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QUANTITATIVE METHODS IN THE RISK ANALYSIS ON THE EXAMPLE OF THE OIL SECTOR

ALICJA BYRSKA-RĄPAŁA

Faculty of Management, AGH University of Science of Technology Krakow

The geological object (deposit) is a specific type of asset of mining enterprise which cannot be quantitatively or qualitatively interchangeably measured. In spite of rapid development of geological and economic sciences, still none of universal method of evaluation of value of resources has been settled yet. In the article, on the example of the oil sector, author described chosen methods of the mathematical statistics and the financial mathematics tools, which are used in the risk assessment related to estimating the fair market value of hydrocarbons. In the article the following subjects are presented:

- the hyperbolic lognormal distribution in the exploration risk assessment,
- the exponential utility function and the evaluation of the real option in the economic risk assessment.

For each area of risk assessment is to recommend software.

Statistical methods and the financial engineering used in estimating the fair value of the properties, are from many years the topic of numerous scientific publications and research. Many of the studies has indicated the importance of mathematical methods in solving problems in the area, where not long ago intuition and experience of evaluator of deposit were mainstream.

Keywords: fair market value, hyperbolic lognormal distribution, exponential utility function, risk tolerance level, equivalent of certainty, real option

1. Introduction

Methods of properties'evaluation establishing nowadays an independent scientific sub-discipline: geological objects'evaluation engeenering. In spite of rapid development of geological and economic sciences, no universal method of evaluation of fair market value (FMV) resources has been fixed yet; hence, tools allowing to make the process of their estimation more objective are still searched for.

The controversy, especially among practitioners, raises the notion of fair market value. In the case of such assets like crude oil or nature gas deposit, can be discussed only about the estimated fair value. It is worth underlining that fair market value is a broader concept than economic value; it is a hypothetic value, reflecting changes of crude oil and natural gas prices, as well as the risk connected with the estimated property. The quality of properties' valuation depends on the correctness of exploitation forecasts and on proper economic assumptions (the prediction of capital requirements, operation cost and taxes, discount rates, as well as the risk and uncertainty connected with the subject of estimation).

Globalization of economy, providing transparency and security of marketing transactions, were the basis for the introduction of International Accounting Standards (IAS) and International Financial Reporting Standards (IFRS). In accordance with the standards, by the fair value shall be understood the amount at which an asset could be exchanged or a liability settled, between interested, well-informed and clearly unrelated to each other parties under the conditions of market transactions. To determine FMV one can apply three approaches: market, profitable and cost. It should be noted that the use of valuation based on market prices is possible only if there is an active market (items of marketing are homogeneous and in any moment one can find willing buyers and sellers, transaction prices are publicly available). In 2000 years, the concept of fair value was first introduced to the Polish law by the Accounting Act. Unfortunately, the provisions of the Act does not explain how to set fair value properly.

The most often quoted definition of FMV of hydrocarbon deposit says that it is "a value established by a willing buyer and willing seller with neither party having a compulsion to buy or to sell and both having reasonable knowledge of relevant facts" [4]. Although very appropriate, is this definition useless from practical point of view. This is why FMV is also determined as a value established by a qualified and impartial estimator using fair and reasonable assumptions for the purpose for which the value is rendered. The typical evaluator is an experienced reservoir engineer with sufficient knowledge of economic and risk issues.

In Poland the problem of valuation fold appeared along with the evolution of the political system. Unfortunately, the pace of the system transformation did not follow up with the legal system.

The statutory definition of fair value draws attention to the presence of an active market and is not useful in determining FMV of hydrocarbon property. There is no active market for practically traded property – no two identical property, there are no continuous quotations, prices of “buy and sell” transactions are often commercial secret. The author proposes a division of methods for assessing the FMV property depicted in Fig. 1. The presented classification combines economic science methods, classification methods proposed by the guidelines for the evaluation of petroleum reserves and resources.

