International Symposium on
**HYDROACOUSTICS AND ULTRASONICS**
**EAA Symposium (formerly 13th FASE Symposium)**
Gdańsk-Jurata, 12-16 May 1997

# GENERATION CONDITIONS AND NONLINEAR SOUND DIRECTED PROPAGATION IN INHOMOGENIOUS MEDIUM

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## ABSTRACT

Boundary (and initial) conditions choice for the effective generation and propagation of nonlinear sound in a stratified medium is studied. The adjustment to a medium type is introduced via a choice of equations of state. Liquid, liquid with bubbles and gas cases are considered. Model propagation equations are derived for the special cases of plane directed waves. The application of the conditions in two-dimensional version (three-dimensional with the cylindric symmetry) is discussed and some test calculations of solutions are performed numerically. A good shape and direction conservation in a weak nonlinear conditions is discovered for a triangular initial wave from low frequency range.

## 1. INTRODUCTION

The problem under consideration is rather complicated in general therefore we begin to deal with one-dimensional models. We construct models that may be applied to liquid, gas, liquid-bubbles and gas-drops mixtures. We wish to investigate how the choice of initial (boundary) conditions influence the effectivity of the process of energy transfer from a source to the hydrodynamical waves. We interest especially directed waves and mean field (e.g.streaming) generation [1]. It should be mentioned as well that the problem is also connected with the general hydrodynamical problems which obtained the name of meteorological field adaptation [2] and dispersion relation branches division of the arbitrary disturbance [1]. We study such division in the week nonlinear evolution regime of the composite taking into account the self- action and interaction effects. Such problem arises and effectively solved for the surface waves beginning from the works of Korteweg and de Vries [3] for the long wave range. In this range the dispersion operator may be approximated by the pure derivatives and the initial problem for a directed waves appears to be integrable that now serves like a classical example of the such mechanical system with many conservations laws, solitons solutions and many other interesting features [4]. The dispersion of nonlinear sound in stratified medium is different from those ones [1]. The simple nonlinear equations in a homogenious one- dimensional medium could be integrated in some other clas sical initial problem. The simplest one (Hopf equation) shows the breakdown of any wave and the Burgers equation introduces stabilizing dissipative effect which give the realistic shock wave model and many interesting features in its multidimensional counterparts [5]. Even this one- dimensional theory crucially changes when we introduce the density inhomogeneity of the propagation medium. The strong dispersion appears and the concurence of it with nonlinearity is not studied well yet [1,6]. Moreover, the realistic physical foundation needs the multicomponents description and the adequate appropriate choice of the thermodynamical state equations. Even in the one-dimensional space we should introduce three dynamical variables (components) and therefore three fundamental types of disturbance (dispersion equation solution). We would also show how the choice of the equations of state changes the dispersion properties of the wave.

## 2. THEORY OF DIRECTED WAVES

Here we stop ourselves on the Euler three- component equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial r} = 0$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g$$

$$\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial r} = -\frac{p}{\rho}\frac{\partial v}{\partial r} \qquad (1)$$

where r- vertical coordinate, height over the Earth surface, t- time, $\rho$ ,p,$\mathcal{E}$,v - density, pressure, internal energy and velocity of gas correspondingly, g - gravity acceleration. We adopt now the linearized equations for internal energy and pressure as a function of density and temperature. The choice of a medium determine the coefficients in those equations. The order of them is strongly depend on the aggregate state of the medium. In the most general case instead of the third equation in system (1) and the state relation connecting $\mathcal{E}$, p and $\rho$, we have the next one:

$$A\frac{\partial p}{\partial t} + B\frac{\partial \rho}{\partial t} = -p_0\frac{\partial v}{\partial r},$$

$p_0$ being the unperturbed underground pressure. For ideal gas we have the well-known: $\mathcal{E} = p / (\rho(\gamma - 1))$ The appropriate choice of A and B coefficients allows to consider the more complicated multiple flows. It may be, for example, water with bubbles or air with droplets [7]. In this case an inhomogeneity is usually accompanied by some stratification.

