoch,
which is JD 2451545.0, that is, 2000 January 1, 12 hours TDB;
a is the length of the satellite orbit's semi-major axis.

Table 4. Variation of the rotational elements for Calypso and comparison with near satellites for their secular terms of the geodetic rotation in Euler angles

Name, *a* (km)	**Archinal et al. (2018)**		**Present paper**	***t***
Tethys (S3) 294,672	α_0 (°)	$40.66 - 0.036T$	$\Delta\psi$ (")	−31.5081
	δ_0 (°)	$83.52 - 0.004T$	$\Delta\theta$ (")	−90.1979
	W (°)	$8.95 + 190.6979085d$	$\Delta\varphi$ (")	541.1442
Telesto (S13) 294,720	α_0 (°)	$50.51 - 0.036T$	$\Delta\psi$ (")	−28.6117
	δ_0 (°)	$84.06 - 0.004T$	$\Delta\theta$ (")	−80.2283
	W (°)	$56.88 + 190.6979332d$	$\Delta\varphi$ (")	541.1450
Calypso (S14)[19] 294,721	α_0 (°)	$36.41 - 0.036T$	$\Delta\psi$ (")	0.2767
	δ_0 (°)	$85.04 - 0.004T$	$\Delta\theta$ (")	−84.1557
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	541.1466
Calypso with α_0 from Tethys	α_0 (°)	$40.66 - 0.036T$	$\Delta\psi$ (")	−3.2464
	δ_0 (°)	$85.04 - 0.004T$	$\Delta\theta$ (")	−81.6924
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	−529.5805
Calypso with α_0, δ_0 from Tethys	α_0 (°)	$40.66 - 0.036T$	$\Delta\psi$ (")	−32.1455
	δ_0 (°)	$83.52 - 0.004T$	$\Delta\theta$ (")	−90.3406
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	−502.3581
Calypso with α_0 from Telesto	α_0 (°)	$50.51 - 0.036T$	$\Delta\psi$ (")	−9.7437
	δ_0 (°)	$85.04 - 0.004T$	$\Delta\theta$ (")	−75.4873
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	−524.7257
Calypso with δ_0 from Telesto	α_0 (°)	$36.41 - 0.036T$	$\Delta\psi$ (")	−18.2542
	δ_0 (°)	$84.06 - 0.004T$	$\Delta\theta$ (")	−90.4678
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	−514.7784
Calypso[20] with α_0, δ_0 from Telesto	α_0 (°)	$50.51 - 0.036T$	$\Delta\psi$ (")	**−28.5499**
	δ_0 (°)	$84.06 - 0.004T$	$\Delta\theta$ (")	−80.2670
	W (°)	$153.51 + 190.6742373d$	$\Delta\varphi$ (")	−507.3759
Calypso[20] with α_0, δ_0,W from Telesto	α_0 (°)	$50.51 - 0.036T$	$\Delta\psi$ (")	**−28.5499**
	δ_0 (°)	$84.06 - 0.004T$	$\Delta\theta$ (")	−80.2670
	W (°)	$56.88 + 190.6979332d$	$\Delta\varphi$ (")	−507.3759

[19] 1Calypso (S14) has the opposite sign for the values ΔW compared to similar values of other satellites of Saturn. (see comment 6 for Table 3).

[20] 2The experiment showed if we replace the polar rotation parameters of Calypso (Archinal et al. 2018) with the corresponding polar rotation parameters of Telesto (still using the coordinates and velocities for Calypso from the ephemerides; Giorgini et al. 1996) to calculate the quantity of the Calypso geodetic precession, then the resulting geodetic precession value is in good agreement with the predicted theory.