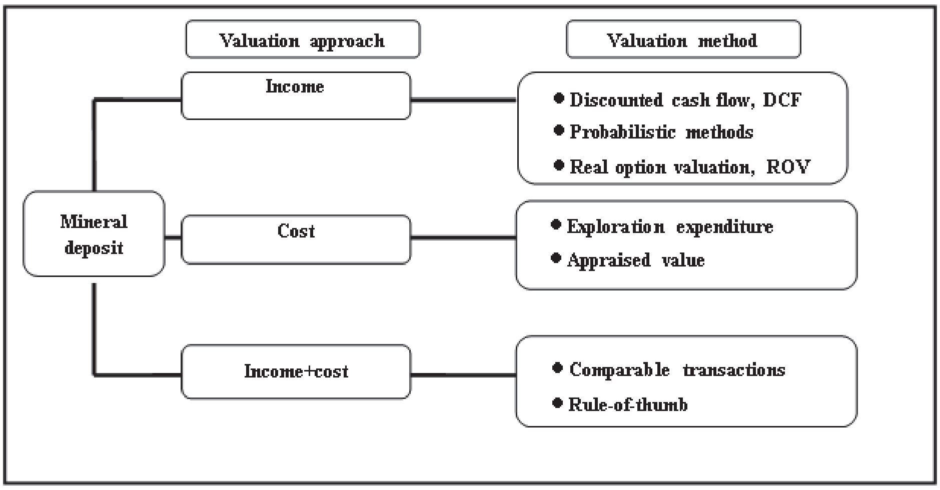

Figure 1. Approaches and methods valuation of fair market value of hydrocarbon deposit

Each one of the FMV estimation methods has to take into account risks and uncertainties. The development of mining technology, computer-aided decision-making, creating online databases cause the fact, that new solutions are still introducing to the quantitative assessment of the risks and uncertainties of mining projects. The purpose of this article is to present the applicability of the hyperbolic lognormal distribution in the exploration risk assessment, the exponential utility function and the evaluation of the real option in the economic risk assessment.

2. Mathematical methods for the assessment of risks and uncertainties

The petroleum industry is a classic case of decision making under risk and uncertainty. Found in the literature sharing risk and uncertainty – for exploratory and economical – stems primarily from:

- random nature of all phenomena and effects related to exploration,

- uncertain quantities of hydrocarbons in a property,
- stochastic nature of decline rate of production,
- the high cost of obtaining hydrocarbons (capital expenditure, operating costs),
- fluctuations in the price of oil and natural gas.

Exploratory risk and uncertainty relates to the existence, size, and quality resources. In the evaluation of quantity and quality of resources, the source of risk exploration is mainly the geological risk (existence of viable accumulation of hydrocarbons). The basic methods for geological risk assessment, i.e. the probability of discovering include:

- intuitive assessment of the probability,
- estimating the probability on the basis of geological materials,
- assessment of probability on the basis history of exploration in the region,
- probability models for oil exploration.

During the 1980s and 1990s, the risk was estimated on the basis of distributions models of deposits for geological basins such as the lognormal distribution, lognormal-hyperbolic distribution and fractal normal percentage [3].

Breakthrough in reservoir engineering is the application of a new generation of methods, i.e. artificial intelligence. This group new methods include mathematical solutions to assess the geological basins, whether individual deposits: expert systems, neural networks, genetic algorithms, fuzzy sets, whether the analysis databases (data base mining) [8].

Economic risk and uncertainty, otherwise the market uncertainty, concerns the future prices, the conditions on the market, future operating costs and capital expenditure. In the 1980s was introduced for economical risk analysis the theory of preference (utility) with the exponential utility function [5].

Practical ways of assessing risk and economic uncertainty in the estimate the value of the investment project shall include:

- application of decision theory (method of decision trees, utilities)
- analysis of the sensitivity,
- Monte Carlo simulation,
- adaptation to risk discount rate (risk premium)
- application of real option theory.