The problem of gasdynamical field components separation relates to the set of adaptation and initialization problems [1] being the subject of

Geophysics [2]. As a rule, it is accepted to separate the wave field to composites distinguishing by dispersion relation (k) - acoustic, internal gravity waves, Rossby waves (in wide variety of meterological problems). Separation by this way has quite clear physical meaning because of every type determination by characteristic region of frequencies and group velocities. Nevertheless it seems very useful to separate wave field to composites of different propagation direction. It allows to analyze the energy and amplitude properties distribution in a gas disturbance evolution especially in the weak nonlinear regime likely the pioneering works of Korteveg - de Vries and many of them till up-to-date ones [4]. The present paper is a prolongation of [6] in which the nonlinear evolution of one- dimensional, exponentially stratified atmosphere initial disturbancies, caused up- or down- directed wave were investigated. In one - dimensional atmosphere model only acoustical waves propagate. The system of gasdynamical equations for one-dimensional gas movement is (1). The main steps for directed wave equations derivation are following [1,6]. First of all from the linearized system (1) one can obtain the twin connection equations for the disturbancies of gasdynamical variables in ( ,k) - representation:

$$v_0' = \rho_0' i\omega / (\rho_{00}(1/2h + ik - \alpha));$$

$$p_0' = \rho_0'(g(1/2h + ik - \alpha) - \omega^2) / (1/2h + ik - \alpha)^2$$

The disturbed values p(r,t), v(r,t) by the following way are connected with the introduced $v_0, p_0, \rho_0$:

$\rho(r,t) = \rho_0'(k)\exp(-r/2h + r)*$ $*\exp(i( t-kr))$, the same for p(r,t) , $v(r,t) = v_0'(k)*$ $\exp(r/2h - \alpha r) *\exp(i( t-kr))$;

where $\omega$ and k are connected by the dispersion relation:

$$\omega^2 = k^2\phi^2 + \psi^2; \quad \phi^2 = (gh - B)/A \ ; \quad \psi^2 = \{B(1/2h - \alpha)^2 + gh(1/4h^2 - \alpha^2) - A(\alpha - 1/2h)g\}/A;$$

$\varphi^2$ and $\psi^2$ being the positive numbers. We consider an exponentially density stratified $\rho_0 = \rho_{00} *\exp(- r/h)$ undisturbed medium, h - scale height, $\gamma = C_p/C_v$ - specific heats ratio. In the case of ideal gas we have a constant internal energy density $\mathcal{E}_0 = gh/(\gamma-1)$. In this case the basic coefficients are: $\alpha = 0$; $A = 1/(\gamma - 1)$; $B = -gh/(\gamma-1)$; $\phi^2 = \gamma gh$ ; $\psi^2 = \gamma gh / 4h^2$ and the main formulas obviously have the more simplier form [6]. By the second step it is possible to obtain the connection equations in (r,t) - presentation by uniting of (k,t) compounds in Fourier integrals corresponding to different signes of and k for up- directed wave, to the same signes for down-directed wave and to = 0 for the stationary contribution.

Thus, the wave field in any time moment is separated to the three independent compounds by the simple way. The full wave field separation such as proposed one:to directed and stationary parts or by the type of dispersion relations is based on Fourier - decomposition of wave field in any evolution moment. It is true only in the case of weak disturbancies described by the linearized gasdynamical system. Thus, the exact classification of wave field composites is possible only in this case. For the clearest explanation let's take the example of wave motions in ideal gas. As one could see, the main formulae for liquid differ only by the coefficients A and B.

We have derived an important conclusion : the directed wave keeps it properties even for essentially large amplitude initial conditions disturbancies - disturbance of velocity up to 150m/sec. The initial conditions are constructed by the linear theory connections, numerical calculation being realized by means of nonlinear Lagrange finite-differencies scheme. Thus, very actual are the following more general aspects of such initialization problem, up to our opinion: a) to separate components moving in different directions and stationary one in any evolution moment; b) to develop an analytical description of their further evolution and to calculate as complete as possible the wave form distortion up to nonlinear effects (not only up to dispersion properties ). It means to construct nonlinear evolution equations for the directed waves; c) to investigate a movement in nonisothermic case; d) to generalize the results to the cases of two- and three-dimensional problem.