One of the software packages used to risk analysis through Monte Carlo Simulation is @RISK of the company Palisade Corporation. @RISK use Monte Carlo simulation to: decision tree analysis, production forecasting, estimation of reserves of oil deposit, production and economic forecasts. It is an add-on MS Excel spreadsheet and Lotus 1-2-3. Fig. 2 shows the standard financial analysis for oil investment for the assumed probability distributions of the input data.

Financial model of discounted Cash flows

Known Input					
Discount rate	12%				
Bonus limit	$30 000		Parameters of distribution		
Uncertain inputs		Distributions	Parameter 1	Parameter 2	Parameter 3
Investment cost	$50 000	Triangular	$40 000	$50 000	$90 000
Year 1 revenue	$100	Triangular	$80 000	$200 000	$110 000
Annual fixed cost	$35 000	Triangular	$32 000	$35 000	$35 000
Annual revenue growth rate	3%	Normal	3%	3%	
Annual variable rate percentage	30%	Normal	30%	2%	

Discounted cash flow calculations

Year	0	1	2	3	4	5	6	7	8	9	10
Investment cost	($50										
Revenue		200000	105000	110250	115763	121551	127628	134010	140710	147746	155133
Fixed cost		35000	35000	35000	35000	35000	35000	35000	35000	35000	35000
Variable cost		50000	52500	55125	57881	60775	63814	67005	70355	73873	77566
Cash flow	($50000)	15000	17500	20125	22881	25775	28814	32005	35355	38873	42566
Output											
NPV	$91 913										
Bonus	$61 913										

Figure 2. Outputs window at the risk model of @RISK
Source: http://www.palisade.com/industry/OilGasModels.asp

The most popular, used by the world's largest oil, gas and mining companies to analyse spreadsheet models and improve decision making, is Crystal Ball software. Like @Risk, Crystal Ball through Monte Carlo Simulation is used to quantify the impact of uncertainties in the financial and economic analysis.

2.1. Exploration risk assessment based on the history of prospecting in geological basin

One of the key factors in decision analysis in exploration and development of hydrocarbon reserves is deposit size. As it remains unknown before an extensive study, it is of great interest to determine the field size distribution function, since it can be used for predictions. The distribution models of size of oil and gas fields in a region used to be and still are the subject of research conducted by many geostatisticians.

When talking about the field size distribution - to avoid any misunderstanding - tree different types of distribution have to be distinguished (Fig. 3):

1) A distribution of all fields created by Nature in a region (which can be called parent population distribution). The evidence suggests that the parent population has a monotone decreasing distribution function with large amount of small fields and small amount of large fields.
2) B distribution of discovered fields (sampling or observed distribution). The characteristics of petroleum exploration process caused that the usual assumption of random sampling is not valid in that case. Empirical evidence indicates that larger deposits are found usually early in exploration process. It is also certain that the process of exploration is influenced by economic factors

which determine the minimum field size which is economically acceptable under specific oil price to oil exploration and development cost ratio (it could be called economic truncation phenomenon). The models of observed distributions are usually unimodal with mode relevant to average field size. According to the economic truncation reasoning the mode is being shifted to the left as a result of oil price rise.

3) C distribution of fields remained to be discovered. Petroleum engineers are eager to get know both models since the difference between A and B distributions gives the distribution type C.

Modelling and predicting hydrocarbon deposit size distribution meet in reality many obstacles which make these problems more connected with theoretical wisdom than with practical applications. There are many reasons for these obstacles.

1) Determining A distribution on the base of B distribution is very complicated mathematically, especially because of the lack of agreement among geostatisticians how the difference between A type and B type distributions looks like.
2) B distribution remodels with the time (see Fig. 3). In the first stage of exploration large and medium, and rather incidentally small fields are discovered. In the second stage medium and small fields are explored, and consequently when the exploration process goes to the next stages more and smaller fields are the matter of exploration.
3) When determining C distribution, a continually changing B distribution has to be subtracted from not precisely described A distribution.
4) There are not many regions in the world known detailed enough to determine how B distribution alters with the time or how really A distribution looks like.
5) Globalisation of petroleum industry as well as supremacy in hydrocarbon markets by the owners of gigantic fields have resulted in abandonment of many small and medium fields as they are regarded as unprofitable.

Unresolved problem of the parent population distribution and distribution of fields remained to be discovered, development of information technology and in particular access to the databases on the history of geological units, to describe the distribution of deposits geostatisticians propose still new models. Preferably used by practitioners and constantly verifying for different regions, is lognormal distributions [3, 9]. Log-normal distribution is considered a classic distribution of petroleum geology, not only for modelling the distribution of deposits in geological basin, but also to model the distribution parameters of the deposit, i.e. the thickness, porosity, area, etc.

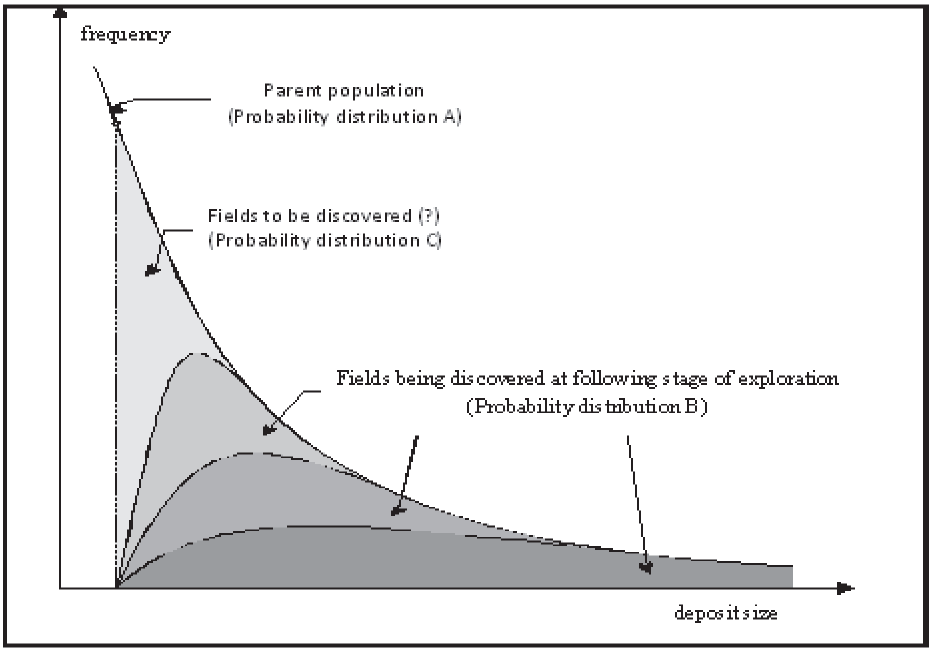

Figure 3. Modelling distributions of deposits of hydrocarbons. Source: based on [1]

The alternative to a log-normal distribution is hyperbolic lognormal distribution (loghyperbolic). Loghyperbolic distribution is not as popular in geostatistic as the log-normal, probably because the estimation of its parameters is more labour-consuming. It has four parameters and its probability density function may be written as follows:

$$p(x;\mu,\sigma,\alpha,\beta) = a(\alpha,\beta,\mu,\sigma)\cdot \exp\left\{-\frac{\sigma}{2}\left[\alpha\sqrt{1+(\frac{x-\mu}{\sigma})^2} - \beta\frac{x-\mu}{\sigma}\right]\right\} \qquad (1)$$

where: x – random variable X, $X = \ln Q$, Q – field size, μ, σ, α, β – parameters of the distribution, $a(\mu, \sigma, \alpha, \beta)$ – norming constant.