## 3. THE NONLINEAR THEORY DEVELOPMENT

Let's solve the a) and b) problems. Problem c) was considered in [6]. About problem d): it was verified by numerical calculation that wave remains quite directed along the axis of cylindrical symmetry disturbance with the radius 3 km [6]. But generally the problem requires further direct development [1].The development of method [8] is general tecnique of wave motion projection in any evolution moment to the three directed parts:

$$\begin{pmatrix} v'(r,t)\exp(-r/2h) \\ p'(r,t)\exp(r/2h) \\ \rho'(r,t)\exp(r/2h) \end{pmatrix} = \begin{pmatrix} v_+ \\ p_+ \\ \rho_+ \end{pmatrix} + \begin{pmatrix} v_- \\ p_- \\ \rho_- \end{pmatrix} + \begin{pmatrix} 0 \\ p_{stat}\exp(r/2h) \\ \rho_{stat}\exp(r/2h) \end{pmatrix}$$

There designed $v_+(r,t) = v\ r,t)_{up}\exp(-r/2h)$; $p_+ = p_{up} * \exp(r/2h)$ and so on, i.e. essentially directed parts of wave motion, $p_{stat}$ being the stationary part of wave motion. In the case of the linearized system of equations the projection operators are obtained in explicit form:

$$P_+ = \begin{pmatrix} 1/2 & l1 & l2 \\ L1/2 & L1*l1 & L1*l2 \\ L2/2 & L2*l1 & L2*l2 \end{pmatrix}; \quad (2)$$

and so on for $P_-$ and $P_{stat}$ [6]. Integrodifferential operators - matrix elements are determined by formulas:

$$L1 = -\frac{\rho_{oo}}{\pi\sqrt{\gamma gh}}\left[-\partial/\partial_r + 1/2h\right] * \int_{-\infty}^{\infty} dr' F(r-r')$$

$$L2 = \frac{\rho_{oo}}{\pi\sqrt{\gamma gh}}\int_{-\infty}^{\infty} dr' \left[\gamma gh*\partial/\partial_{r'} + g(1-\gamma/2)\right] F(r-r')$$

$$l1 = \frac{\sqrt{\gamma gh}}{2\pi\rho\ \ \gamma gh}\int_{-\infty}^{\infty} dr' F(r-r')* \left[-\partial/\partial_{r'} + 1/2h\right]$$

$$l2 = -\frac{\sqrt{\gamma gh}}{2\pi\rho_{oo}\gamma h}\int_{-\infty}^{\infty} dr' F(r-r'),$$

where $F(r)=2\ (I_0(r/2h)\ -\ L_0(r/2h))$ [ ] $I_0$ - modified Bessel function of zero order, $L_0$-Struve function. The base of calculation is the connection equations for each pair from $v, p, \rho$ of every component [6]. The obtained operators (2) have ordinary properties of projection operators: $P_+ + P_- + P_{stat} = 1$; $P_+ * P_- = P_+ * P_{stat} = 0$; $P_+ * P_+ = P_+, \ldots$

To obtain in any moment up-directed field, for example, it is sufficient to apply $P_+$ to the total field by the ordinary way.

The possibilities of the method are described in [6,8]. There are: the calculation of the wave energy parts for every component possesses at any time, choice of initial conditions for the preferential generation of the needed type component . Let's consider the expansion of linear problem projection operators, pointed out in b). Evolutionary equation for $v_-$ derived for the self-interaction case has the form:

$$\frac{\partial v_-}{\partial t} - \frac{\sqrt{\gamma g h}}{\pi} \int_{-\infty}^{\infty} dr' F(r-r') * [\partial^2/\partial r'^2 - 1/4h^2] v_-(r',t) = \quad (3)$$