In Table 1 and on Fig. 4 have been presented the comparing results of surveys for lognormal and loghyperbolic distributions in the Polish Carpathian Mountains geological province. The obtained results indicate that both distribution models – lognormal and loghyperbolic – show good fit to the empirical data in the Carpathians basin. The results of χ^2 test at 5-percent significance level were satisfying.

On the ground of empirical and theoretical number of fields, geostatisticians can estimate the probability of exploration (geological risk). Of course the assessment of geological risk has to respond to a series of additional information: geology, the order of discovery of deposits, etc.

To estimate parameters of the loghyperbolic distribution used Levenberg-Marquardt metod from software Statgraphics, based on the least-squares procedure. You can also use the software BestFit of the company Palisade Corporation.

Table 1. Observed and estimated number of fields in the Carpathians

	Class intervals [milin TOE}	Number of fields		
		Observed	Lognormal distribution	Loghyperbolic distribution
I	below 0.006	8	5.3	8.1
II	0.006 – 0.016	8	6.5	7.6
III	0.016 – 0.043	8	12.6	10.7
IV	0.043 – 0.116	10	15.6	12.5
V	0.116 – 0.311	12	10.3	11.9
VI	0.311 – 0.837	12	10.1	9.6
VII	0.837 – 2.250	10	5.0	6.8
VIII	above 2.250	3	5.5	4.3
	Total	71	70.9	71.5

Source: based on [1]

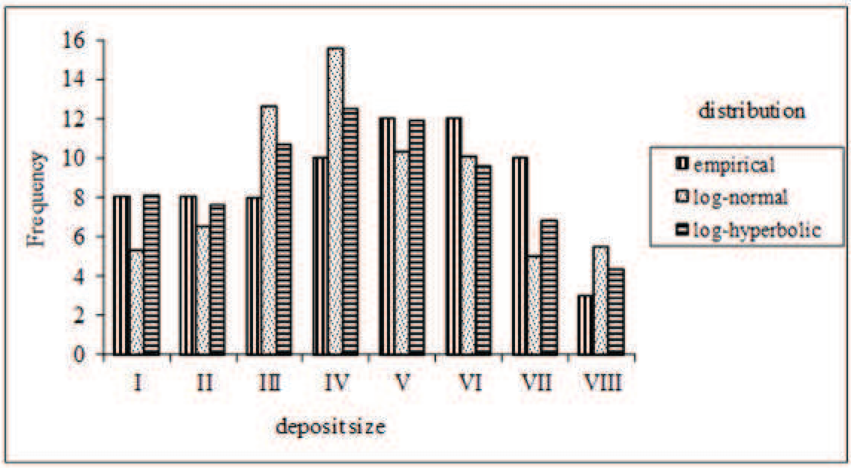

Figure 4. Observed and estimated number of fields in the Carpathian basin.
Source: based on [1]

2.2. Mathematical statistics rules in the economic risk measurement

Expected value is one of the basic mathematical measures and found a permanent place in the economic analysis. The expected monetary value (*EMV*) for the investment project shall be determined on the basis of available information as to the possible gains (losses) from enterprise and the probability of their occurrence. Most commonly used formula is:

$$EMV = p_s \cdot NPV + p_p \cdot (-K_n); \quad p_s + p_p = 1 \tag{2}$$

where:
p_s – probability of discovery field (success),
p_p – probability of undiscovery (lost),
K_n – cost drilling of negative object,
NPV – net present value in the event of discovery field, which can be calculated by the formula:

$$NPV = p_s \sum_{t=1}^{T} CF_t \cdot a_{i,t} - K_0 \quad \text{or} \quad NPV = P - K_0 \tag{3}$$

where:
K_0 – drilling cost of positive object,
$\{CF_1, CF_2, \ldots, CF_T\}$ – expected future cash flows in case of correct drilling site,
$a_{i,t} = \frac{1}{(1+i)^t}$ – discount rate for the year *t* and interest rate,

$P = p_s \sum_{t=1}^{T} CF_t \cdot a_{i,t}$

The supplement to the *EMV* analysis, may be three coefficients, more accurately characterized the risk and the probability of success of financial investments:
- the variance σ^2 or standard deviation σ of financial results,
- volatility *v,*
- probability *P*(*FMV**) that fair value *FMV* of project will be greater or equal to the specified value *FMV**.