$$-1/2 v_-(\partial/\partial r + 1/2h) v_- \exp(r/2h) +$$

$$\exp(r/2h)(\partial/\partial r - 1/2h) I_0(r,t) *$$

$$(\gamma h \partial/\partial r + (1-\gamma/2))(\partial/\partial r - 1/2h) *$$

$$I\ (r,t)\ +$$

$$1/(2\pi^2 \gamma h) \int_{-\infty}^{\infty} dr' F(r-r') \exp(r'/2h)$$

$$* \Big\{ \partial^2 v_-/\partial r'^2 \Big[ -(\gamma^2-2) h \partial/\partial r' - 1 - \gamma + \gamma^2/2 \Big] + \partial v_-/\partial r' (\gamma^2-\gamma-2) \Big[ -h * \partial^2/\partial r'^2 + \partial/\partial r' - 1/4h \Big] +$$

$$v_- \Big[ \gamma h \partial^2/\partial r'^2 + (\gamma^2/2-1)\partial/\partial r' + (\gamma-1)(1-\gamma/2)/2h \Big] \partial/\partial r' \Big\} I_0(r',t);$$

where

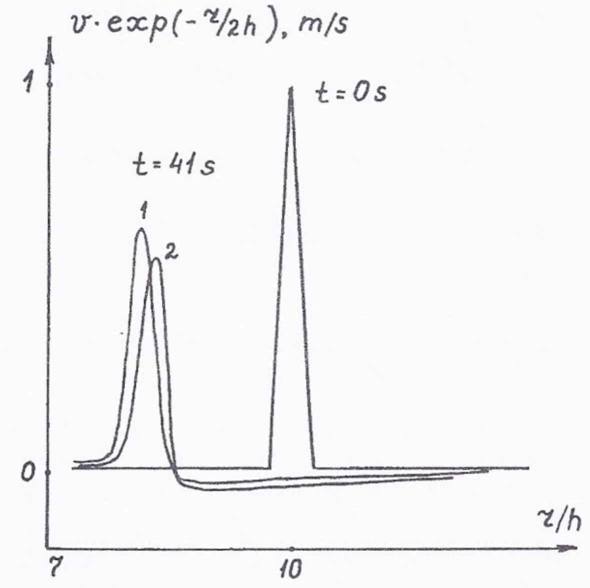
$$I_0(r,t) = \int_{\infty} dr' F(r-r') v_-(r',t)$$

The difficulties of this equation analytical investigation are apparent. Therefore the evolutionary equations have been investigated numerically. For calculations $\gamma = 1.4$, h =6.8 km, g = 9.8 m/sec have been accepted. As initial disturbance of velocity the localized function was choosen: $v * \exp(-r/2h) =$

$$-4v^*(r-10h-\tfrac{h}{4})/h; \quad 10h \le r \le 10h+h/4$$

$$4v^*(r-10h+\tfrac{h}{4})/h; \quad 10h-h/4 \le r \le 10h$$

$v^* = 1$ m/s that has the maximum of v= 150 m/s at the height r = 10h. This initial condition has width of h/2 and its evolution cannot be described neither short- nor long-wave approach. Calculations were made for all three nonlinear improvements from (3) and in the case of each one separately (see fig.). It was revealed the preferential role of the first nonlinear addenda which has the most simple form. At the figure the initial disturbance and its evolution are shown at the t = 41 sec from the beginning of disturbance in linearized case ( curve 1 - the distortion caused only by dispersion properties of the atmosphere) and in general case of all nonlinear improvements (curve 2) taken into account. One can clearly see the joint influence to the wave form of nonlinear and dispersion effects : the back




front became more steep. Dispersion effects are connected with inhomogeneity of the background atmosphere (exponentially stratified), nonlinear - with the usage of nonlinear equations of gas dynamics. Moreover, nonlinear distortions of up- and down- directed waves have the same character - the right-hand side additions are the same signes for directed waves ( up to numerical calculations). In general, method has a practical application. For large- scale disturbancies it is possible: 1) to separate gasdynamical field disturbancies in any moment to three composites: up-, down-propagated and stationary one. This fact permits to estimate simply the part of wave energy wich possesses every composite even in case of strong initial disturbancies. (The most interesting are height sources generated composites of all three types) 2) to describe further evolution of composites with account of their interaction. Some other possibilities arise. Evolutionary equations of such type have to describe the propagation of electromagnetic waves in waveguides, this fact being caused by the similarity of initial systems of equations [1,9].

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