The variance or standard deviation shall be calculated by the formulas:

$$\sigma^2 = m_2 - EMV \quad \text{or} \quad \sigma^2 = p_s \cdot p_p \cdot P^2 \tag{4}$$

where:
m_2 – second moment of the project value, calculated by the formula:

$$m_2 = p_s \cdot (NPV - K_0)^2 + p_p \cdot K_0{}^2 \tag{5}$$

A measure of risk is often assigned by the volatility, defined by $v = \sigma / EMV$, which evaluates the stability of the estimated mean value *EMV*. A small volatility implies that there is little uncertainty in the expected value, while a large volatility implies a considerable uncertainty in the expected value.

The third indicator $P(FMV^*)$, shall be calculated by the formula (6) with the assumption that the *FMV* of the deposits has normal distribution $N(EMV; \sigma)$ [6]:

$$P(FMV \geq FMV^*) = \frac{1}{\sigma\sqrt{2\pi}} \int_{FMV^*}^{\infty} \exp[-\frac{1}{2}(\frac{x-EMV}{\sigma})^2]dx \quad (6)$$

Each measure uses a different combination of the expected value and standard deviation – this allows to look at the value of the field/project and risks to different perspective.

2.3. Utility theory in the risk assessment

Measure the profitability of investment projects, such as net present value (*NPV*), internal rate of return (*IRR*), payback period (*DPB*) and discounted expected value *EMV*, do not take human factors into project valuation – assume that all investors have equal attitude to risk. Consideration of the decision makers attitude to the risk is possible by applying the utility/preference theory. The theory provides a practical basis for decision maker to formulate and implement a consistent risk policy in the form of a mathematical function. The exponential utility function is often used to measure risk preferences of the decision maker. The mathematical form of the exponential function $u(x)$ represents the formula [9]:

$$u(x) = e^{-x/RT} \quad (7)$$

where:
x – variable of interest,
RT – risk tolerance level of the decision maker corporation; the RT value represents the sum of money such that the decision makers are indifferent as a company investment to a 50 – 50 chance of winning that sum and losing half of that sum,
e – exponential constant.

Knowledge of the utility function and RT, makes opportunity to calculate a new index – the equivalent of certainty/risk-adjusted value RAV [5]. Assuming a discrete probability distribution of few possible outcomes of financial investment project, the RAV can be apply by the formula:

$$RAV = -RT \cdot \ln(\sum_{i=1}^{n} p_i \cdot e^{-x_i/RT}) \quad (8)$$

where:
p_i – probability of the i-th financial result,
x_i – value of the i-th financial result.

Equivalent of certainty RAV can be interpreted as the value of the investment minus the risk premium. Decision maker chooses this project, which ensures maximum equivalent confidence.

2.4. Real options as the dynamic method to determine *FMV*

Real options were first used in oil industry to estimate the value of oil fields in tenders for geological leases. Optional approach considers reacting to future investment activities and makes the development schedule and exploitation of the reserve more flexible. Taking into account the static and dynamic aspects of the valuation of the investment project, the general formula for *FMV* can be written as:

$$FMV = \overbrace{NPV}^{\text{static value}} + \overbrace{\text{option}}^{\text{dynamic value}} \tag{9}$$

Binomial tree models are most often used to evaluate the value of real options. The classic model of a binomial tree shows the price levels which a share (basic tool) can reach within the validity period of the option. The binomial tree assumes that price fluctuations happen by leaps and bounds. Fig. 5 shows the rules according to calculate the option value (H) for a three-period binomial tree, where:

V_t – oil price in time t ($t = 0, 1, 2, 3$),
C – development cost,
H – option value,
i – discount rate,
p – probability of price fluctuation in the next period, calculated by formula:

$$p = \frac{e^{i\sqrt{\Delta t}} - d}{u - d} \tag{10}$$

u and d – coefficients of price V increase/fall, calculated by formula:

$$u = e^{\sigma\sqrt{\Delta t}} \quad \text{and} \quad d = \frac{1}{u} \tag{11}$$

Δt – the time interval (the period of validity of the option T divided into n periods, $\Delta t = T / n$),
σ – volatility of oil price.

The value of option H at each note of the binomial tree informs us about the possible value of reserve development in future. The value of undeveloped reserves calculated with real option method is positive and is bigger than the *NPV* value. It is particularly visible for low oil prices, when the *NPV* is negative. The deciding rule for *NPV* method is clear: projects having negative current net value should not be developed – the *NPV* method does not consider the 'optional ' value of reserves. The difference between *NPV* value and real option value shows the scope of the reserves owner's decision flexibility. When the oil price goes up, the difference between *NPV* and real option values decreases, because the 'optional' value of reserves also goes down.

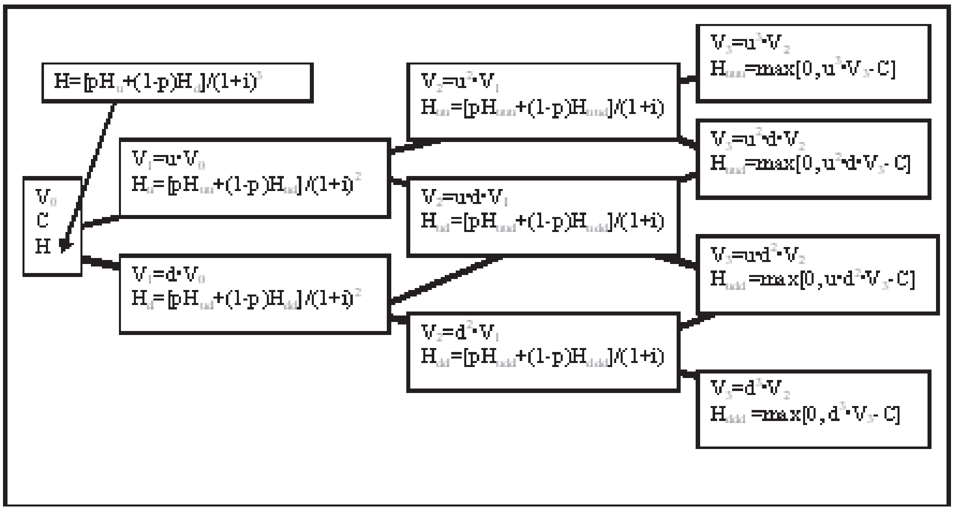

Figure 5. The binomial option pricing formula

3. Conclusion

The evaluation engineering offers many methods for determining fair-market-value of a hydrocarbon property. The proper *FMV* determinations should consider the time element of the revenue stream and the technological, economic and political uncertainties.

The size of geological deposit is the main parameter of the *FMV*, which is usually modeled in the standard distribution of petroleum geology, i.e. the lognormal distribution. The alternative to the lognormal distribution might be the more labour-consuming, loghyperbolic distribution. Both of the above mentioned types of distribution, could be considered as equivalent.

The key role in the process of estimating the *FMV* of the deposits takes a "human factor", i.e. the decision maker attitude towards risk and uncertainty. Proposal to quantify the "human factor" could be utility theory with the exponential function, the equivalent of certainty and the method of real options.

Present-day software should be considered as a tool, more and more excellent, but still determined by the geological and economic knowledge. Increasingly powerful computational algorithms could not replace experience, and above all, common sense.

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